How different can the ν_μ and ν_e cross sections be?

Artur M. Ankowski
SLAC, Stanford University

based on Phys. Rev. C 96, 035501 (2017)

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Outline

1) **Motivation**

- Why do we need precise cross sections for ν_e's?
- How are the ν_e and ν_μ cross sections related?

2) **When is the ν_e cross section higher then the ν_μ one?**

- Fermi gas with and without Pauli blocking
- Shell model and the spectral function

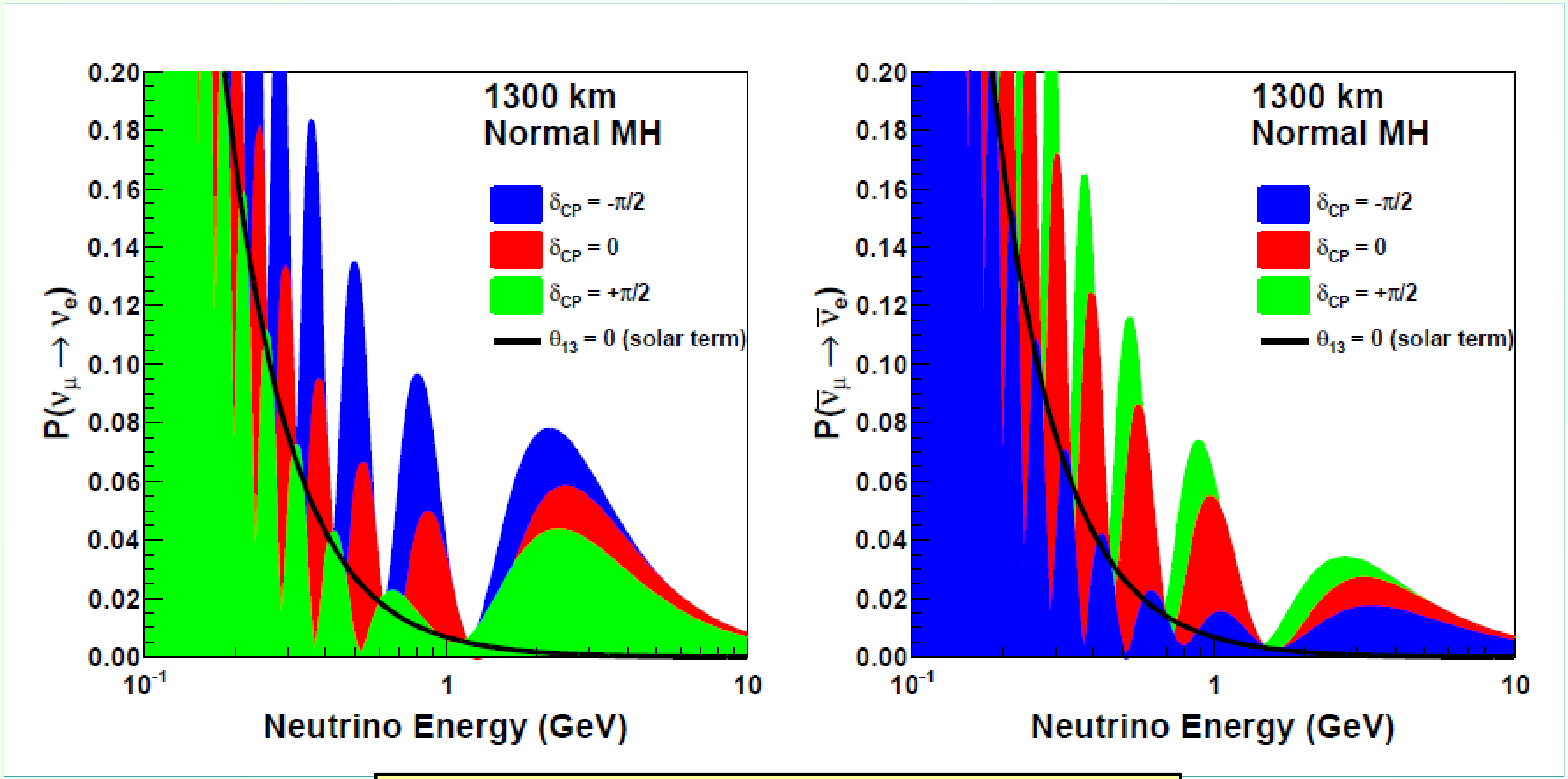
3) **Should we be concerned about model differences?**

4) **Summary**

Motivation

δ_{CP} from $\overset{(-)}{\nu}_e$ event distributions

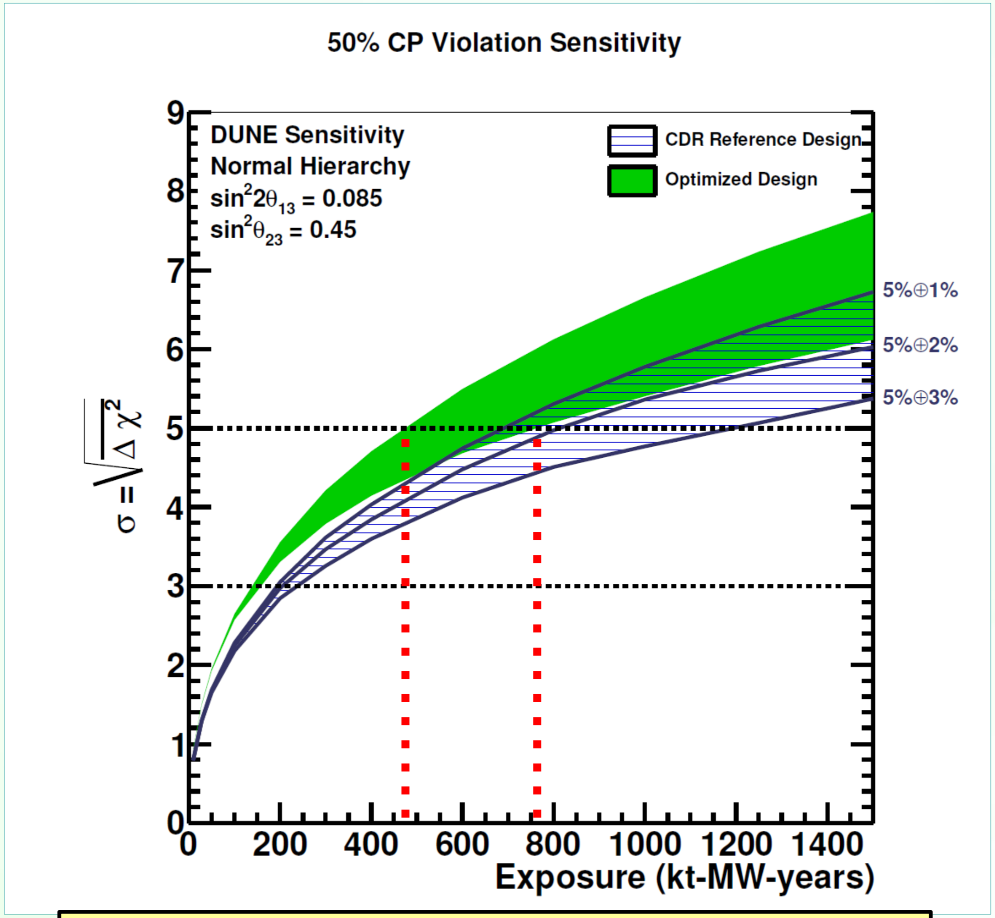
To find $P(\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_e)$ from event distributions, precise $\overset{(-)}{\nu}_e$ cross sections are necessary.



Acciari *et al.* (DUNE), arXiv:1512.06148

How relevant is the precision?

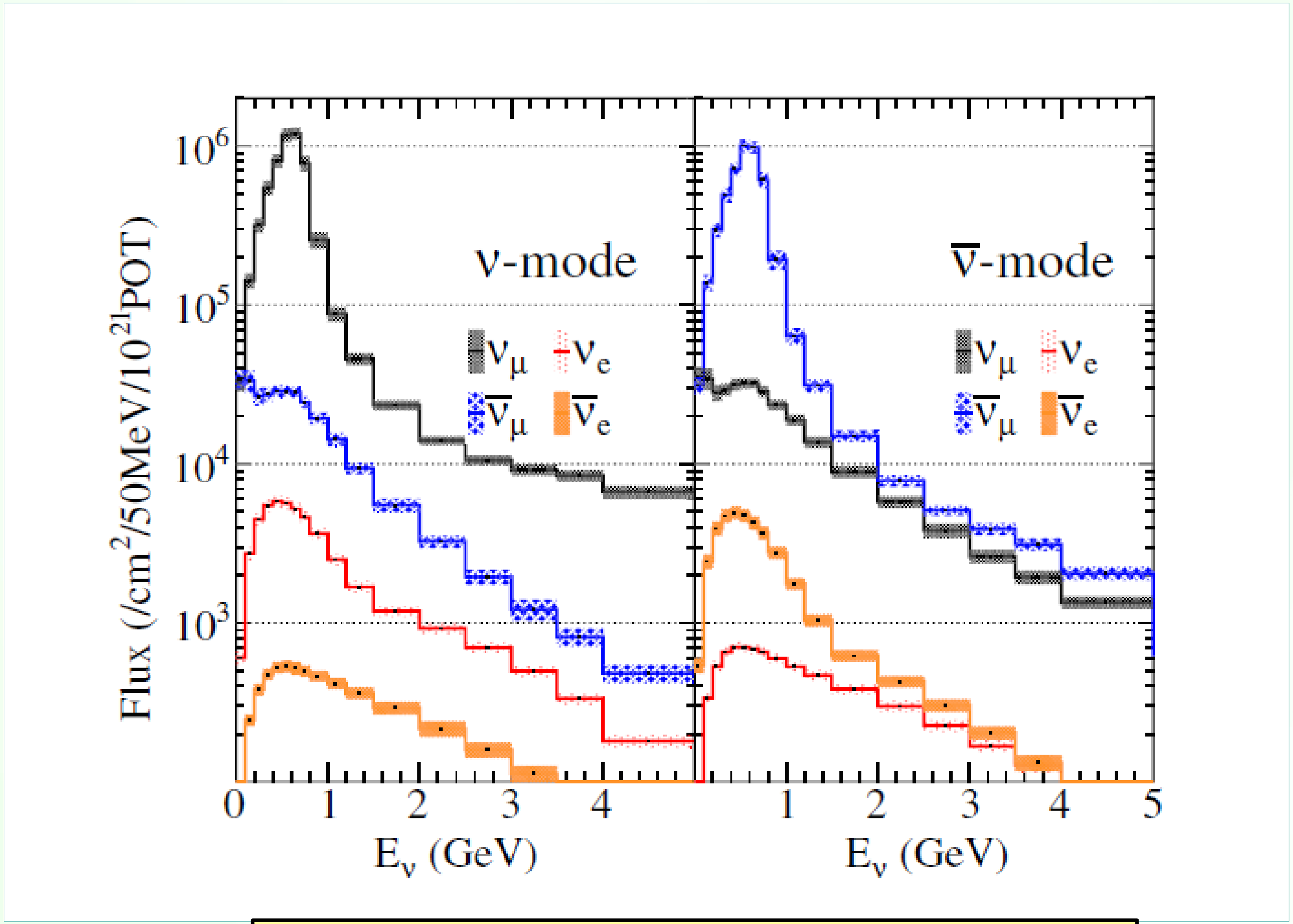
Dependence of DUNE's ~~CP~~ sensitivity on exposure for $\sigma(\nu_e)/\sigma(\nu_\mu)$ uncertainty between 1% and 3%.



Acciari *et al.* (DUNE), arXiv:1512.06148

Measurement in near detector

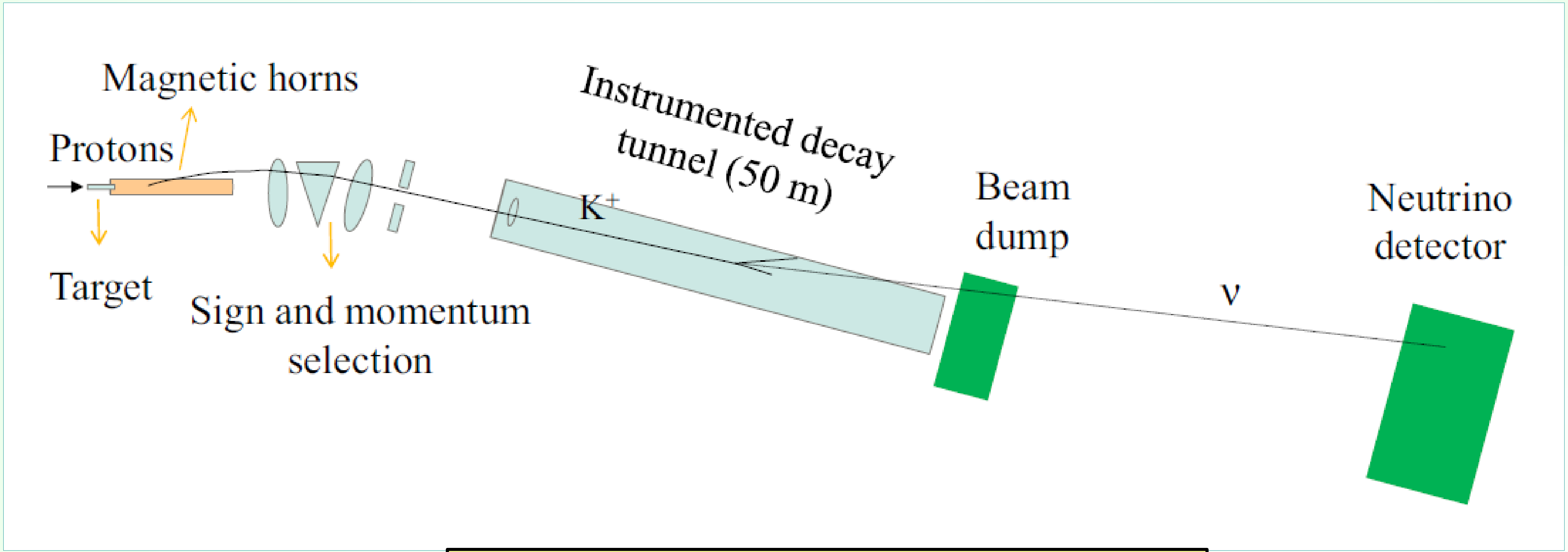
Event statistics lower ~100 times for ν_e's then for ν_μ's.
Higher flux and detector-response uncertainties.



Abe *et al.* (T2K), PRL 116, 181801 (2016)

Measurement in near detector

New concept of tagging $K^+ \rightarrow e^+\nu_e\pi^0$ events should allow the ν_e cross section determination with 1% uncertainty.



