Theory of neutrino pion production

NuFact18, Virginia Tech, Aug 18. RGJ, Univ. Complutense Madrid

Theory of neutrino pion production

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**Nuclear Physics Group,
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*The 20th International Workshop on Neutrinos from Accelators (NuFact18),
Virginia Tech, Blacksburg, 12-18 August, 2018*

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Tom Van Cuyck (Ghent)

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TW Donnelly (MIT)

Outline

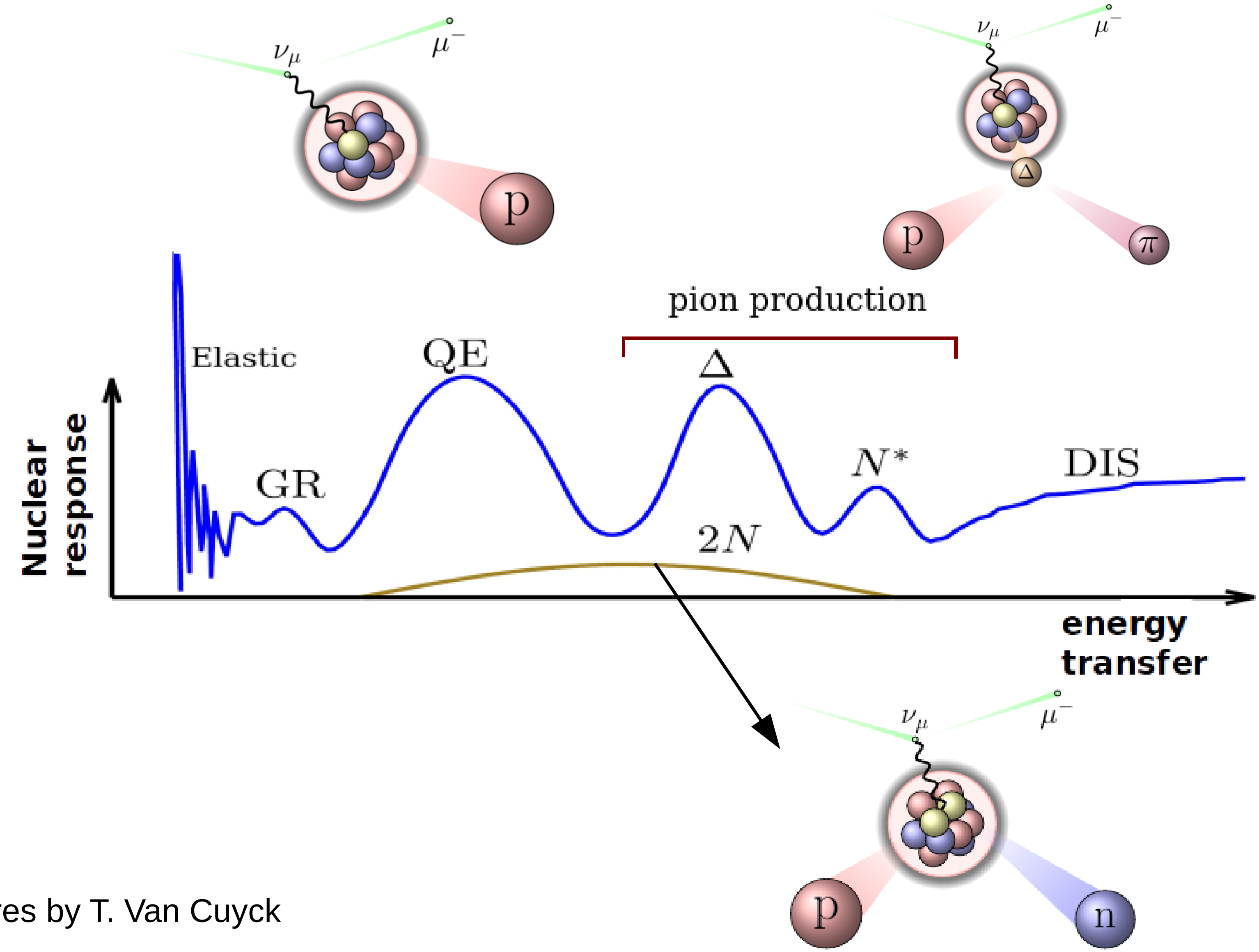
I Kinematics

II Interaction model

III Nuclear effects

IV Conclusions

What we know from (e,e')



Figures by T. Van Cuyck

Two different approaches:

