Global analysis of neutrino oscillation experiments

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Neutrino oscillations
Parametrization of the mixing matrix

- Neutrino oscillation probability is given by

\[ P(\alpha \rightarrow \beta; E, L) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* e^{i \frac{\Delta m_{kj}^2}{2E} L} \]
Neutrino oscillation probability is given by

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For a given energy \( E \) and distance \( L \) the probability depends on:

- Two mass splittings \( \Delta m_{21}^2, \Delta m_{31}^2 \)
- The entries of the matrix \( U \)
Parametrization of the mixing matrix

• The mixing matrix can be parametrized as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
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- There are also two more majorana phases, but oscillation experiments are blind to them.
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- Different types of experiments are sensitive to different parameters
Global fit to neutrino oscillations


https://globalfit.astroparticles.es/
Global fit to neutrino oscillations

- Solar experiments measure disappearance \( (P_{ee}) \) and conversion \( (P_{ex}) \) of electron neutrinos created in the sun.
Global fit to neutrino oscillations

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Global fit to neutrino oscillations

- Solar experiments measure disappearance ($P_{ee}$) and conversion ($P_{ex}$) of electron neutrinos created in the sun.
- They depend mainly on $\theta_{12}$ and $\Delta m_{21}^2$ and sub-dominantly on $\theta_{13}$.
- The solar parameters are measured also by the long baseline reactor experiment KamLAND.
Global fit to neutrino oscillations

- Data included:
  - SK I-IV
  - Borexino: Beryllium data
  - SNO I-III
  - Sage
  - Gallex+GNO
  - Chlorine
  - KamLAND
Global fit to neutrino oscillations

- Result of solar experiments and KamLAND
Global fit to neutrino oscillations

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- $\theta_{12}$ better measured by solar data
Global fit to neutrino oscillations

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Global fit to neutrino oscillations

- Result of solar experiments and KamLAND
  - $\theta_{12}$ better measured by solar data
  - $\Delta m_{21}^2$ better measured by KamLAND
- Maximal mixing is highly disfavored
- Mismatch between solar and KamLAND data for mass splitting
Global fit to neutrino oscillations

- Reactor experiments measure disappearance of electron antineutrinos ($P_{\nu\bar{\nu}}$) created at reactors
Global fit to neutrino oscillations

- Reactor experiments measure disappearance of electron antineutrinos ($P_{\bar{e}\bar{e}}$) created at reactors
- The main dependence of short baseline reactors is on $\theta_{13}$ and $\Delta m_{31}^2$
Global fit to neutrino oscillations

• Data included:
  – 1230 day Daya Bay spectrum
  – 1500 day RENO spectrum
  – 461 day (FII) and 212 day (FII) Double Chooz spectrum
Global fit to neutrino oscillations

- Data included:
  - 1230 day Daya Bay spectrum
  - 1500 day RENO spectrum
  - 461 day (F1) and 212 day (FII) Double Chooz spectrum

- Older reactors are not included, because they only provide upper limits on $\theta_{13}$
Global fit to neutrino oscillations

- Result of reactor experiments
Global fit to neutrino oscillations

- Result of reactor experiments

- Reactor analysis is dominated by Daya Bay
Global fit to neutrino oscillations

- Result of reactor experiments

- Reactor analysis is dominated by Daya Bay
- RENO starts being competitive
Global fit to neutrino oscillations

- Atmospheric neutrino experiments are mostly focused on the disappearance of muon neutrinos ($P_{\mu\mu}$) and antineutrinos ($P_{\bar{\mu}\bar{\mu}}$) created in the atmosphere.
Global fit to neutrino oscillations

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- They measure the atmospheric parameters $\Delta m_{31}^2$ and $\theta_{23}$. 

IceCube at the South Pole
Global fit to neutrino oscillations

- Data included:
  - 863 days of ANTARES data
  - 953 days of IceCube DeepCore data
  - SK I-IV
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Global fit to neutrino oscillations

- **Data included:**
  - 863 days of ANTARES data
  - 953 days of IceCube DeepCore data
  - SK I-IV

- **In the case of SK we add the grid provided by the collaboration**
  - 14 datasets, 4 times 520 bins and 155 systematic errors with possible correlations among them, make it impossible to reproduce the results outside the collaboration
Global fit to neutrino oscillations

- Result of atmospheric experiments
Global fit to neutrino oscillations

- Result of atmospheric experiments

- Atmospheric experiments start to be competitive with long baseline experiments as we will see now
Global fit to neutrino oscillations

- Long baseline experiments measure disappearance of muon neutrinos \( P_{\mu\mu} \) and antineutrinos \( P_{\bar{\mu}\bar{\mu}} \) and appearance of electron neutrinos \( P_{\mu\bar{e}} \) and antineutrinos \( P_{\bar{\mu}\bar{e}} \) created at accelerator experiments.
Global fit to neutrino oscillations