Longhin *et al.*, EPJ C 75, 155 (2015)

Charged-Current Cross section

Well-known dependence on the charged-lepton's mass

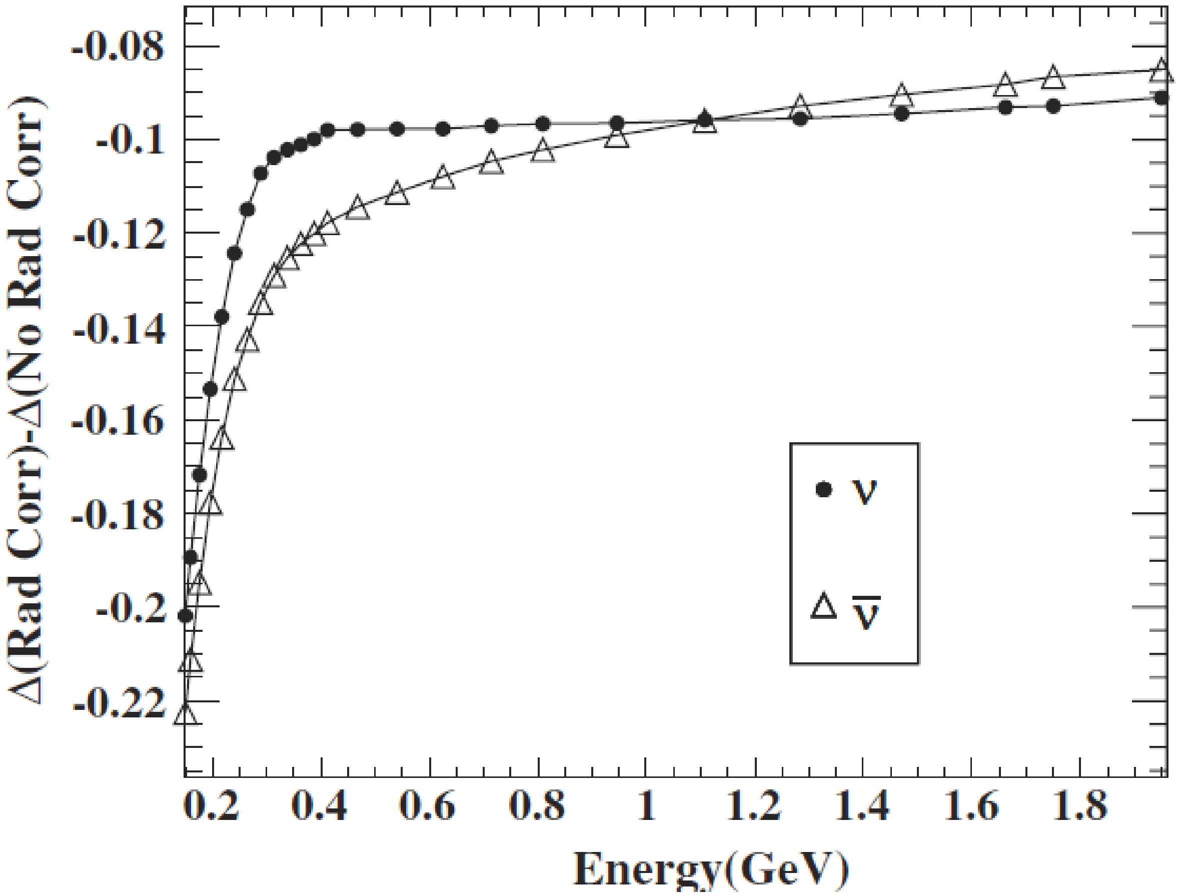
$$\frac{d\sigma}{d\omega d|\mathbf{q}|} = \frac{(G_F \cos\theta_C)^2}{2\pi} \frac{|\mathbf{q}|}{|\mathbf{k}|^2} \Big[v_{CC} R_{CC}(\omega, |\mathbf{q}|) + v_{CL} R_{CL}(\omega, |\mathbf{q}|) + v_{LL} R_{LL}(\omega, |\mathbf{q}|) + v_T R_T(\omega, |\mathbf{q}|) + v_{T'} R_{T'}(\omega, |\mathbf{q}|) \Big]$$

$$\begin{aligned} v_{CC} &= E_k E_{k'} + k_x k'_x + k_z k'_z, \\ v_{CL} &= -2(E_k k'_z + E_{k'} k_z), \\ v_{LL} &= E_k E_{k'} - k_x k'_x + k_z k'_z, \\ v_T &= E_k E_{k'} - k_z k'_z, \\ v_{T'} &= 2(E_{k'} k_z - E_k k'_z), \end{aligned}$$

$$\begin{aligned} k_x &= \frac{|\mathbf{k} \times \mathbf{k}'|}{|\mathbf{q}|} = k'_x, \\ k_y &= 0 = k'_y, \\ k_z &= \frac{|\mathbf{k} \cdot \mathbf{q}|}{|\mathbf{q}|}, \\ k'_z &= \frac{|\mathbf{k}' \cdot \mathbf{q}|}{|\mathbf{q}|}, \end{aligned}$$

See e.g. Amaro *et al.*, PRC 71, 015501 (2005)

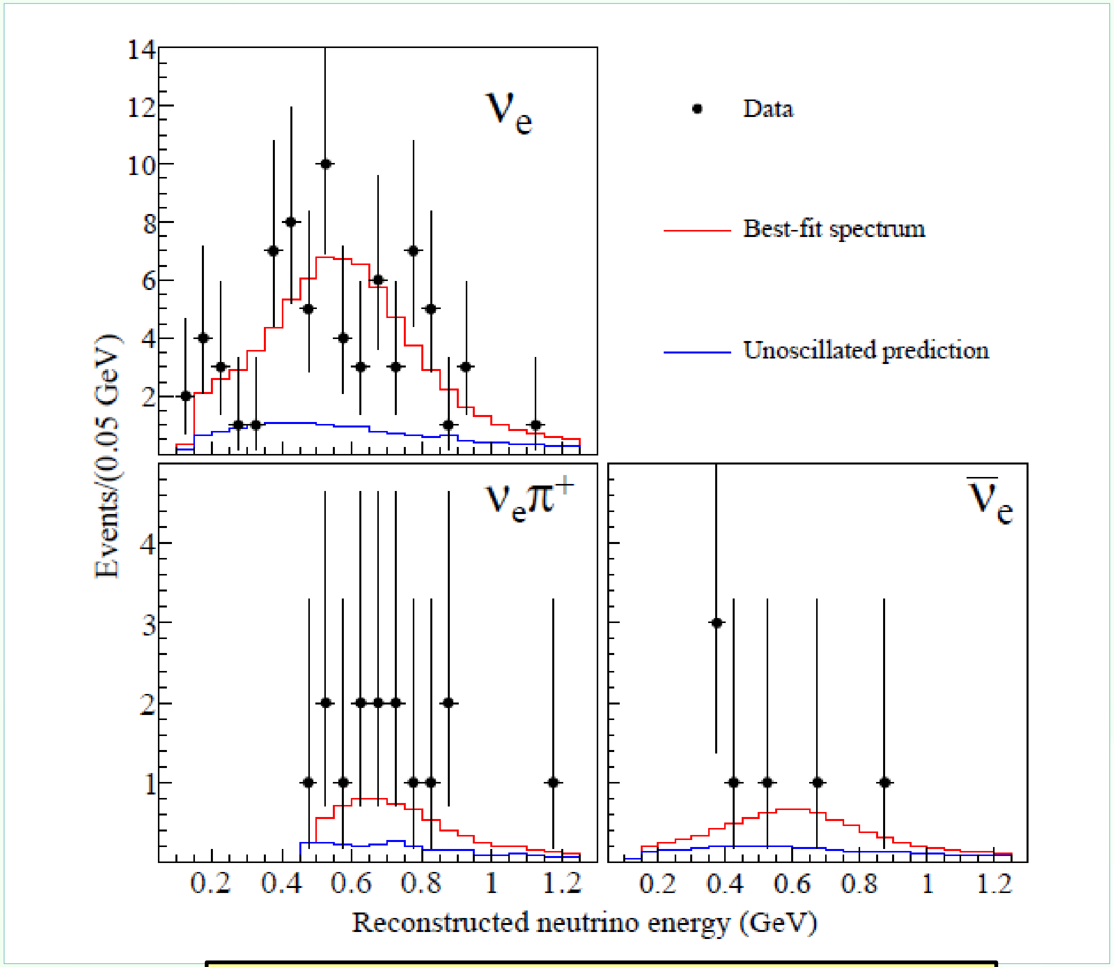
Radiative corrections



$$\Delta = \frac{\sigma(\nu_\mu) - \sigma(\nu_e)}{\sigma(\nu_e)}$$

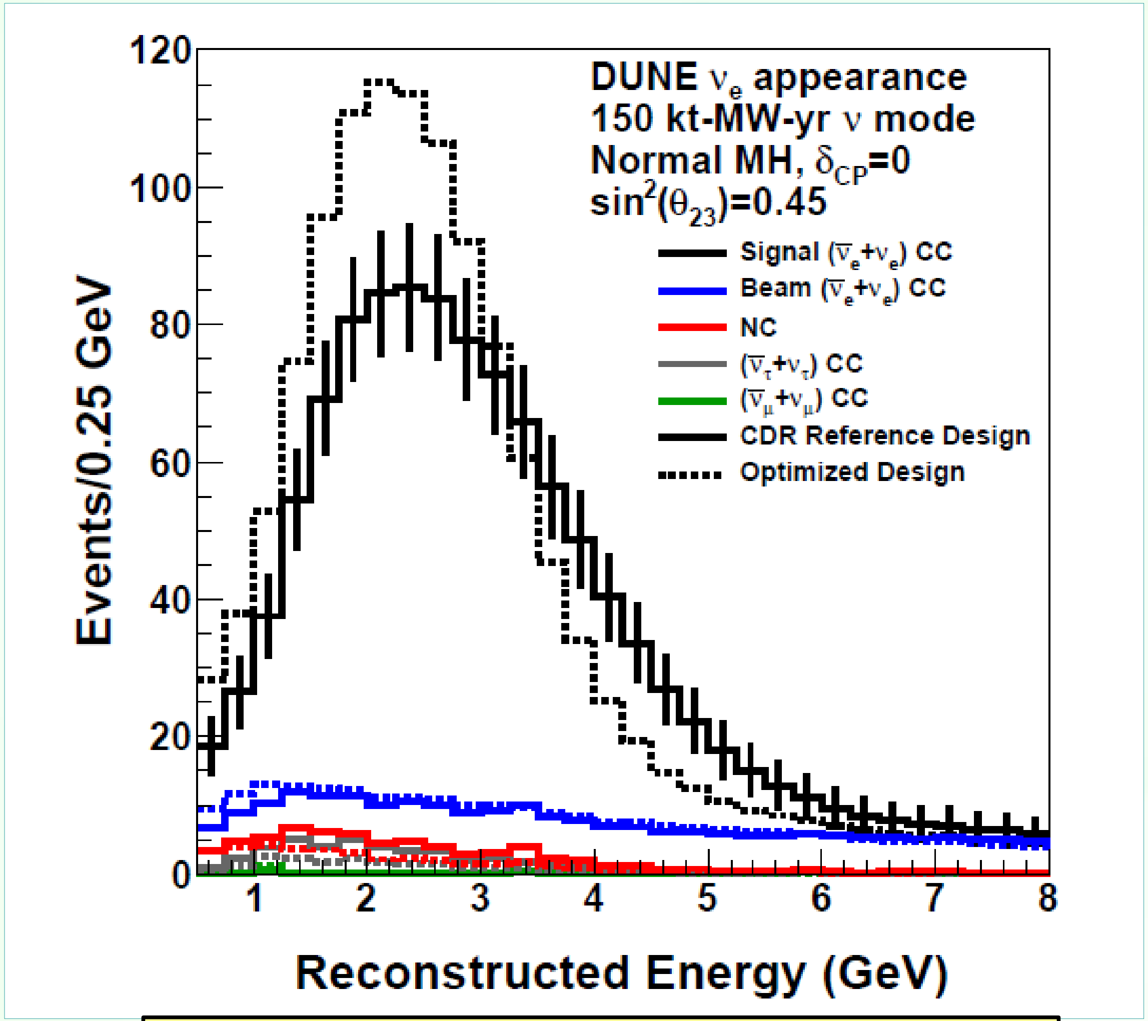
Day & McFarland,
PRD 86, 053003 (2012)