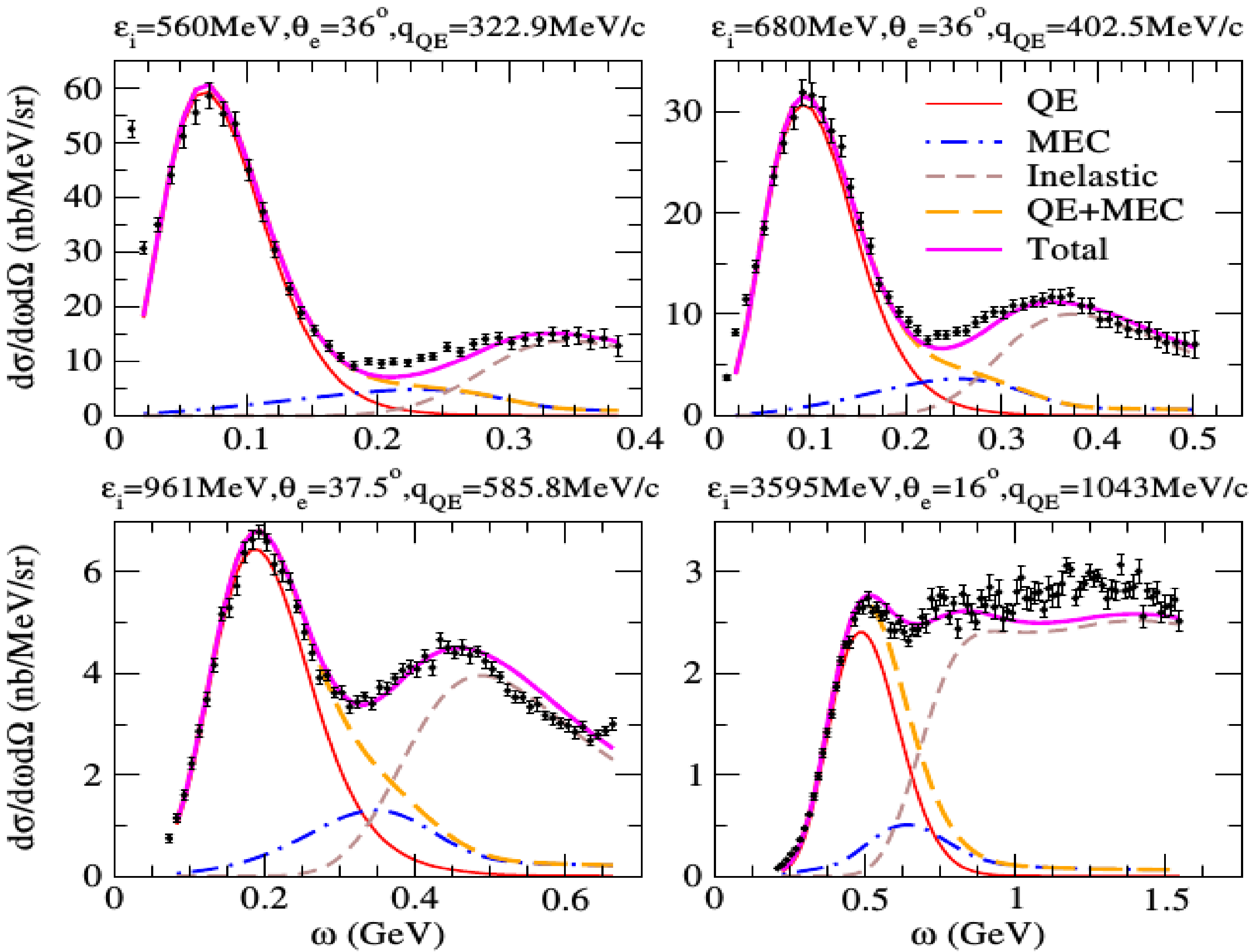
1) Models for inclusive processes (only lepton detected)

2) Models for exclusive processes (hadron(s) are detected)

1) Models for inclusive data (only lepton detected)

What we know from (e,e')

Superscaling approach (PRD 91, 073004 (2015); 94, 013012 (2016))



