Lattice QCD and Neutrinos

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NuFact | Virginia Tech
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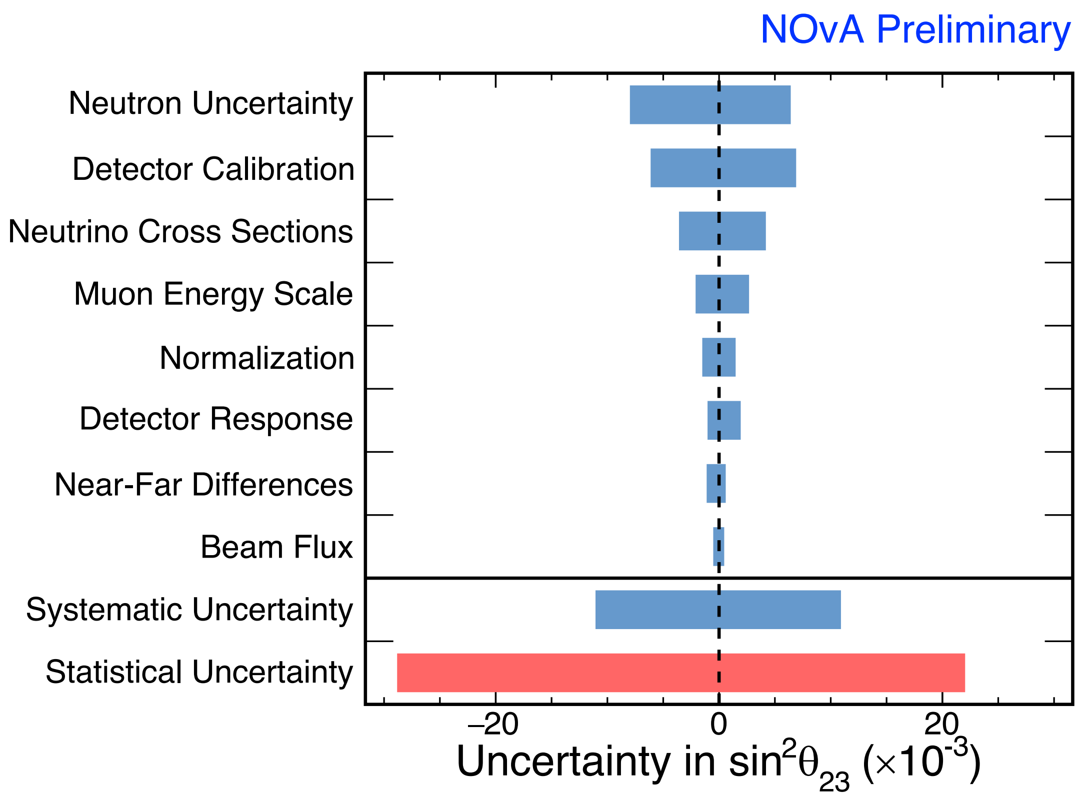
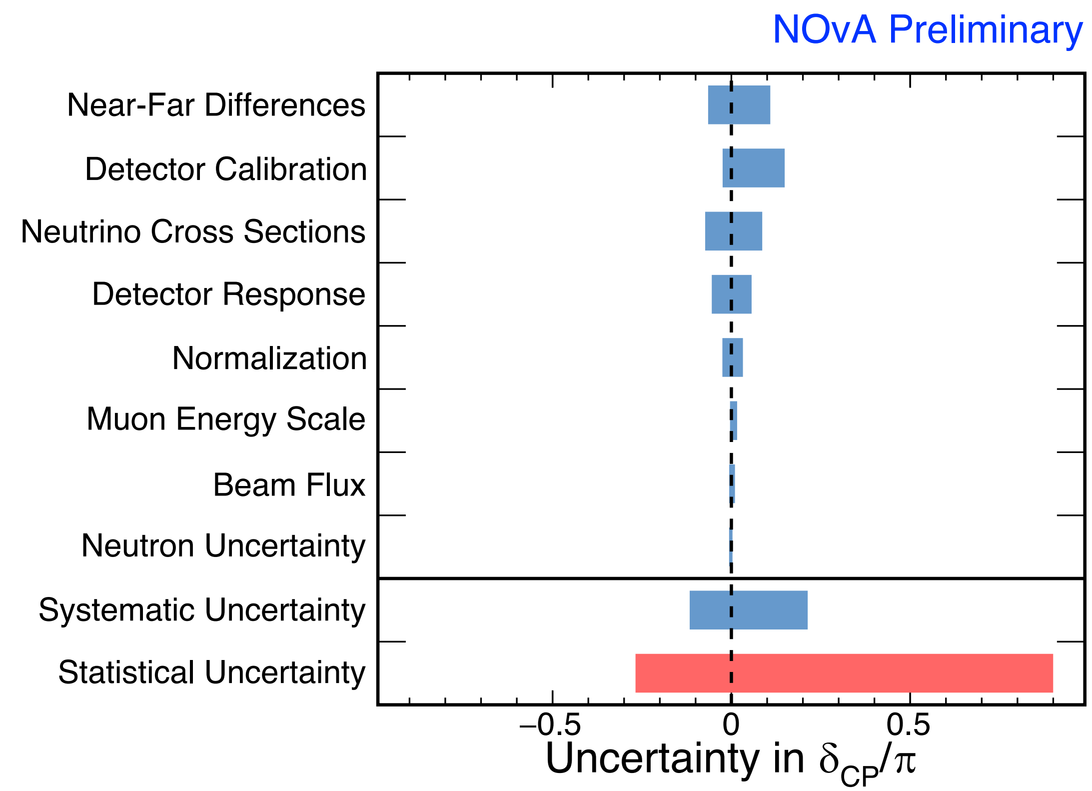
Neutrino Physics $\subset$ Particle Physics

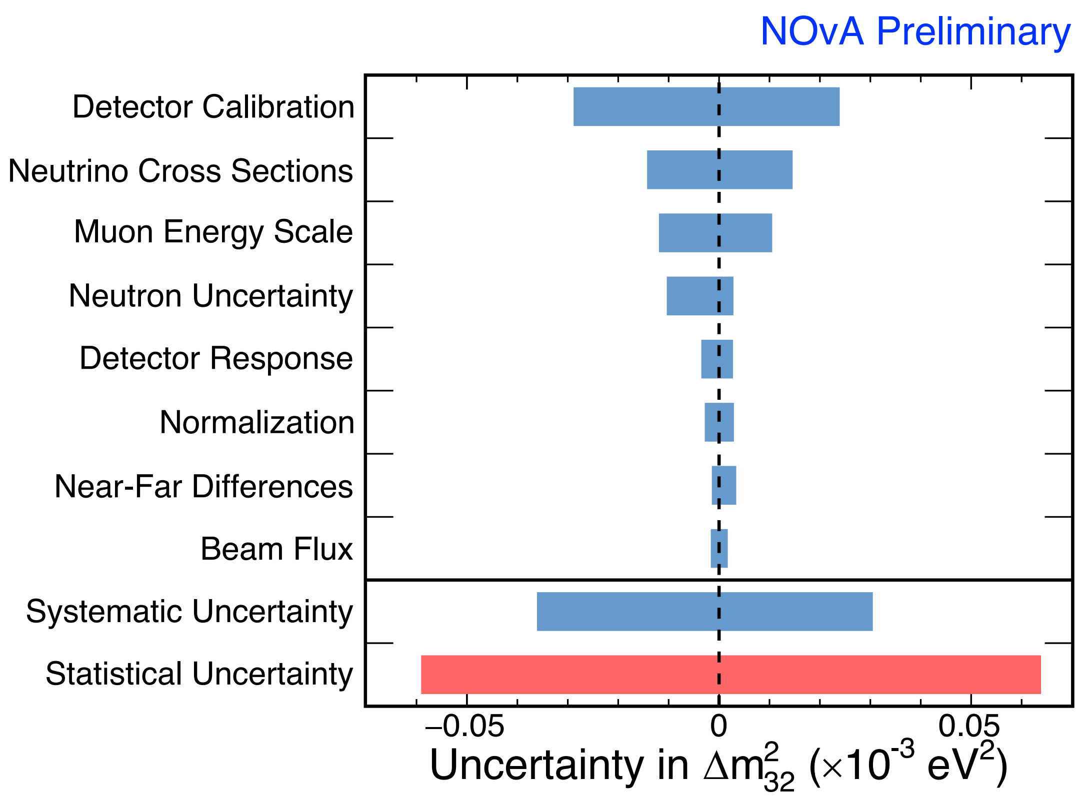
- Neutrinos have mass, change flavor (“oscillate”), and may violate CP.
- Taxpayers are eager to find out about CP and to test the three-family paradigm:
  - $\$10^9$ for LBNF/DUNE;
  - numerous other experiments.
- Oscillation experiments aim a(n) (anti)neutrino beam at nuclei such as ^{12}C, ^{16}O, ^{40}Ar.
- A nucleus can be thought of as a collection of nucleons.
- “A riddle, wrapped in a mystery, inside an enigma”—
  - a flavor change, wrapped in a nucleon, inside a nucleus.

Alex Himmel, June 15 @ Fermilab

Systematic Uncertainties



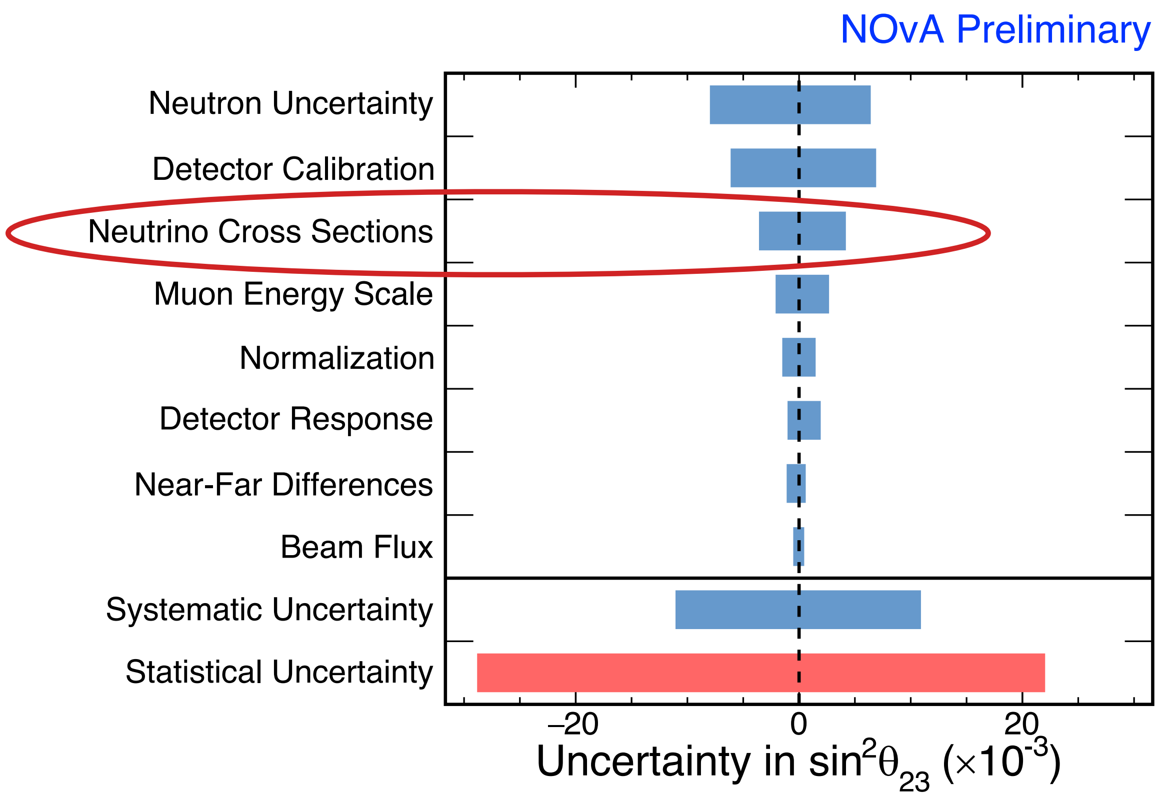


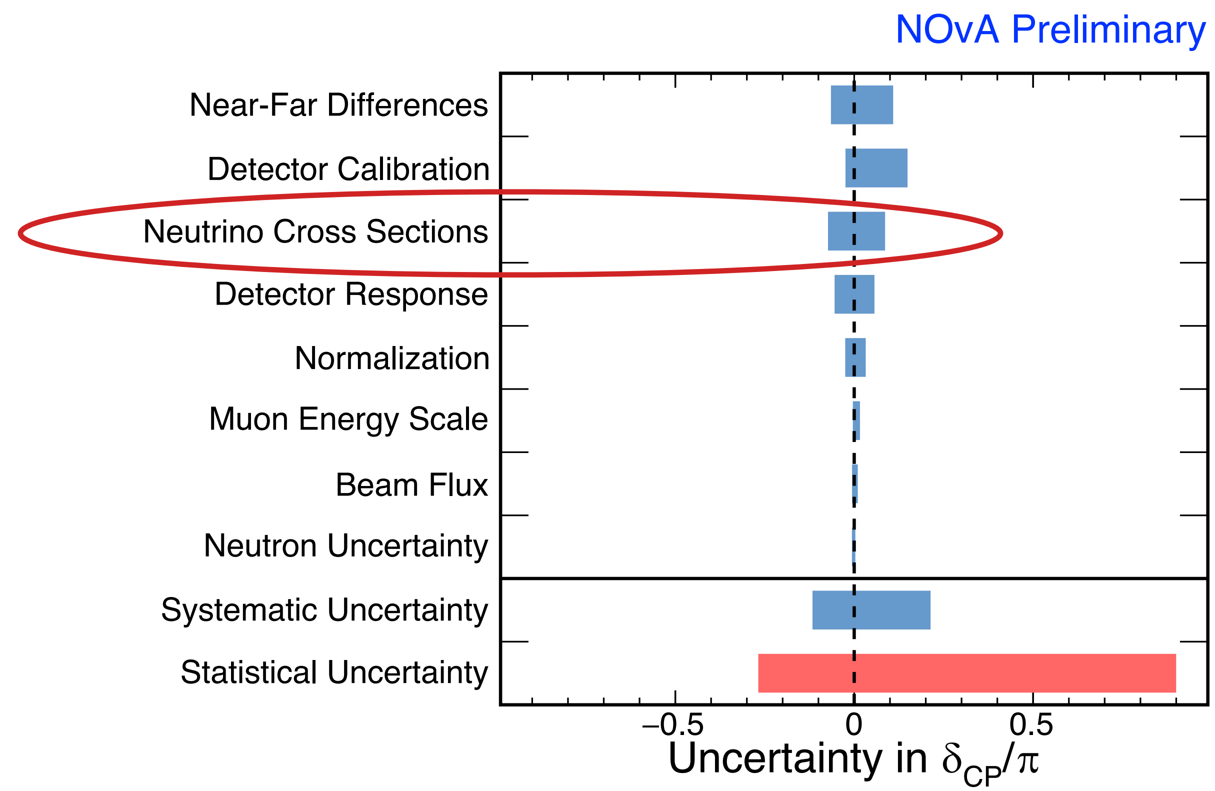


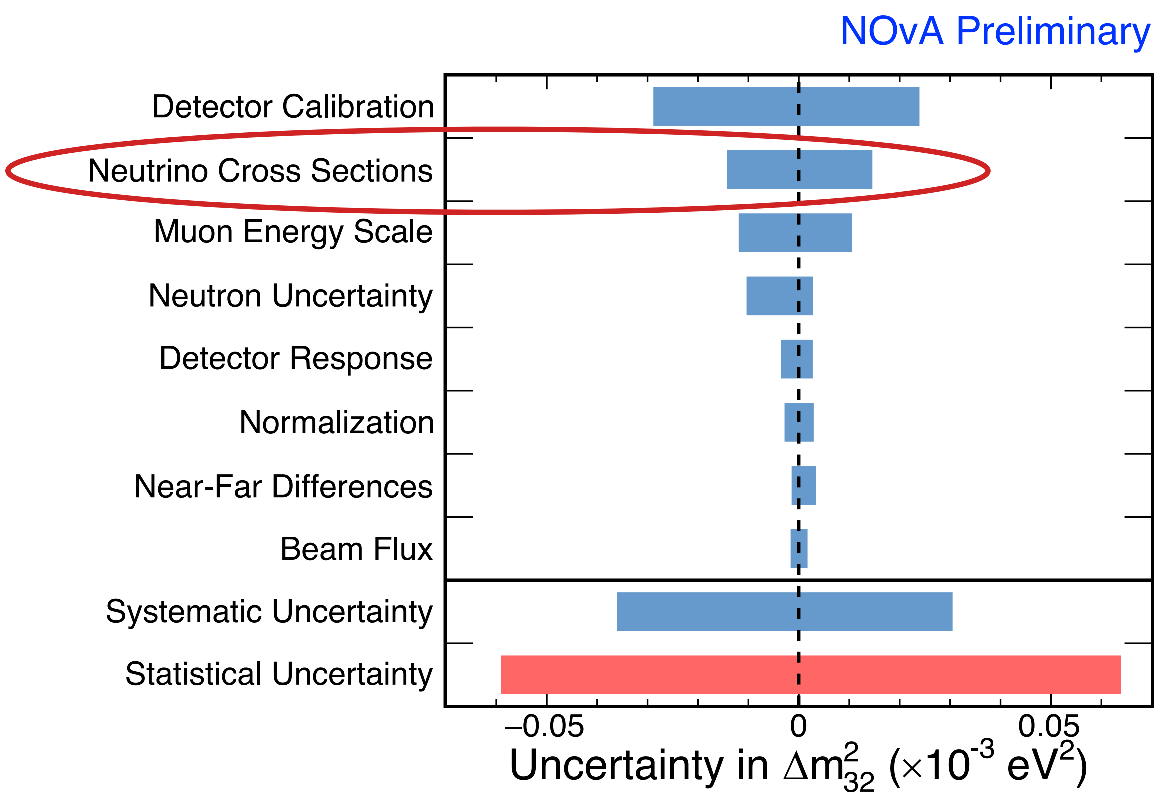
Most important systematics:

