Phases of GLSMs and transition defects

Ilka Brunner

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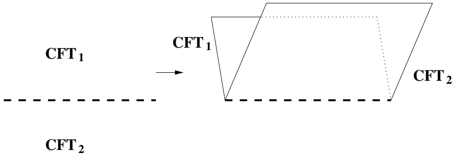
Introduction

- Gauged linear sigma model: 2 dimensional SUSY Abelian gauge theory.
- Several phases: Landau-Ginzburg phase, geometric phase.
- **Question:** How can we transport "data" from one phase to another? From UV to IR?
- **Data?:** For example boundary conditions/D-branes.
- **Tool:** Defects.

`with Fabian Klos, Daniel Roggenkamp, to appear`

Defects

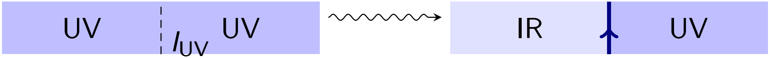
- One dimensional lines which separate two possibly different CFTs/TFTs
- On the defect, there are in general some additional defect degrees of freedom that couple to the bulk.
- They can be regarded as boundaries of a folded theory



- But they are more than just boundaries for folded theories.
- They can be moved, merged, intersect . . .

Defects and flows, general theories

- Very special class of defects: Flow defects (RG domain wall).
- Defect that separates UV and IR theory
- Obtain them by starting with an initial (UV) theory and restricting the perturbation to a subdomain $U \subset \Sigma$.
- Defect will build up at the boundary of the subdomain ∂U.

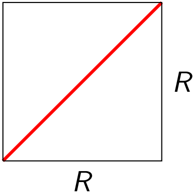


- Functors between category of boundary conditions

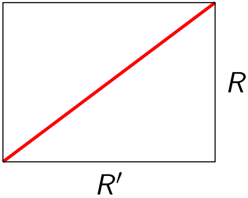
- Merging RG defect with UV boundary condition $\rightarrow$ IR boundary condition.

RG defect in folded picture

- Toy example: Identity interface for free boson on a circle S^1, radius R
- Diagonal brane on torus $S^1 \times S^1$

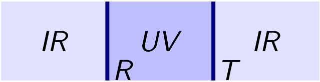


- Deformation of radius $\Rightarrow$ Deformed identity



Features of RG defects

- They are not topological. Fusion with other defects is highly singular
- Study them ideally in some topological subsector of a full theory
- Fusion in one direction yields identity:



- . . . and a projector in the other direction

Linear sigma models

- UV theory: $U(1)^k$ gauge theory, charged matter multiplets, superpotential, $N = (2, 2)$ supersymmetry
- Potential for scalars

$$U = \sum_{i=1}^{n} \left| \sum_{a=1}^{k} Q_i^a \sigma_a x_i \right|^2 + \frac{e^2}{2} \sum_{a=1}^{k} \left(\sum_{i=1}^{n} Q_i^a |x_i|^2 - r^a \right)^2 + \sum_{i=1}^{n} \left| \frac{\partial W}{\partial x_i}(x_1, \dots, x_n) \right|^2 . \qquad (1)$$

- Classical vacuum manifold: $U = 0$ modulo gauge transformations.
- Depends on r^a.
- Calabi-Yau examples: Fields P, X_i, $W = P(X_1^3 + X_2^3 + X_3^3)$, $W = P(X_1^5 + \cdots + X_5^5)$
- $r >> 0 \rightarrow$ Geometric phase: torus/quintic hypersurface in projective space
- $r << 0 \rightarrow$: stringy phase: $P = 1$, Landau-Ginzburg orbifold, orbifold group $\mathbb{Z}_3/\mathbb{Z}_5$.

Non-CY examples

- 2 chiral matter multiplets P, X, superpotential $W = P^{d-n}X^d$, 2 different LG phases
- charges of fields $q(X) = d - n, \quad q(P) = -d$

$$U = |\sigma|^2 \left((d-n)^2|X|^2 + d^2|P|^2\right) + \frac{e^2}{2}\left((d-n)|x|^2 - d|p|^2 - r\right)^2 + d|P^{d-n}X^d|^2 + (d-n)|P^{d-n-1}X^{d-1}|^2.$$

- $r > 0$: X must not vanish, gets expectation value, LG model with $W \sim P^{d-n}$, gauge symmetry broken to $\mathbb{Z}_{d-n}$
- $r < 0$: LG model with $W \sim X^d$
- Quantum effects: r gets renormalized.
- IR phase: $W \sim P^{d-n}$ and n massive vacua on the Coulomb branch.
- RG flow drives the model to this phase.

