Measurement of Phase Space Density Evolution in MICE

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for the MICE Collaboration

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Muon Ionization Cooling Experiment

- Demonstration of ionization cooling in a setting relevant to muon accelerators
  - measure performance in various modes of operation and beam conditions, thereby investigating the limits and practicality of cooling
  - study aspects critical to performance (multiple scattering, energy loss, emittance evolution)
  - validate design & simulation tools
- Concept
  - Track each muon before & after cooling hardware
  - Can form virtual beams in offline software
  - Designed for measuring relative change in emittance to 1%
  - Accelerator R&D in the form of a particle physics experiment

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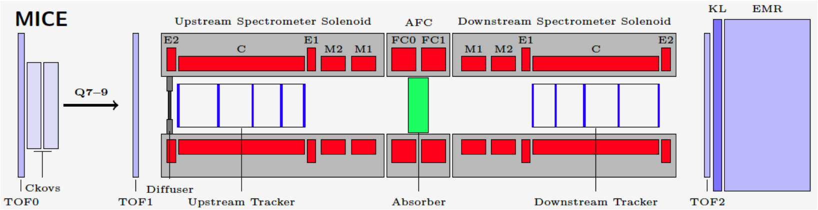
Layout

Particle ID – Momentum measurement – Cooling – Momentum measurement – Particle ID



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- solenoid (same sign coils) and (sign) flip optics modes

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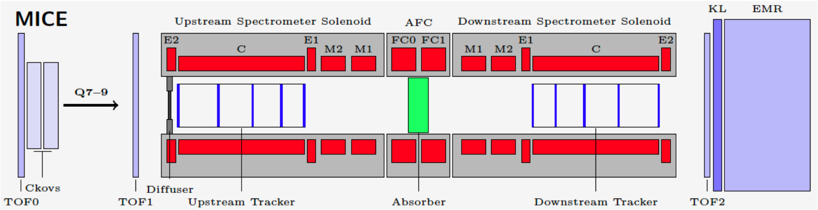

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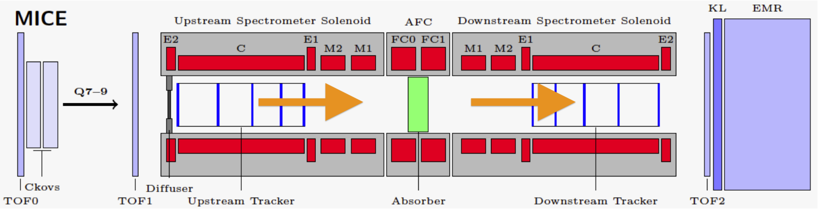

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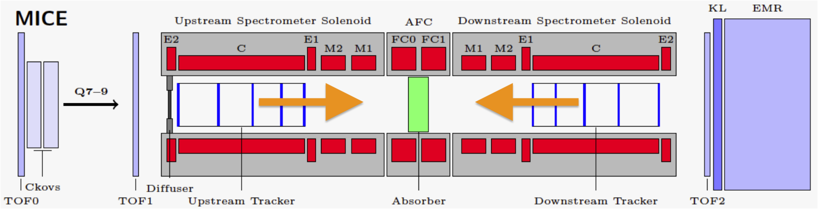

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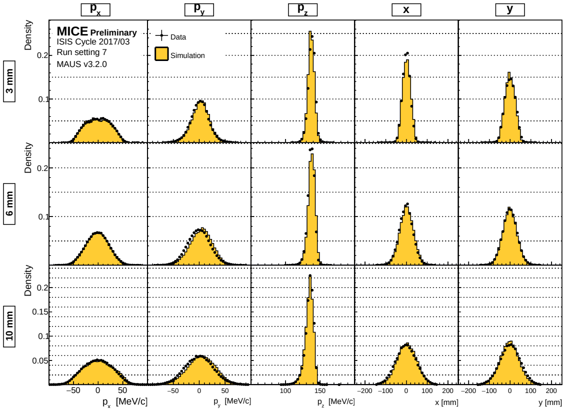
Analysis status

- Full suite of tools with detailed material and field distributions
- Excellent agreement for beam profiles (upstream/downstream), transmission, optical functions