- Detector Calibration
  - Will be improved by the 2019 test beam program
- Neutrino cross sections
  - Particularly nuclear effects (RPA, MEC)
- Muon energy scale
- Neutron uncertainty – **new** with $\bar{\nu}$'s

Systematic Uncertainties







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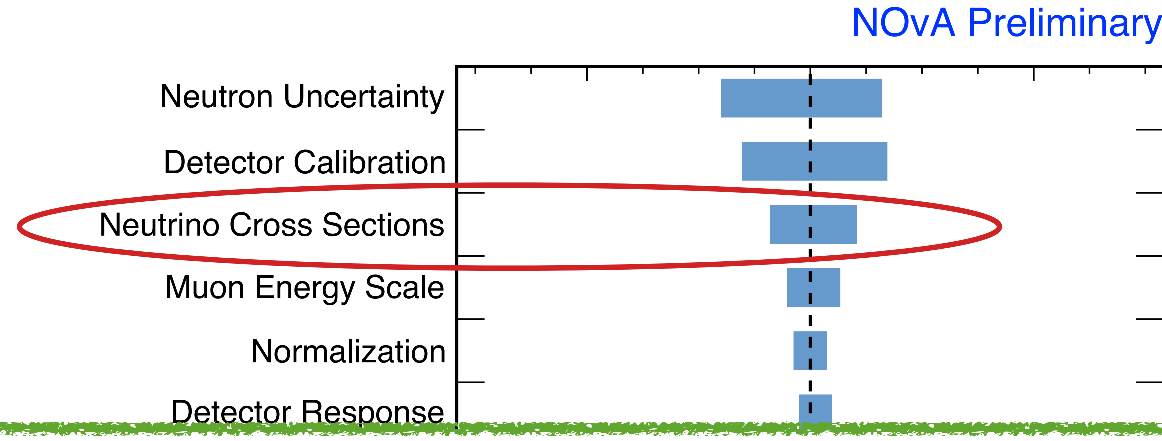
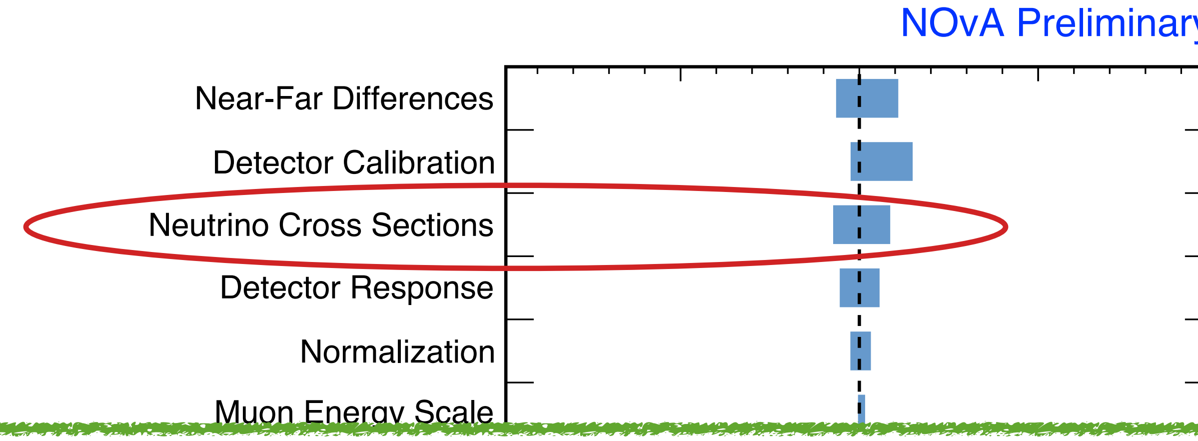
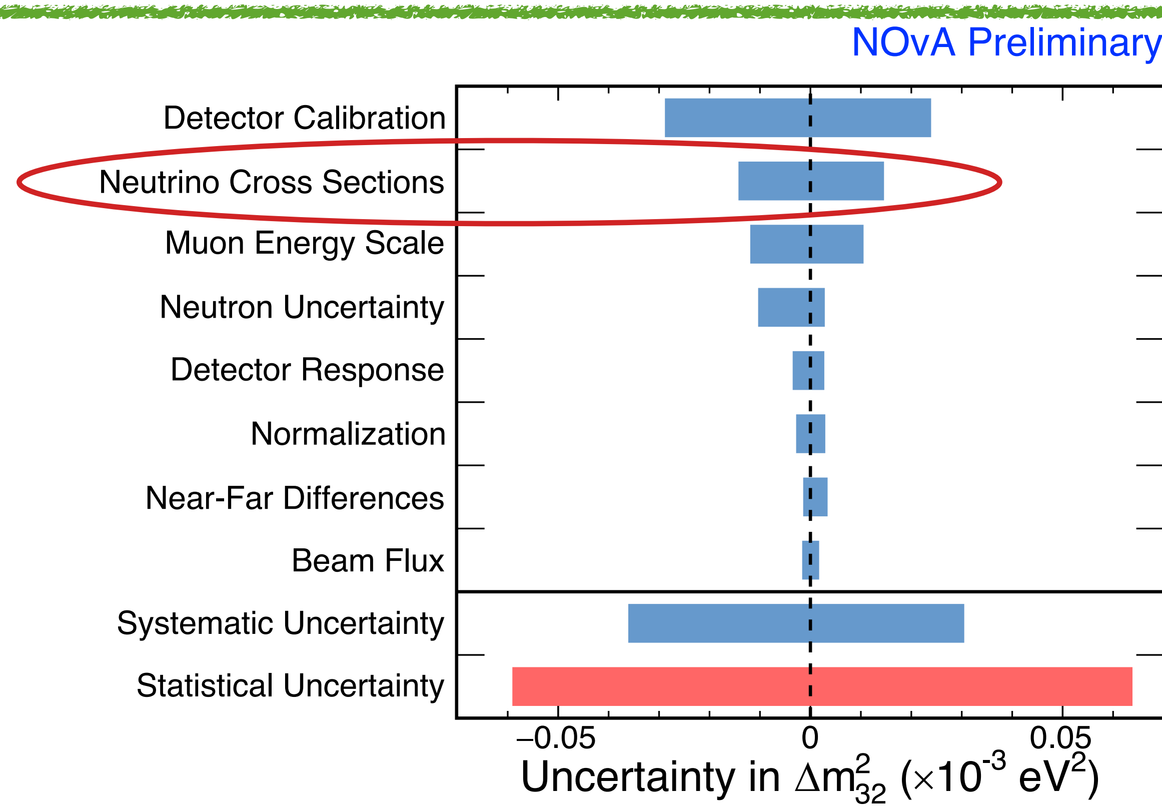




Uncertainty in Δm^2_{lj}, θ_{lj}, δ_{CP}, α_i owing to imperfect knowledge of nucleus?



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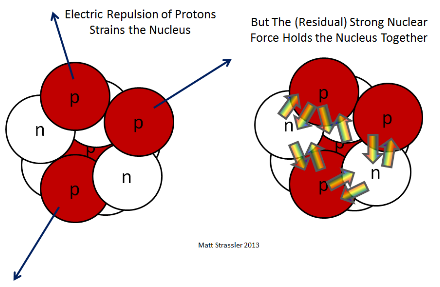
Neutrino-Nucleus Collisions

- At the energies used by experiments measuring oscillation parameters, the nucleus is treated as a collection nucleons:
  - use nuclear theory for nuclear structure;
  - use *ab initio* (i.e., derived from QCD) nucleon-level calculations for neutrino-nucleon transition matrix elements including
    - $\nu N \to eX$, $X = N, N', \Delta, N\pi$, i.e., matrix elements $\langle N|V-A|N\rangle$;
    - and, eventually, two-body effects: $\langle NN|V-A|NN\rangle$;
  - use nuclear transport (e.g., GiBUU) theory for post-collision absorption/ secondary production of pions.

Nuclear Structure



- A huge challenge:
  - many approaches, some called “*ab initio*” —
  - means that a unified formalism describes many things and can be systematically improved—
    - can be cast as a chiral effective theory (nucleons in a pion cloud);
    - not QCD though.
- Full error budgets are yet to come and may be impossible.
- Measure “data over here” to constrain nuclear model; then trust validated models “over there” where oscillation parameters are determined.

Hadronic Physics—νN Scattering

- Hard, but easy compared to nuclear theory.
- Trap to avoid: nuclear model using nucleonic ingredients inconsistent with QCD:
  - even if tuned up “here” such a thing would be scary “over there”.
- Therefore, aim to get scattering amplitudes from first principles.
- Two approaches:
  - $\bar{\nu}p$ or νd scattering experiments in the Tokai, NuMI, or LBNF beam;
  - lattice QCD with error budgets as comprehensive as those for CKM.

Outline

- Introduction & Motivation
- Paradigm of Lattice QCD
- Results (not for νA scattering)
- Results (for νN and νA scattering)
- Outlook

One thing I like about neutrino experiments is that they don’t rely on some damn lattice calculation!

Heidi Schellman

Lattice QCD for Beginners

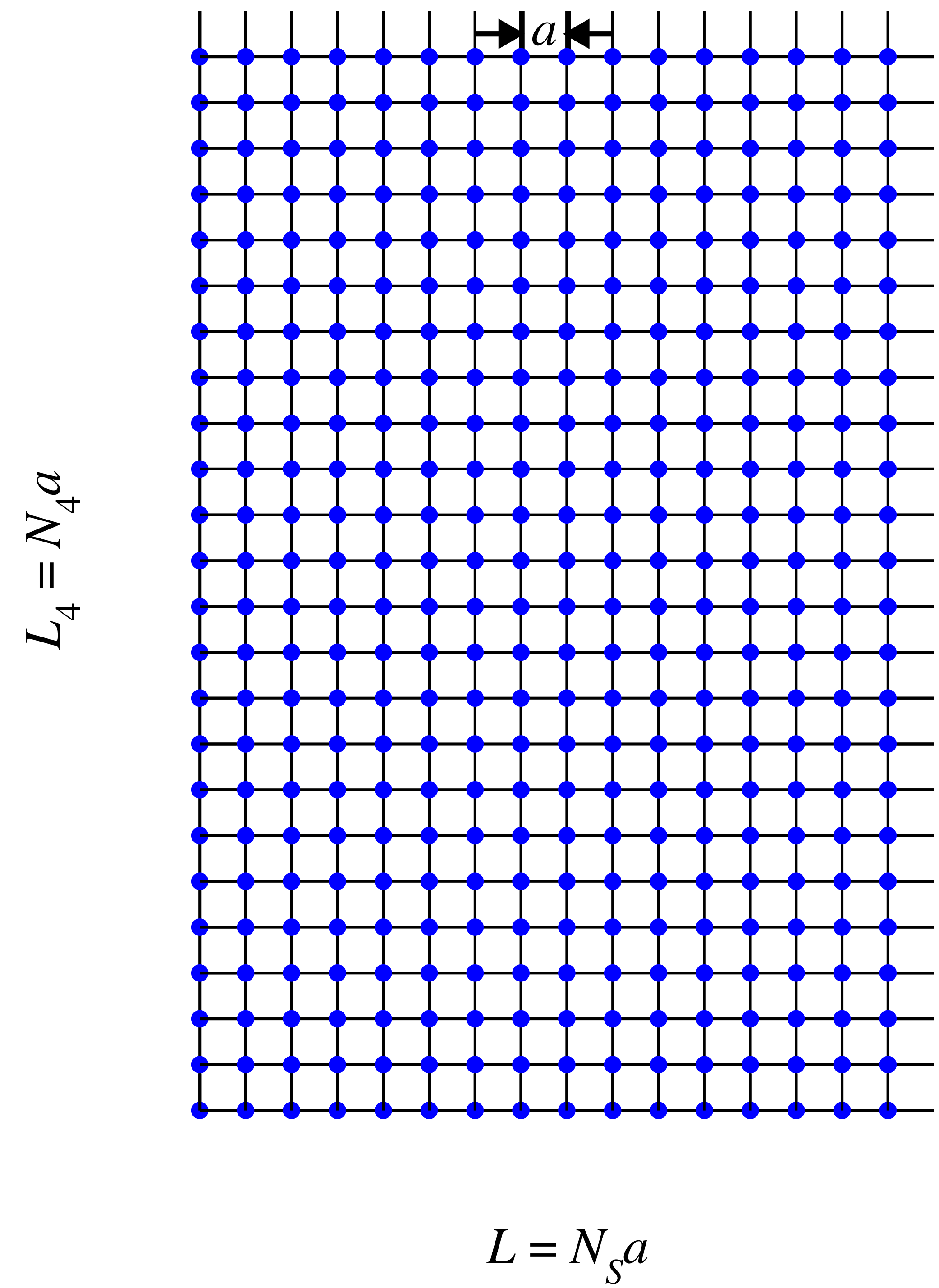
Lattice Gauge Theory

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U\, \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) \left[\bullet\right]$$

- Infinite lattice: countably many; used to define QFT;

- Finite lattice: finite dimension ~ 10^8, so compute integrals numerically.