Spectra in T2K



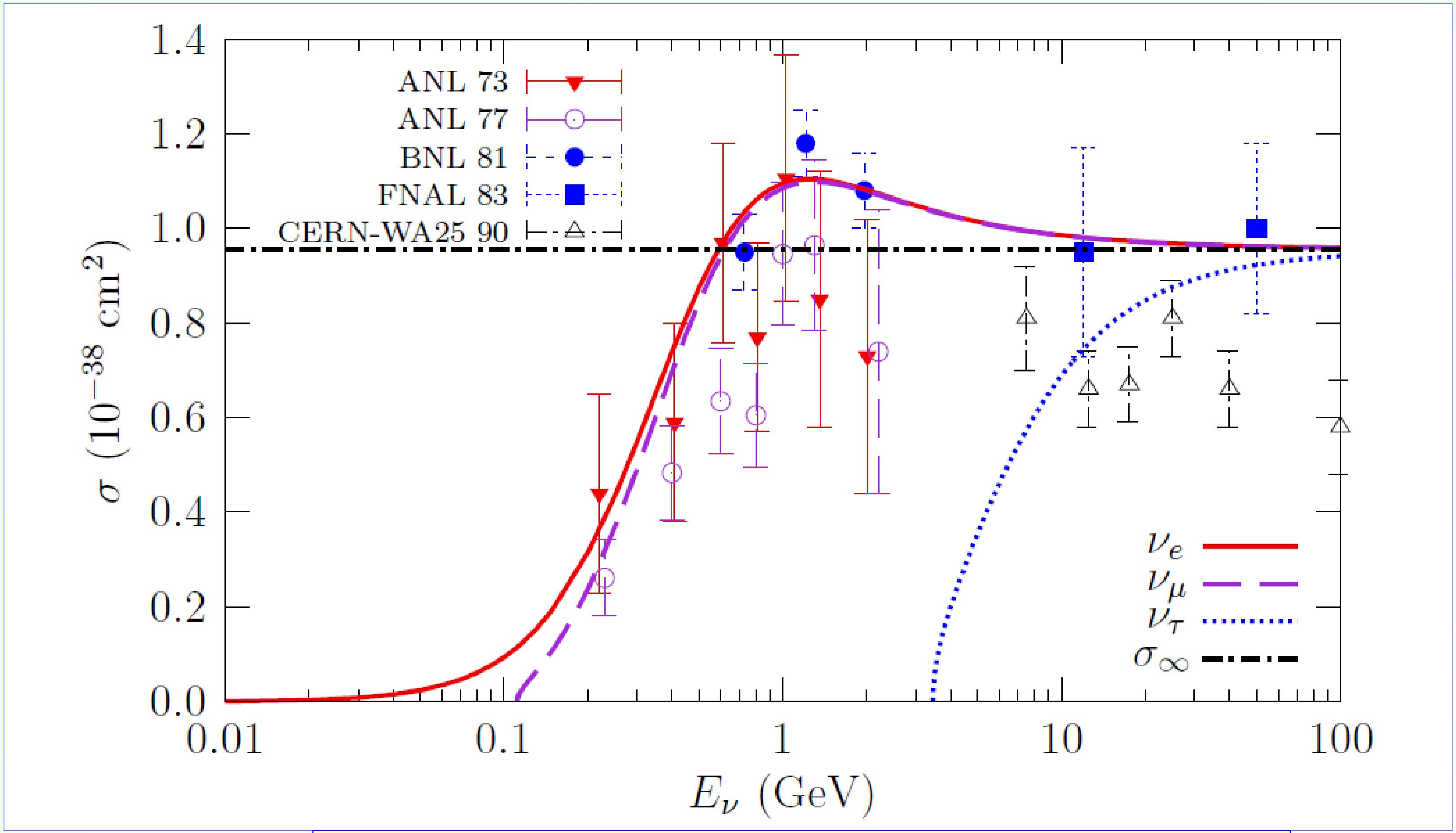
Abe *et al.* (T2K), arXiv:1807.07891

Expected spectra in DUNE



Acciari *et al.* (DUNE), arXiv:1512.06148

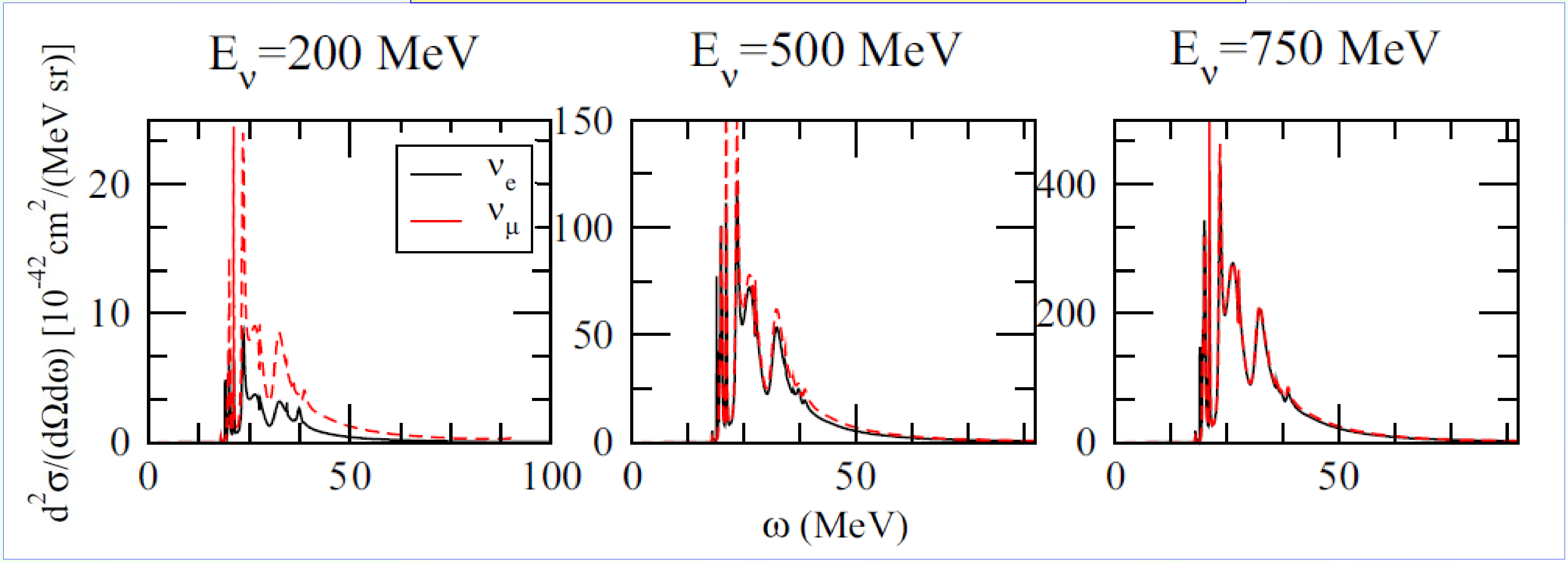
Free nucleon CC QE cross section



AMA, Acta Phys. Pol. B **37**, 0377 (2006)

CCQE ν_μ and ν_e cross sections at 5°

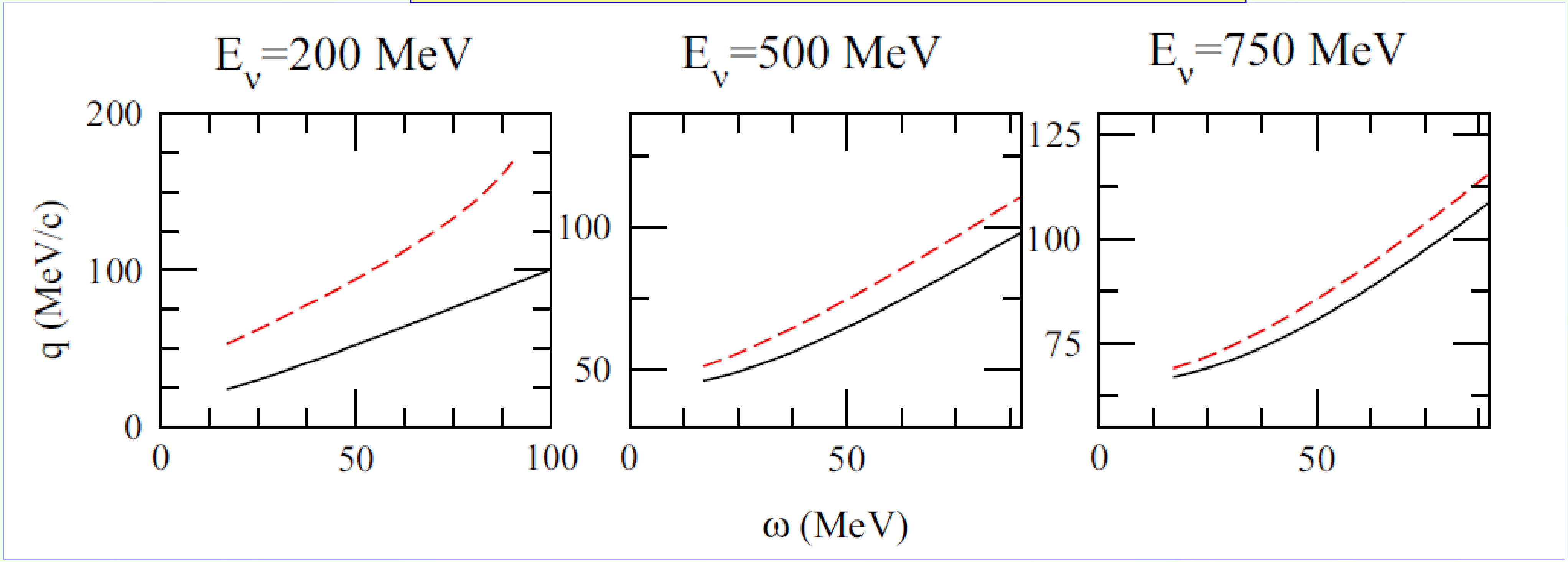
M. Martini *el al.*, PRC **94**, 015501 (2016)



Conclusion: While at higher energies the ν_μ and ν_e cross sections practically coincide, at low energies and small scattering angles the ν_μ cross section is **higher** than the ν_e one.

CCQE ν_μ and ν_e cross sections at 5°

M. Martini *el al.*, PRC **94**, 015501 (2016)

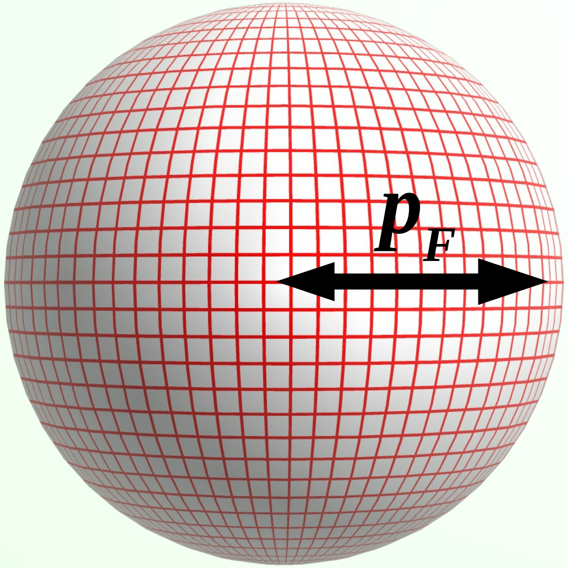


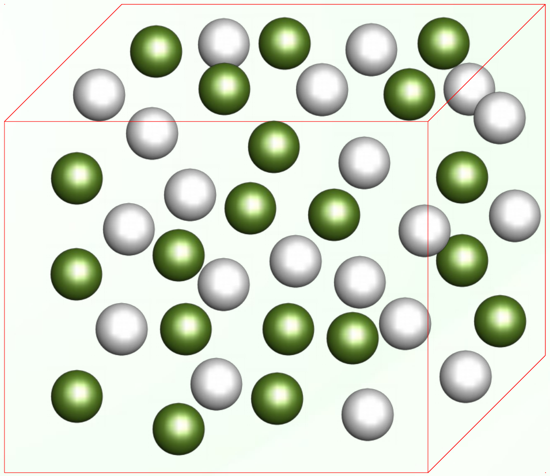
Conclusion: this behavior is related to the differences in the momentum transfer between the ν_μ and ν_e scattering.

When is the ν_e cross section higher then the ν_μ one?

Fermi gas model

Nucleus treated as a fragment of non-interacting infinite nuclear matter of constant density. Eigenstates have definite momenta and energies $E_p = \sqrt{M^2 + \mathbf{p}^2} - \epsilon$.





Momentum space

Coordinate space

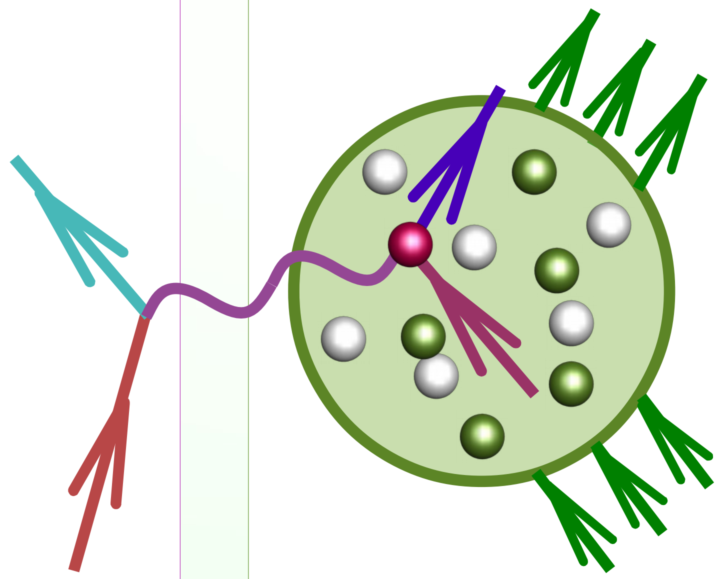
Relativistic Fermi gas

Lepton kinematics

$$\omega = E_\nu - E_\ell,$$
$$\mathbf{q} = \mathbf{k}_\nu - \mathbf{k}_\ell,$$

Nucleon kinematics

$$\omega = \sqrt{M^2 + \mathbf{p}'^2} - \sqrt{M^2 + \mathbf{p}^2} - \epsilon,$$
$$\mathbf{q} = \mathbf{p}' - \mathbf{p},$$



Relativistic Fermi gas

Lepton kinematics

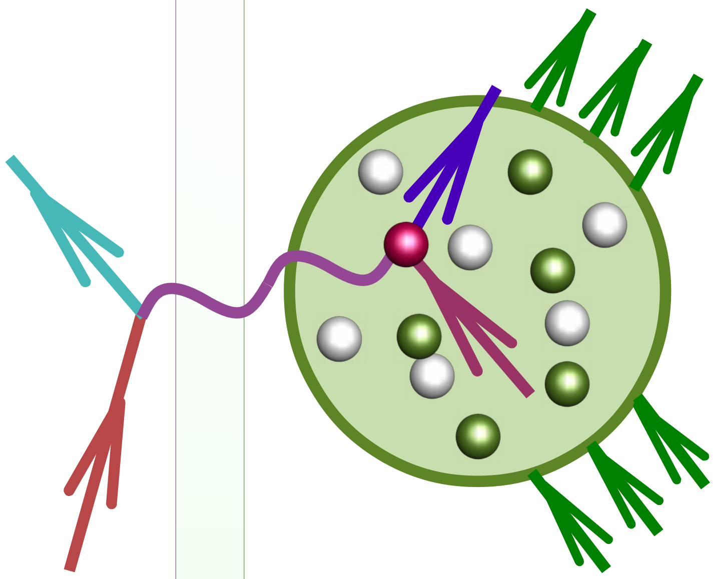
$$|\mathbf{q}| = \sqrt{E_\nu^2 - 2E_\nu|\mathbf{k}'|\cos\theta + |\mathbf{k}'|^2},$$

$$|\mathbf{k}'| = \sqrt{(E_\nu - \omega)^2 - m^2}.$$

Nucleon kinematics

$$|h - p_F| \le |\mathbf{q}| \le h + p_F,$$

$$h = \sqrt{(\omega - \epsilon + E_F)^2 - M^2} \text{ and } E_F = \sqrt{M^2 + p_F^2}$$