- Long baseline experiments measure disappearance of muon neutrinos ($P_{\mu\mu}$) and antineutrinos ($P_{\bar{\mu}\bar{\mu}}$) and appearance of electron neutrinos ($P_{\mu e}$) and antineutrinos ($P_{\bar{\mu}\bar{e}}$) created at accelerator experiments.
- They measure the parameters $\Delta m_{31}^2, \theta_{23}, \theta_{13}$ and $\delta$. 
Global fit to neutrino oscillations

- Data included:
  - $14.7 \times 10^{20}$ POT in neutrino mode at T2K
  - $7.6 \times 10^{20}$ POT in antineutrino mode at T2K
  - $8.85 \times 10^{20}$ POT in neutrino mode at NOvA
  - MINOS: full accelerator data set
  - K2K: full data set
Global fit to neutrino oscillations

- Result of long-baseline experiments
Global fit to neutrino oscillations

- Result of long-baseline experiments

- Analysis dominated by T2K and NOvA
Results of the combined analysis
The solar plane

- The solar parameters are measured by solar experiments and KamLAND

- Best fit: $\sin^2 \theta_{12} = 0.320$, $\Delta m^2_{21} = 7.55 \times 10^{-5} \text{eV}^2$
The atmospheric plane

- Measurement of atmospheric parameters dominated by the combination of LBL and reactor experiments

- Best fit: \( \sin^2 \theta_{23} = 0.547, \Delta m_{31}^2 = 2.50 \times 10^{-3} \text{eV}^2 \) (NO)
  \( \sin^2 \theta_{23} = 0.551, \Delta m_{31}^2 = -2.42 \times 10^{-3} \text{eV}^2 \) (IO)
The reactor angle and the CP phase

- For the first time we can exclude big part of the parameter space for $\delta$

- Best fit: $\sin^2 \theta_{13} = 0.02160, \delta = 1.32\pi$ (NO)
  $\sin^2 \theta_{13} = 0.02220, \delta = 1.56\pi$ (IO)
The reactor angle

- The measurement of the reactor angle is dominated by the short baseline reactors
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- LBL+ATM might start being competitive in the near future
The CP phase

- Best sensitivity to $\delta$ comes from T2K
The CP phase

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- Constraint on $\theta_{13}$ improves sensitivity to $\delta$ at all experiments significantly
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- Best sensitivity to $\delta$ comes from T2K
- Constraint on $\theta_{13}$ improves sensitivity to $\delta$ at all experiments significantly
- This results in exclusion of values around $0.5\pi$ at $>4\sigma$
The mass ordering and the role of SK

- Inverted mass ordering is now disfavored at more than 3σ, with $\Delta\chi^2 = 11.7$

SK does not change regions:
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- If we exclude SK from the fit we obtain $\Delta \chi^2 = 7.7$
- This is due to the combination of LBL+Reactors, since LBL alone gives $\Delta \chi^2 = 2.0$
- SK “only” improves the sensitivity to the mass ordering

SK does not change regions:
Summary of the global fit

- $\Delta \chi^2$ vs $\sin^2 \theta_{12}$
- $\Delta \chi^2$ vs $\sin^2 \theta_{23}$
- $\Delta \chi^2$ vs $\sin^2 \theta_{13}$
- $\Delta m_{21}^2 \ [10^{-5} \text{ eV}^2]$ vs $\delta/\pi$
# Summary of the global fit

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit ± 1σ</th>
<th>3σ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{21}^2 \ [10^{-5}\text{eV}^2]$</td>
<td>$7.55^{+0.20}_{-0.16}$</td>
<td>7.05–8.14</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m_{31}^2</td>
<td>\ [10^{-3}\text{eV}^2]$ (NO)</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m_{31}^2</td>
<td>\ [10^{-3}\text{eV}^2]$ (IO)</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}/10^{-1}$</td>
<td>$3.20^{+0.20}_{-0.16}$</td>
<td>2.73–3.79</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$ (NO)</td>
<td>$5.47^{+0.20}_{-0.30}$</td>
<td>4.45–5.99</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$ (IO)</td>
<td>$5.51^{+0.18}_{-0.30}$</td>
<td>4.53–5.98</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-2}$ (NO)</td>
<td>$2.160^{+0.083}_{-0.069}$</td>
<td>1.96–2.41</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-2}$ (IO)</td>
<td>$2.220^{+0.074}_{-0.076}$</td>
<td>1.99–2.44</td>
</tr>
<tr>
<td>$\delta/\pi$ (NO)</td>
<td>$1.32^{+0.21}_{-0.15}$</td>
<td>0.87–1.94</td>
</tr>
<tr>
<td>$\delta/\pi$ (IO)</td>
<td>$1.56^{+0.13}_{-0.15}$</td>
<td>1.12–1.94</td>
</tr>
</tbody>
</table>
More on mass ordering

JCAP 1803 (2018) no.03, 011, S. Gariazzo, M. Archidiacono, P.F. de Salas, O. Mena, CAT, M. Tórtola