Lattice Gauge Theory

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hand

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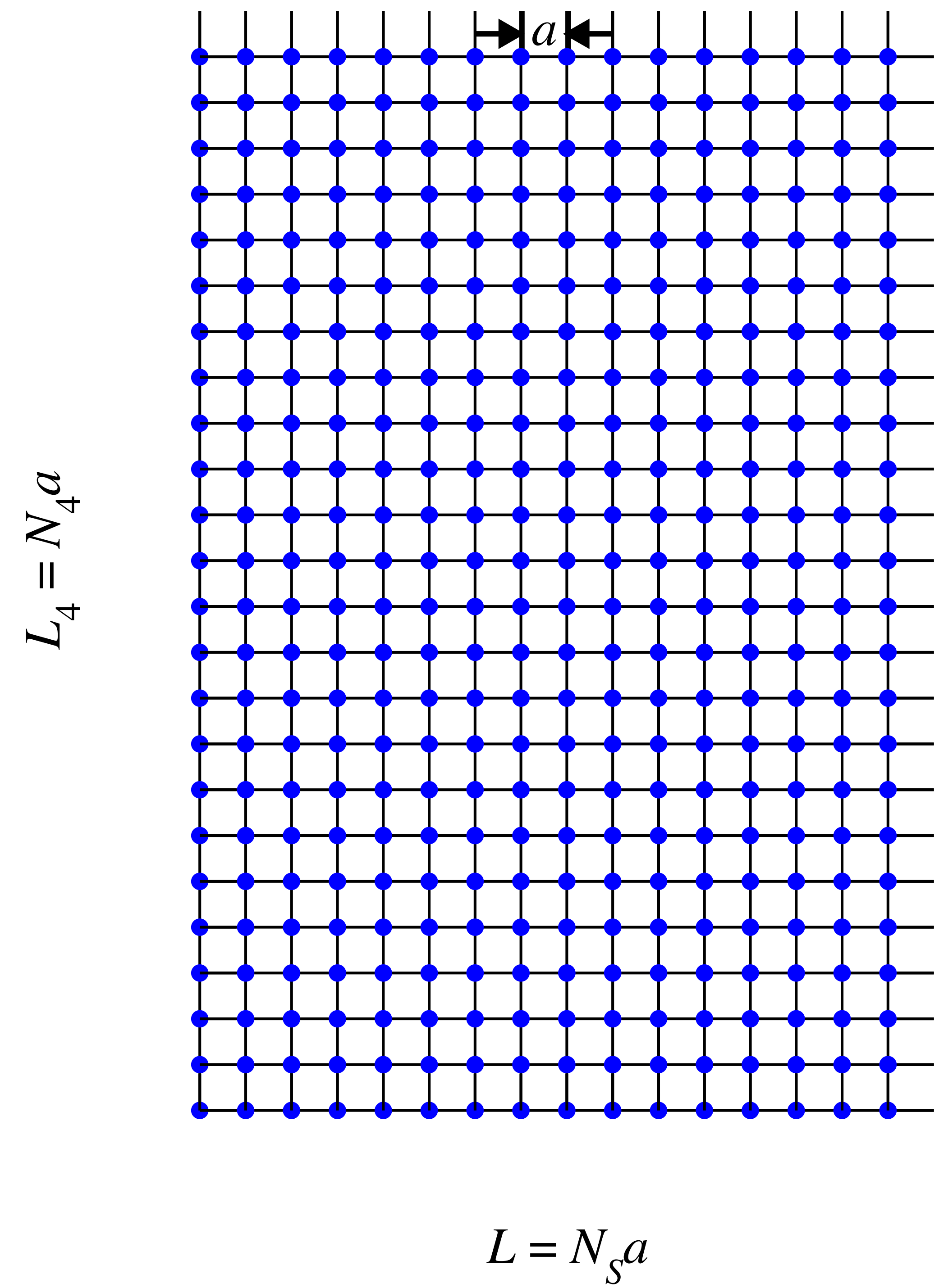

Lattice Gauge Theory

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);

$$\langle \bullet \rangle = \frac{1}{Z} \int \underbrace{\mathcal{D}U}_{\text{MC}} \underbrace{\mathcal{D}\psi \mathcal{D}\bar{\psi}}_{\text{hand}} \exp\left(-S\right) \left[\bullet\right]$$

MC hand

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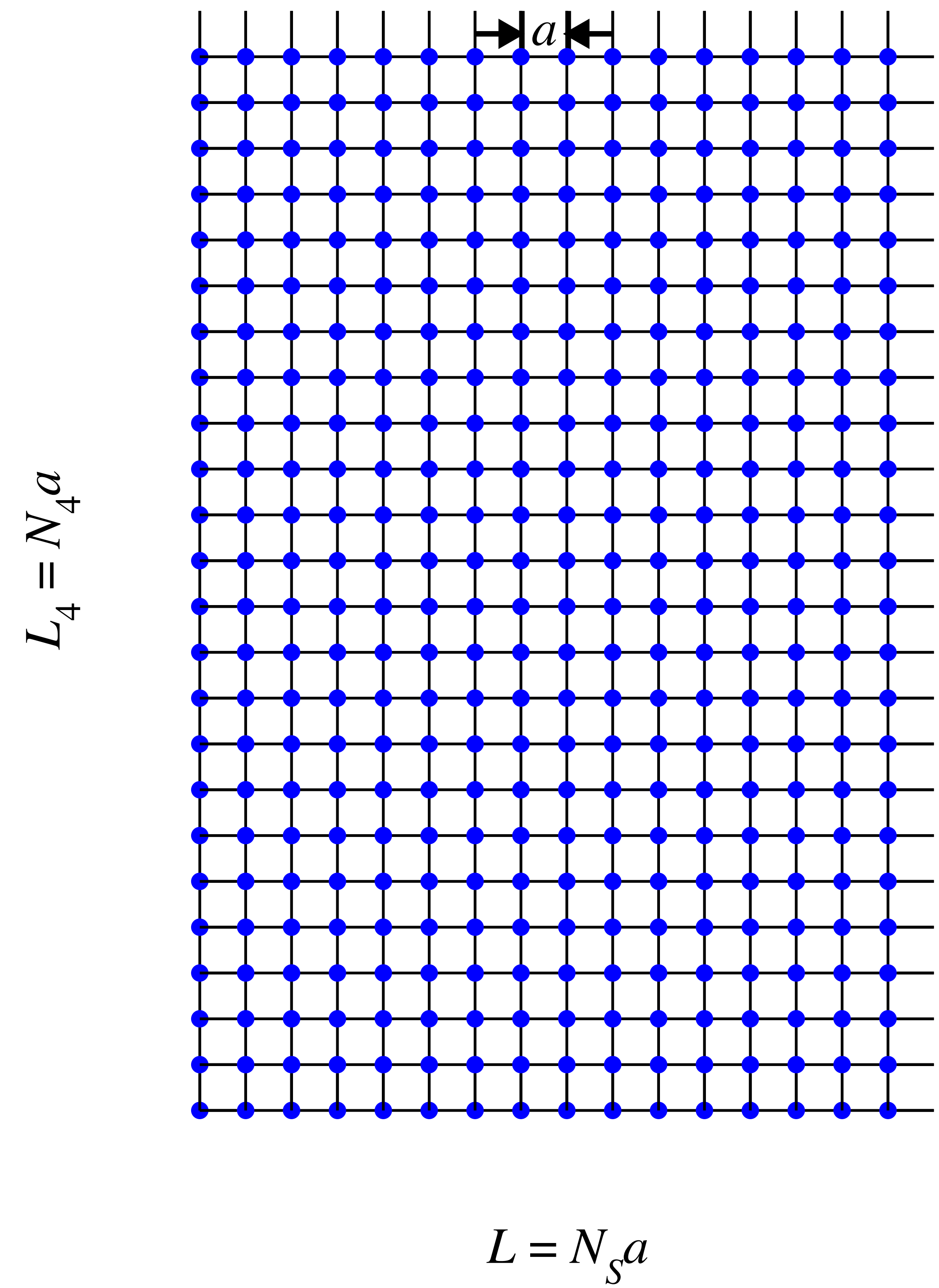


The Steps

- Use random number generator to create gauge fields distributed $\sim e^{-S}$.
- Solve $(\not{D} + m)_{xz} G_{zy} = b_x$ for quark propagators in these gauge fields.
- Fit hadron correlation functions to get masses and matrix elements.
- Repeat several times while varying bare gauge coupling and bare masses.
- Find a trajectory with constant pion, kaon, D_s, B_s, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.
- Convert units to MeV.
- Comprehensive analysis of uncertainties!

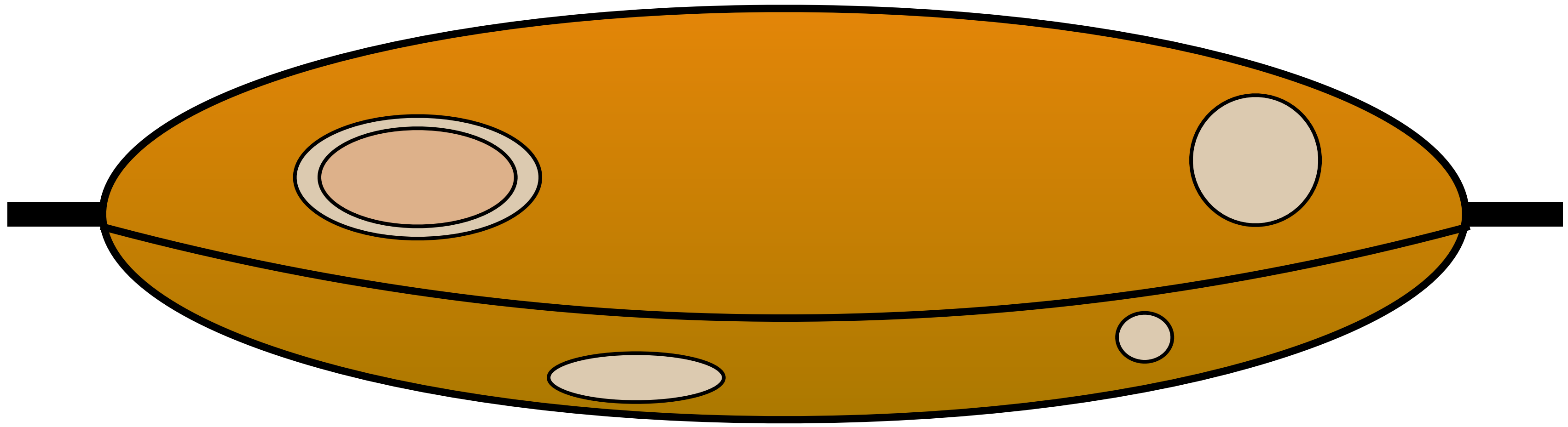
Error Budget

TABLE I. Error budget for strange-, charm- and bottom-quark masses, their ratios, and the HQET matrix element $\overline{\Lambda}_{\mathrm{MRS}}$. See the text for the description.

Error (%)	m_b/m_c	m_c/m_s	m_b/m_s	$m_{s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$	$\overline{m}_c$	$\overline{m}_b$	$\overline{\Lambda}_{\mathrm{MRS}}$
Statistics and EFT fit	0.10	0.09	0.12	0.43	0.30	0.29	4.5
Two-point correlator fits	0.08	0.02	0.10	0.13	0.08	0.03	1.1
Scale setting and tuning	0.02	0.08	0.10	0.12	0.03	0.02	0.2
Finite-volume corrections	0.02	0.04	0.06	0.07	0.02	0.01	0.1
Topological charge distribution	0.00	0.00	0.00	0.00	0.00	0.00	0.0
Electromagnetic corrections	0.11	0.11	0.01	0.02	0.08	0.01	0.1
α_s	0.01	0.00	0.01	0.56	0.75	0.18	2.9
$f_{\pi,\mathrm{PDG}}$	0.03	0.07	0.10	0.13	0.04	0.02	0.3

- Quark-flavor results compiled by Flavor Lattice Averaging Group (FLAG).
- With meson form factors, mixing matrix elements, etc., it is now common to publish correlation matrices as well as error budgets.

Sea Quarks



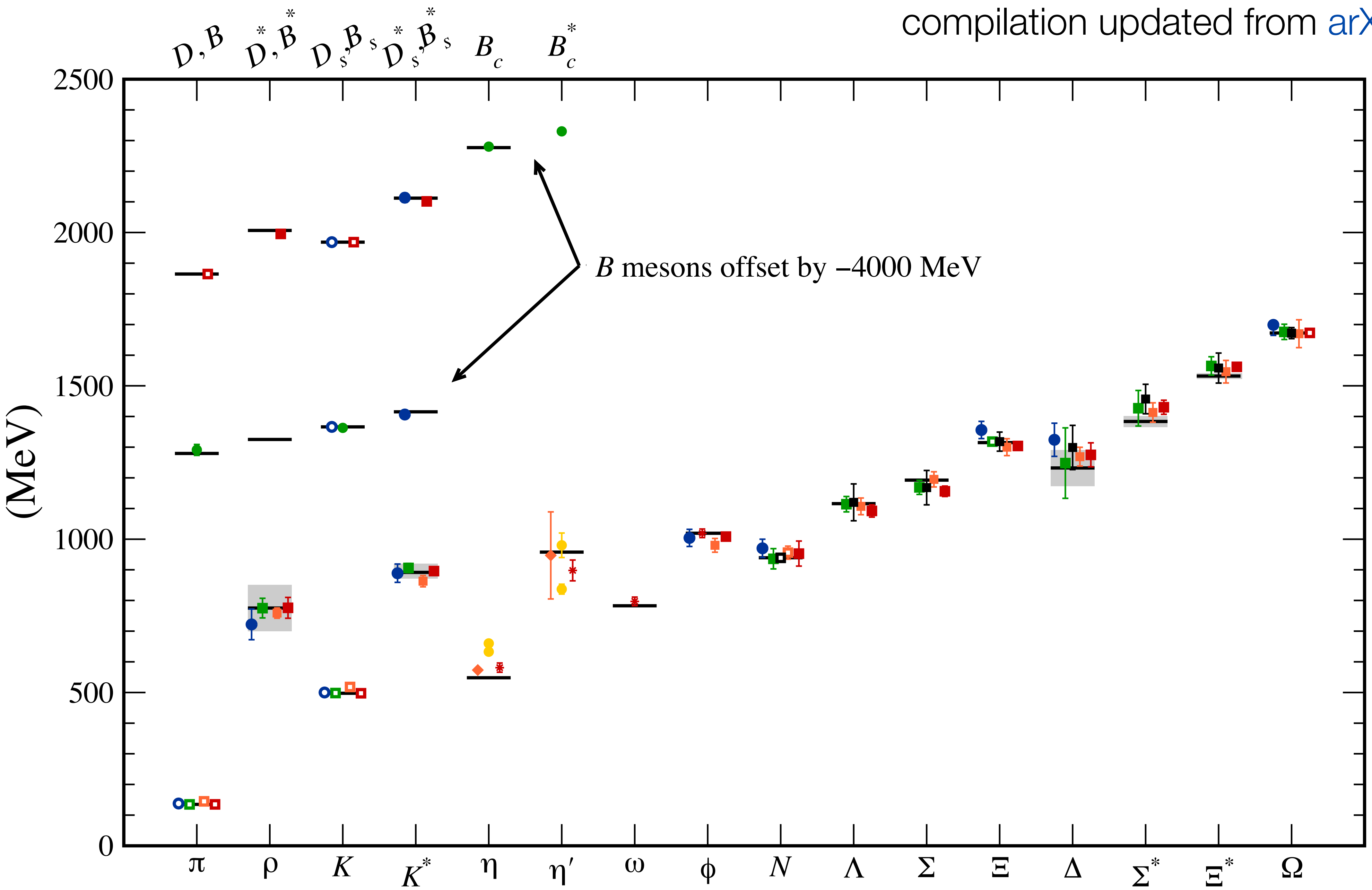
- Computationally the most demanding part.
- So often charmed sea is omitted, sometimes strange sea is omitted.
- Sometimes sea quarks take physical masses; often larger than up/down.
- Most calculations of small nuclei have sea quarks such that the “pion” in the computer has mass 800 MeV.

