Standard Model prediction for muon g-2

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talk at NuFACT 2018 @ Virginia Tech

August 14, 2018

Based on collaboration with
Alex Keshavarzi and Thomas Teubner **(KNT)**
Phys. Rev. D97 (2018) 114025 [arXiv:1802.02995]

Muon g-2: introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad (\vec{s} = \frac{1}{2}\vec{\sigma} \text{ (spin)}, \qquad g = 2 + 2F_2(0))$$

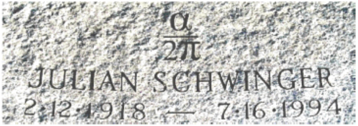
where

$$\overline{u}(p+q)\Gamma^{\mu}u(p) = \overline{u}(p+q)\left(\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

★ One of the **most precisely measured** quantities:

$$\boxed{a_{\mu}^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}} \quad [0.5\text{ppm}] \qquad \text{(Bennett *et al*)}$$

★ **Extremely useful** in **probing/constraining physics beyond the SM**

KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995, PRD (in press)]

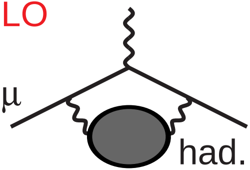
	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

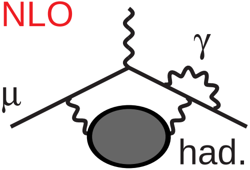
Hadronic Contributions

There are several hadronic contributions:

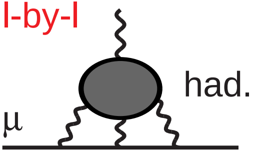
LO



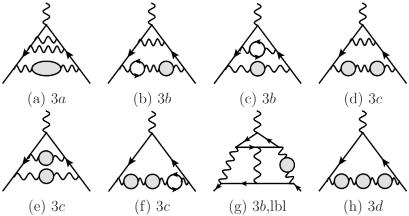
NLO



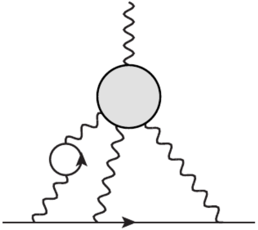
l-by-l



LO: Leading Order (or Vacuum Polarization) Hadronic Contribution
NLO: Next-to-Leading Order Hadronic Contribution
l-by-l: Hadronic light-by-light Contribution



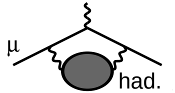
NNLO Hadronic Contributions



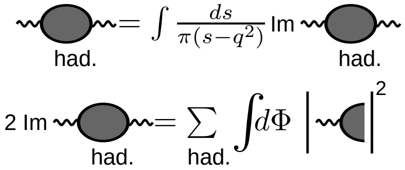
Hadronic l-by-l NLO Contrib.

LO Hadronic Vacuum Polarization Contribution

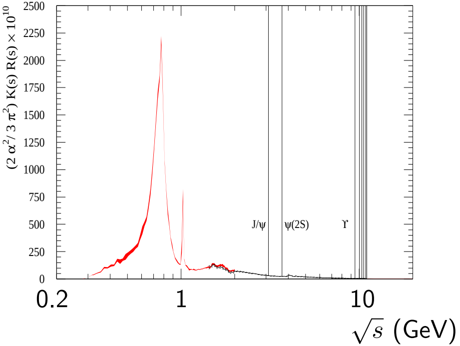
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

Main improvements between HLMNT11 and KNT18

- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
- Improvements in the estimates of uncertainties due to radiative corrections (Vacuum Polarization Radiative Corrections & Final State Radiations)
- Improvements in data-combination method

Channel	Energy range [GeV]	$a_\mu^{\rm had, LO\,VP} \times 10^{10}$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \times 10^4$	New data
		Chiral perturbation theory (ChPT) threshold contributions		
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	$\cdots$
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	$\cdots$
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	$\cdots$
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	$\cdots$
		Data based channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	$\cdots$
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\rm no\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	$\cdots$
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\rm no\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	$\cdots$
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	$\cdots$
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	$\cdots$
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to {\rm npp})K\bar{K}_{{\rm no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
		Estimated contributions ($\sqrt{s} \le 1.937$ GeV)		
$(\pi^+\pi^-3\pi^0)_{\rm no\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	$\cdots$
$(\pi^+\pi^-4\pi^0)_{\rm no\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	$\cdots$
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	$\cdots$
$\omega(\to {\rm npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	$\cdots$
$\omega(\to {\rm npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	$\cdots$
$\omega(\to {\rm npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	$\cdots$
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	$\cdots$
		Other contributions ($\sqrt{s} > 1.937$ GeV)		
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	$\cdots$	6.26 ± 0.19	7.07 ± 0.22	$\cdots$
ψ'	$\cdots$	1.58 ± 0.04	2.51 ± 0.06	$\cdots$
$\Upsilon(1S-4S)$	$\cdots$	0.09 ± 0.00	1.06 ± 0.02	$\cdots$
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	$\cdots$
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	$\cdots$