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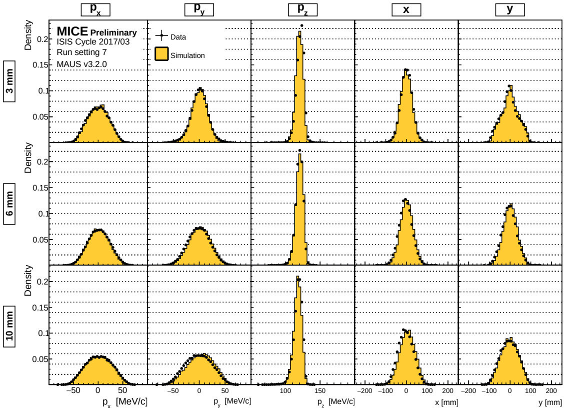

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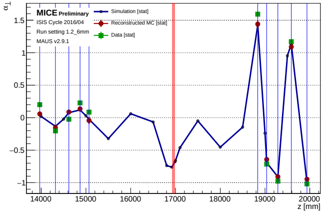

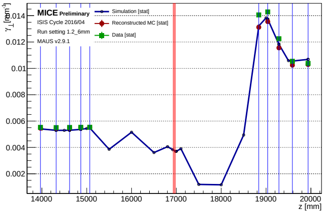

Measuring beam cooling

- Transverse normalized emittance commonly used to characterize phase space volume
  - works well for Gaussian beam through linear optics with no losses
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  - high precision required for detailed comparison in a wide range of beam and optics parameters
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- Use quantities that are robust and relevant
  - transverse amplitude
  - subemittance, fractional emittance
  - phase space density, core volume

Emittance

Normalized RMS transverse 4D emittance $\epsilon_\perp$

$$\epsilon_\perp = \frac{1}{m_\mu c} |\Sigma|^{1/4}$$

defined through phase space covariance matrix Σ

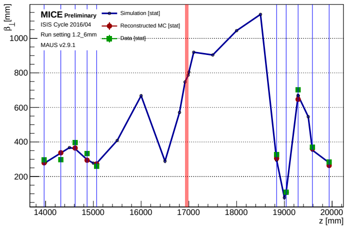
$$\Sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xp_x} & \sigma_{xy} & \sigma_{xp_y} \\ \sigma_{p_x x} & \sigma_{p_x p_x} & \sigma_{p_x y} & \sigma_{p_x p_y} \\ \sigma_{yx} & \sigma_{yp_x} & \sigma_{yy} & \sigma_{yp_y} \\ \sigma_{p_y x} & \sigma_{p_y p_x} & \sigma_{p_y y} & \sigma_{p_y p_y} \end{pmatrix}, \quad \sigma_{ab} = \langle (a - \langle a \rangle)(b - \langle b \rangle) \rangle$$

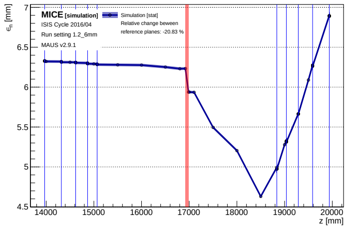
corresponds to volume V of 4D rms ellipsoid and indicates an average phase space density

$$\rho = \frac{N}{V} = \frac{2}{\pi^2} \frac{N}{|\Sigma|^{1/2}}$$

Evolution of RMS emittance

- solenoid mode optics, LiH absorber, 6-mm 140-MeV/c input beam
- limited transmission + betatron motion
  $\Rightarrow$ large apparent cooling at downstream tracker plane
- rms emittance is a poor indicator in this case





Transverse single-particle amplitude

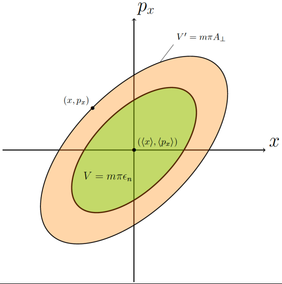
Defined as

$$A_\perp = \epsilon_\perp \, \mathbf{u}^T \Sigma^{-1} \, \mathbf{u}$$

for centered phase space coordinates

$$\mathbf{v} = (x, p_x, y, p_y)$$

$$\mathbf{u} = \mathbf{v} - \langle \mathbf{v} \rangle$$



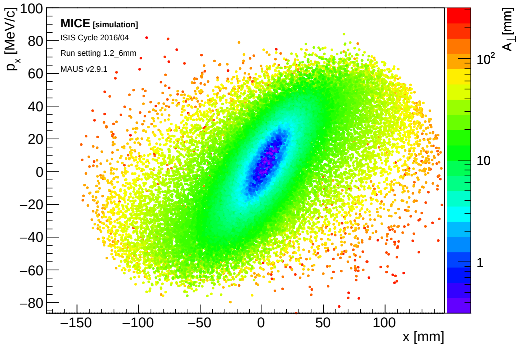
- Associated with phase space volume similar to rms ellipsoid (emittance)
- Provides density estimate at every sample point