Lattice QCD for Textbooks

QCD Hadron Spectrum

π…Ω: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, *B*: Fermilab, HPQCD, Mohler&Woloshyn

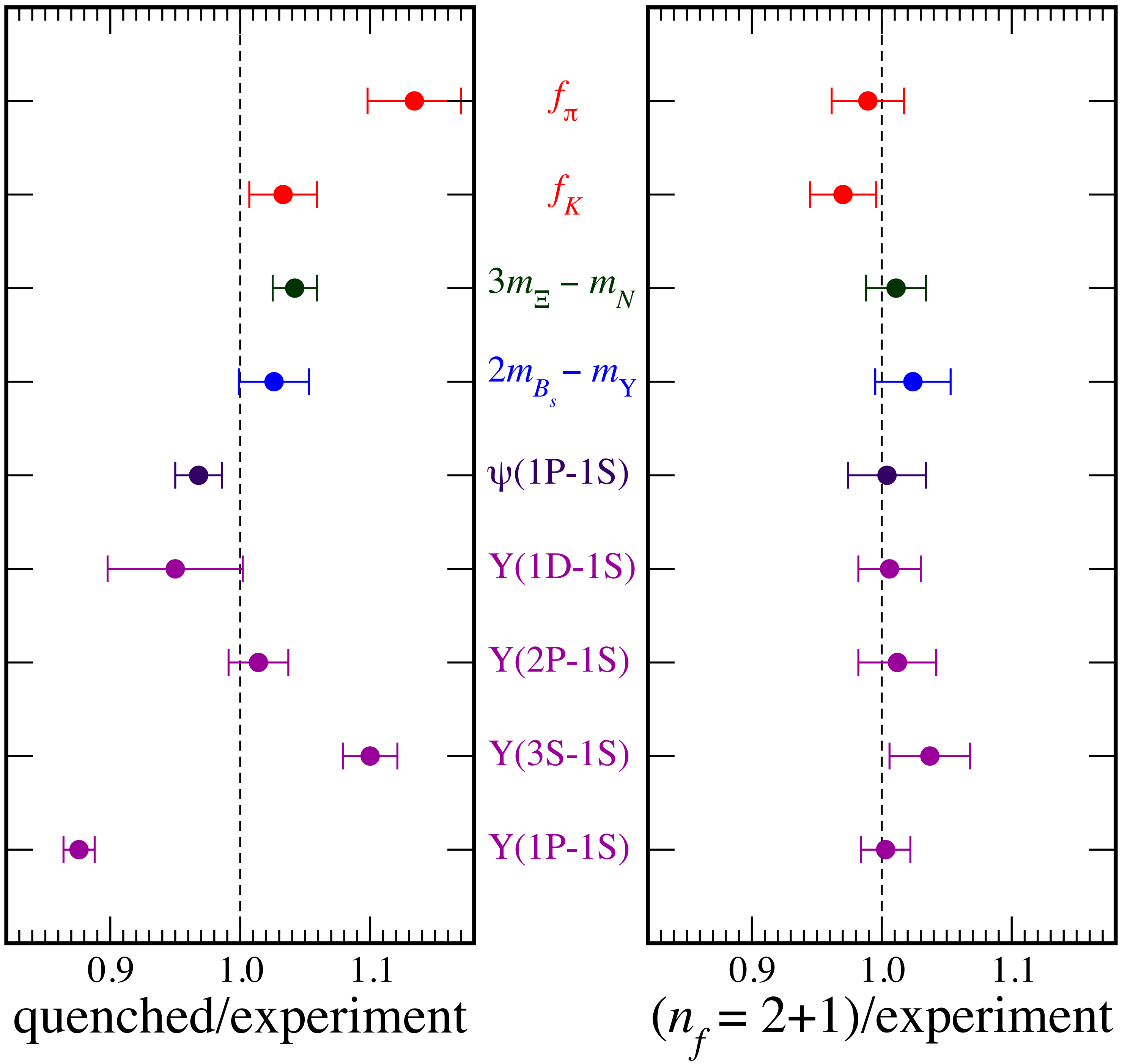
compilation updated from arXiv:1203.1204



numerous quarkonium omitted

Postdictions

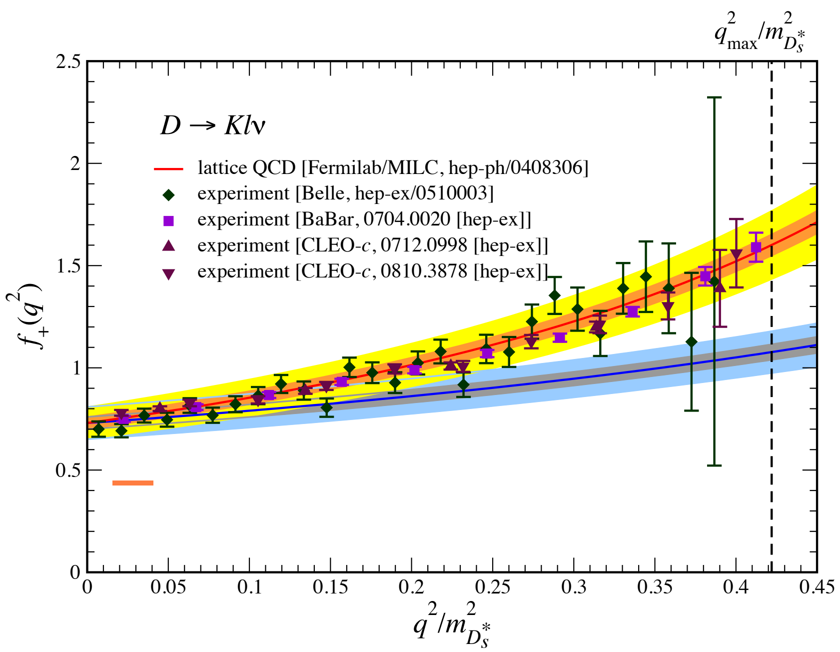
HPQCD, MILC, Fermilab Lattice, hep-lat/0304004

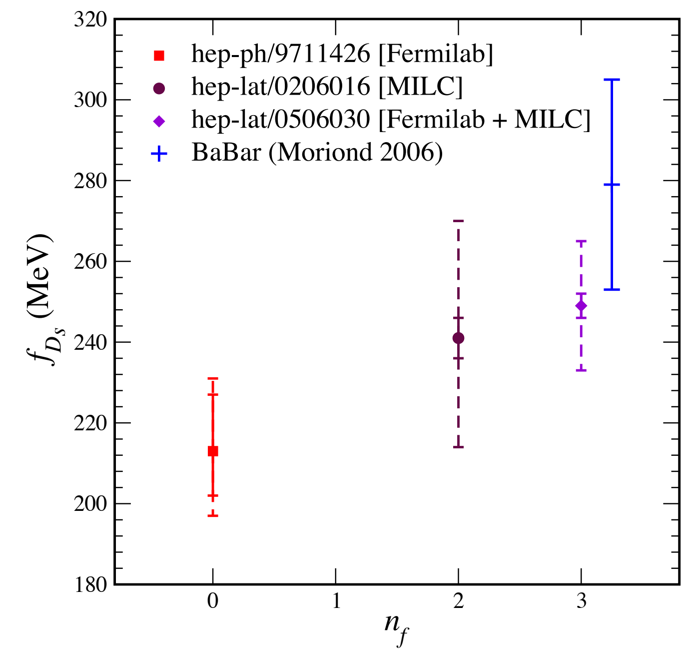


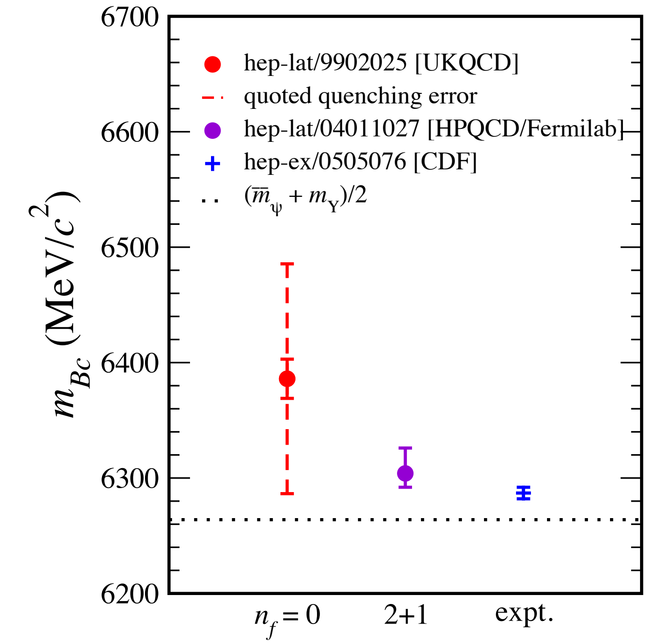
- From 2003!
- $a = 0.12$ & 0.09 fm;
- $O(a^2)$ improved: asqtad;
- FAT7 smearing;
- $2m_l < m_q < m_s$;
- π, K, Υ(2S-1S) input.

Predictions

Fermilab Lattice, MILC, HPQCD





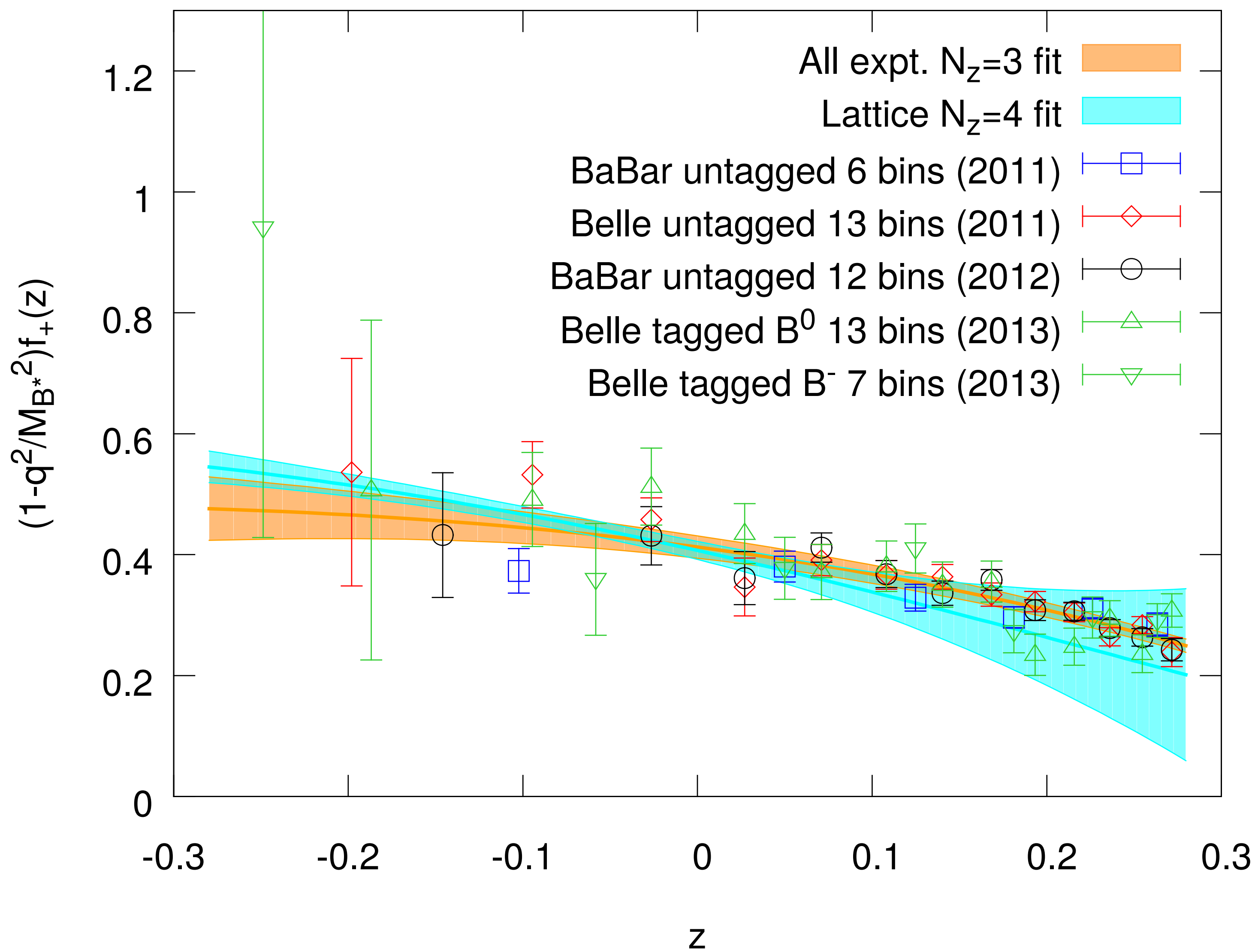


- Semileptonic form factor for $D \to Kl\nu$: hep-ph/0408306—updates: arXiv:1008.4562 (normalization), arXiv:1305.1462 (shape);
- Charmed-meson decay constants: hep-lat/0506030—update below;
- Mass of B_c meson: hep-lat/0411027—updates: arXiv:0909.4462 ($M_{B^*} = 6330 \pm 9$ GeV), arXiv:1010.3848.

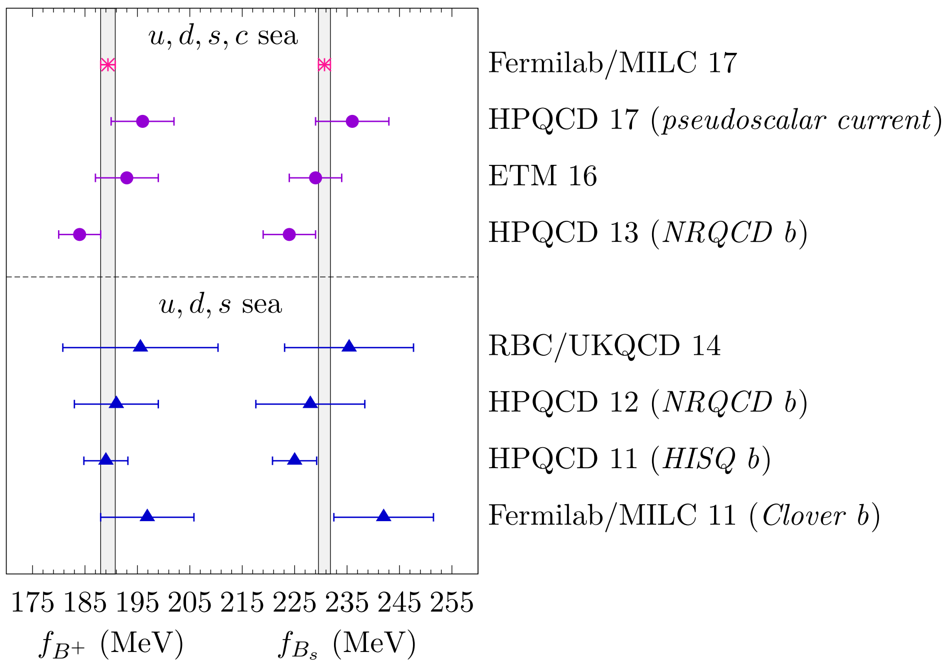
Semileptonic $B \to \pi l \nu$ for $|V_{ub}|$

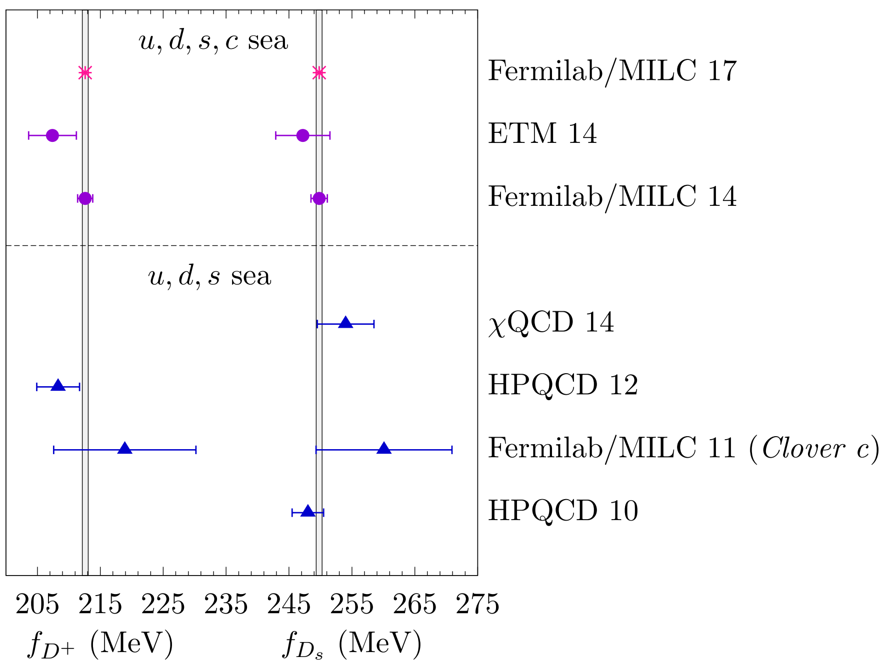
$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