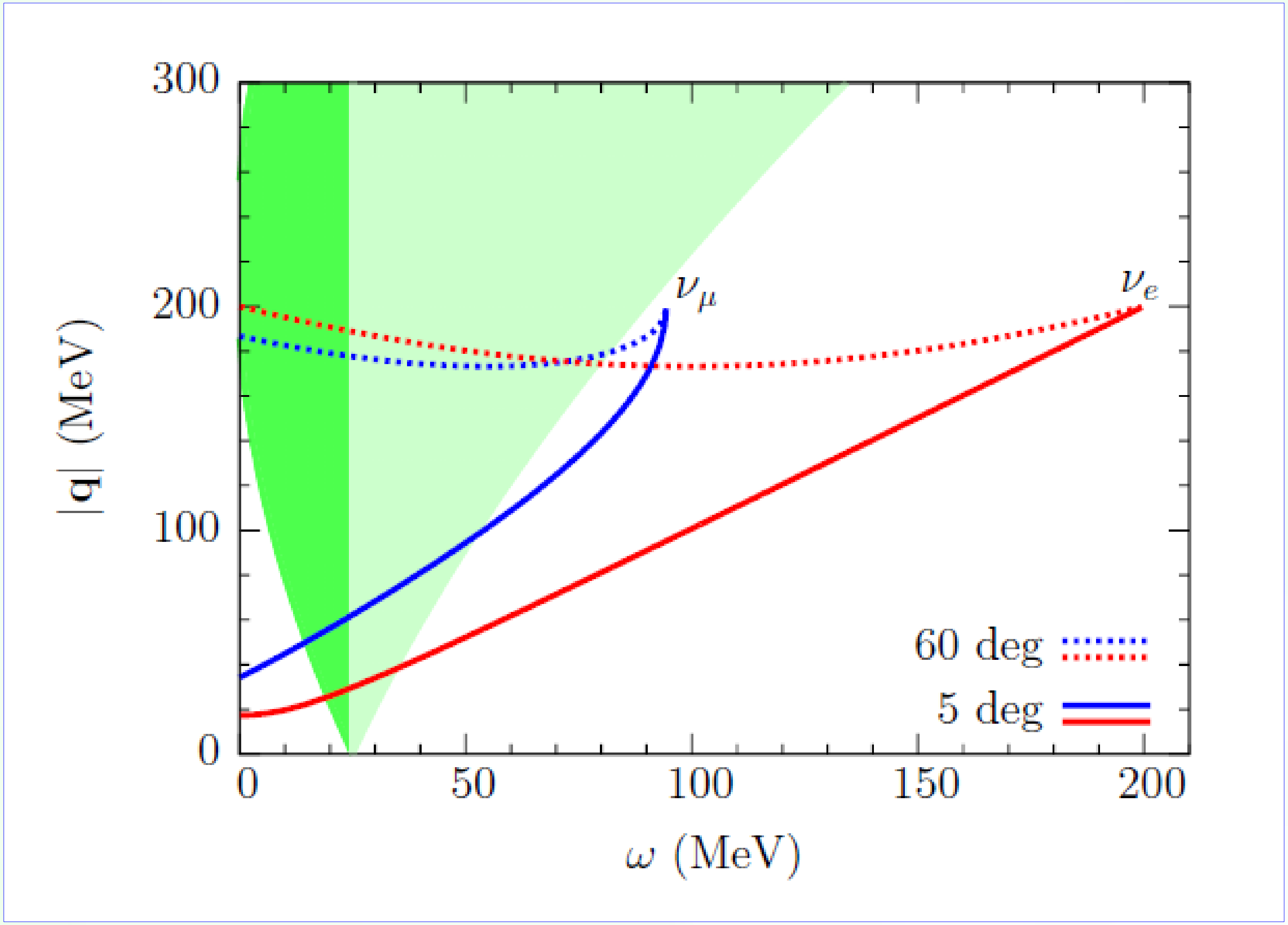
w/o Pauli blocking

$$\omega_{\min} = M - E_F + \epsilon$$

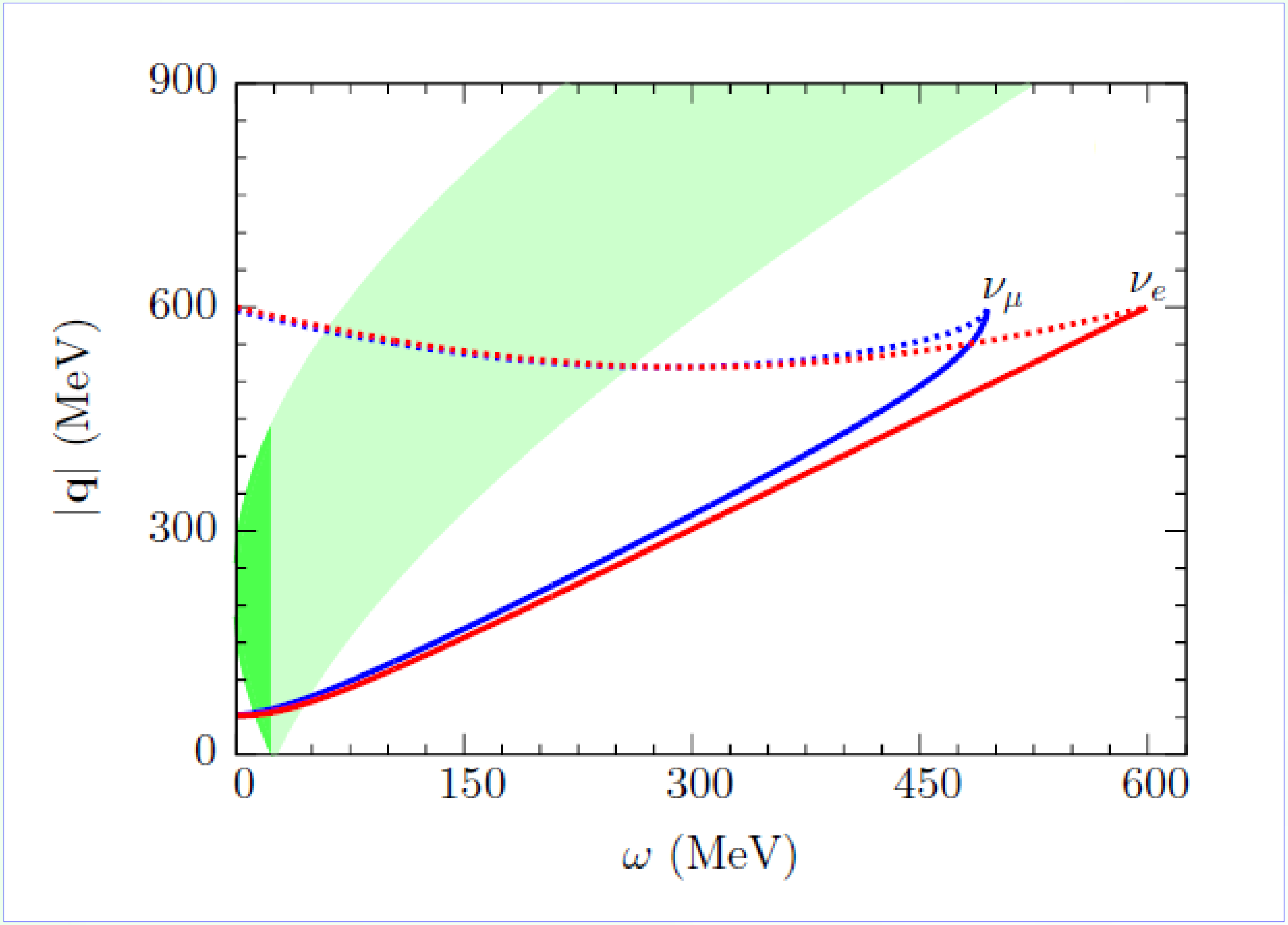
w/ Pauli blocking

$$\omega_{\min} = \epsilon$$

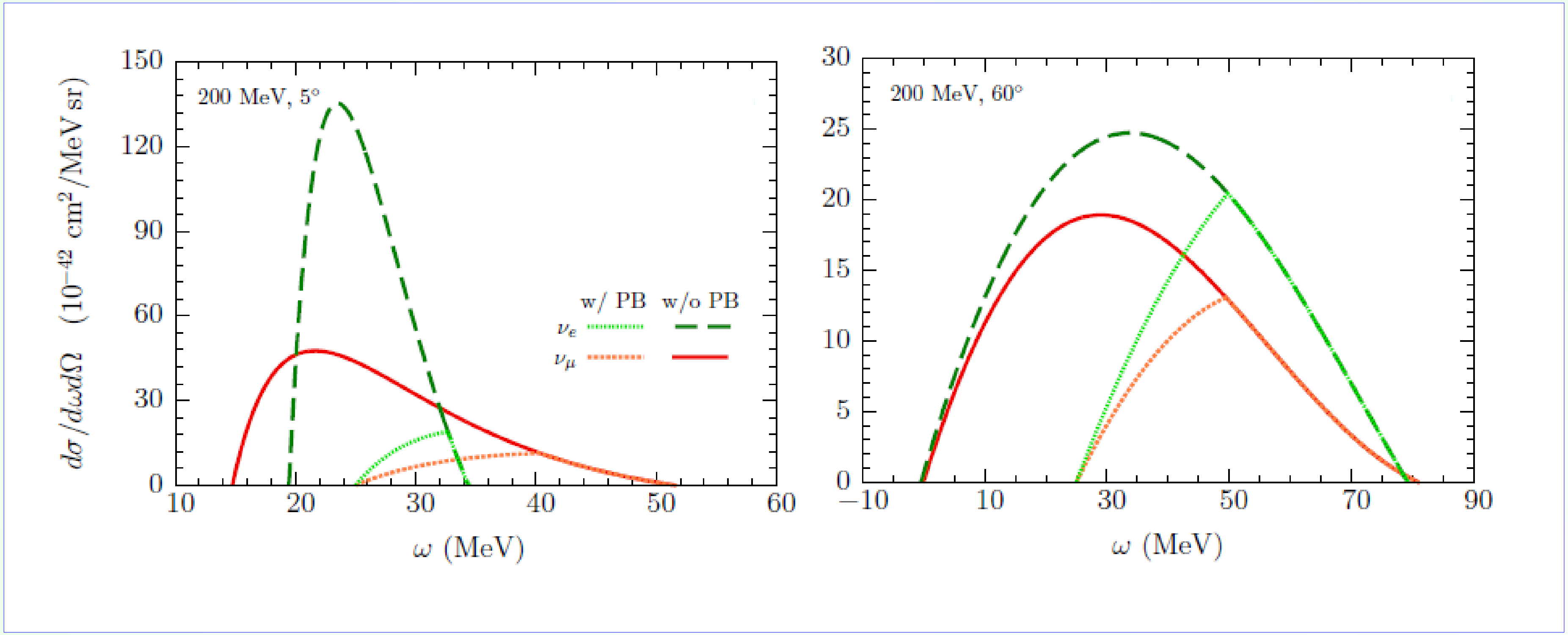
RFG, CCQE scattering at 200 MeV



RFG, CCQE scattering at 600 MeV



CCQE scattering at 200 MeV



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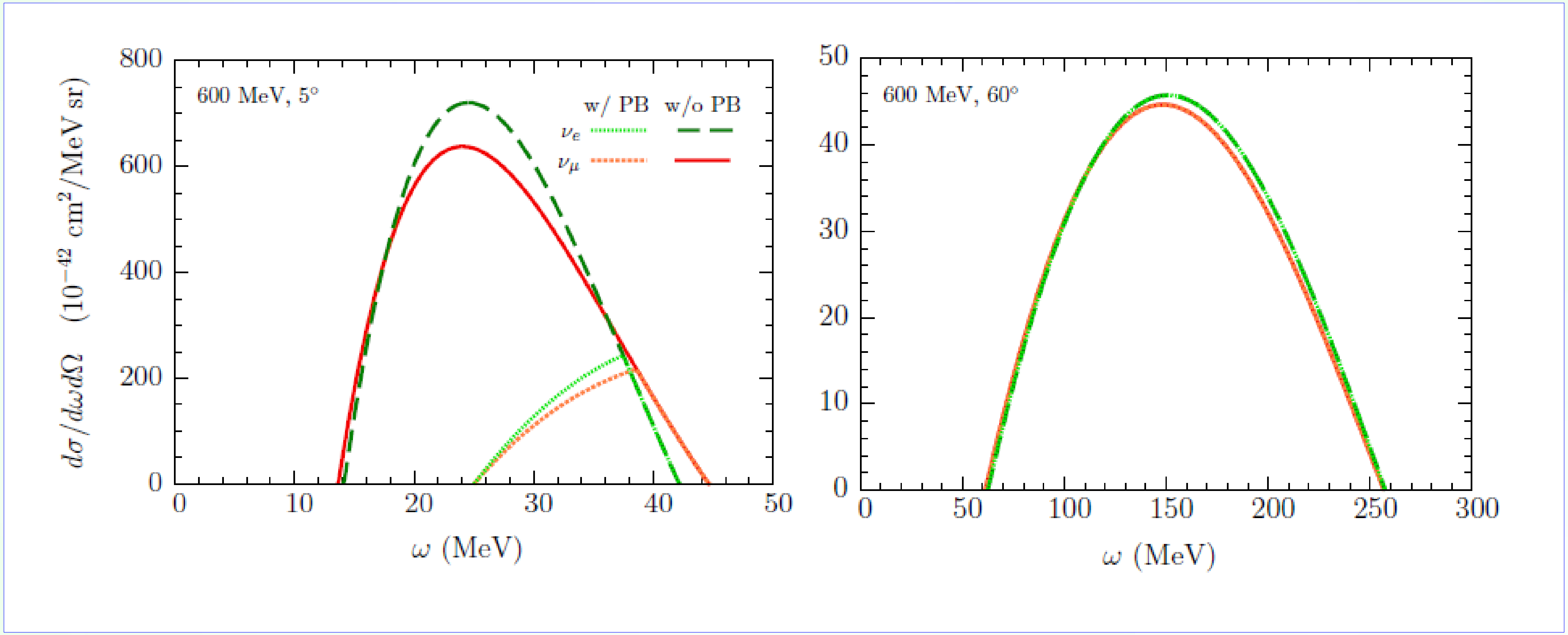


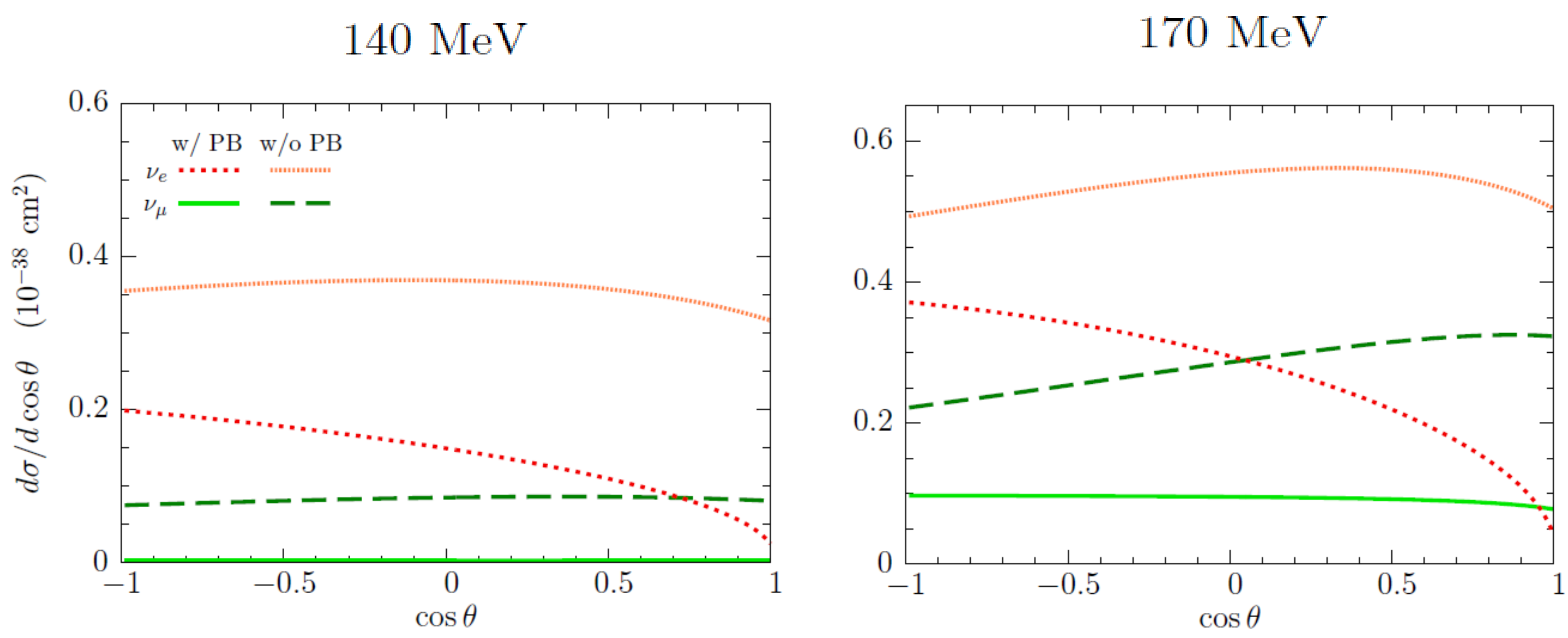
Cross-sections' ratio $\frac{d\sigma(\nu_\mu)}{d\cos\theta} \Big/ \frac{d\sigma(\nu_e)}{d\cos\theta}$

	$E_\nu = 200$ MeV		$E_\nu = 600$ MeV	
	5°	60°	5°	60°
RFG w/ PB	1.57	0.62	1.03	0.97
RFG w/o PB	0.73	0.71	0.96	0.97

Details of nuclear model can qualitatively change the mass dependence of the cross section.