$$\rho(\mathbf{v}_i) = \rho_0 \, exp\left[-\frac{1}{2}\frac{A_\perp}{\epsilon_\perp}\right]$$

- Allows identification of
  low $A_\perp \Leftrightarrow$ high ρ core
  high $A_\perp \Leftrightarrow$ low ρ tail
- Highest amplitude particles can be removed iteratively to prevent bias

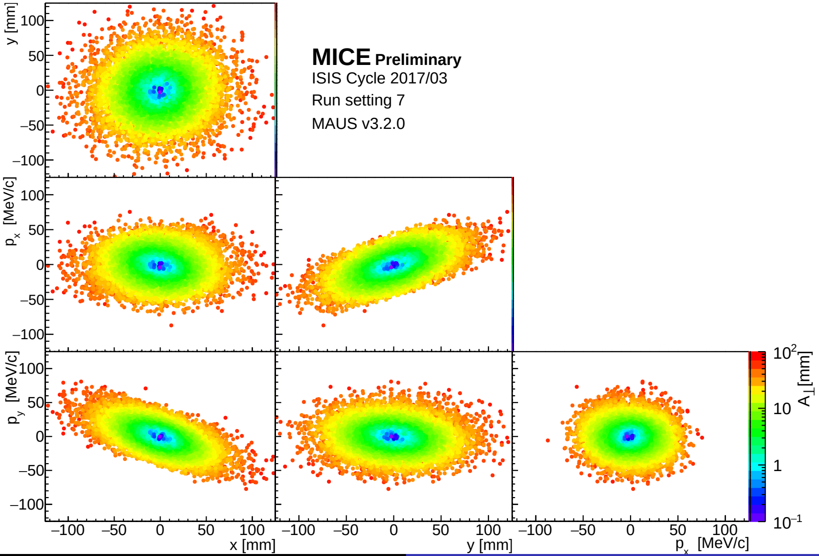
Amplitude reconstruction example [simulation]

- 6-mm 140-MeV/c input beam, solenoid mode optics
- Last (most downstream) measurement plane

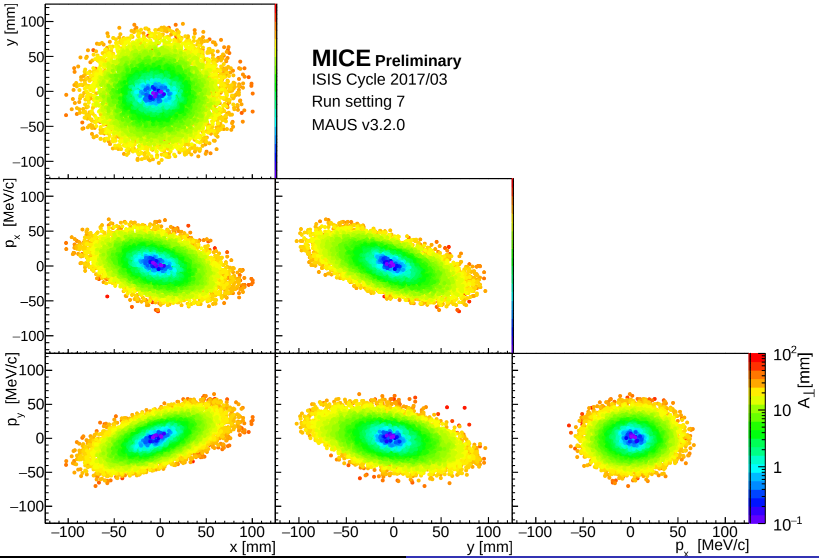


- High-density low-amplitude (cool) Gaussian core
- Low-density high-amplitude (hot) tails