- Much more precise than 2008.
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$ is 4.1%:
  - $10^3|V_{ub}| = 3.72 \pm 0.16$



Leptonic $B \to \tau\nu$ for $|V_{ub}|$, $D \to l\nu$ for $|V_{cd}|$





- Fermilab Lattice & MILC [arXiv:1712.09262]:

$$f_{D^0} = 211.5(0.3)_{\rm stat}(0.3)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

$$f_{D^+} = 212.6(0.3)_{\rm stat}(0.3)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

$$f_{D_s} = 249.8(0.3)_{\rm stat}(0.3)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

$$f_{B^+} = 189.4(0.8)_{\rm stat}(1.1)_{\rm syst}(0.3)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

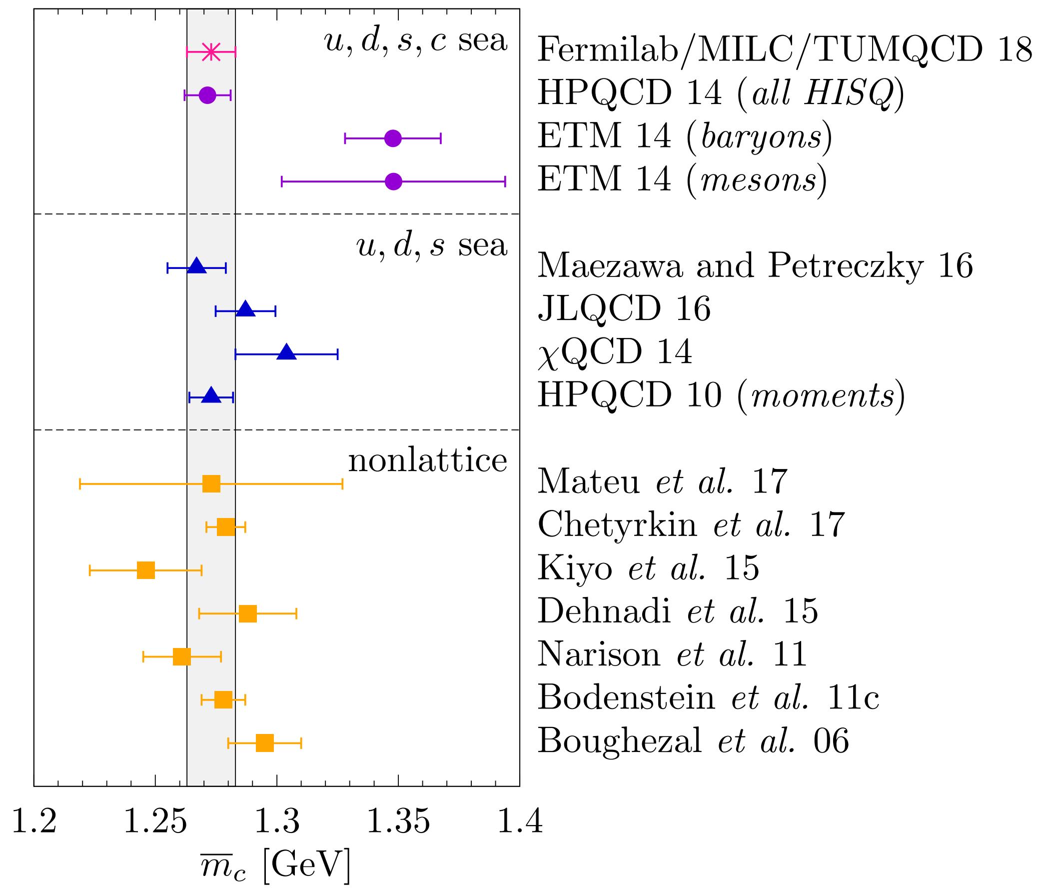
$$f_{B^0} = 190.5(0.8)_{\rm stat}(1.0)_{\rm syst}(0.3)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

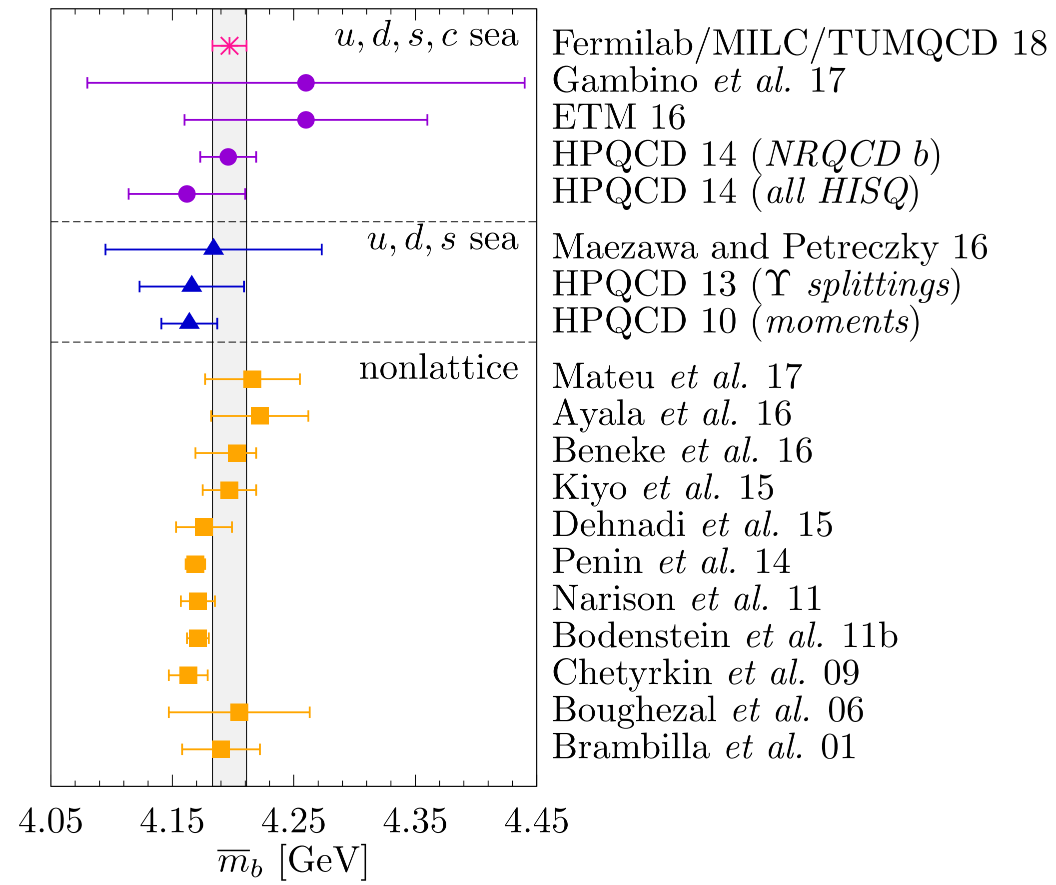
$$f_{B_s} = 230.7(0.8)_{\rm stat}(0.8)_{\rm syst}(0.2)_{f_{\pi,\rm PDG}}\ \text{MeV}$$

- Overall uncertainty: ~0.2% for D mesons, ~0.7% for B mesons.

Quark Masses

- Results from arXiv:1802.04248:





- To our knowledge, first results w/ order-α_s^5 running & order-α_s^4 matching.
- 0.3% for bottom, 0.5% for charm, 0.7% for strange, 1% for down, 2% for up.

Lattice QCD for Neutrino Scattering

Form Factors

- Hadronic matrix elements of the weak currents are decomposed into Lorentz covariant forms, multiplied by form factors:

$$\langle p(\boldsymbol{p})|\mathcal{V}^{\mu}_{\bar{u}d}|n(\boldsymbol{k})\rangle = \bar{u}_p(\boldsymbol{p})\gamma^{\mu}u_n(\boldsymbol{k})\,F_1(q^2) + \frac{q_\nu}{2M_N}\bar{u}_p(\boldsymbol{p})i\sigma^{\mu\nu}u_n(\boldsymbol{k})\,F_2(q^2),$$

$$\langle p(\boldsymbol{p})|\mathcal{A}^{\mu}_{\bar{u}d}|n(\boldsymbol{k})\rangle = \bar{u}_p(\boldsymbol{p})\gamma^{\mu}_{\perp}\gamma^5 u_n(\boldsymbol{k})\,F_A(q^2) + \frac{2M_N q^{\mu}}{q^2}\bar{u}_p(\boldsymbol{p})\gamma^5 u_n(\boldsymbol{k})\,F_P(q^2),$$

$$q^{\mu} = k^{\mu} - p^{\mu}$$

- The q^2 dependence is constrained by unitarity and analyticity, leading to a model-independent parametrization known as the "z expansion".

- Lattice-QCD calculations plus z expansion is a key ingredient in CKM determinations, *e.g.*, $|V_{ub}|$.

Vector Form Factor from Lattice QCD

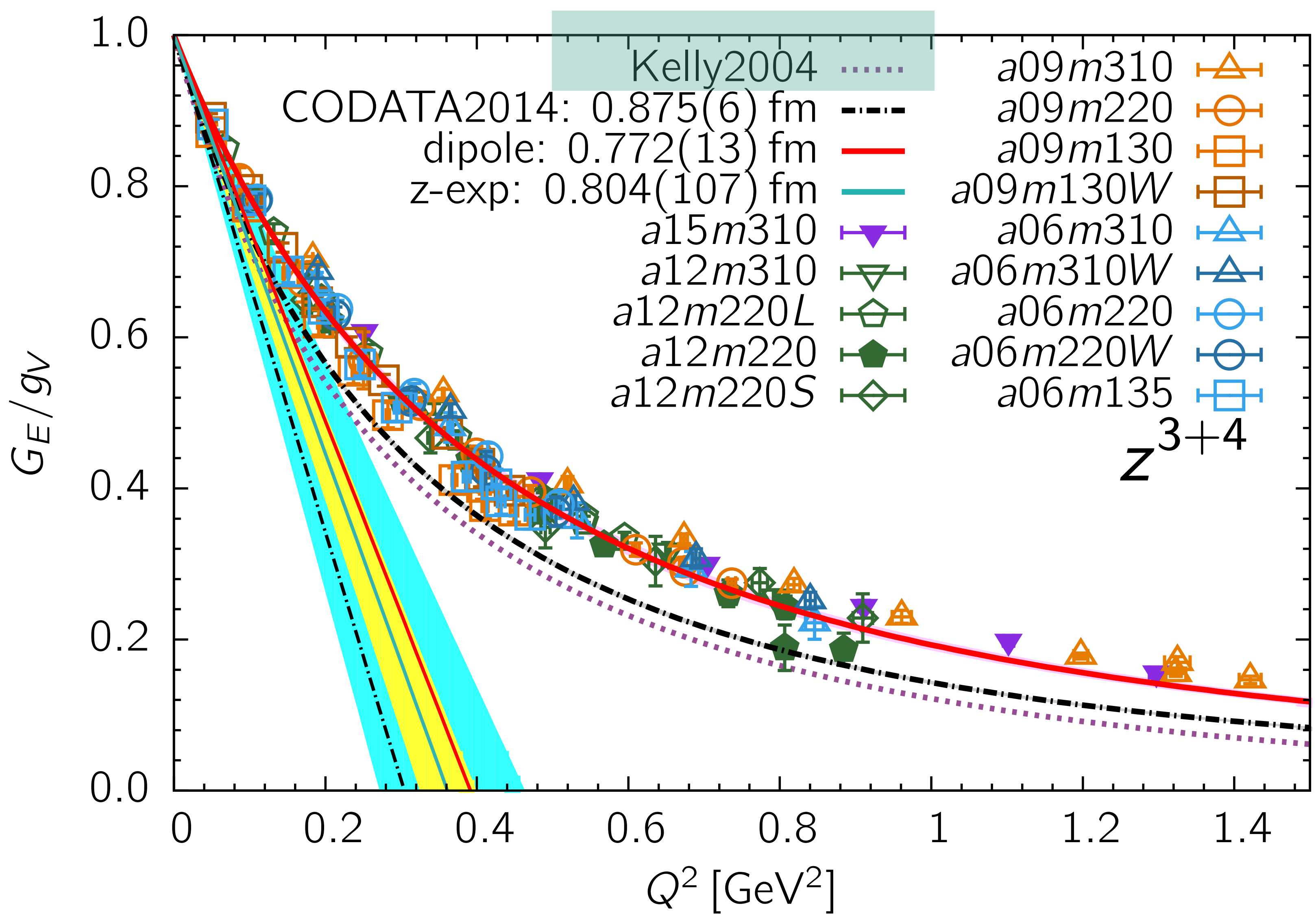
- Lattice QCD too high?
- Even continuum limit:

  $r_E = 0.80(11)$ fm,

  vs CODATA:

  $r_E = 0.875(6)$ fm.

- $G_E = F_1 - Q^2 F_2/M^2$.



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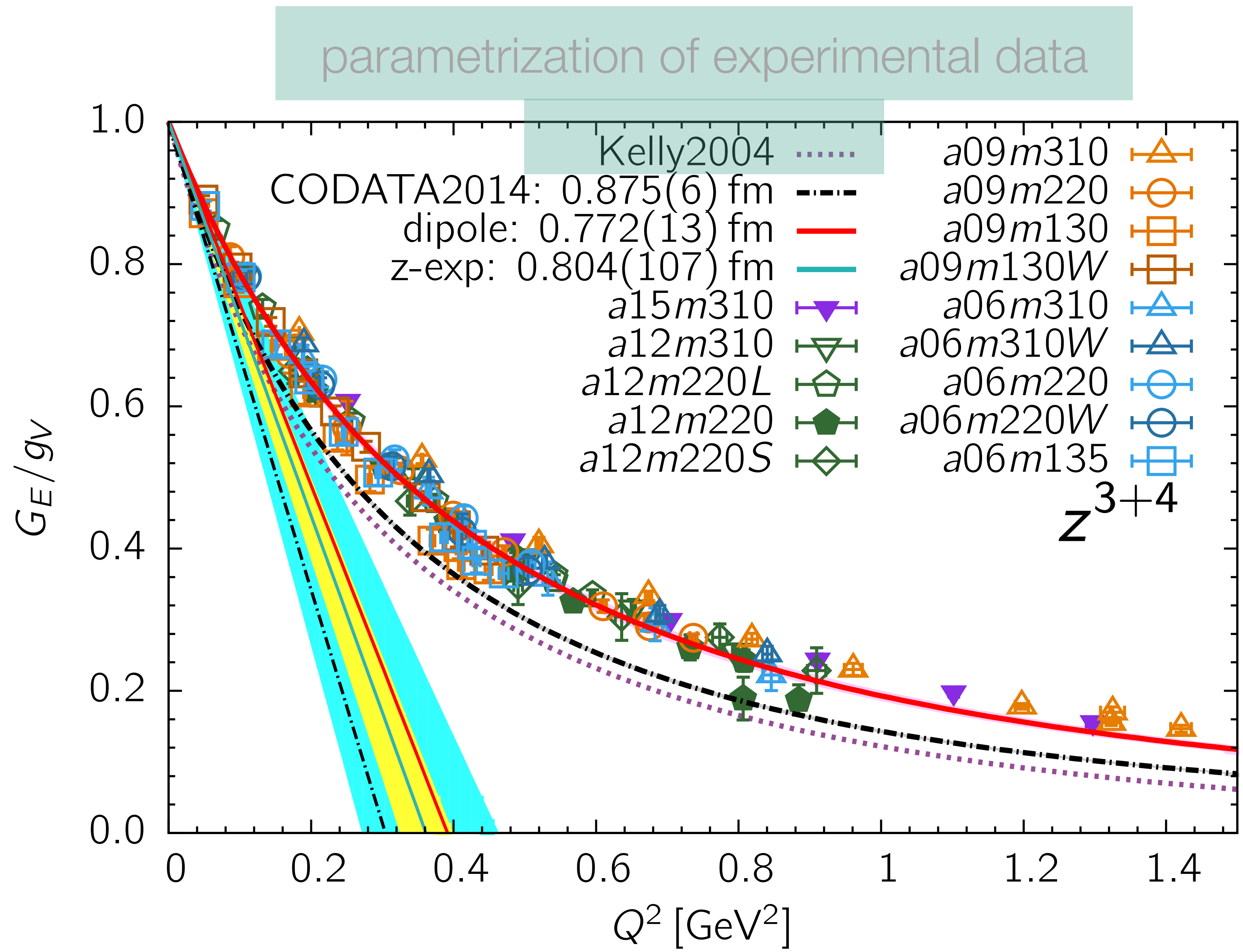