The behavior is driven by the phase-space availability, rather than the kinematics.

Differential cross section *dσ/d*cos*θ*



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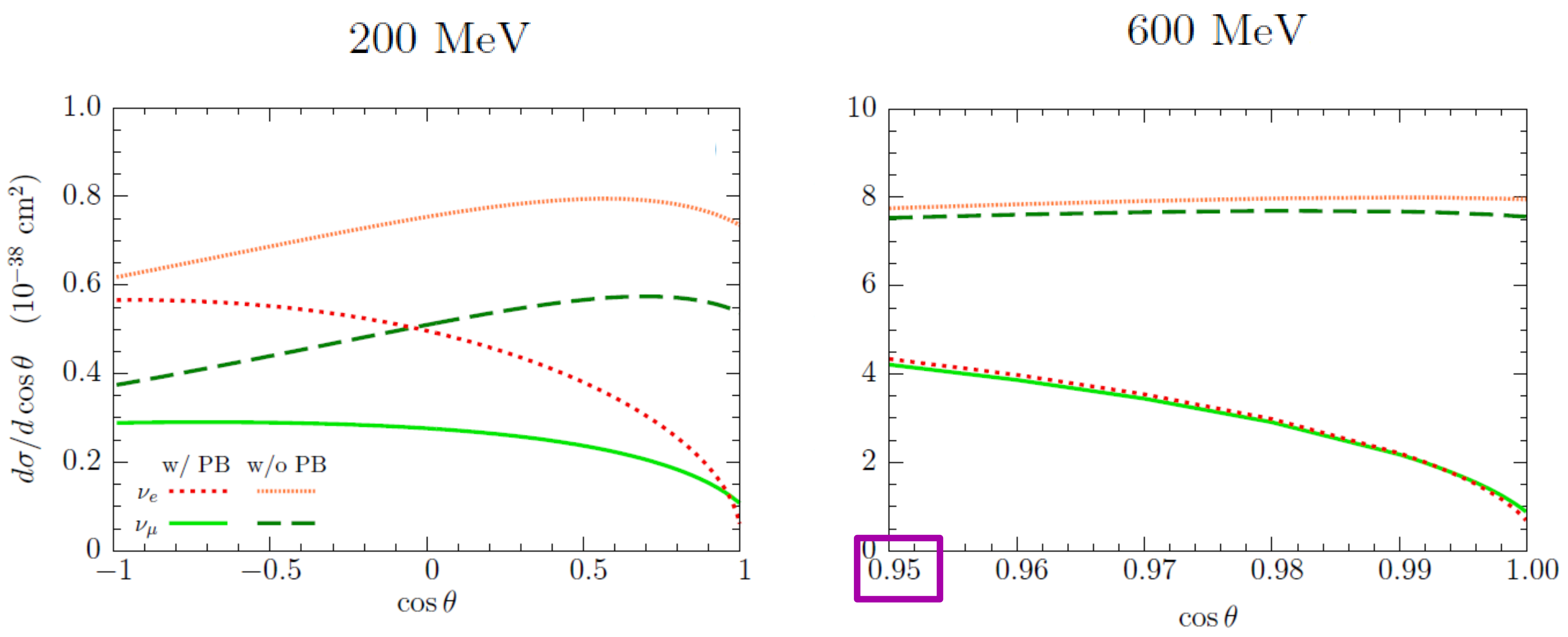


Relativistic Fermi gas

Reducing the available phase space, Pauli blocking changes the behavior of the cross section.

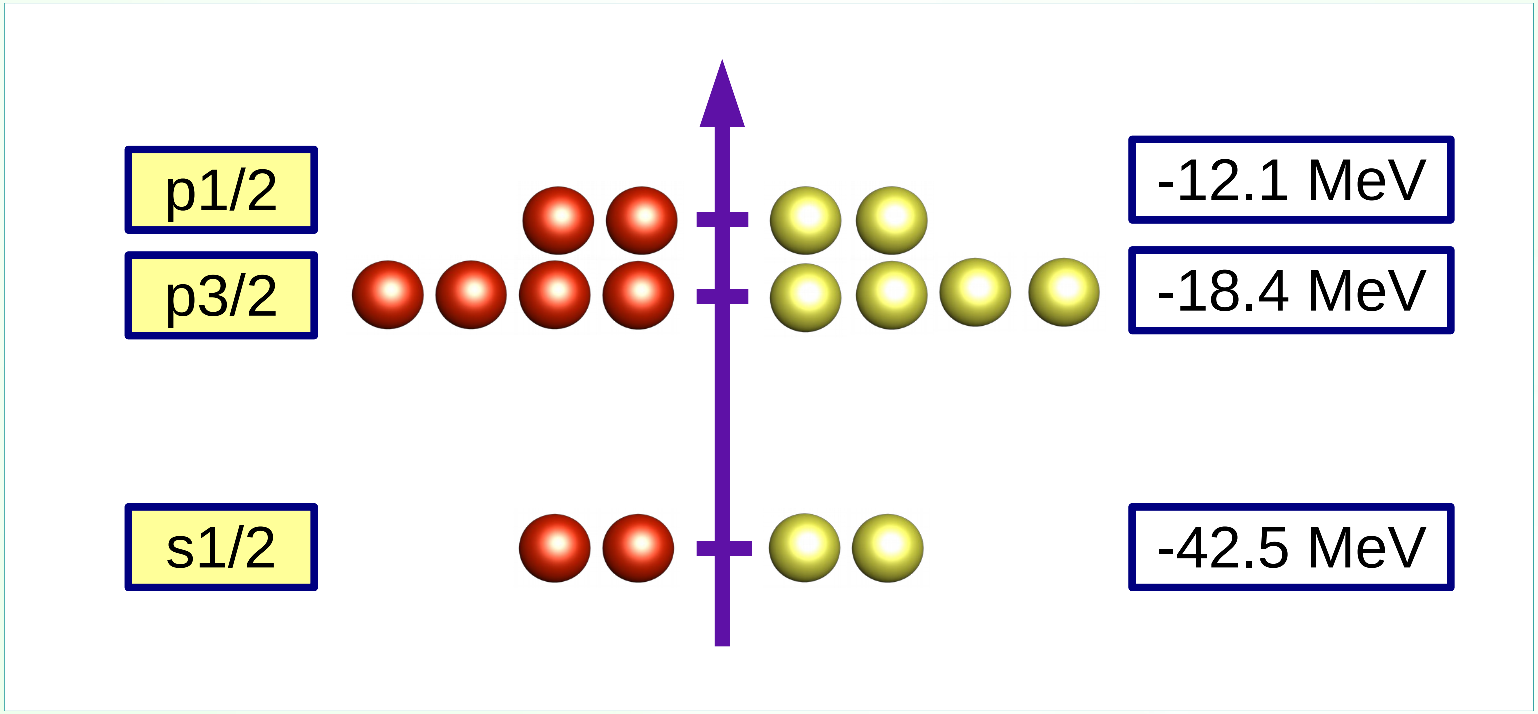
- Without Pauli blocking, the ν_e cross section converges to the ν_μ one from below, when energy increases.
- With Pauli blocking, close to the threshold the ν_μ cross section is lower than the ν_e one for any angle. At higher energies, there is a range of angles where

$$d\sigma(\nu_\mu)/d\cos\theta > d\sigma(\nu_e)/d\cos\theta.$$

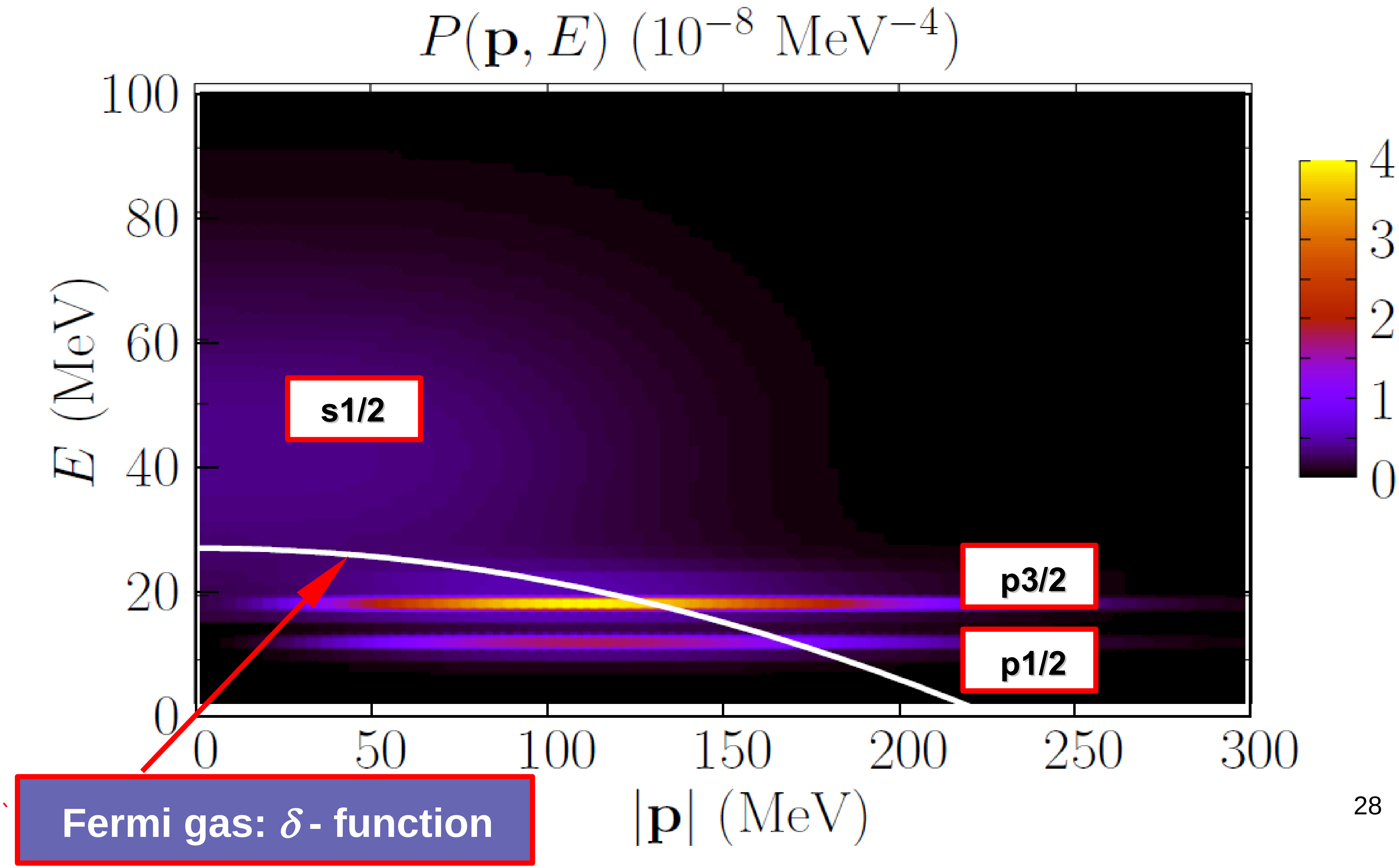
Shell model

In a spherically symmetric potential, the eigenstates correspond to definite values of the total angular momentum.

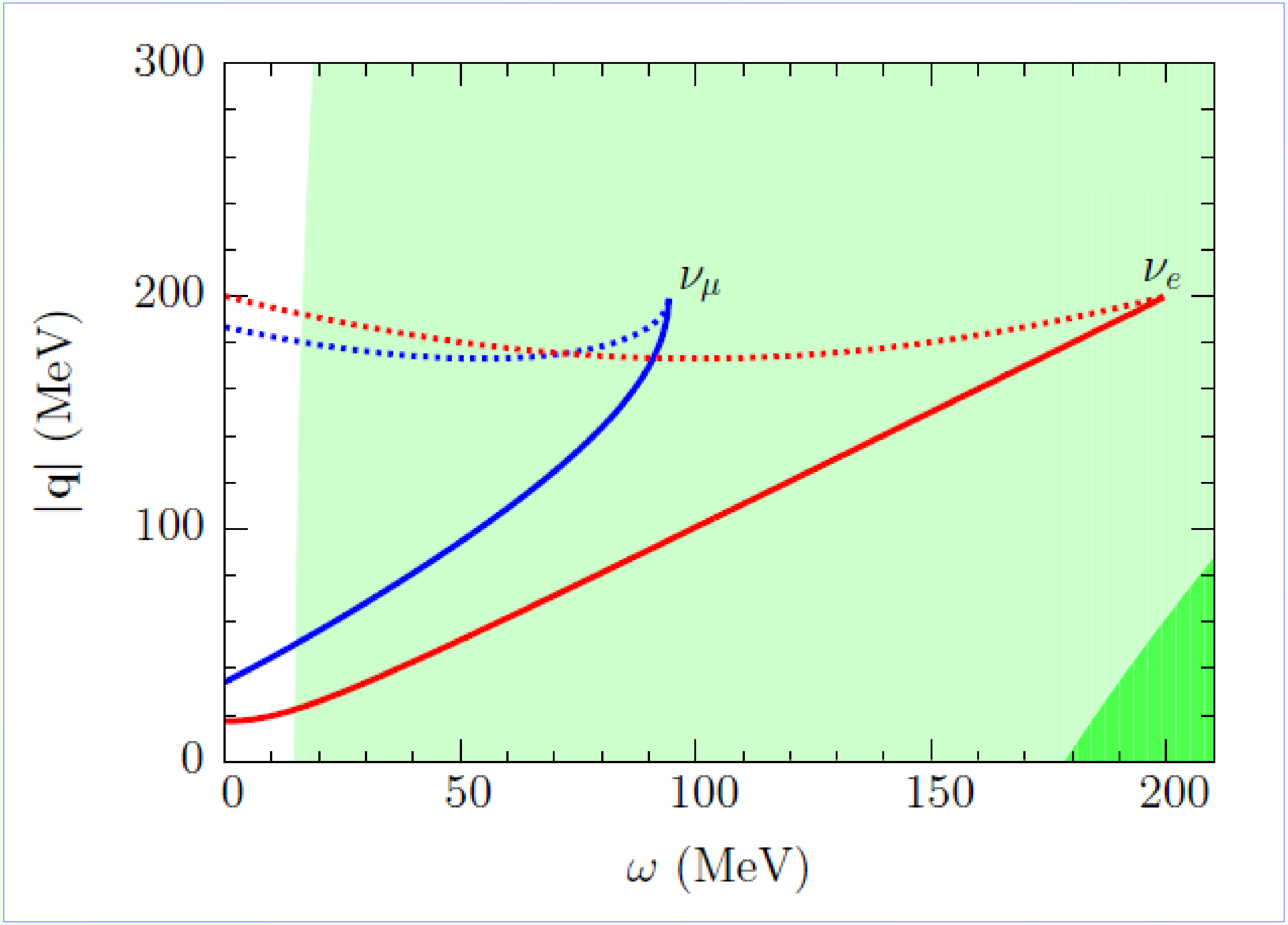
No correspondence between nucleon momentum and energy, unlike in the Fermi gas model.