Poincaré sections [data] (upstream)
6-mm 140-MeV/c beam – flip mode – LiH



Poincaré sections [data] (downstream)
6-mm 140-MeV/c beam – flip mode – LiH



Subemittance

- For a parent beam of n particles, select a fraction α from the core
- α-amplitude A_α is the largest amplitude in the α-sample

$$A_\alpha = \epsilon_\perp \text{ at } \alpha\text{=9\% for Gaussian beam in 4D}$$

- 9% is the 1-σ volume fraction in 4D
- α-subemittance e_α is defined as the rms emittance of the α-sample

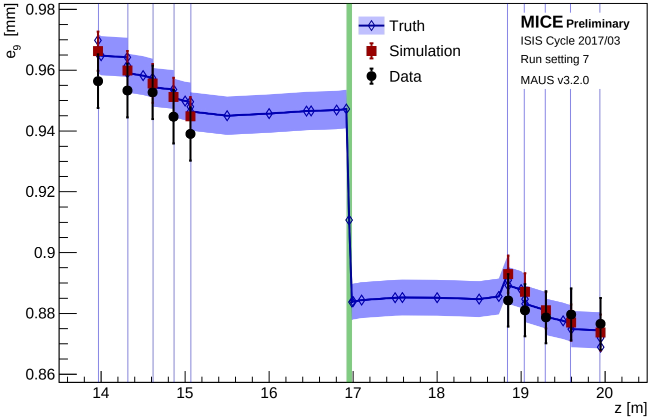
$$e_\alpha \leq \epsilon_\perp$$

- If an identical fraction α is selected upstream and downstream

$$\frac{\Delta A_\alpha}{A_\alpha} = \frac{\Delta e_\alpha}{e_\alpha} = \frac{\Delta \epsilon_\perp}{\epsilon_\perp}$$

for Gaussian core with full transmission

Subemittance evolution
6-mm 140-MeV/c beam – flip mode – LiH

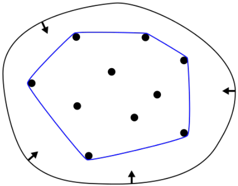


Fractional emittance

- The α-fractional emittance ϵ_α is defined as the phase space volume occupied by the core fraction α of the parent beam.
- Found by calculating the volume of the convex hull of the α-sample (smallest convex set containing all the points)

- For $\alpha = 9\%$

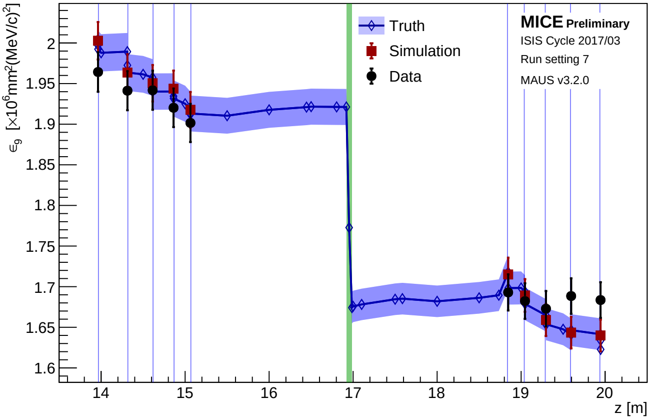
$$\epsilon_\alpha = \frac{1}{2}\left(\pi\, m\, c\, \epsilon_\perp\right)^2$$



- For small change

$$\delta = \frac{\Delta\epsilon_\perp}{\epsilon_\perp} \ll 1 \rightarrow \frac{\Delta\epsilon_\alpha}{\epsilon_\alpha} \simeq 2\delta$$

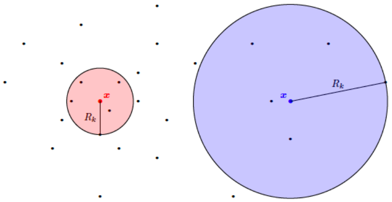
Fractional (9%) emittance evolution
6-mm 140-MeV/c beam – flip mode – LiH