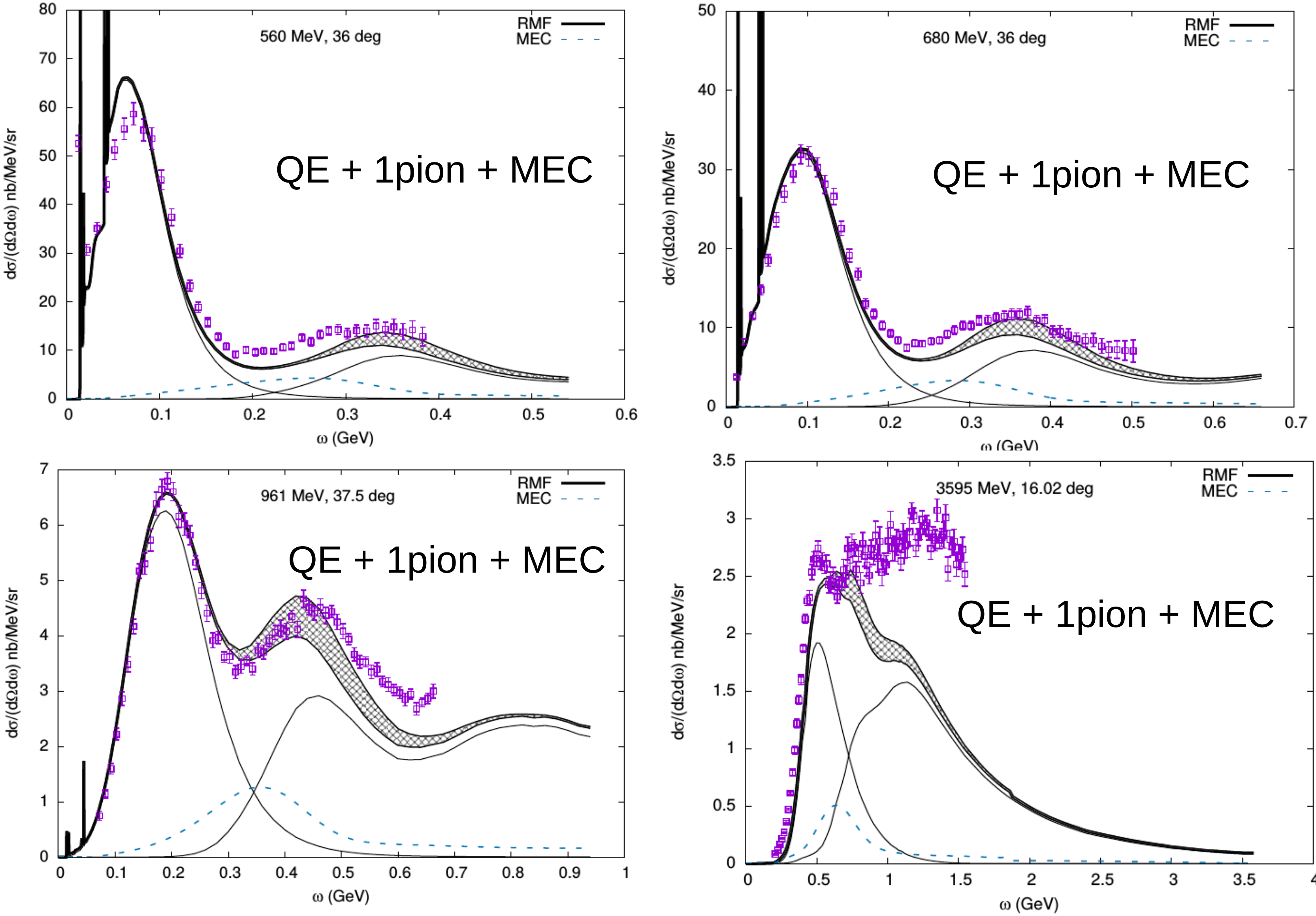
$^{12}C(e,e')$

2) Models for semi-inclusive or exclusive data (hadron(s) detected in coincidence with the lepton)

Ideally, after integrating over hadronic variables, one should be able to reproduce the inclusive data

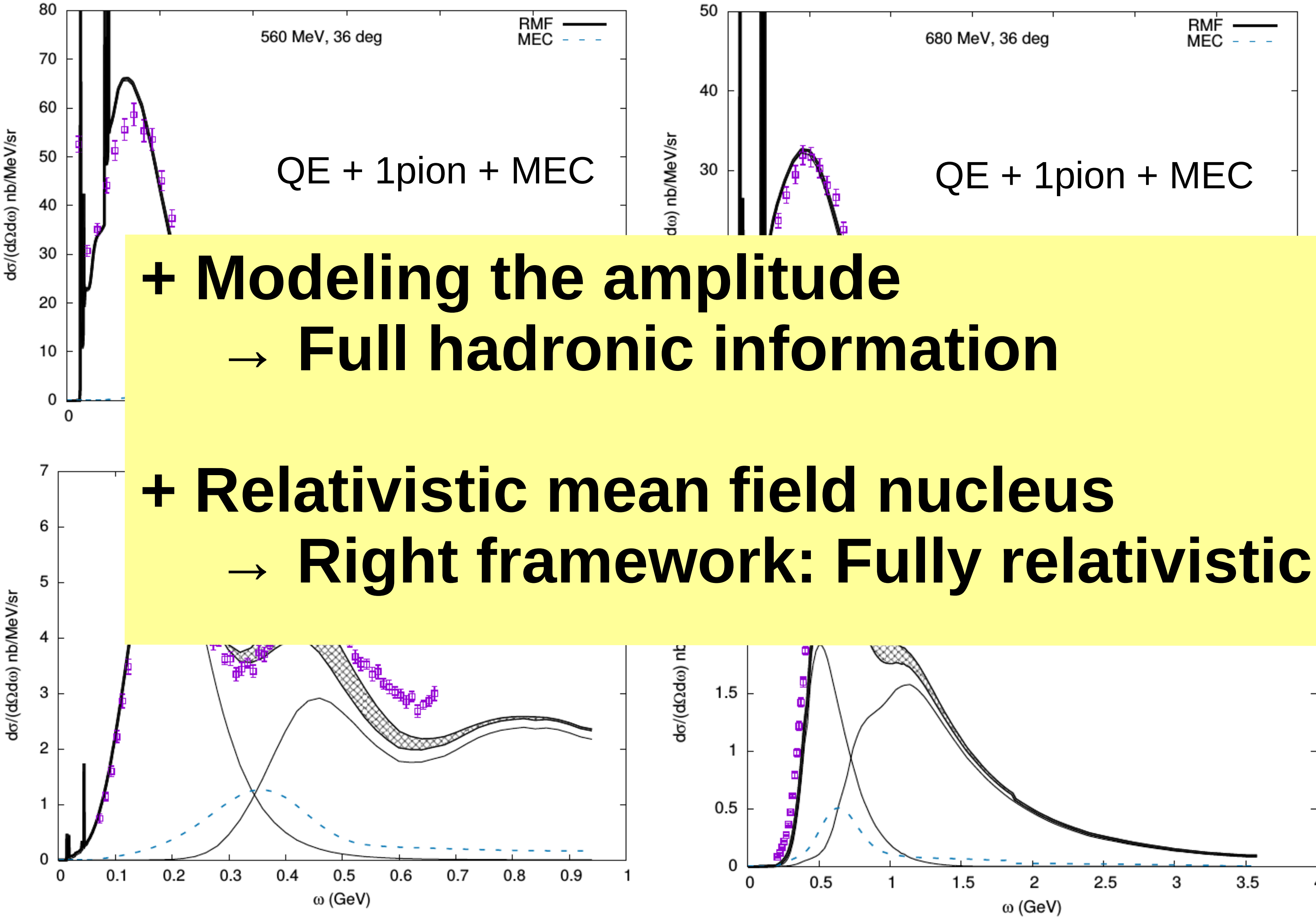
What we know from (e,e')

$^{12}C(e,e')$



What we know from (e,e')