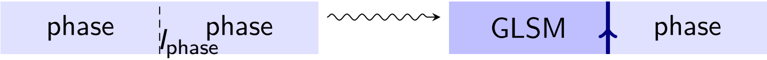
Clingempeel-leFloch-Romo

Setting

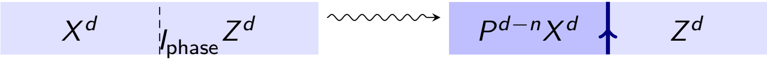
- In addition to the IR phase, we can tune parameters, such that other phases are realized at some energy scale.
- In the example, we regard the LG model with $W \sim X^d$ as the "UV" phase.
- Decouple gauge degrees of freedom, go to topological sector.
- GLSM: $U(1)^k$ equivariant Landau Ginzburg model.
- In general different phases with (partially) broken gauge symmetry.
- In the example, two LG orbifold phases, UV phase, IR phase
- Arguments apply to any phase where we control the identity defect well enough.

Transition defects in GLSM

- Starting point: Identity defect in a phase.
- "Lift" on one side to GLSM



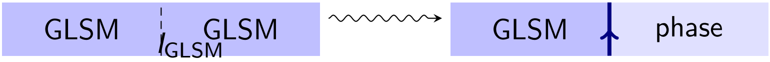
- Similar to discussion Herbst, Hori, Page for branes
- LG example



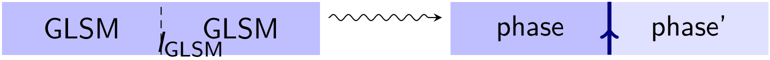
- Lift involves a choice $a \in \mathbb{Z}$, transition defects T_a
- Defect can act on branes and lifts them to the GLSM.

Flowing from the GLSM identity

- Other starting point: GLSM identity defect



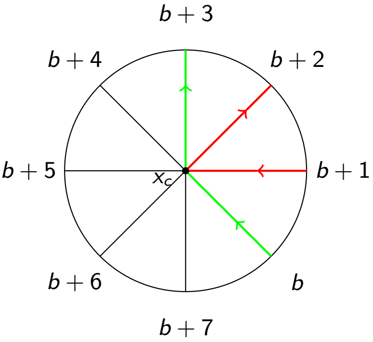
- This constructs the right transition defects T_a.
- We can also construct transition defects between different phases in this way.



- In our concrete example: Obtain RG defects from $W = X^d$ to $W = P^{d-n}$.

Mirror perspective: A-branes in LG models

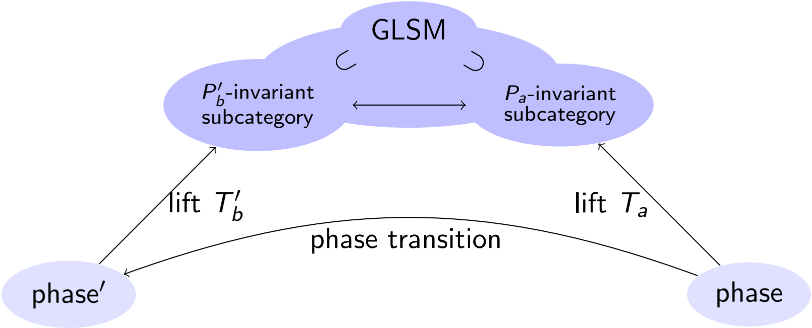
- LG orbifold $X^d/\mathbb{Z}_d$ is mirror to LG model with $W = X^d$.
- B-branes get mapped to A-branes
- A-branes: described by straight lines emanating from a critical point, reality condition on W. Hori, Iqbal, Vafa
- RG flow: relevant perturbation by lower order polynomial



RG flows: Mirror perspective

- Under a perturbation, the critical point splits up and some (elementary) branes decouple.
- Topological defect describing the flow contains precisely the information on which "wedges" decouple. Roggenkamp-IB
- Here: We obtained a lift of known flows to the GLSM model.