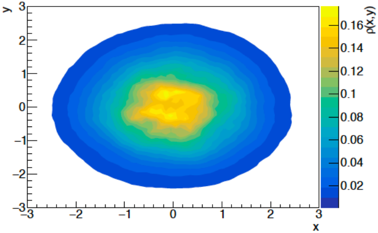


Non-parametric density estimation

- Amplitude based methods work well for Gaussian core, small fraction of a nonlinear beam
- Non-parametric density estimators can be used to extend the analysis
- Several methods considered including
  - optimally binned histograms
  - k-nearest neighbors (kNN)
  - tessellation density estimators (TDEs)
  - kernel density estimation (KDE)
- kNN and KDE examples follow

k-Nearest neighbor algorithm





kNN density of 10^3 Gaussian points

To find the density $\rho(\mathbf{x})$ at a point $\mathbf{x}$ in phase space, identify nearby data points $\mathbf{x}_i$. Using the distance R_k to the kth-nearest point

$$\rho(\mathbf{x}) = \frac{k}{V(R_k)}$$

where $V(R_k)$ is the volume of the 4-ball with radius R_k

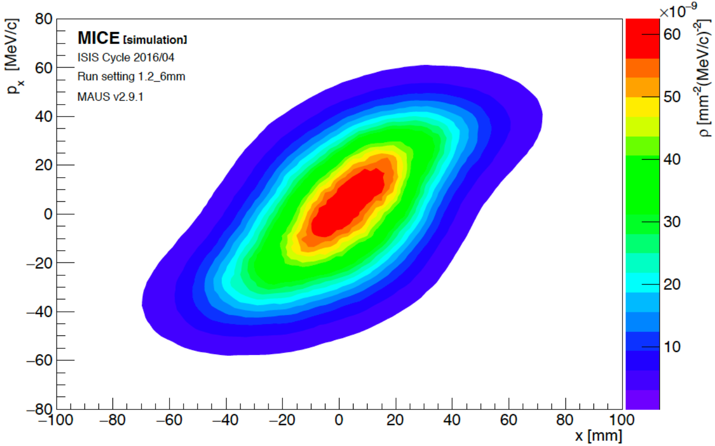
$$V = \pi^2 R_k^4 / 2$$

Near optimal results for

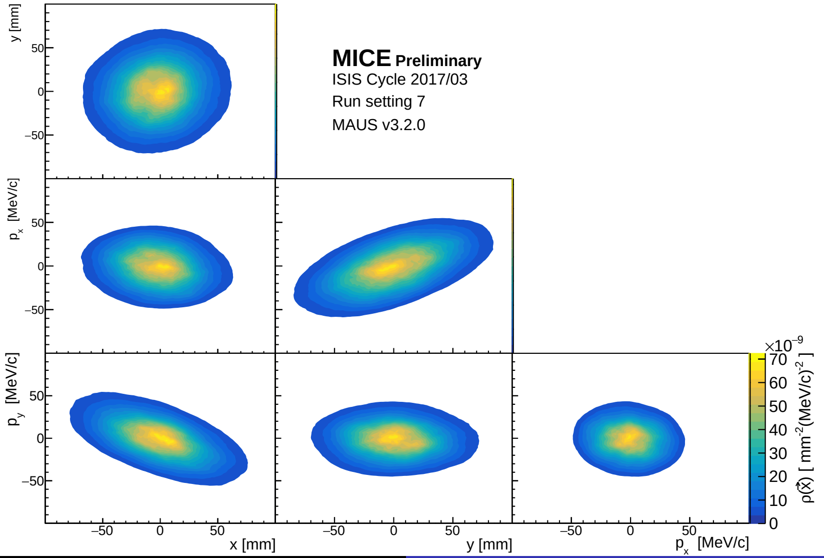
$$k = \sqrt{n}$$

kNN density estimate [simulation]

- 6-mm 140-MeV/c input beam, solenoid mode optics
- Last (most downstream) tracker plane
- reconstructed 4D density projected to $(y, p_y) = (0, 0)$



Poincaré sections [kNN + data] (upstream)
6-mm 140-MeV/c beam – solenoid mode – LiH