Table from KNT18, Phys. Rev. D97 (2018) 114025

Breakdown of contributions to a_μ(had, LO VP) from various hadronic final states

We have included new data sets from $\sim$ 30 papers, in addition to those included in the HLMNT11 analysis

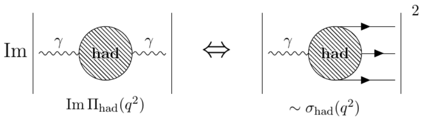
We have included $\sim$ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At higher energies $\gtrsim 11$ GeV, we use pQCD

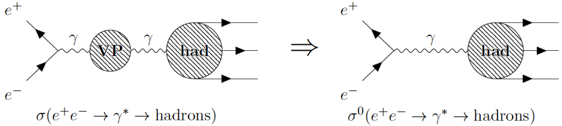
$\sigma^0_{\mathrm{had},\gamma}$: vacuum polarisation corrections

⇒ Reconsider the optical theorem:



⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\mathrm{had,\,VP}}$

→ Must subtract VP:



⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0], available for distribution

→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\mathrm{had}}(s) = \sigma_{\mathrm{had}}(s)|1 - \Pi(s)|^2$

⇒ If correcting data, apply corresponding radiative correction uncertainty

→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

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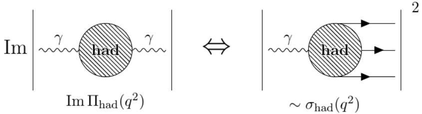
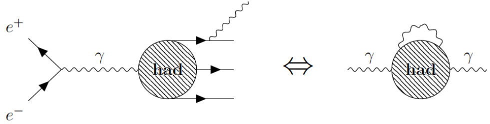
$\sigma^0_{\text{had},\gamma}$: final state radiation corrections

⇒ Reconsider the optical theorem:



⇒ Photon FSR formally higher order corrections to $a_\mu^{\text{had, VP}}$



⇒ Cannot be unambiguously separated, not accounted for in HO contributions

→ Must be included as part of 1PI hadronic blobs

⇒ Experiment may cut/miss photon FSR → Must be added back

⇒ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For higher multiplicity states, difficult to estimate correction

∴ Apply conservative uncertainty

Need new, more developed tools to increase precision here

(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

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d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

We first consider an observable x whose true value is 1. Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%.
Now, assume that this experiment measured x twice:

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \,,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \,.$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \\ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} \,,$$

$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} \,.$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? In the previous page,

1st result: $0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}}$,
2nd result: $1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}}$.

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$\text{(cov.)} = \begin{pmatrix} 0.1^2 + (0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2 + (0.1\bar{x})^2 \end{pmatrix} ,$$

$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} \text{(cov.)}^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} .$$

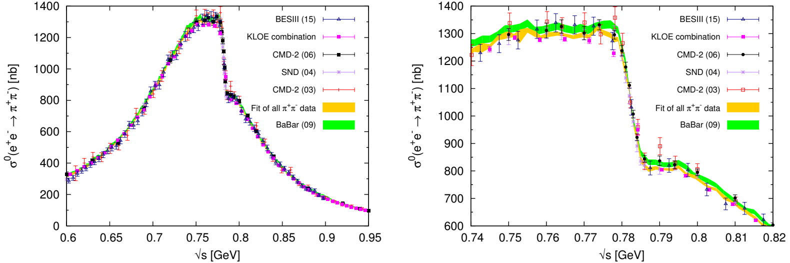
χ^2 takes its minimum at $x = 1.00$: Unbiased!

In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization. R.D.Ball et al, JHEP 1005 (2010) 075.