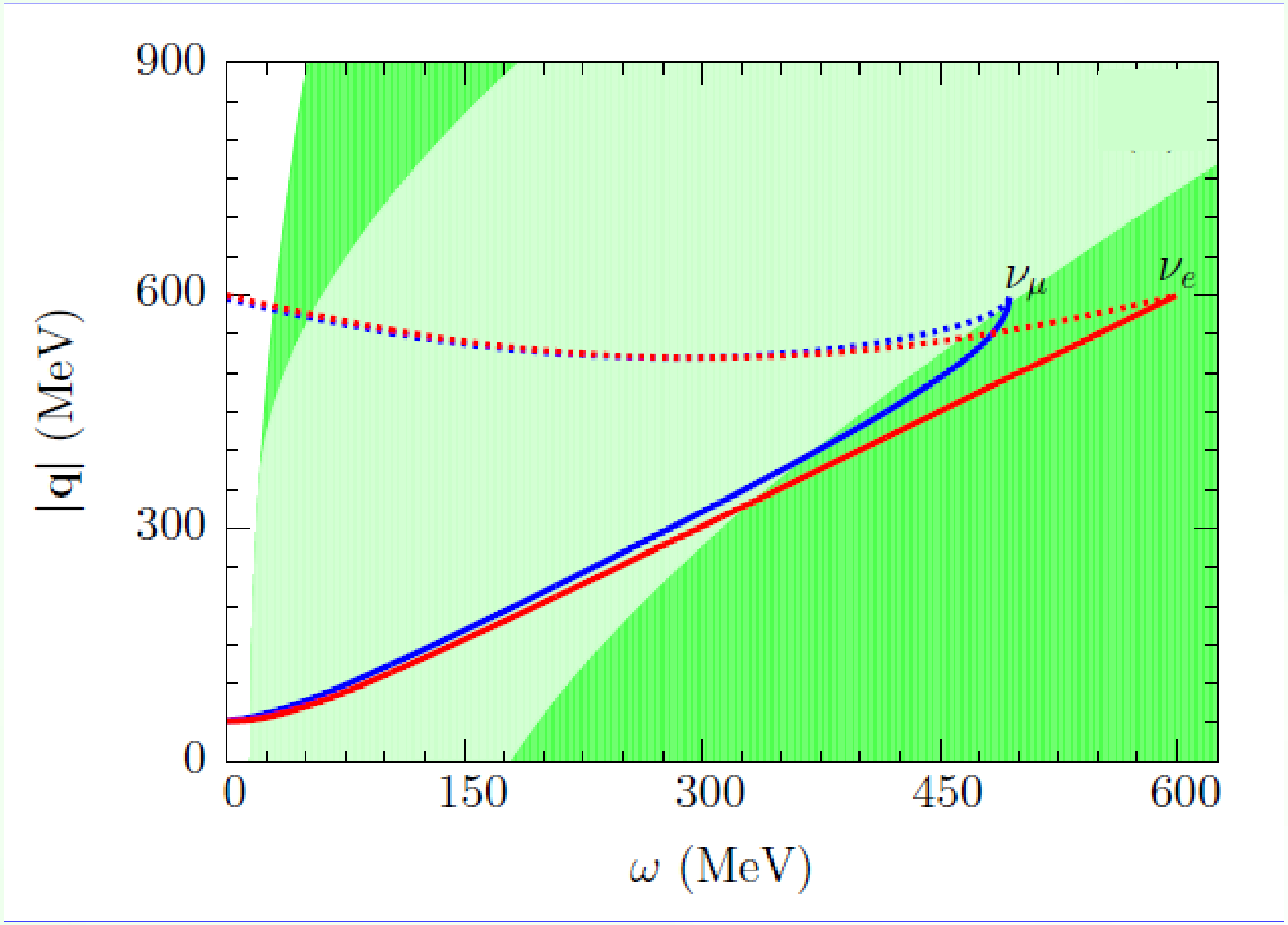
Example: spectral function of oxygen



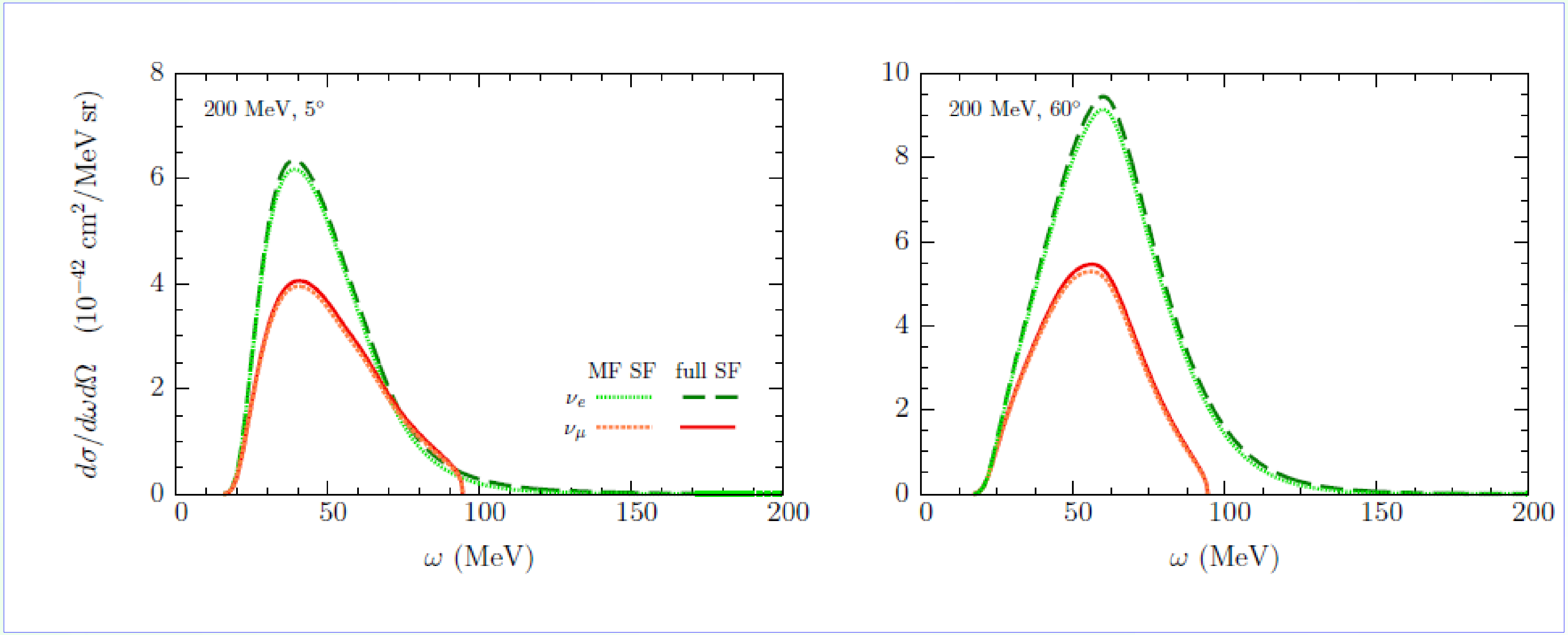
SF, CCQE scattering at 200 MeV



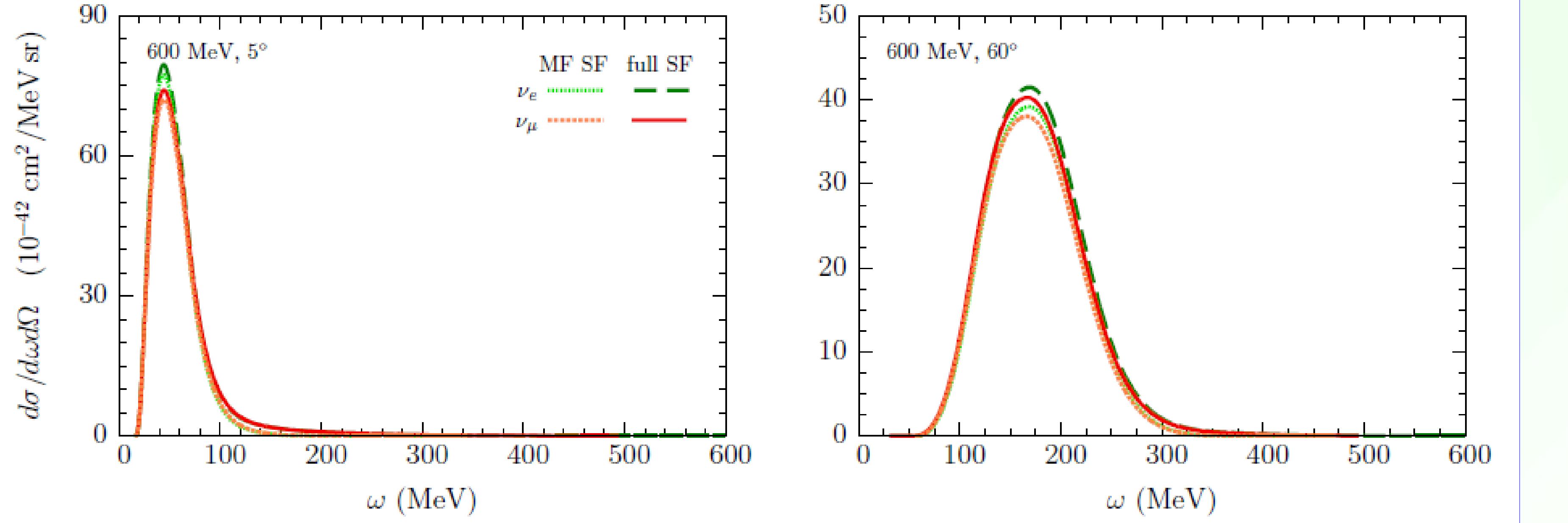
SF, CCQE scattering at 600 MeV



CCQE scattering at 200 MeV



CCQE scattering at 600 MeV



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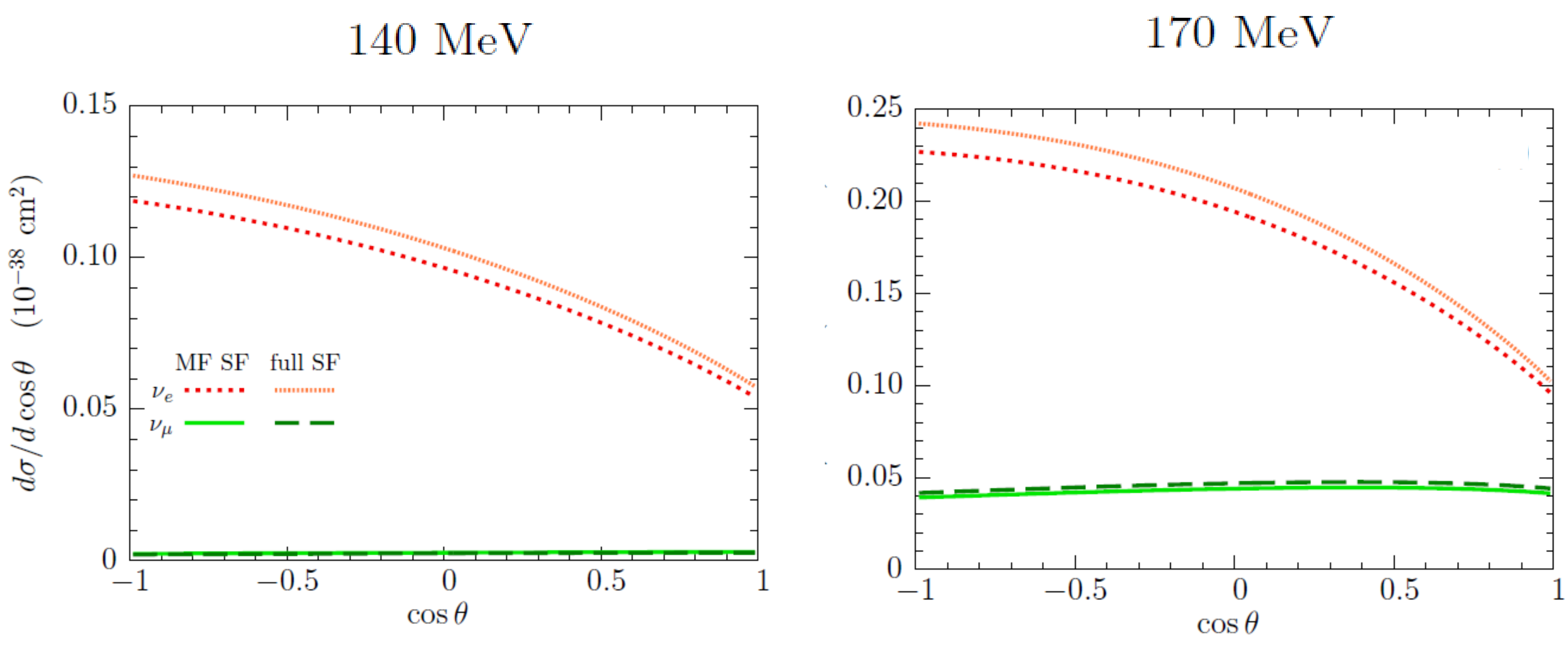
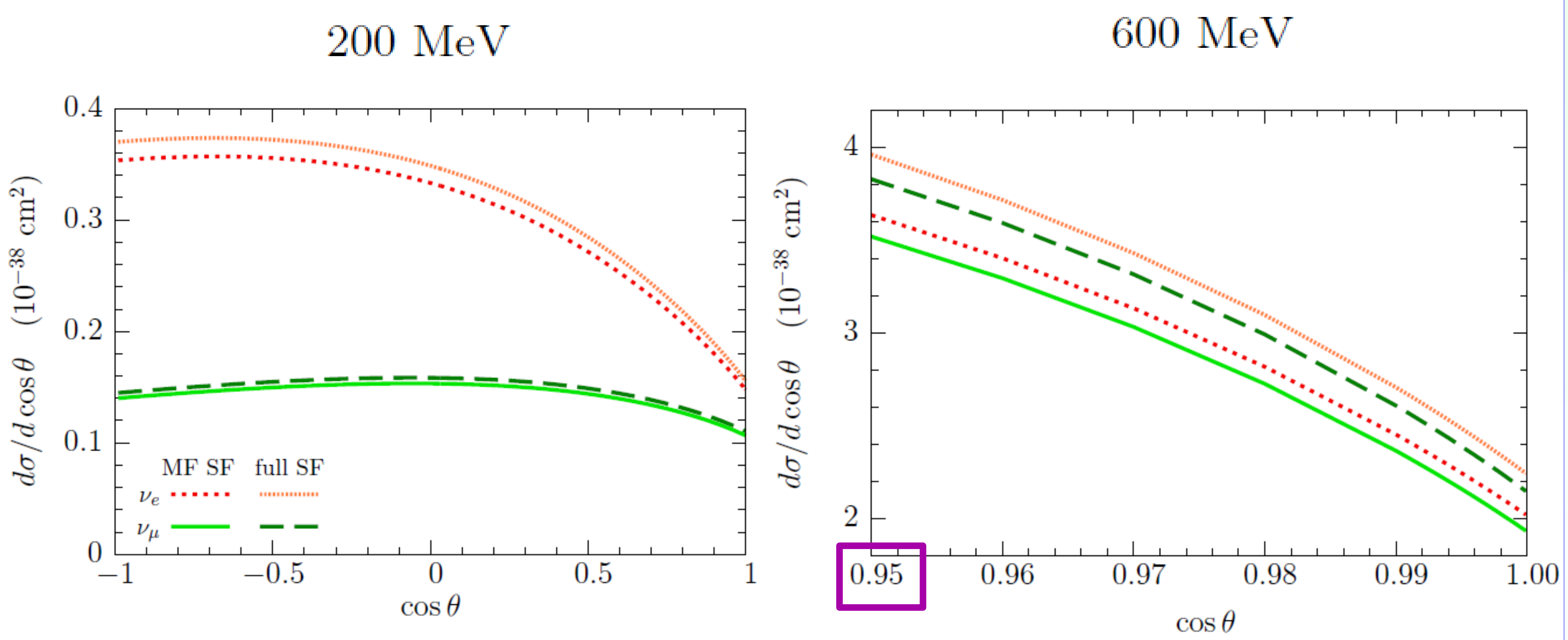
RFG w/o PB	0.73	0.71	0.96	0.97
Mean-field SF	0.72	0.53	0.96	0.97
Full SF	0.71	0.52	0.96	0.97

The ratio is governed mostly by the shell contribution, extracted from (*e*,*e'p*) data.