Summary



GLSM
P_b'-invariant subcategory
P_a-invariant subcategory
lift T_b'
lift T_a
phase transition
phase′
phase

Comparison

- Brane transport in GLSMs was discussed before `Herbst-Hori-Page, Hori-Romo, Knapp-Romo-Scheidegger, Clingempeel-le Floch-Romo,`
- "Grade restriction rule"
- Here: T_a lifts branes to a "window".
- One can also interpret the procedure of "binding trivial branes to GLSM branes to find a representative in a window" in terms of defects.
- Results in agreement with previous works, our arguments are different.
- We construct a concrete defect: Explicit functor.
- At the heart of the whole construction is the identity defect of the GLSM.

Identity defect

- Defects in LG models are matrix factorizations of the difference of the superpotential of the two theories, $p_1 p_0 = W_1(X_i) - W_2(X'_i)$.

$$D : P_1 \underset{p_0}{\overset{p_1}{\rightleftarrows}} P_0$$

- Identity defect
  "$(X_i - X'_i) \times polynomial(X_i, X'_i) = W(X_i) - W(X'_i)$"

$$d_I = \sum_{i=1}^{n} \left[(X_i - X'_i) \cdot \theta_i^* + \partial_i^{X,X'} W \cdot \theta_i \right]$$

with $\partial_i^{X,X'} W = \dfrac{W(X'_1, ..., X'_{i-1}, X_i, ..., X_n) - W(X'_1, ..., X'_i, X_{i+1}, ..., X_n)}{X_i - X'_i}$

$$P_1 \oplus P_0 = \bigwedge \left(\bigoplus_{j=1}^{n} \mathbb{C}[X_i, X'_i] \theta_j \right)$$

Identity defect and orbifolds

- Orbifold group G, G finite group

$$I_{\text{orb.}} = \bigoplus_{g \in G} {}_g I_{\text{non-orb.}}, \qquad {}_g I_{\text{non-orb.}} : \text{ symmetry defect}$$

- Action of the group: regular representation.
- Extension to continuous group?
- For LG phase, $\mathbb{Z}_d$: Pick a basis, such that the group action is diagonal.
- Matrices can be rewritten in terms of a new bosonic field α with $\alpha^d = 1$, $(\mathbb{Z}_d, \mathbb{Z}_d)$-charges $(+1, -1)$.
- identity defect can be rewritten using α.
- Module:

$$\bigoplus_{g \in G} \left(\bigwedge \left(\bigoplus_{j=1}^{n} \mathbb{C}[X_i, X_i'] \theta_j \right) \right) \to \bigwedge \left(\bigoplus_{j=1}^{n} \mathbb{C}[X_i, X_i', \alpha] \theta_j \right) \tag{2}$$

- Differential of the non-orbifolded theory, replacing X_i' by $\alpha^{Q_i} X_i'$.

Identity defect and orbifolds

- Example: single chiral superfield, $W_1 - W_2 = Z^d - X^d$, $\eta^d = 1$

$$p_1 = \begin{pmatrix} Z-X & 0 & \dots & 0 & 0 \\ & Z-\eta X & & & \\ 0 & 0 & Z-\eta^2 X & & \\ \vdots & & \ddots & \ddots & \\ 0 & & & 0 & Z-\eta^{d-1}X \end{pmatrix} \to \begin{pmatrix} Z & 0 & \dots & 0 & -X \\ -X & Z & & & \\ 0 & -X & Z & & \\ \vdots & & \ddots & \ddots & \\ 0 & & & -X & Z \end{pmatrix}$$

- This construction now generalizes to continuous orbifold groups.
- Get identity for the $U(1)$ equivariant LG model (GLSM).
- Operations such as “pushing down” to a phase are explicit.
- We can explicitly merge defects in LG orbifolds.

Conclusions

- Discussion of functors between brane categories in different phases of a GLSM.
- Functors are given in terms of defects, e.g. T_a between phase and GLSM.
- Provides an alternative point of view on brane transport.
- Example: 2 LG phases; Open but doable: Models with a geometric phase
- Construction of identity defect for continuous orbifolds with the help of a new field.
- More on $U(1)$ equivariant Landau-Ginzburg theories?