$^{12}C(e,e')$



+ Modeling the amplitude
→ Full hadronic information

+ Relativistic mean field nucleus
→ Right framework: Fully relativistic

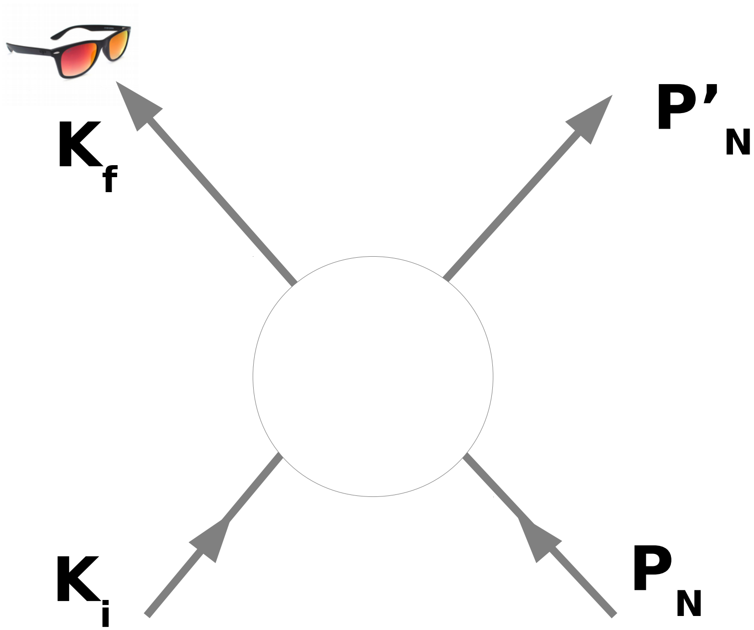
Kinematics

For more:

Donnelly, Prog. Part. Nucl. Phys. 13, 183-236 (1985)

Van Orden, Donnelly, Moreno, PRD 96, 113008 (2017)

The mass of the final hadronic system is known, e.g., **elastic electron-proton scattering**.



4 x 4-momentum → **16 variables**

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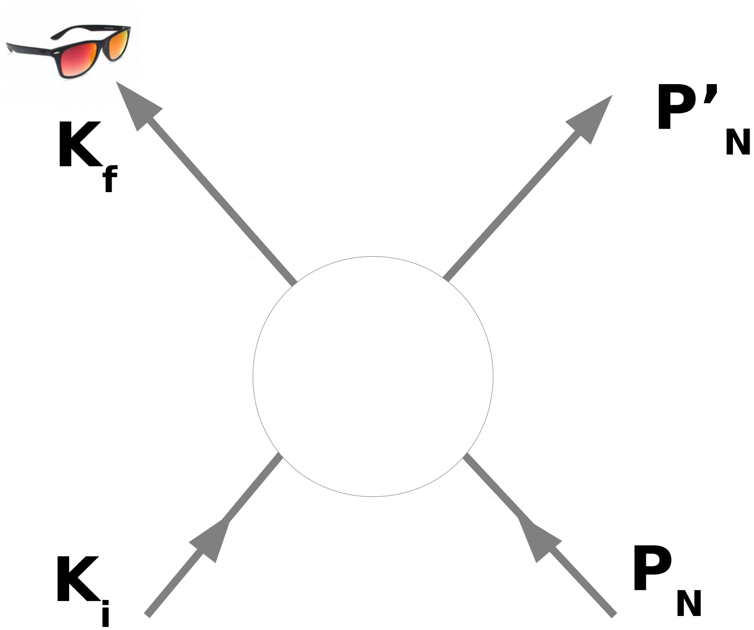

4 x 4-momentum	→	**16 variables**
1 x 4-mom. conserv.	→	**-4 constraints**
4 x ($E^2=M^2+p^2$)	→	**-4 constraints**

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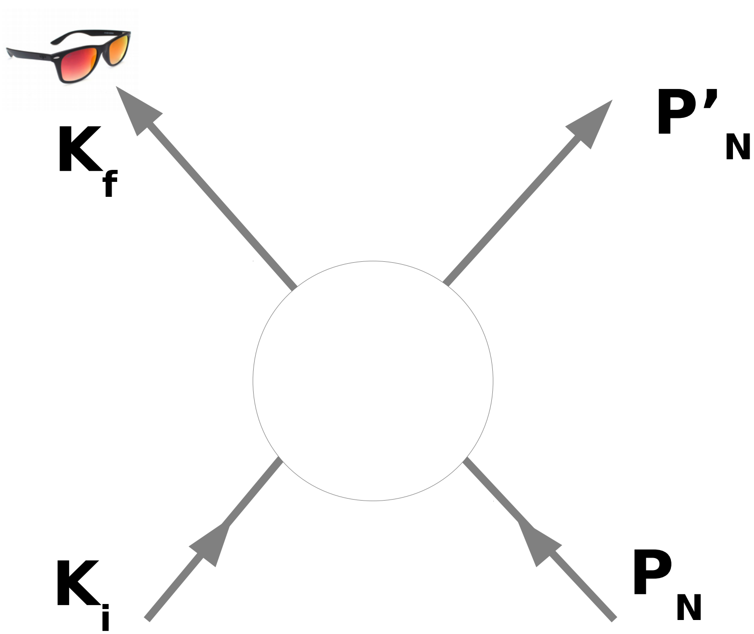


4 x 4-momentum → 16 variables

1 x 4-mom. conserv. → -4 constraints

4 x ($E^2=M^2+p^2$) → -4 constraints

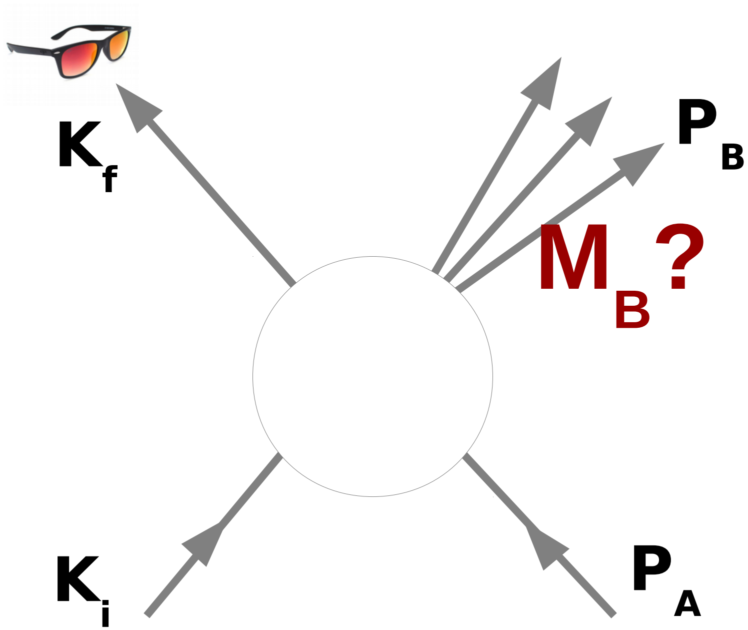
3-mom. of the beam → -3 known

3-mom. of the target → -3 known

Independent variables left → 2

$$\theta_f \text{ and } \phi_f \longrightarrow \frac{d^2\sigma}{d\cos\theta_f d\phi_f}$$