Axial Form Factor from Lattice QCD

- Gupta, Jang, Lin, Yoon, Bhattacharya (PNDME) [arXiv:1705.06834]:

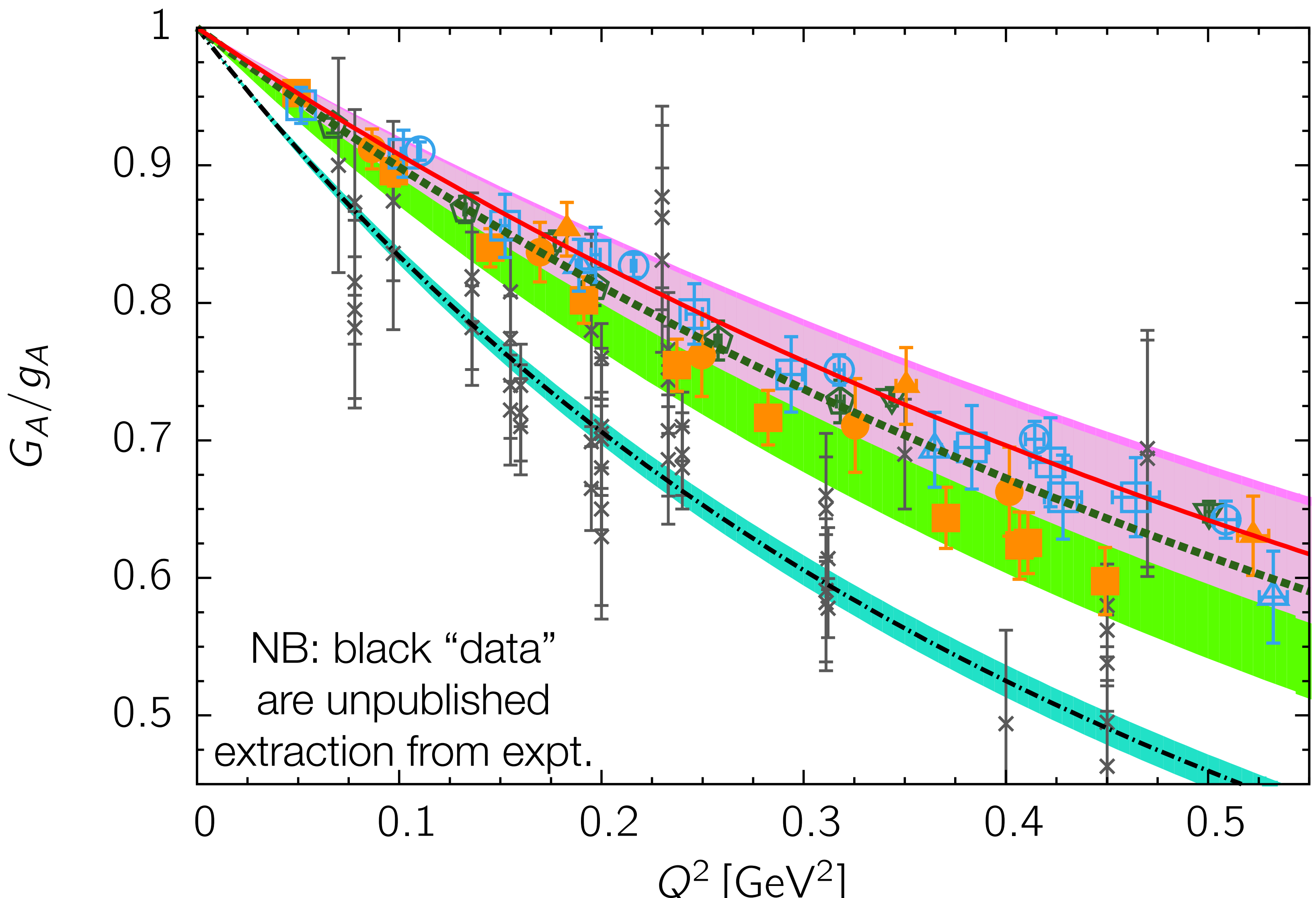
- $a \neq 0$ slope smaller than deuterium & other data.

- Even continuum limit:

$$r_A = 0.46(6)\ \text{fm},$$

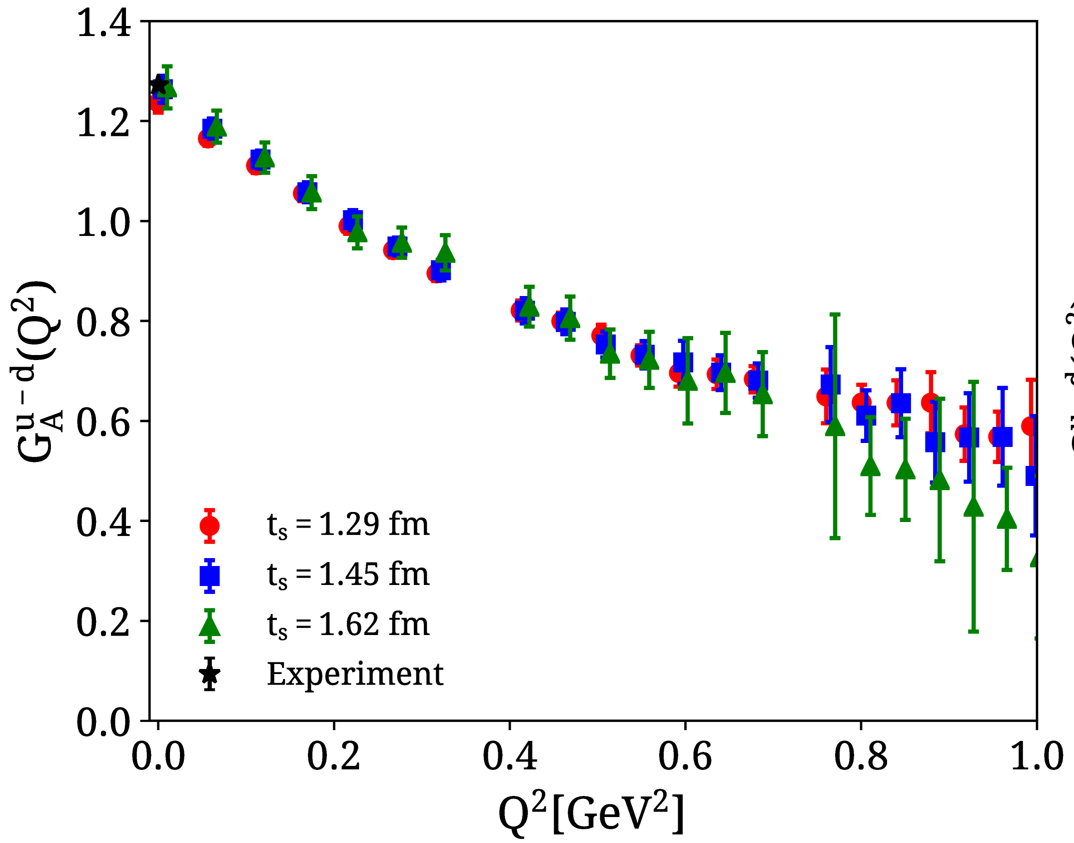
vs Meyer *et al*.:

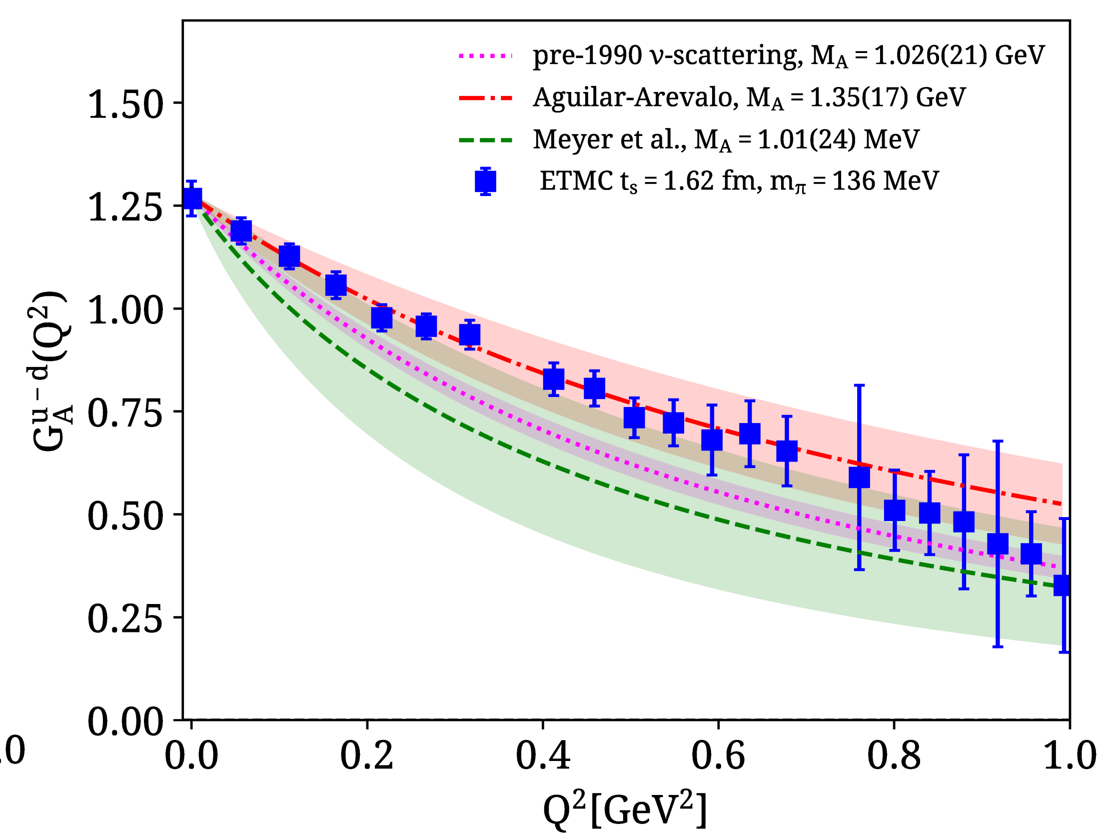
$$r_A = 0.68(16)\ \text{fm}.$$

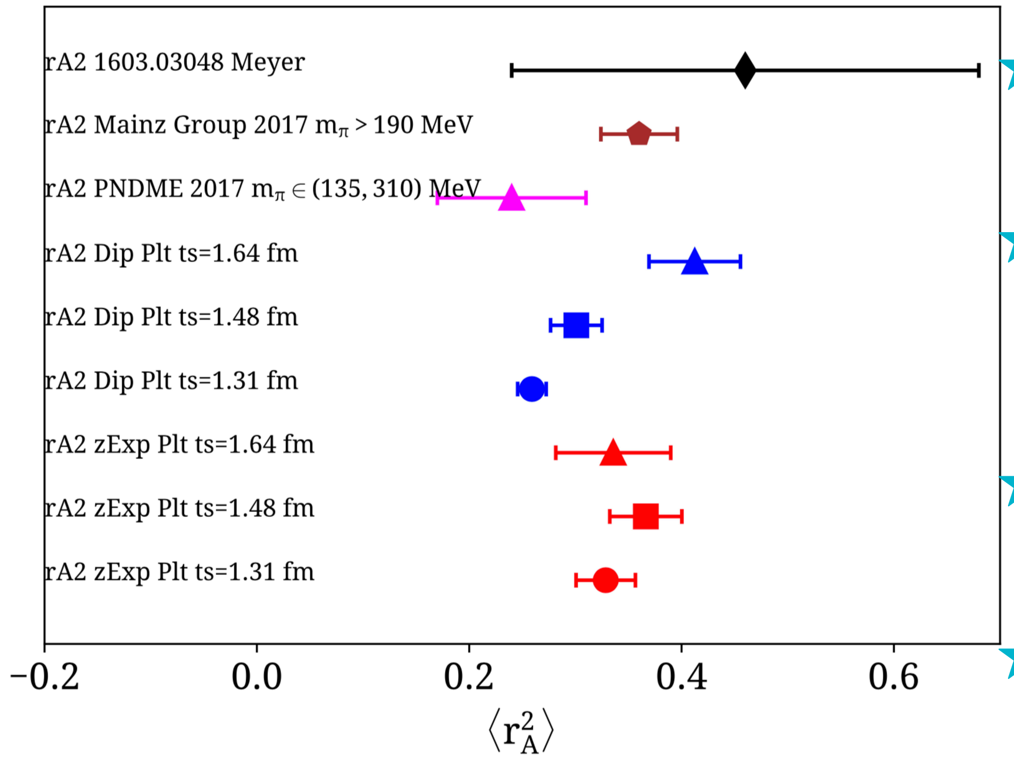


- What would 0.46 fm say about deuteron?