$\pi^+\pi^-$ channel

⇒ Large improvement for 2π estimate

→ BESIII [Phys.Lett. B753 (2016) 629-638] and KLOE combination [arXiv:1711.03085] provide downward influence to mean value



⇒ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 502.97 \pm 1.14_{\text{stat}} \pm 1.59_{\text{sys}} \pm 0.06_{\text{vp}} \pm 0.14_{\text{fsr}}$$
$$= 502.97 \pm 1.97_{\text{tot}}$$

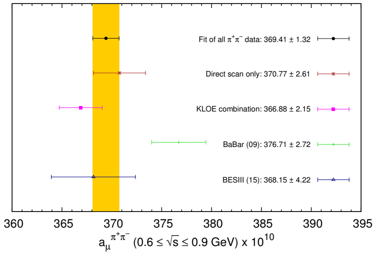
⇒ 15% local $\chi^2_{\text{min}}/\text{d.o.f.}$ error inflation due to tensions in clustered data

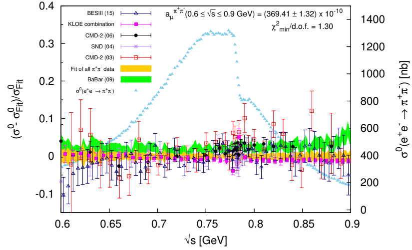
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018

$\pi^+\pi^-$ channel

⇒ Tension exists between BaBar data and all other data in the dominant ρ region.

→ Agreement between other radiative return measurements and direct scan data largely compensates this.





BaBar data alone ⇒ $a_\mu^{\pi^+\pi^-}$(BaBar data only) $= 513.2 \pm 3.8.$

Simple weighted average of all data ⇒ $a_\mu^{\pi^+\pi^-}$(Weighted average) $= 509.1 \pm 2.9.$
(i.e. - no correlations in determination of mean value)

BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

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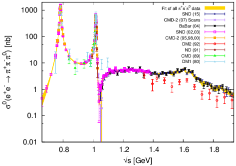
Other notable exclusive channels [KNT18: arXiv:1802.02995, PRD (in press)]

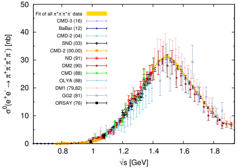
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

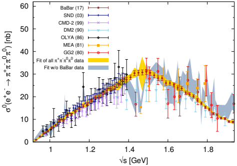
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

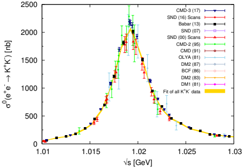
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

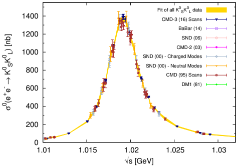
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

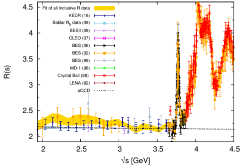
$K^0_S K^0_L$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)



HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

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KNT18 $a_\mu^{\text{had, VP}}$ update

HLMNT(11): 694.91 ± 4.27

$\downarrow$

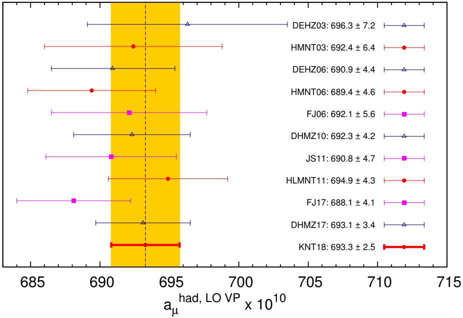
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\rm stat} \pm 2.01_{\rm sys} \pm 0.22_{\rm vp} \pm 0.71_{\rm fsr}$

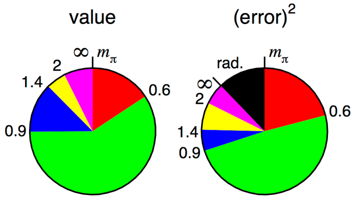
$= 693.27 \pm 2.34_{\rm exp} \pm 0.74_{\rm rad}$

$= 693.27 \pm 2.46_{\rm tot}$

$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\rm tot}$

$\Rightarrow$ Accuracy better then 0.4%
(uncertainties include all available correlations)