Inclusive scattering on a target at rest: only scattered lepton is detected. We know nothing about final hadronic system



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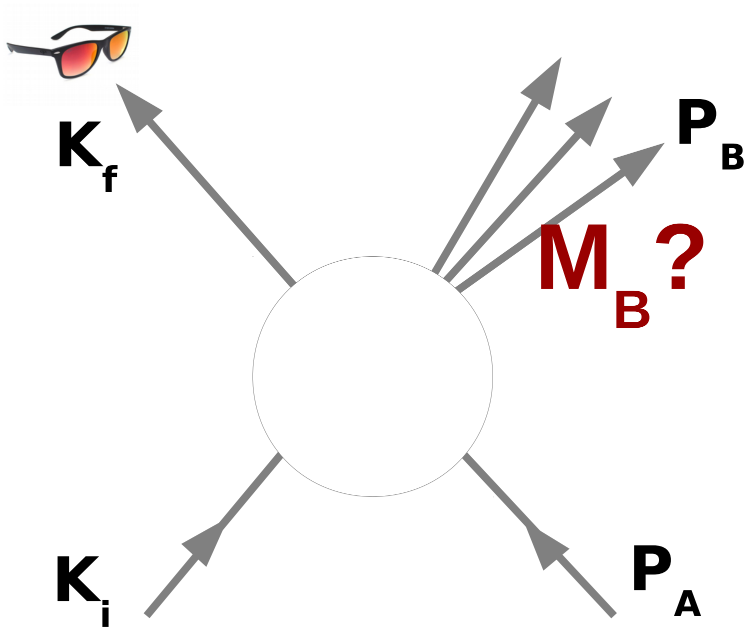

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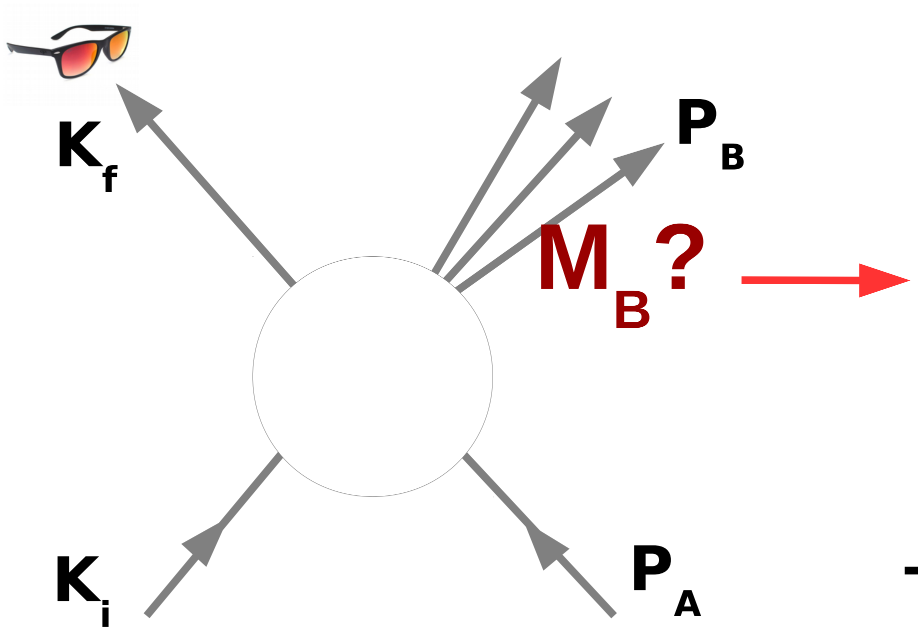

4 x 4-momentum	→	**16 variables**
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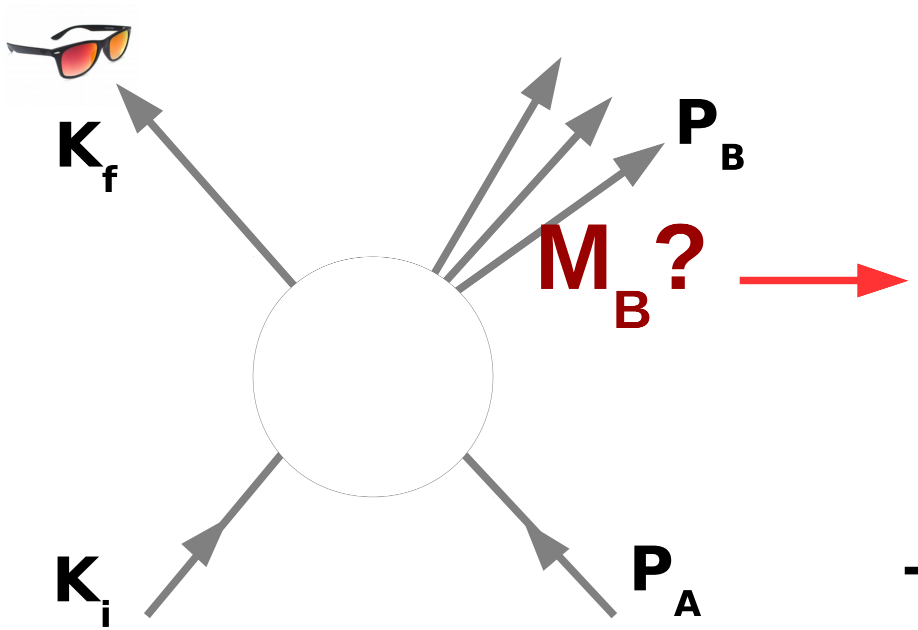