Differential cross section *dσ/d*cos*θ*



Differential cross section *dσ/d*cos*θ*

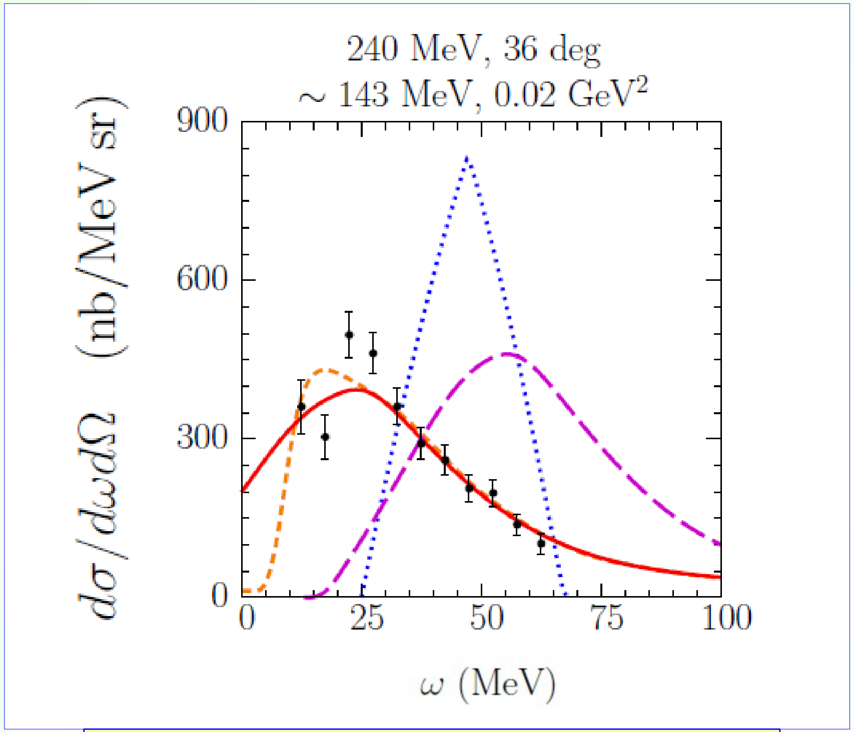


Shell model/Spectral function

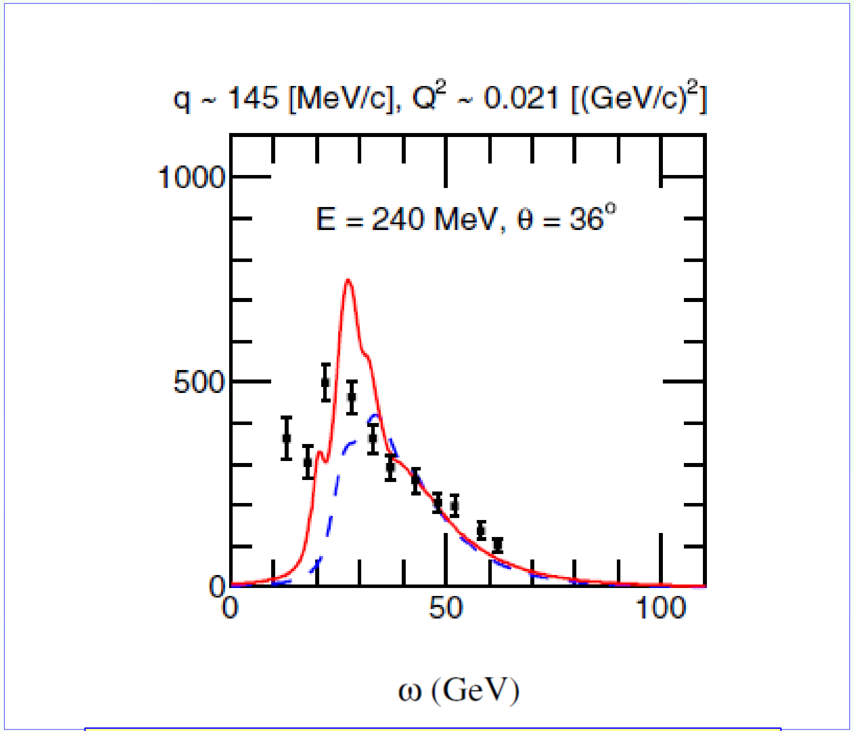
- The available phase space is broad due to energy and momentum distributions of the shell states.
- The ν_μ cross section is lower than the ν_e one **for any angle**, but converges to it as energy increases.

Should we be concerned about model differences?

Nuclear models are converging



AMA *et al.*,
PRD **91**, 033005 (2015)



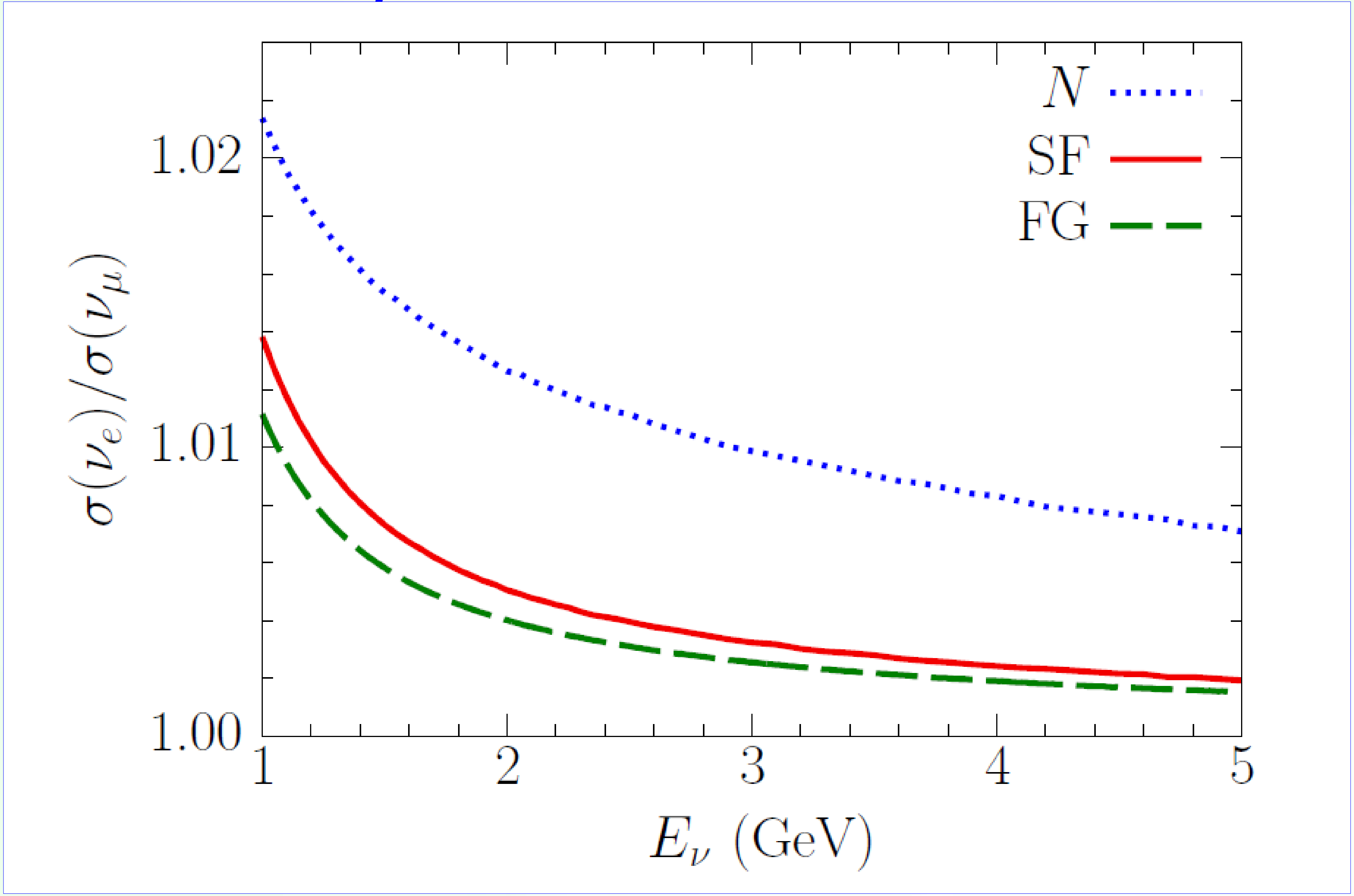
V. Pandey *et al.*,
PRC **92**, 024606 (2015)

See also: G. D. Megias *et al.*, PRC **94**, 013012 (2016);
J. E. Sobczyk, PRC **96**, 045501 (2017)

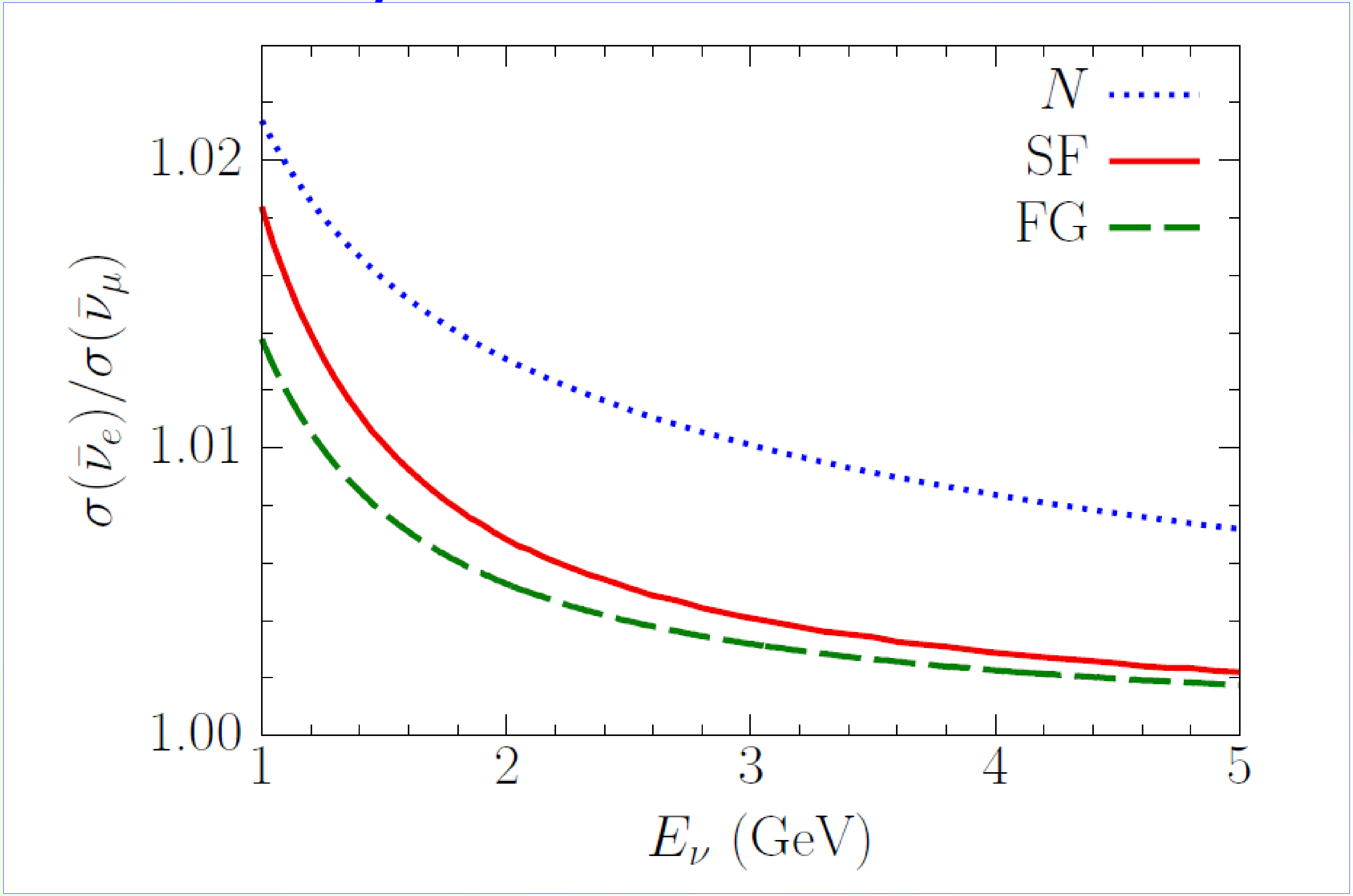
Model differences

- Models developed to reproduce inclusive electron-scattering data may give similar results starting from different physics assumptions.
- Treating the initial states differently, they lead to different exclusive cross sections (hadron distributions).
- For long-baseline neutrino experiments, particularly those using calorimetric energy reconstruction, exclusive cross sections are essential.