Poincaré sections [kNN + data] (downstream)
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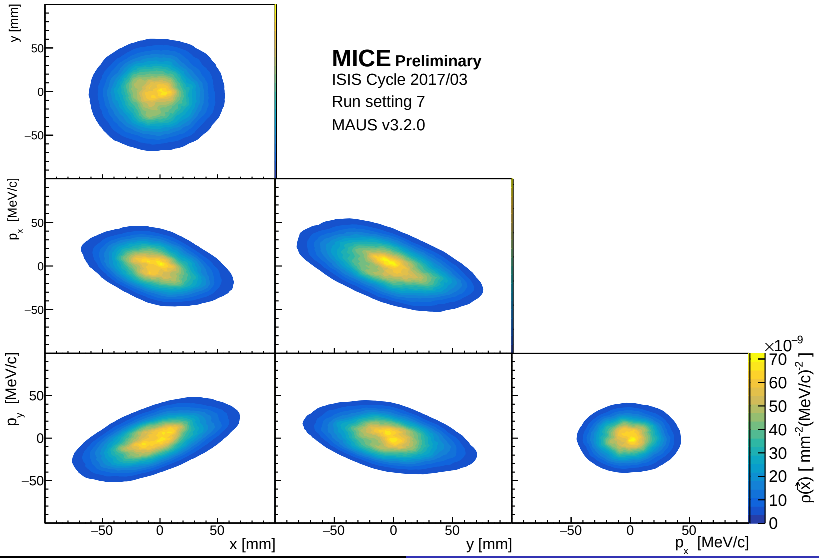
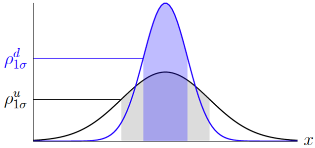


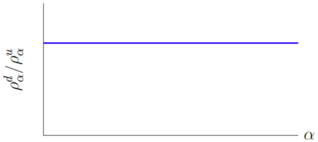
Contour levels

- Given the cumulative distribution function F for the beam
- find the density level ρ_α (α-quantile, inverse of CDF) for the contour that encloses core fraction α of the beam

$$\rho_\alpha = \rho(F^{-1}(\alpha))$$

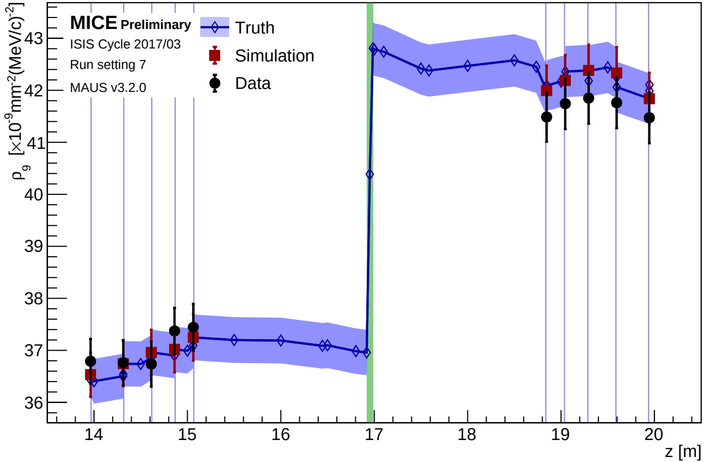
- The evolution of ρ_α shows cooling (ratio independent of α in any dimension for purely Gaussian input/output beams)





- Can also use the volume of phase space V_α that has $\rho > \rho_\alpha$

(9%) Contour density evolution (kNN)
6-mm 140-MeV/c beam – LiH – flip



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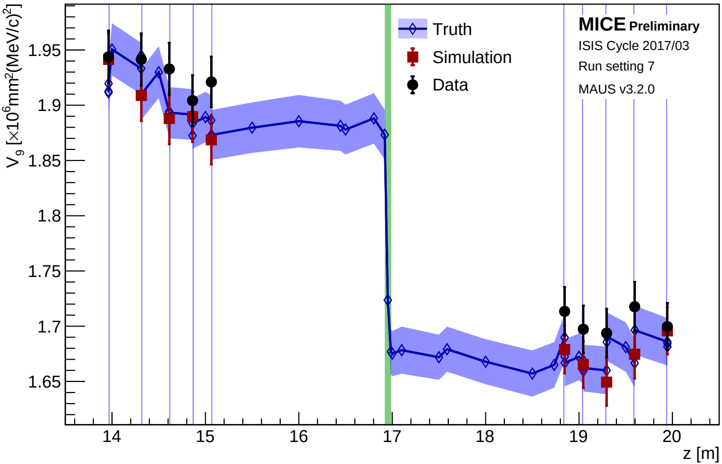


Kernel density estimation

Density estimate ρ at point x

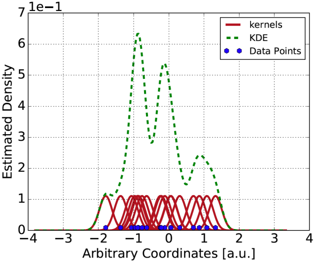
$$\rho(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