4 x 4-momentum	→	**16 variables**
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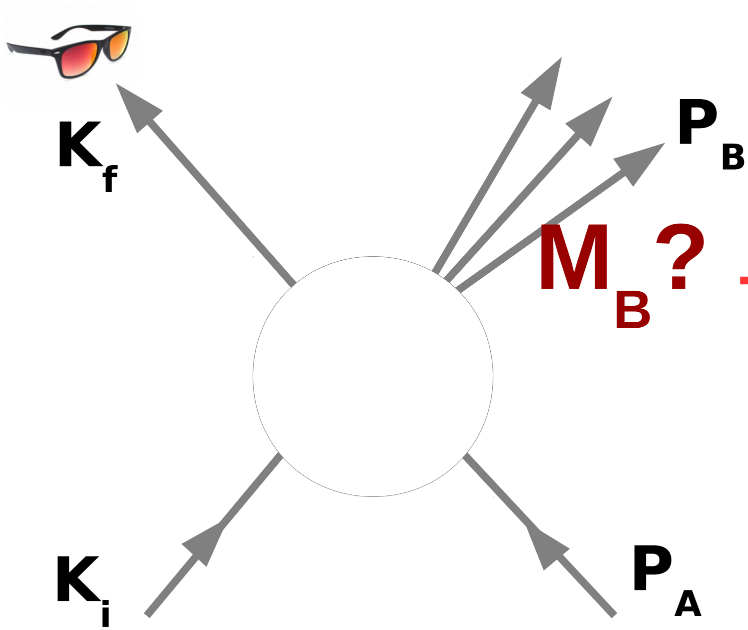
Independent variables left → 3

Inclusive scattering on a target at rest: only scattered lepton is detected. We know nothing about final hadronic system



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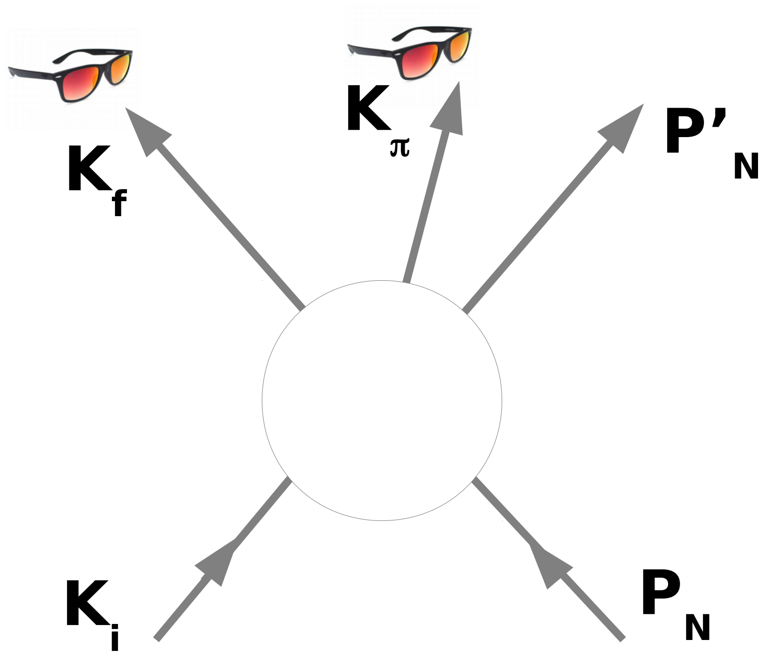
1 x 4-mom. conserv.	→	**-4 constraints**
3 x (E^2=M^2+p^2)	→	**-3 constraints**
3-mom. of the beam	→	**-3 known**
3-mom. of the target	→	**-3 known**
Independent variables left	→	**3**

We can choose the variables that we like the most as the independent variables. We like the lab variables of the final state:

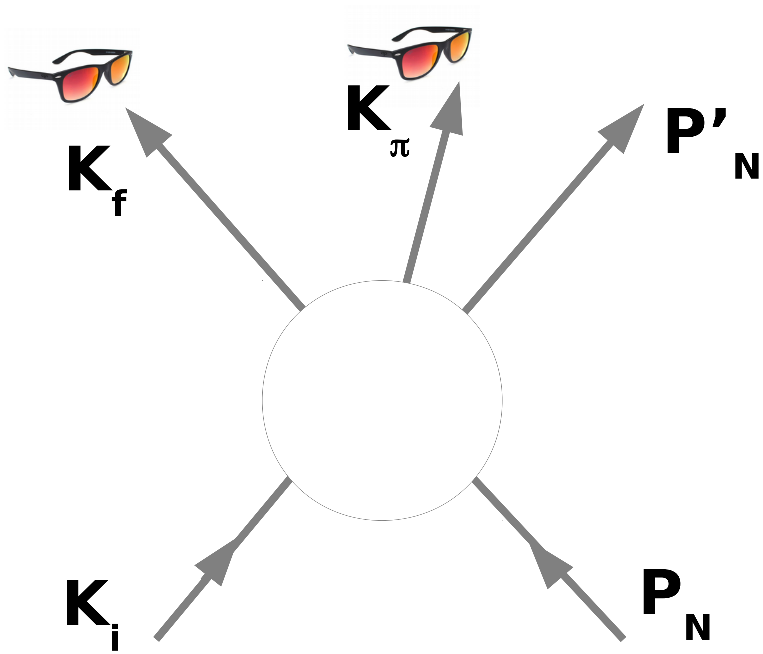
$$\varepsilon_f,\ \theta_f \text{ and } \phi_f \longrightarrow \frac{d^3\sigma}{d\varepsilon_f \cos\theta_f d\phi_f}$$

1-π production off the nucleon.