Nucleon axial form factors



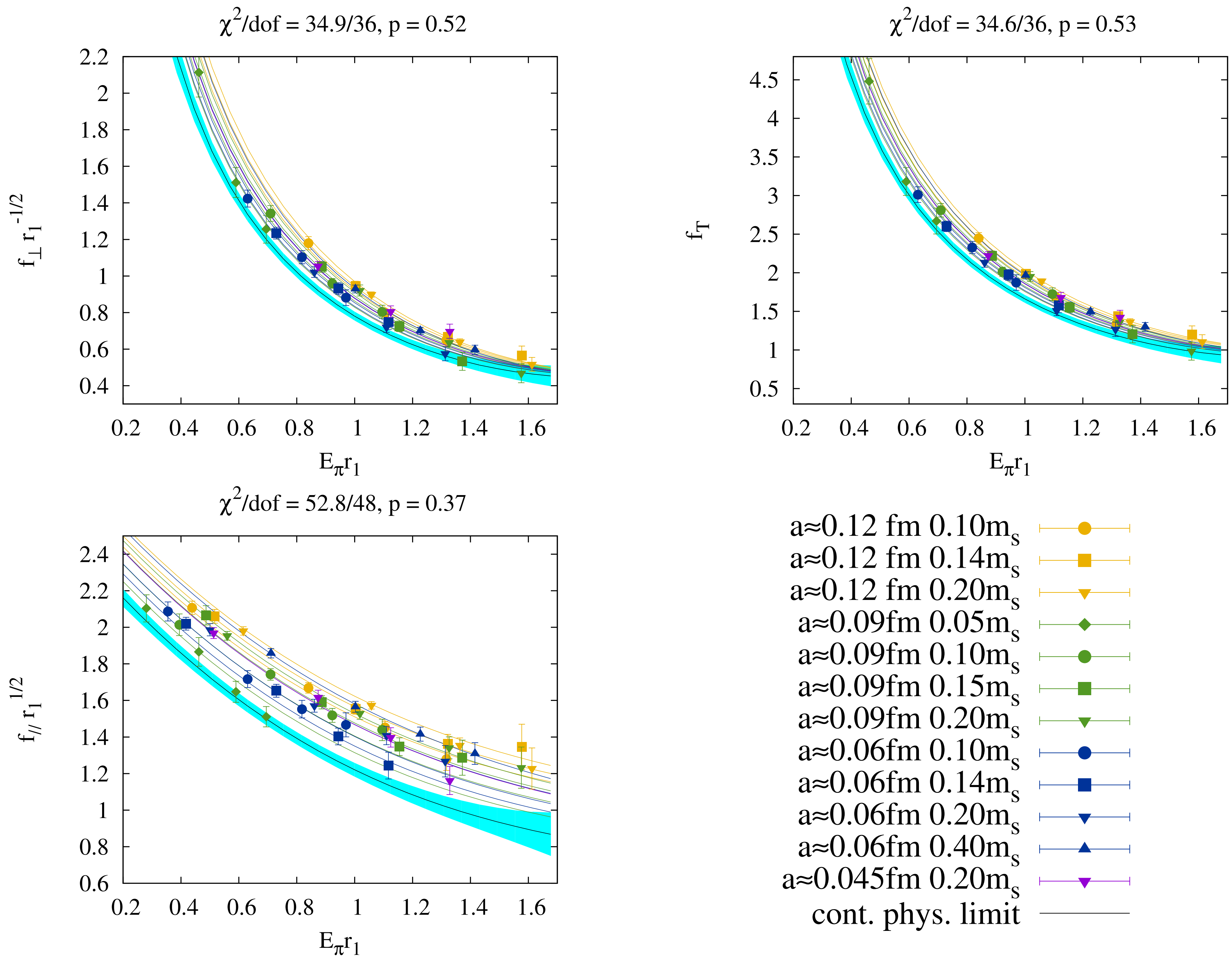




- **Left: Data for $T_{\rm sink} > 1.2{\rm fm}$ compatible, but slope different**
- **Right: Lattice data compatible with upper range of neutrino-nucleus cross sections (green band) and with MiniBooNE (red band)**
- **Parametrization of lattice data:**
  - **Dipole fit, z-expansion**
- **Deviations on $\langle r_A \rangle$ from different methods**

Continuum Limit (for $B \to \pi l \nu$)

- Compute $f(\boldsymbol{k}, m_s, m_s, a)$
- Combine data with Symanzik EFT & chiral PT:
  - $m_l \to \frac{1}{2}(m_u + m_d)$;
  - $a \to 0$.
- Limited range: $|\boldsymbol{k}|a \ll 1$.



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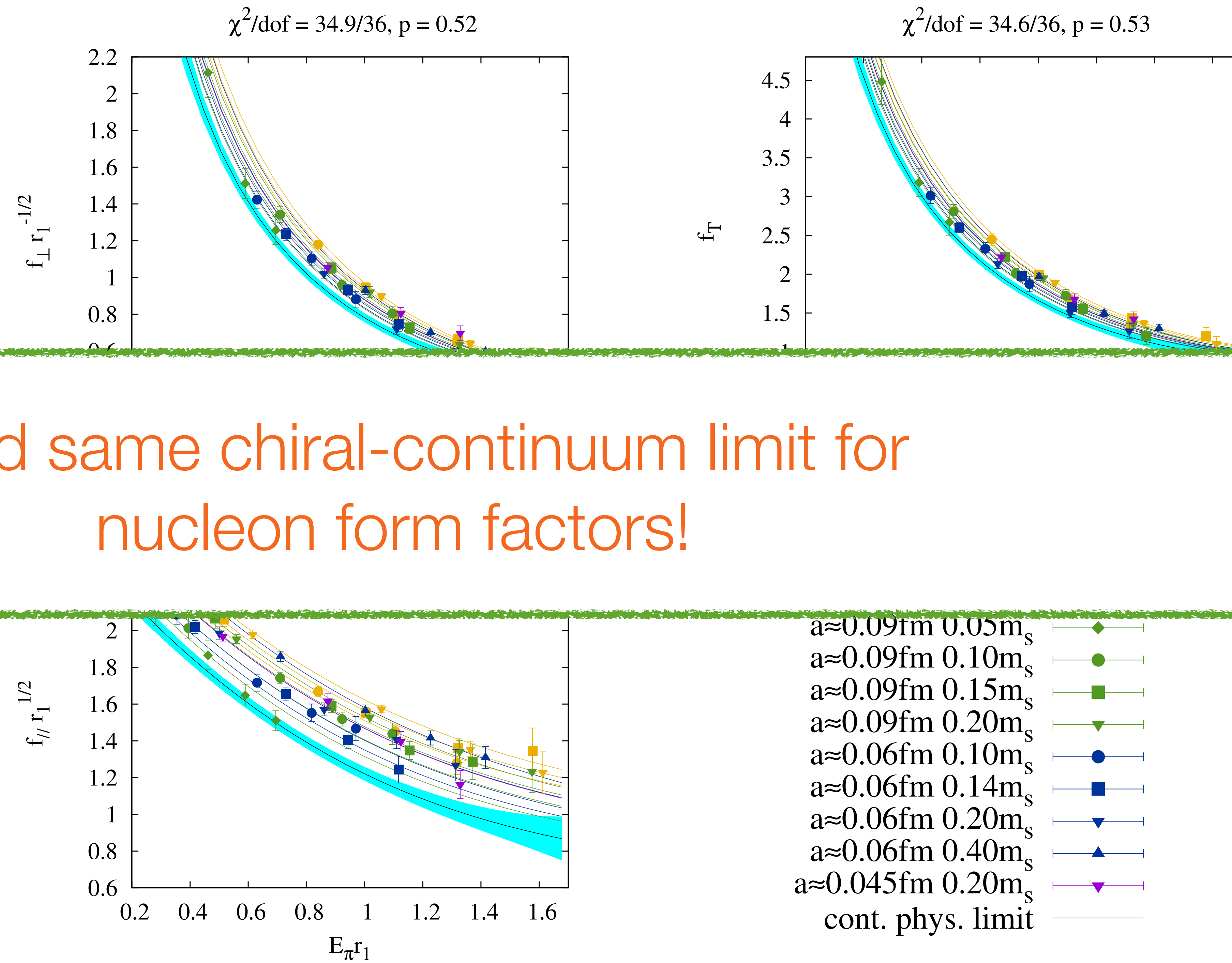
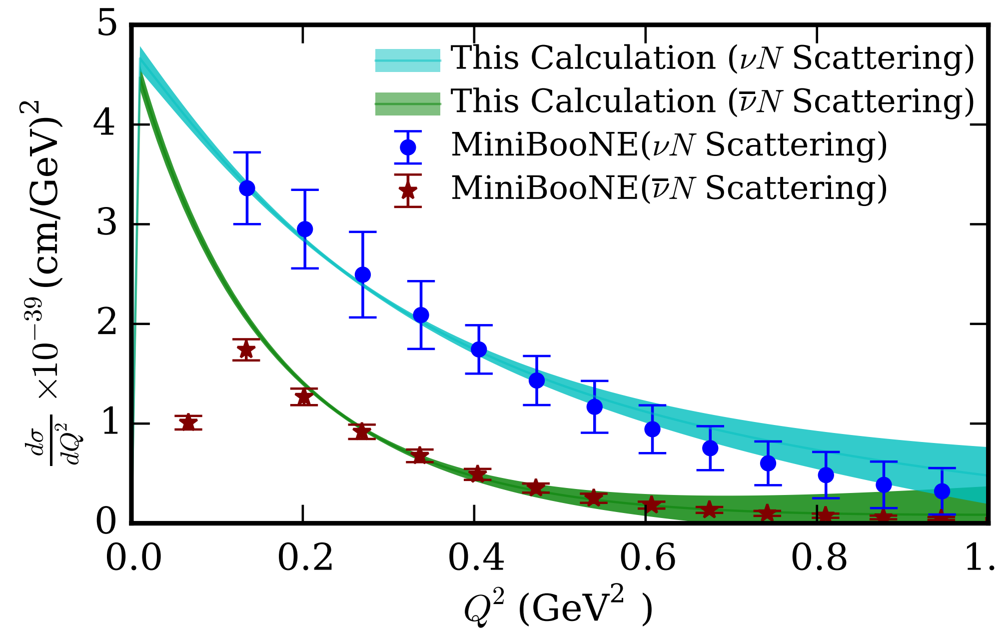
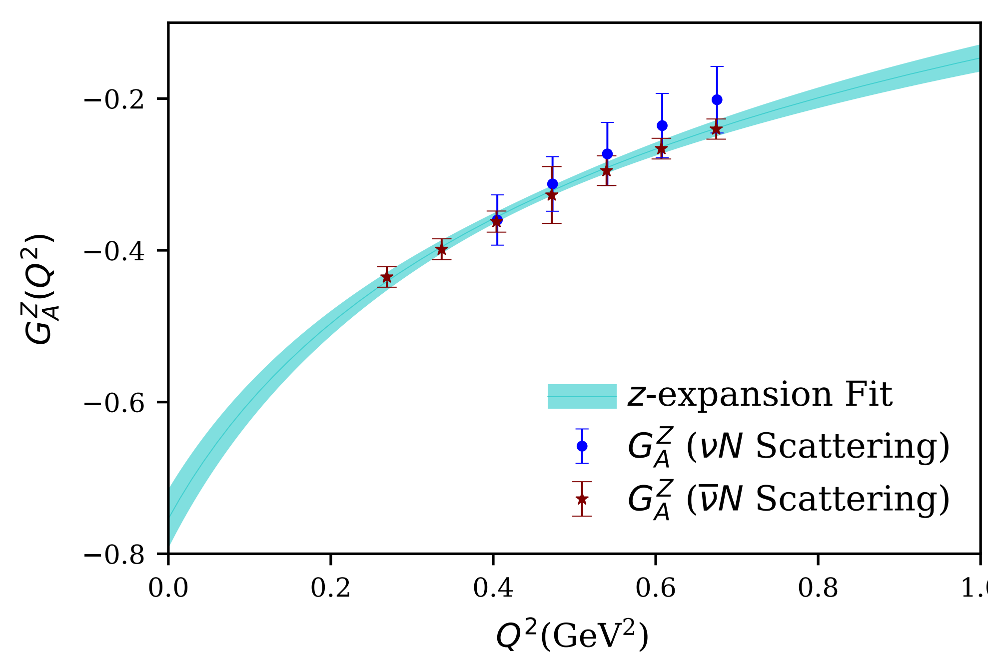


Need same chiral-continuum limit for nucleon form factors!

Neutrino Phenomenology

- Pragmatic inputs:
  - vector form factors from *ep* scattering (*z* parametrization) [Ye *et al.*, arXiv:1707.09063];
  - $s\bar{s}$ form factor from lattice QCD [χQCD, arXiv:1705.05849];
  - extract $F_A(Q^2(z))$ from MiniBooNE;
  - compare with BNL E734.
- Sufian, Liu, Richards, Lattice 2018.





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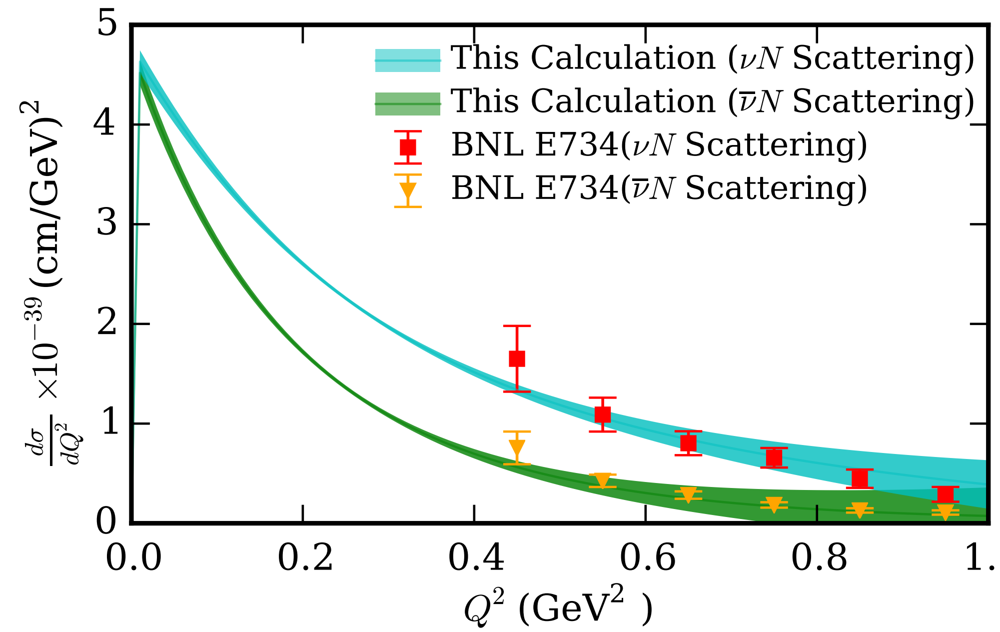
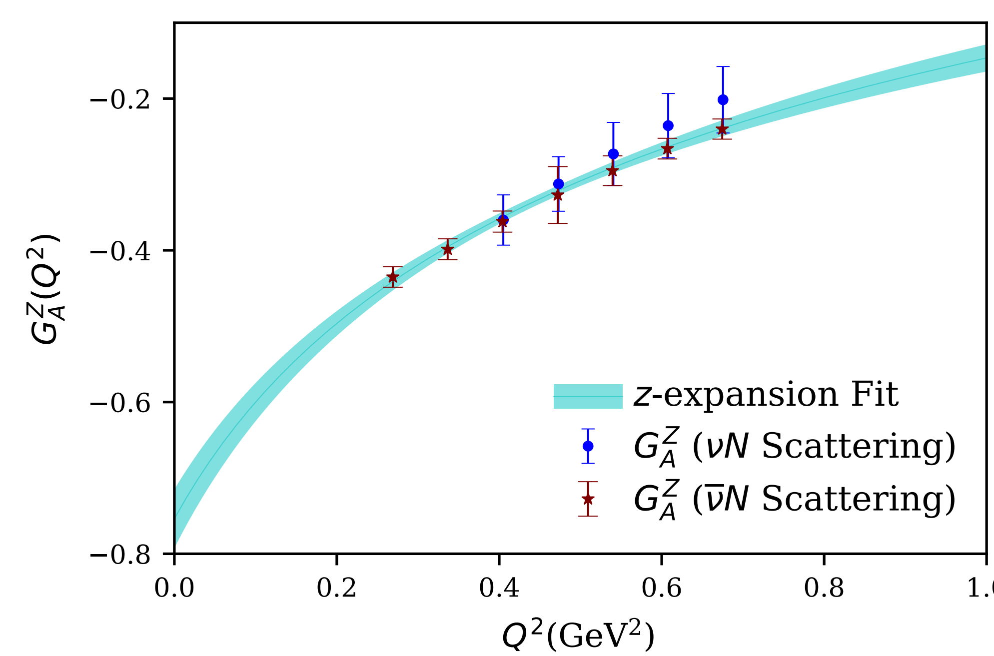
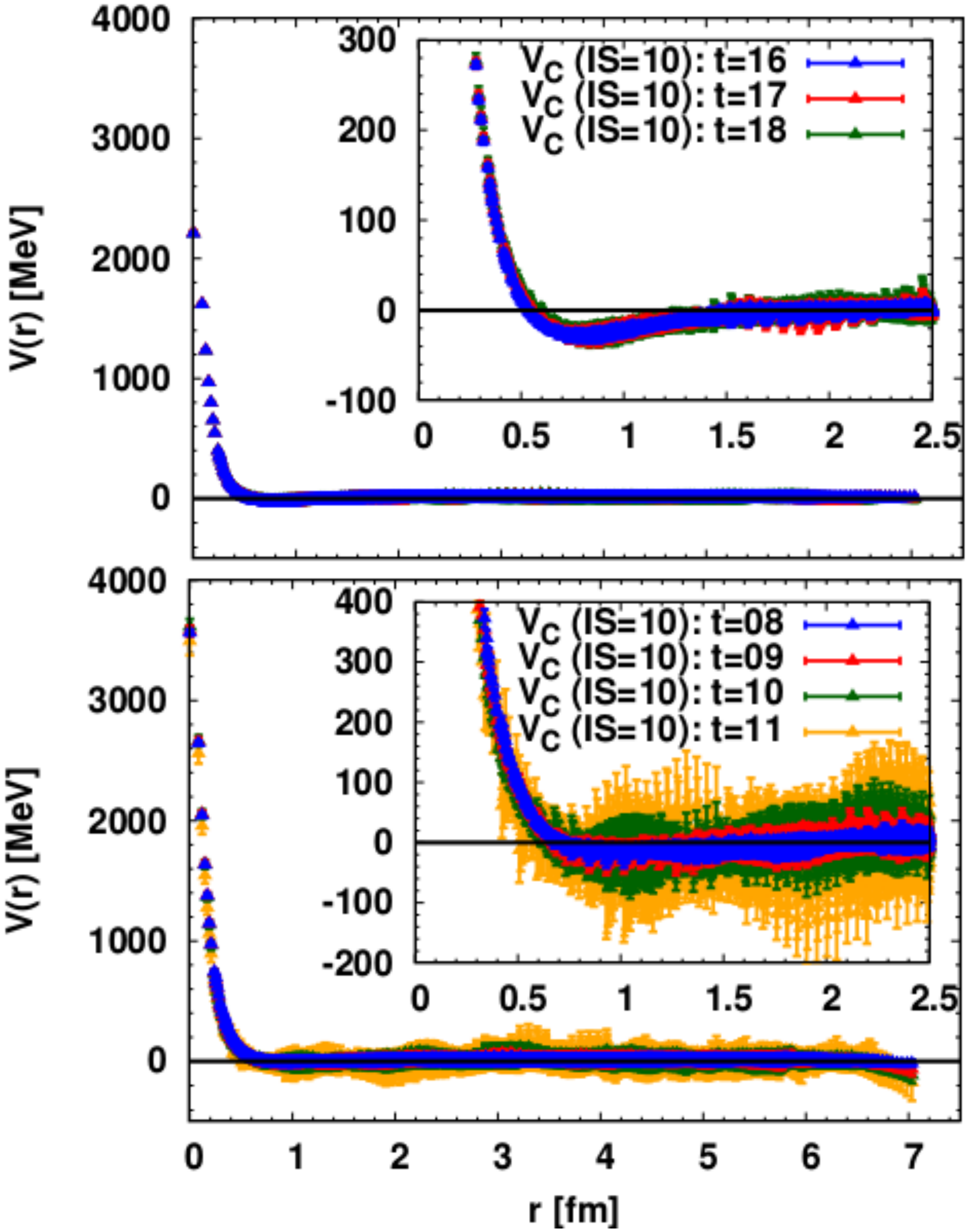