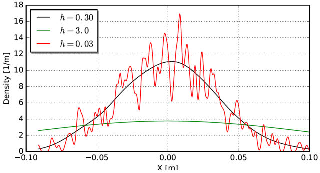
where K is called the kernel function

$$\int K(x)\, dx = 1$$

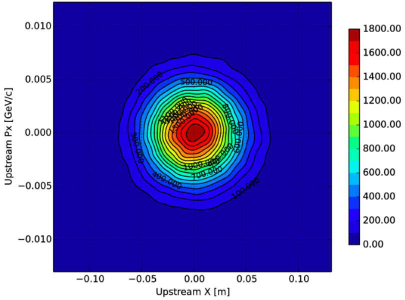
and h, the bandwidth parameter. For d-dimensional phase space, use Gaussian kernel

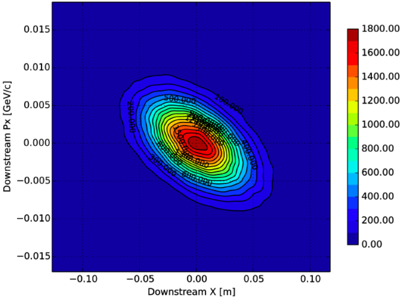
$$\rho(\mathbf{x}) \propto \sum_i exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{x}_i)^T \Sigma^{-1} (\mathbf{x} - \mathbf{x}_i)\right]$$

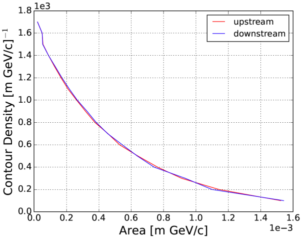




KDE example

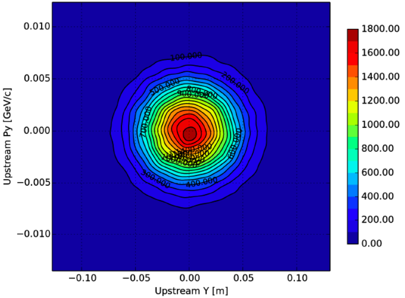


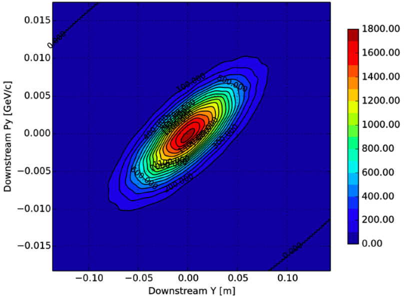


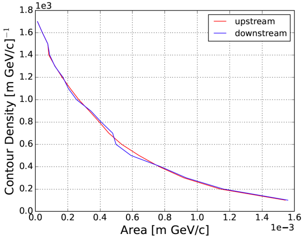


- Gaussian beam with 100k muons through quadrupole
- evaluated on 1k x 1k grid
- 2D contour density and area conserved

KDE example







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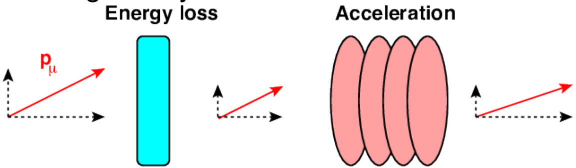
Emittance exchange

- Cooling mainly transverse in a linear channel



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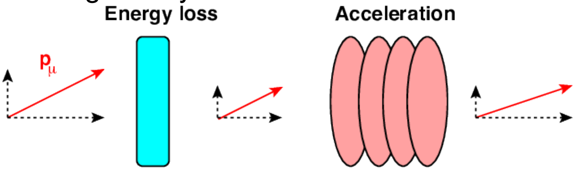
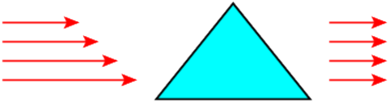


- Longitudinal cooling requires momentum-dependent path-length through the energy absorbers



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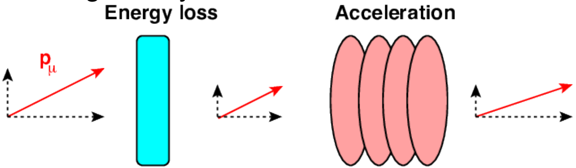