More on mass ordering

- We can perform a Bayesian analysis combining several datasets
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- In this case we compare NO and IO by means of model selection techniques
More on mass ordering

- We can perform a Bayesian analysis combining several datasets
- In this case we compare NO and IO by means of model selection techniques
- We calculate the Bayesian evidence for both mass orderings

\[ Z = p(d|\mathcal{M}) = \int_{\Omega_{\mathcal{M}}} p(d|\theta, \mathcal{M})p(\theta|\mathcal{M})d\theta \]
More on mass ordering

- We can perform a Bayesian analysis combining several datasets
- In this case we compare NO and IO by means of model selection techniques
- We calculate the Bayesian evidence for both mass orderings

\[
Z = p(d|\mathcal{M}) = \int_{\Omega_\mathcal{M}} p(d|\theta, \mathcal{M})p(\theta|\mathcal{M})d\theta
\]

- Then we compute the Bayes factor

\[
B_{NO,IO} = \frac{Z_{NO}}{Z_{IO}} \Rightarrow \ln B_{NO,IO} = \ln Z_{NO} - \ln Z_{IO}
\]
More on mass ordering

- The preference for one ordering is then given by the Jeffreys’ scale

<table>
<thead>
<tr>
<th>$\ln B_{NO,IO}$</th>
<th>Odds</th>
<th>strength of evidence</th>
<th>$N\sigma$ for the mass ordering</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 1.0$</td>
<td>$\lesssim 3 : 1$</td>
<td>inconclusive</td>
<td>$&lt; 1.1\sigma$</td>
</tr>
<tr>
<td>$\in [1.0, 2.5]$</td>
<td>$(3 - 12) : 1$</td>
<td>weak</td>
<td>$1.1 - 1.7\sigma$</td>
</tr>
<tr>
<td>$\in [2.5, 5.0]$</td>
<td>$(12 - 150) : 1$</td>
<td>moderate</td>
<td>$1.7 - 2.7\sigma$</td>
</tr>
<tr>
<td>$\in [5.0, 10]$</td>
<td>$(150 - 2.2 \times 10^4) : 1$</td>
<td>strong</td>
<td>$2.7 - 4.1\sigma$</td>
</tr>
<tr>
<td>$\in [10, 15]$</td>
<td>$(2.2 \times 10^4 - 3.3 \times 10^6) : 1$</td>
<td>very strong</td>
<td>$4.1 - 5.1\sigma$</td>
</tr>
<tr>
<td>$&gt; 15$</td>
<td>$&gt; 3.3 \times 10^6 : 1$</td>
<td>decisive</td>
<td>$&gt; 5.1\sigma$</td>
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Parameterization and data
Parameterization and data

- Apart from oscillation data we include: Temperature and high-l polarization data from Planck (as of 2015) + a prior on the optical depth (2016), BAO, and a prior on $H_0$
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- Also included is data from the decay experiments: KamLAND-ZEN, EXO200 and GERDA
Parameterization and data

- The result can depend drastically on the choice of parametrization and priors
Parameterization and data

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- There are for example several ways to parametrize the neutrino masses

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<th></th>
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<td>Range</td>
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Parameterization and data

- This can give a biased preference for normal ordering
Parameterization and data

- For the present analysis we use the most robust and optimal parameterization to scan efficiently the neutrino parameter space.
Parameterization and data

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- The priors we use are:

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<td>$\tau$</td>
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<tr>
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<td>$5 \times 10^{-5}$ – $10^{-4}$</td>
<td>$n_s$</td>
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<tr>
<td>$\Delta m_{31}^2$/eV$^2$</td>
<td>$1.5 \times 10^{-3}$ – $3.5 \times 10^{-3}$</td>
<td>log($10^{10} A_s$)</td>
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<tr>
<td>log$<em>{10}(m</em>{\text{lightest}}$/eV)</td>
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- with a logarithmic prior on the lightest neutrino mass.
Results
Results

- Different data sets are considered
Results

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  - Only oscillation data

Results

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  - Only oscillation data
  - OSC plus decay data

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  - As before, but with an prior on the Hubble constant

Results

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  - Only oscillation data
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  - OSC+0νββ plus CMB plus BAO
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- Strong preference for NO in all cases (driven by oscillation data)

Conclusions
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Conclusions

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- The solar parameters, the reactor angle and absolute value of the atmospheric mass splitting are very well measured (errors 5% and below)
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- The octant problem remains unsolved, although the value now tends towards the second octant
Conclusions

- Neutrino oscillation experiments are entering the precision era
- The solar parameters, the reactor angle and absolute value of the atmospheric mass splitting are very well measured (errors 5% and below)
- We exclude a large part for of the parameter space for the CP phase
- The octant problem remains unsolved, although the value now tends towards the second octant
- By combining several datasets, including cosmological observations and $0\nu\beta\beta$-data we disfavor inverted mass ordering with $3.5\sigma$
Stay tuned for the future!
Thank you!