$\Rightarrow$ 2π dominance

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Comparison with HLMNT11

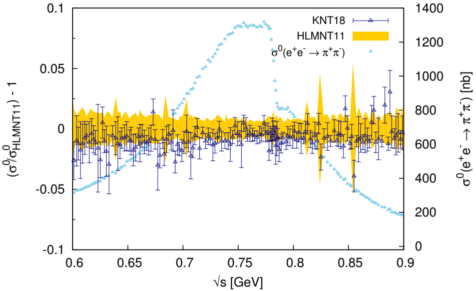
Channel	This work (KNT18)	HLMNT11	Difference
$\pi^+\pi^-$	502.99 ± 1.97	505.77 ± 3.09	-2.78 ± 3.66
$\pi^+\pi^-\pi^0$	47.82 ± 0.89	47.51 ± 0.99	0.31 ± 1.33
$\pi^+\pi^-\pi^+\pi^-$	15.17 ± 0.21	14.65 ± 0.47	0.52 ± 0.51
$\pi^+\pi^-\pi^0\pi^0$	19.80 ± 0.79	20.37 ± 1.26	-0.57 ± 1.49
K^+K^-	23.05 ± 0.22	22.15 ± 0.46	0.90 ± 0.51
$K^0_S K^0_L$	13.05 ± 0.19	13.33 ± 0.16	-0.28 ± 0.25
Inclusive channel	41.27 ± 0.62	41.40 ± 0.87	-0.13 ± 1.07
Total	693.27 ± 2.46	694.91 ± 4.27	-1.64 ± 4.93

⇒ Biggest difference in 2π channel
  → large reduction in mean and uncertainty

⇒ Tensions with HLMNT11 analysis for both two-kaon channels

⇒ Overall agreement with HLMNT11

⇒ Notable improvement of about one third in uncertainty



Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018

Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40 ± 3.24
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50 ± 1.70
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31 ± 0.36
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12 ± 0.92
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19 ± 0.47
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22 ± 0.31
$1.8 \le \sqrt{s} \le 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09 ± 0.86
Total	693.3 ± 2.5	693.1 ± 3.4	0.2 ± 4.2

$\Rightarrow$ Total estimates from two analyses in very good agreement

$\Rightarrow$ Masks much larger differences in the estimates from individual channels

$\Rightarrow$ Unexpected tension for 2π considering the data input likely to be similar

$\rightarrow$ Points to marked differences in way data are combined

$\rightarrow$ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

$\Rightarrow$ Compensated by lower estimates in other channels

$\rightarrow$ For example, the choice to use pQCD instead of data above 1.8 GeV

$\Rightarrow$ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

$\rightarrow$ Much lower mean value, but in agreement within errors

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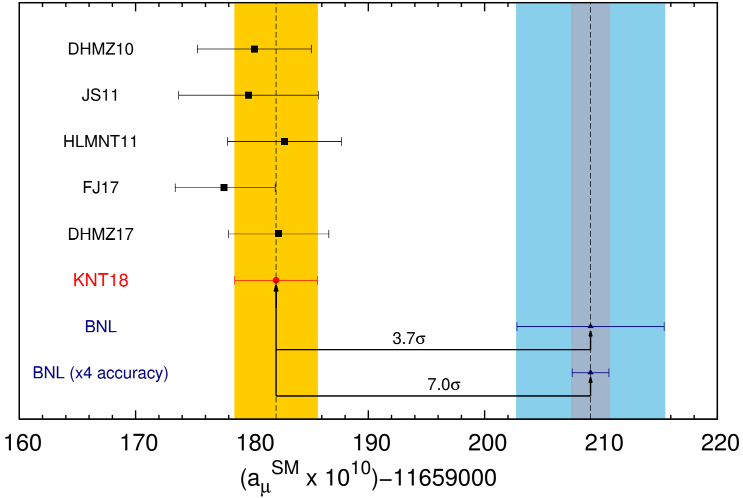
KNT18 a_μ^{SM} update

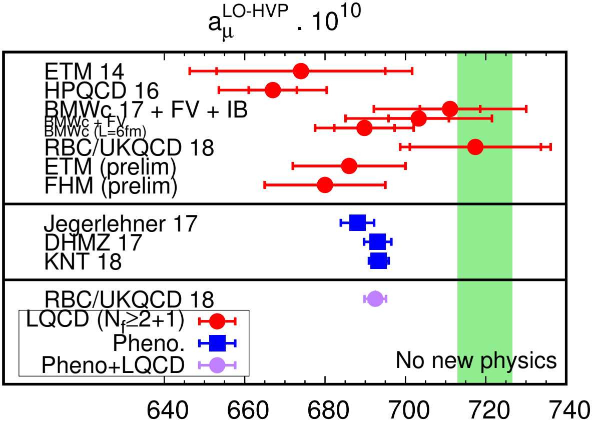
	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018