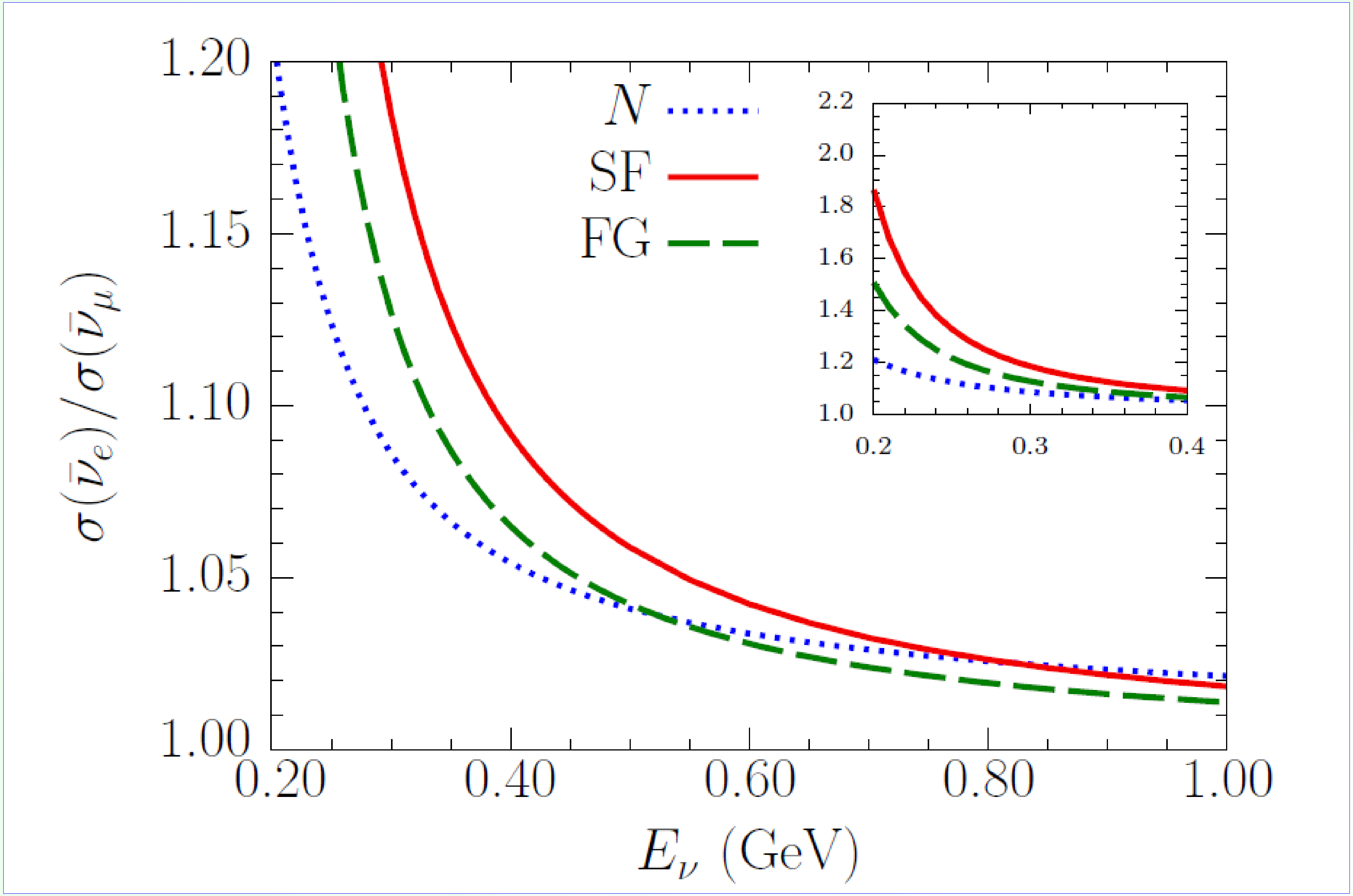
ν_e to ν_μ cross sections' ratio



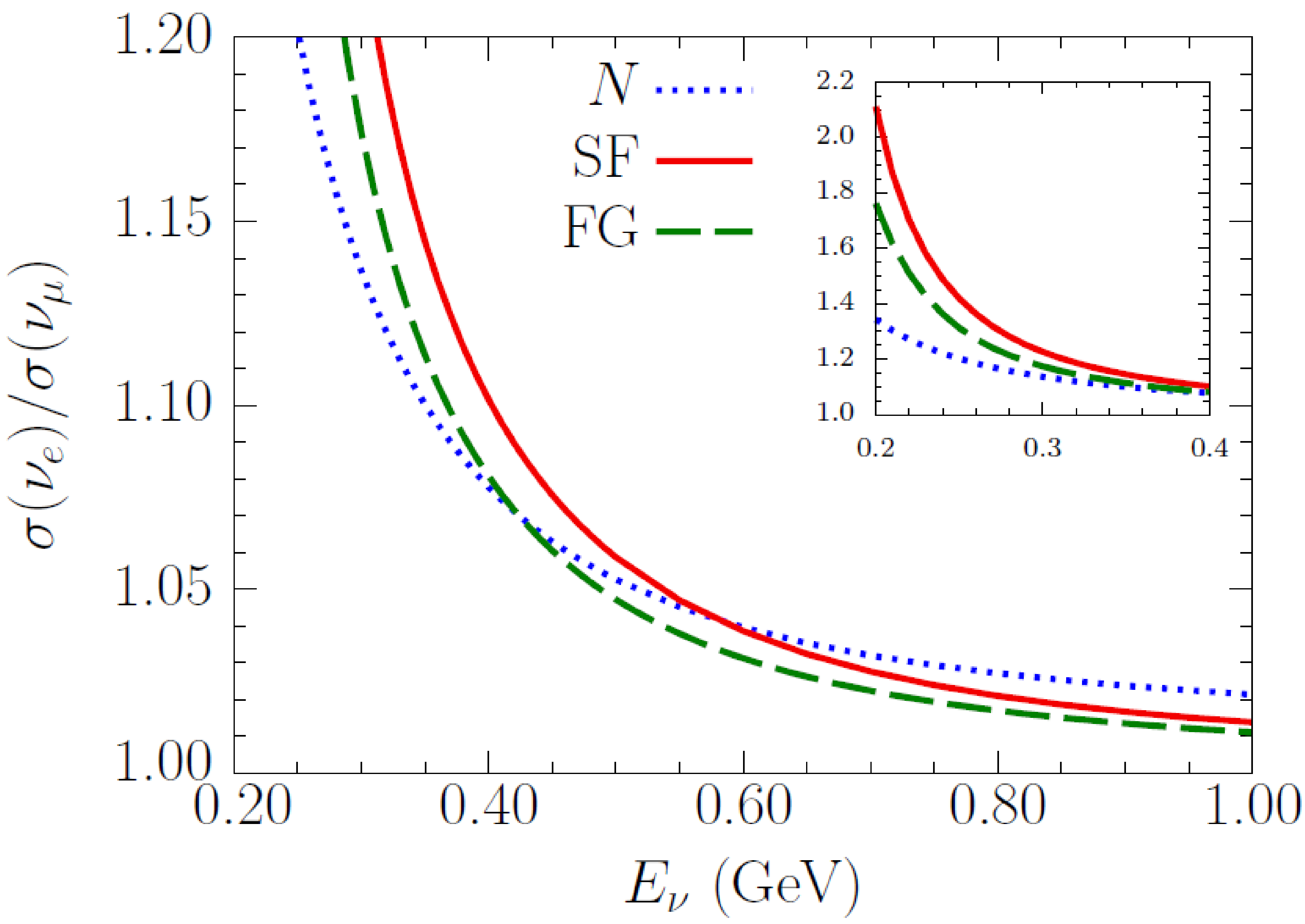
$\bar{\nu}_e$ to $\bar{\nu}_\mu$ cross sections' ratio



$\bar{\nu}_e$ to $\bar{\nu}_\mu$ cross sections' ratio



ν_e to ν_μ cross sections' ratio



Summary

- Next-generation appearance experiments require $\sigma(\nu_\mu)/\sigma(\nu_e)$ known with challenging precision.

- The ratio's precise measurement currently not possible. It may be necessary to rely on input from theory.

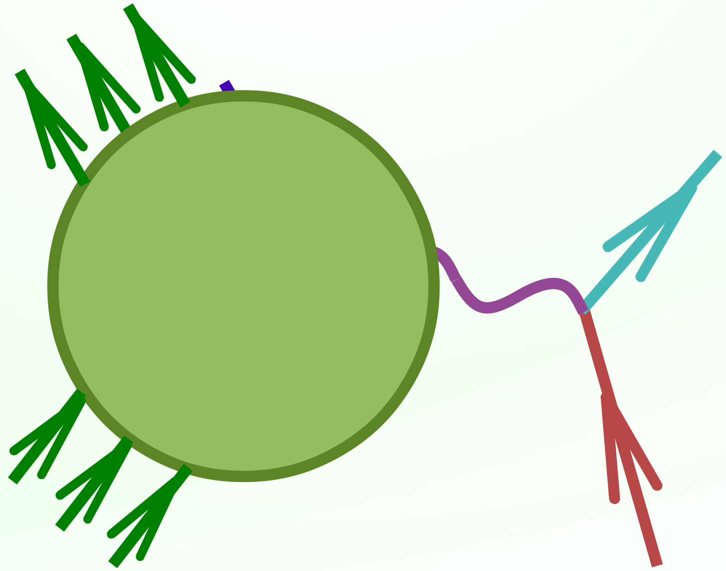
- For differential cross sections at low energies, different nuclear models may yield qualitatively different results.

- The $\sigma(\nu_\mu) - \sigma(\nu_e)$ difference small above 1 GeV, but significant at energies below ~600 MeV.

Backup slides

Impulse approximation

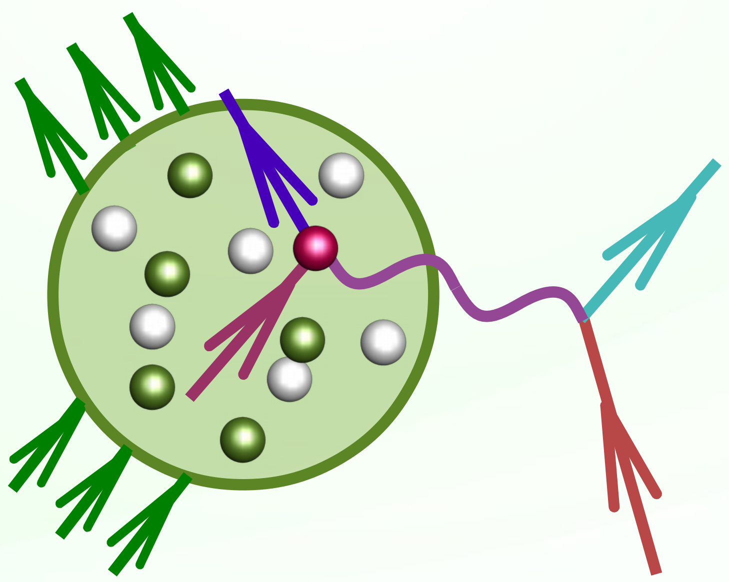
Assumption: the dominant process of lepton-nucleus interaction is **scattering off a single nucleon**, with the remaining nucleons acting as a spectator system.



Impulse approximation

Assumption: the dominant process of lepton-nucleus interaction is **scattering off a single nucleon**, with the remaining nucleons acting as a spectator system.

It's valid when the momentum transfer |**q**| is high enough, as the probe's spatial resolution is ~1/|**q**|.



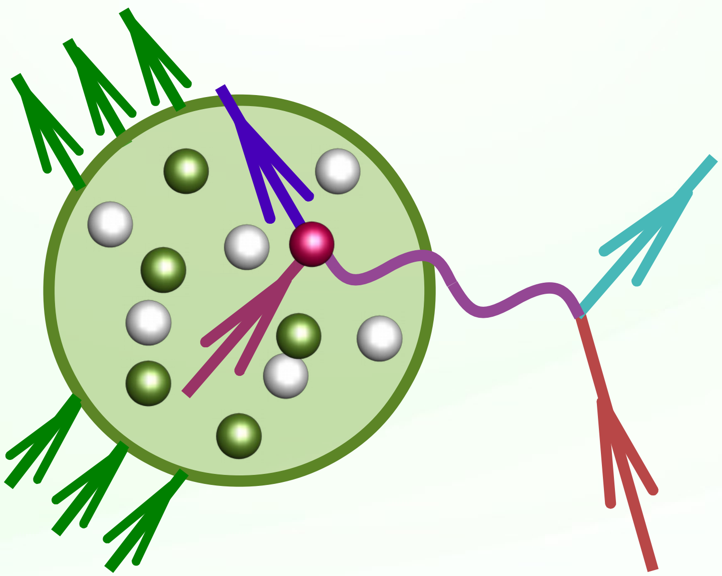
Impulse approximation

$$\frac{d\sigma_{\ell A}}{d\omega d\Omega} = \sum_N \int d\omega'\, d^3p\, dE\, P^N_{\mathrm{hole}}(\mathbf{p}, E)\, \frac{M}{E_{\mathbf{p}}} \frac{d\sigma^{\mathrm{elem}}_{\ell N}}{d\omega' d\Omega}\, P^N_{\mathrm{part}}(\mathbf{p}', \mathcal{T}', \omega')$$

Hole spectral function

Particle spectral function

Elementary cross section



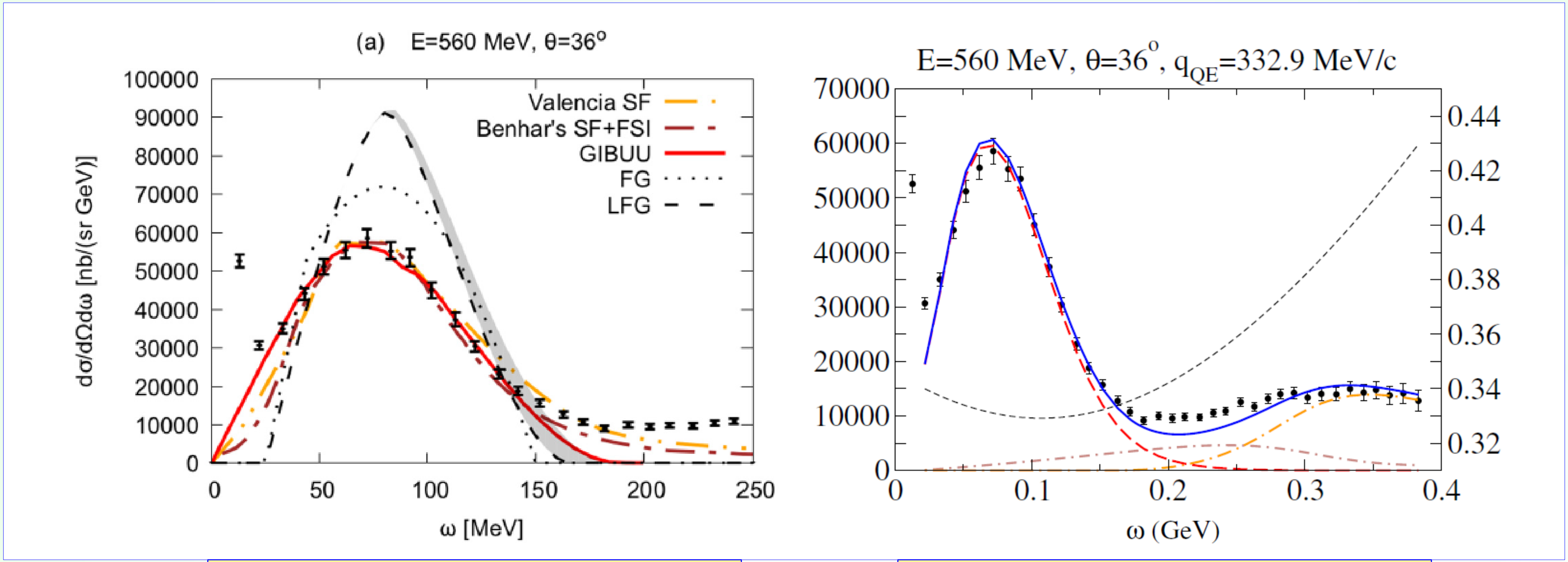
Impulse approximation

For scattering in a given angle, neutrinos and electrons differ only due to **the elementary cross section**.

In neutrino scattering, uncertainties come from
(i) interaction dynamics and (ii) **nuclear effects.**

It is **highly improbable** that theoretical approaches unable to reproduce *(e,e′)* data would describe nuclear effects in neutrino interactions at similar kinematics.

Nuclear models are converging



J. E. Sobczyk,
PRC **96**, 045501 (2017)

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See also: AMA *et al.*, PRD **91**, 033005 (2015)
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