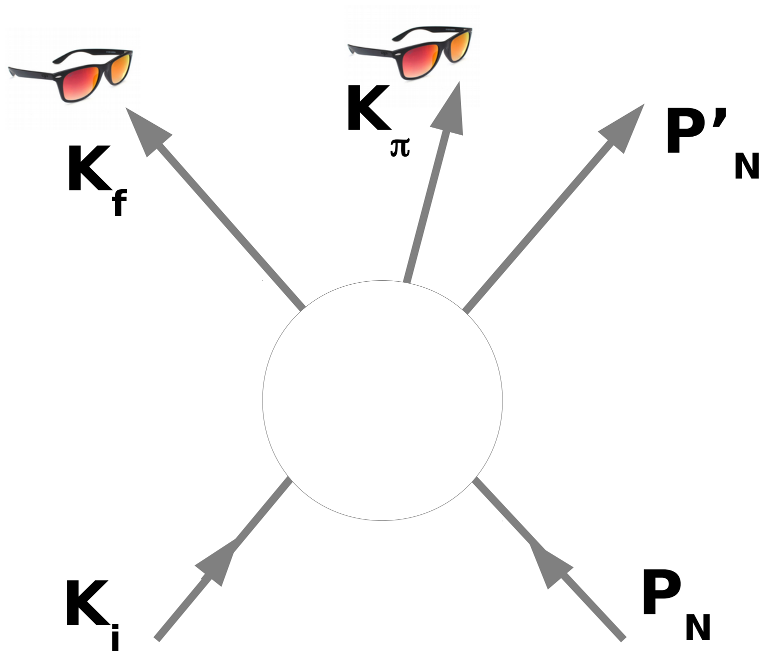
5 x 4-momentum → 20 variables

1-π production off the nucleon.



5 x 4-momentum	→	20 variables
1 x 4-mom. conserv.	→	-4 constraints
5 x ($E^2=M^2+p^2$)	→	-5 constraints

1-π production off the nucleon.

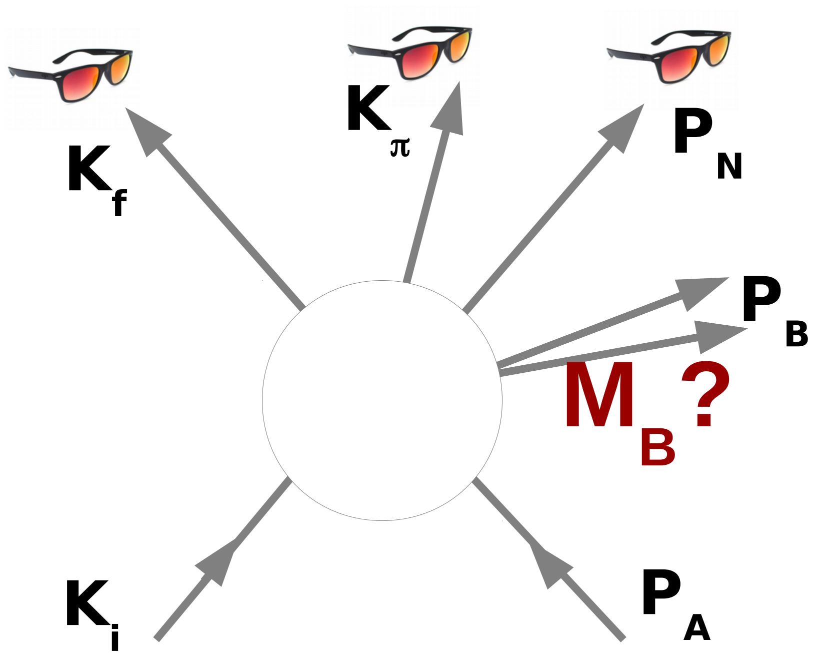


5 x 4-momentum	→	20 variables
1 x 4-mom. conserv.	→	-4 constraints
5 x ($E^2=M^2+p^2$)	→	-5 constraints
3-mom. of the beam	→	-3 known
3-mom. of the target	→	-3 known

Independent variables left → 5

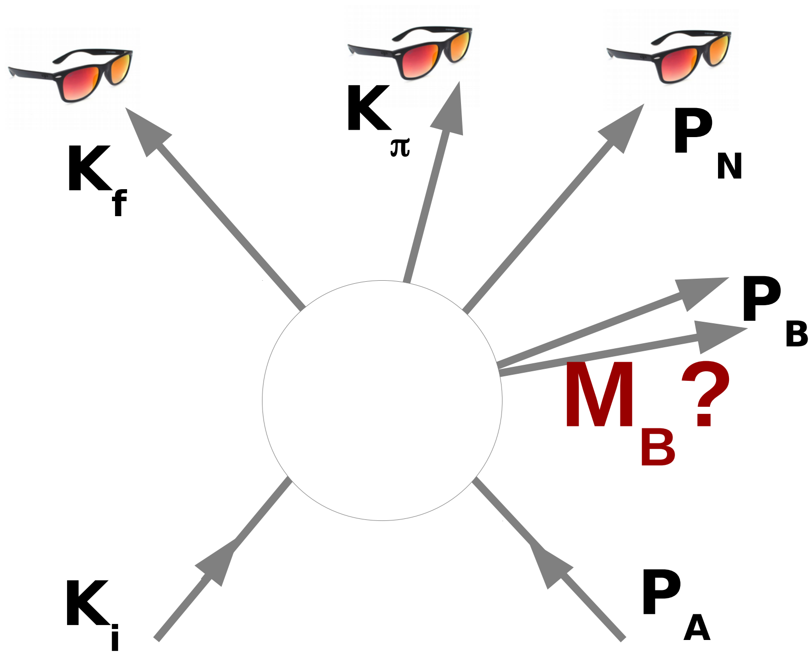
$$\frac{d^5\sigma}{d\varepsilon_f d\Omega_f d\Omega_\pi}$$

1-π production on the nucleus.