KNT18 a_μ^{SM} update



Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018



- Lattice errors $\sim$ 2% vs phenomenology errors $\sim$ 0.4%
- Some lattice results suggest new physics others not but all compatible with phenomenology

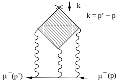
Slide by L. Lellouch (Marseille) at 'Muon g-2 Workshop' at Mainz, June 18-22, 2018

Summary

- Standard Model prediction for $(g-2)_\mu$: $\gtrsim 3.5\sigma$ deviation from measured value $\Longrightarrow$ New Physics?
- Recent data-driven evaluations of hadronic vacuum polarization contributions seem convergent (Similar mean values from KNT18 and Davier et al with slightly smaller uncertainty from KNT18.)
- To better establish the $g-2$ anomaly, better data for $e^+e^- \to \pi^+\pi^-$ welcome (from Belle II !?)
- Lattice calculations still suffer from large uncertainties

Backup Slides

HLbL in muon $g-2$: summary of selected results (model calculations)



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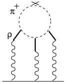
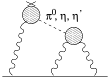
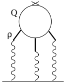


$+ \cdots +$


$+ \cdots +$ Exchange of other resonances $(f_0, a_1, f_2 \ldots)$ $+$


$+ \cdots$

de Rafael '94:

Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

Contribution to $a_\mu \times 10^{11}$:

BPP: +83 (32)	-19 (13)	+85 (13)	-4 (3) [f_0, a_1]	+21 (3)
HKS: +90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) [a_1]	+10 (11)
KN: +80 (40)		+83 (12)		
MV: +136 (25)	0 (10)	+114 (10)	+22 (5) [a_1]	0
2007: +110 (40)				
PdRV: +105 (26)	-19 (19)	+114 (13)	+8 (12) [f_0, a_1]	+2.3 [c-quark]
N,JN: +116 (39)	-19 (13)	+99 (16)	+15 (7) [f_0, a_1]	+21 (3)
	ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

Pseudoscalars: numerically dominant contribution (according to most models !).

Recall (in units of 10^{-11}): $\delta a_\mu(\text{HVP}) \approx 40$; $\delta a_\mu(\text{exp [BNL]}) = 63$; $\delta a_\mu(\text{future exp}) = 16$

BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; "Glasgow consensus"); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

Recent reevaluations of axial vector contribution lead to much smaller estimates than in MV '04: $a_\mu^{\text{HLbL;axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). Would shift central values of compilations downwards:

$a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11}$ (N, JN).

Slide by A. Nyffeler (Mainz) at 'Muon g-2 lbyl Workshop' at Connecticut, March 12-14, 2018

Model calculations of HLbL: recent developments

- Most calculations for neutral pion and all light pseudoscalars agree at level of 15%, but full range of estimates (central values) much larger:

$$a_\mu^{\mathrm{HLbL};\pi^0} = (50-80)\times 10^{-11} = (65\pm 15)\times 10^{-11} \quad (\pm 23\%)$$
$$a_\mu^{\mathrm{HLbL;P}} = (59-114)\times 10^{-11} = (87\pm 27)\times 10^{-11} \quad (\pm 31\%)$$

- New estimates for axial vectors (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15):

$$a_\mu^{\mathrm{HLbL;axial}} = (6-8)\times 10^{-11}$$

  Substantially smaller than in MV '04 !
- First estimate for tensor mesons (Pauk, Vanderhaeghen '14):

$$a_\mu^{\mathrm{HLbL;tensor}} = 1\times 10^{-11}$$

- Open problem: Dressed pion-loop
  Potentially important effect from pion polarizability and a_1 resonance
  (Engel, Patel, Ramsey-Musolf '12; Engel '13; Engel, Ramsey-Musolf '13):

$$a_\mu^{\mathrm{HLbL};\pi-\mathrm{loop}} = -(11-71)\times 10^{-11}$$

  Maybe large negative contribution, in contrast to BPP '96, HKS '96.
  Not confirmed by recent reanalysis by Bijnens, Relefors '15, '16. Essentially get again old central value from BPP, but smaller error estimate:

$$a_\mu^{\mathrm{HLbL};\pi-\mathrm{loop}} = (-20\pm 5)\times 10^{-11}$$

- Open problem: Dressed quark-loop
  Dyson-Schwinger equation approach (Fischer, Goecke, Williams '11, '13):

$$a_\mu^{\mathrm{HLbL;quark-loop}} = 107\times 10^{-11} \qquad \text{(still incomplete !)}$$

  Large contribution, no damping seen, in contrast to BPP '96, HKS '96.

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