Nucleon Forces

- HALQCD Collab computes baryon-baryon potentials in lattice QCD [nucl-th/0611096, arXiv:1711.01952].
- Implicitly assumes smoothness.
- $\Xi\Xi$ (top); NN (bottom).
- Hard-core repulsion obvious.
- Attraction between 0.5 fm and 1.0 fm (almost) visible.



Outlook

Outlook

- Neutrino-nucleus scattering is a difficult problem.
- Understanding it better is of paramount importance for neutrino exp'ts.
- A "parton model" approach:
  - compute the elementary amplitudes from first principles;
  - use models, parametrizations, data to constrain nucleus.
- The amplitudes can be computed in lattice QCD.

Priority vs Difficulty

$f_+^{K\to\pi}$ $f_+^{B\to\pi}$ F_A^{CC} F_A^{NC}

$\langle NN|J|NN\rangle$

Baroni, Lattice 2018

$\langle d|J^\mu|d\rangle$

$s\bar{s}$ in N

$\nu N \to N', \Delta, N\pi, \ldots$

NN forces

πN forces

F_P^{CC} F_P^{NC}

importance to neutrino oscillations (my evolving vague impression)

difficulty in lattice QCD (let's discuss)

Thank you!

Backup

A Word on Parametrizations

- In the old days, we used a variety of parametrizations, from goofy to well-motivated but still presumptuous.
- Now everyone adopts the "z expansion": $z = \dfrac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$.
- Form factors take the form

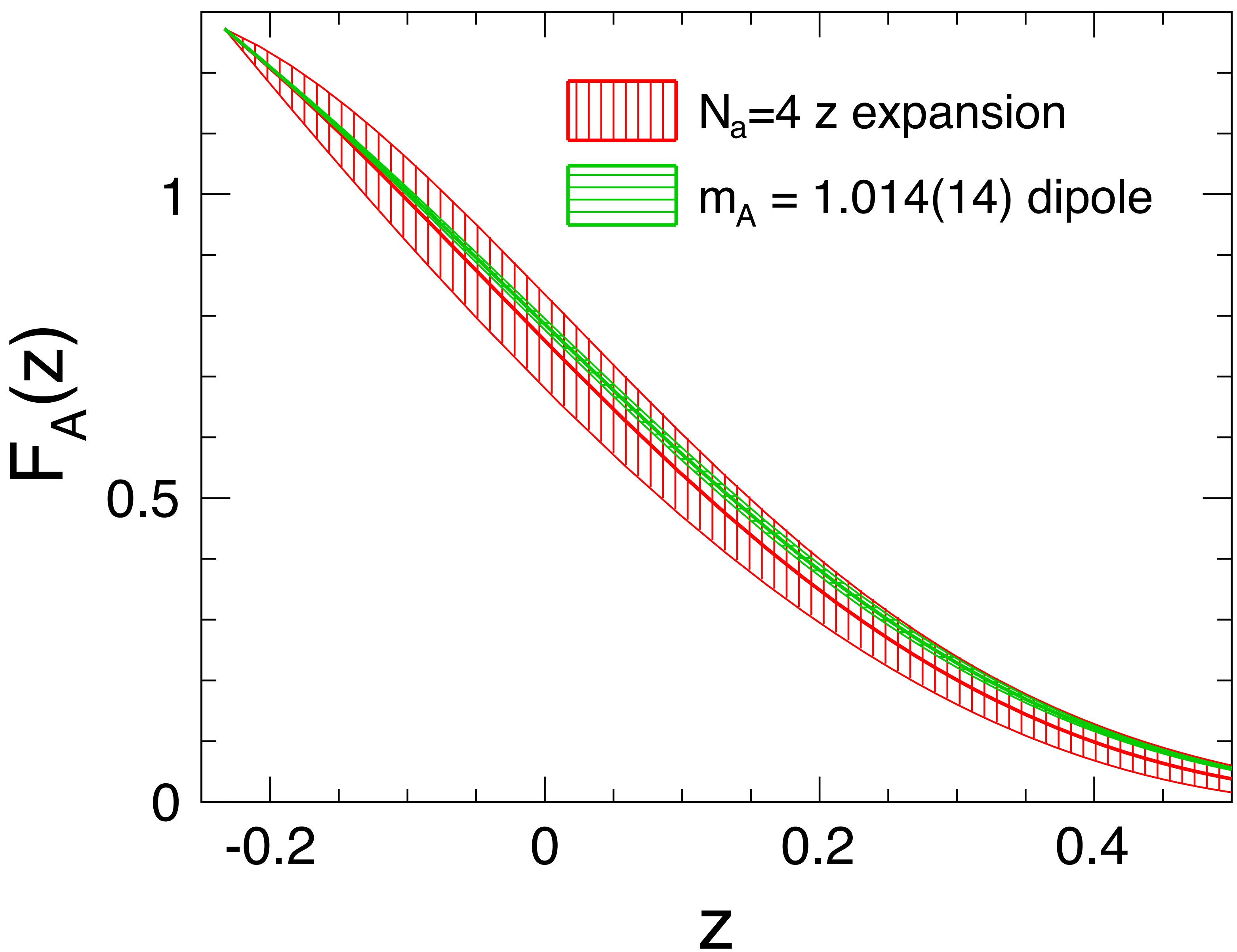
$$f(t(z)) \propto \sum_n a_n z^n$$

- Inspired by analyticity; bounded by unitarity.
- Preferred over "dipole"

$$f(t) = \frac{g_A}{\left(1 - t/M_A^2\right)^2}$$

Axial Form Factor from Deuterium

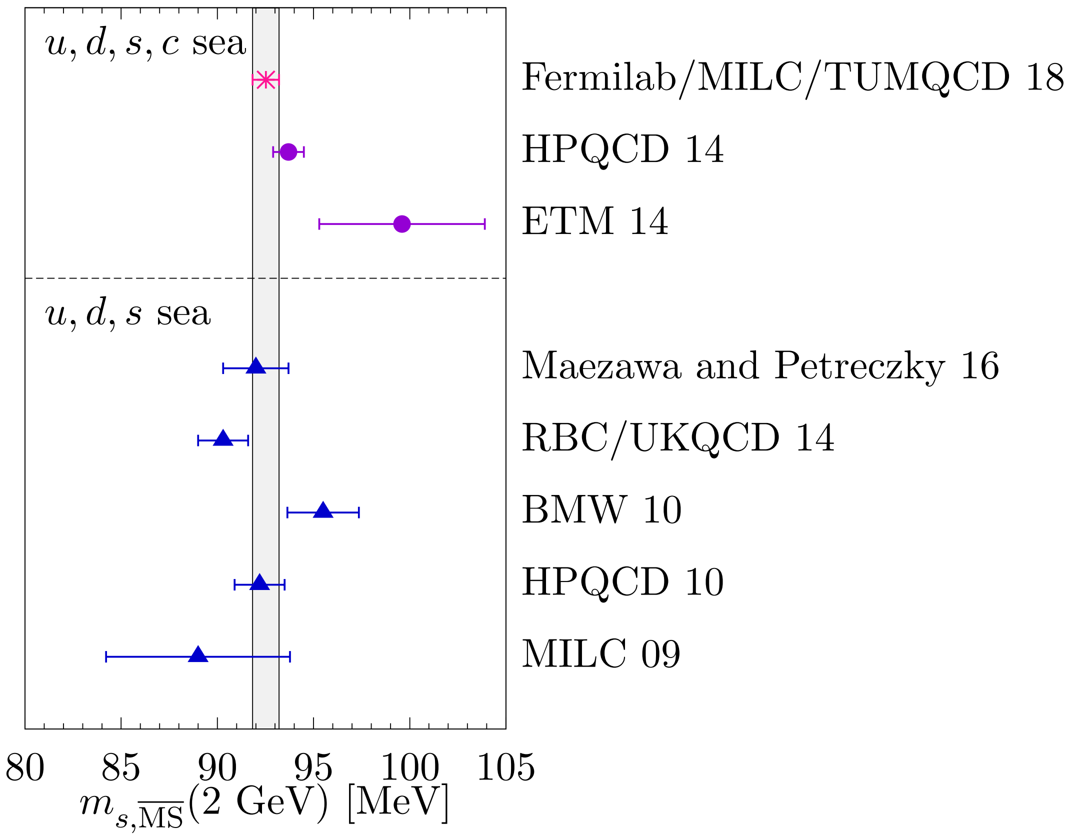
- Meyer, Betancourt, Gran, Hill [arXiv:1603.03048]:

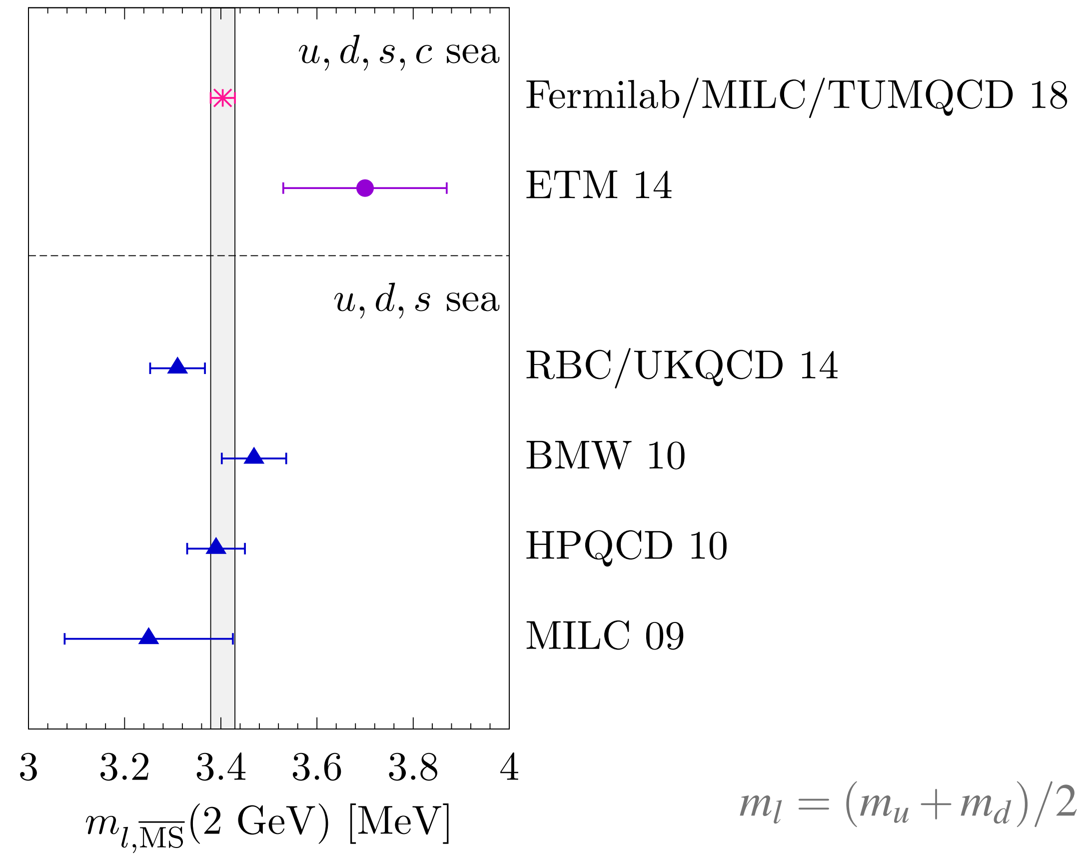


dipole underestimates uncertainty!

Results & Comparisons 2

- With mass ratios from light pseudoscalar mesons:





$m_l = (m_u + m_d)/2$

- Most precise s-, d-, and u-quark masses to date; e.g., m_u to 2%.

- All quarks except top.

Results & Comparisons 3

- Masses in numerical form:

$$m_{l,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 3.404(14)_{\mathrm{stat}}(08)_{\mathrm{syst}}(19)_{\alpha_s}(04)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$
$$m_{u,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 2.118(17)_{\mathrm{stat}}(32)_{\mathrm{syst}}(12)_{\alpha_s}(03)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$
$$m_{d,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 4.690(30)_{\mathrm{stat}}(36)_{\mathrm{syst}}(26)_{\alpha_s}(06)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$
$$m_{s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 92.52(40)_{\mathrm{stat}}(18)_{\mathrm{syst}}(52)_{\alpha_s}(12)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$
$$m_{c,\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 984.3(4.2)_{\mathrm{stat}}(1.6)_{\mathrm{syst}}(3.2)_{\alpha_s}(0.6)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$
$$m_{b,\overline{\mathrm{MS}}}(m_{b,\overline{\mathrm{MS}}}) = 4203(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$

- Mass ratios:

$$m_c/m_s = 11.784(11)_{\mathrm{stat}}(17)_{\mathrm{syst}}(00)_{\alpha_s}(08)_{f_\pi,\mathrm{PDG}}$$
$$m_b/m_s = 53.93(7)_{\mathrm{stat}}(8)_{\mathrm{syst}}(1)_{\alpha_s}(5)_{f_\pi,\mathrm{PDG}}$$
$$m_b/m_c = 4.577(5)_{\mathrm{stat}}(7)_{\mathrm{syst}}(0)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}$$