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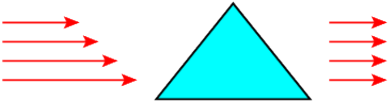
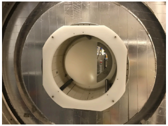
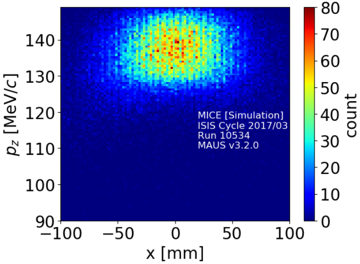
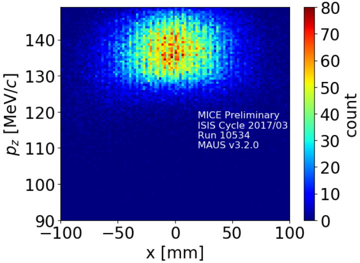


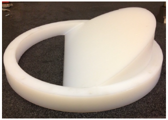


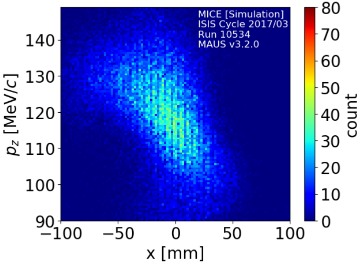
- Wedge shaped polyethylene absorber for demonstration of (reverse) emittance exchange in MICE

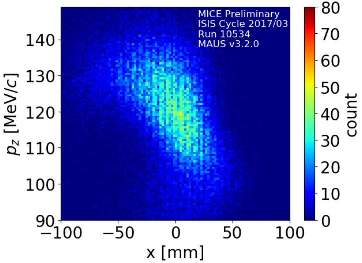
Reverse emittance exchange
6mm 140-MeV/c beam – polyethylene wedge











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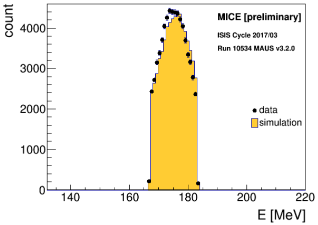

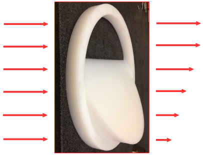

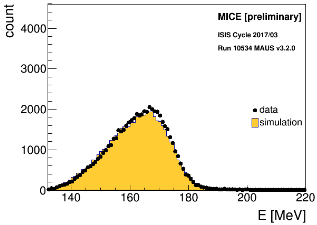

- No RF $\rightarrow$ longitudinal space is 1D (E)
  - Longitudinal heating

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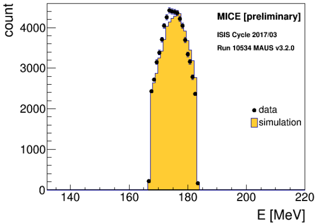
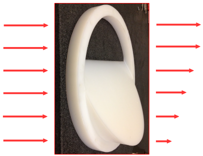
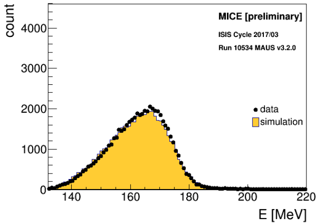
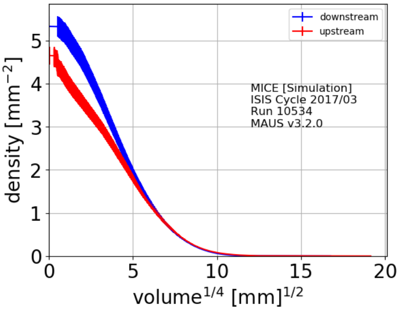
6mm 140-MeV/c beam – polyethylene wedge









- No RF $\rightarrow$ longitudinal space is 1D (E)
  - Longitudinal heating
- 4D transverse phase space $(x, p_x/\langle p\rangle, y, p_y/\langle p\rangle)$
  - Transverse cooling

Outlook

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  - Amplitude based analysis used to avoid artifacts due to nonlinear transport
  - Core density/volume used for selecting the portion of the beam that is transmitted
  - Non-parametric density estimators substantially independent of the underlying distribution

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