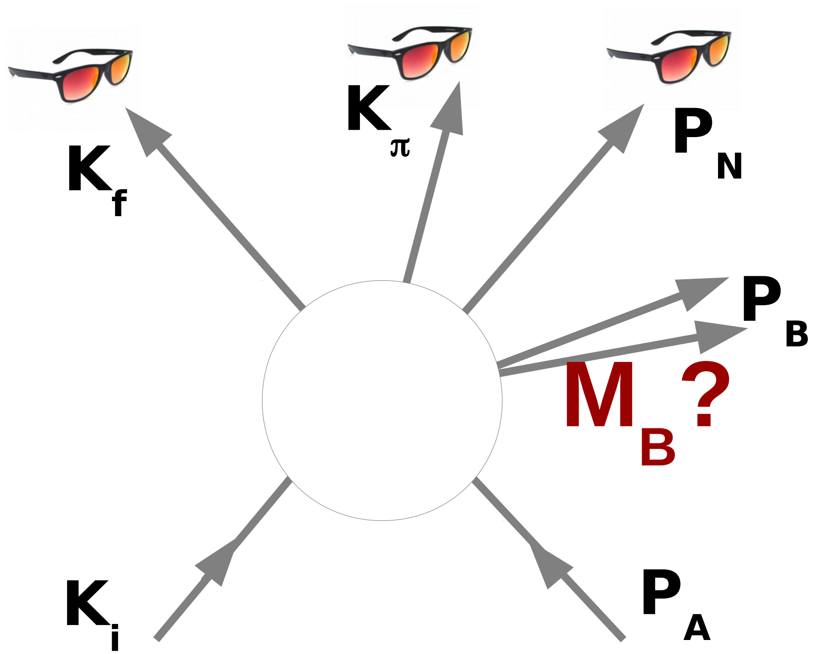
6 x 4-momentum → 24 variables

1-π production on the nucleus.



6 x 4-momentum	→	24 variables
1 x 4-mom. conserv.	→	-4 constraints
5 x ($E^2=M^2+p^2$)	→	-5 constraints

1-π production on the nucleus.



6 x 4-momentum → 24 variables

1 x 4-mom. conserv. → -4 constraints

5 x ($E^2=M^2+p^2$) → -5 constraints

3-mom. of the beam → -3 known

3-mom. of the target → -3 known

Independent variables left → 9

$$\frac{d^9\sigma}{d\varepsilon_f d\cos\theta_f d\phi_f\, dE_\pi d\cos\theta_\pi d\phi_\pi\, d\cos\theta_N d\phi_N\, dE_m}$$

$$\frac{d^9\sigma}{d\varepsilon_f d\Omega_f\, dE_\pi d\Omega_\pi\, d\Omega_N\, dE_m} \propto \ell_{\mu\nu} H^{\mu\nu}$$

$$\begin{aligned} \ell_{\mu\nu} &= \overline{\sum}(j_\mu)^* j_\nu \\ H^{\mu\nu} &= \overline{\sum}(J^\mu)^* J^\nu \end{aligned}$$

$$\begin{aligned} j_\mu &= j_\mu(\varepsilon_i, q, \omega)\,, \\ J^\mu &= J^\mu(q, \omega, E_\pi, \theta_\pi, \phi_\pi, \theta_N, \phi_N, E_m) \end{aligned}$$

$$\frac{d^9\sigma}{d\varepsilon_f d\Omega_f\, dE_\pi d\Omega_\pi\, d\Omega_N\, dE_m} \propto \ell_{\mu\nu} H^{\mu\nu}$$

$$\ell_{\mu\nu} = \overline{\sum}(j$$

$$H^{\mu\nu} = \overline{\sum}$$

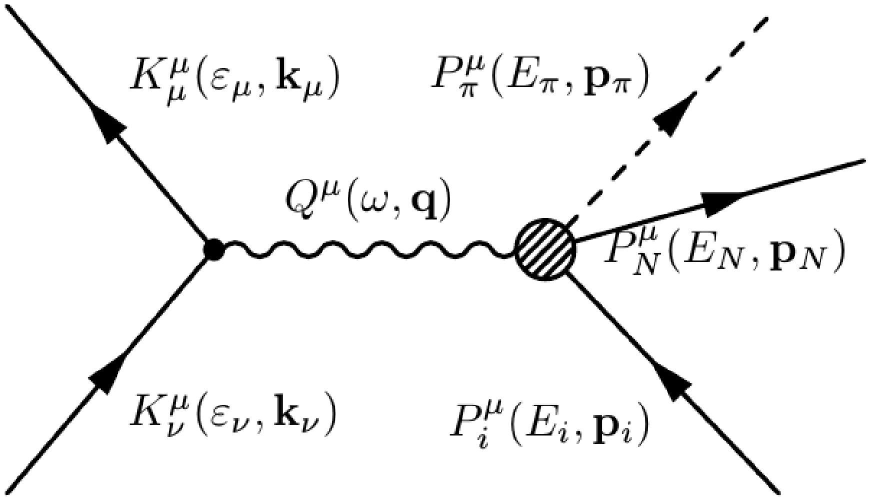
8 indep. variables.
If we have fewer...
better think about it!

$$j_\mu = j_\mu(\varepsilon_i, q, \omega)\,,$$

$$J^\mu = J^\mu(q, \omega, E_\pi, \theta_\pi, \phi_\pi, \theta_N, \phi_N, E_m)$$

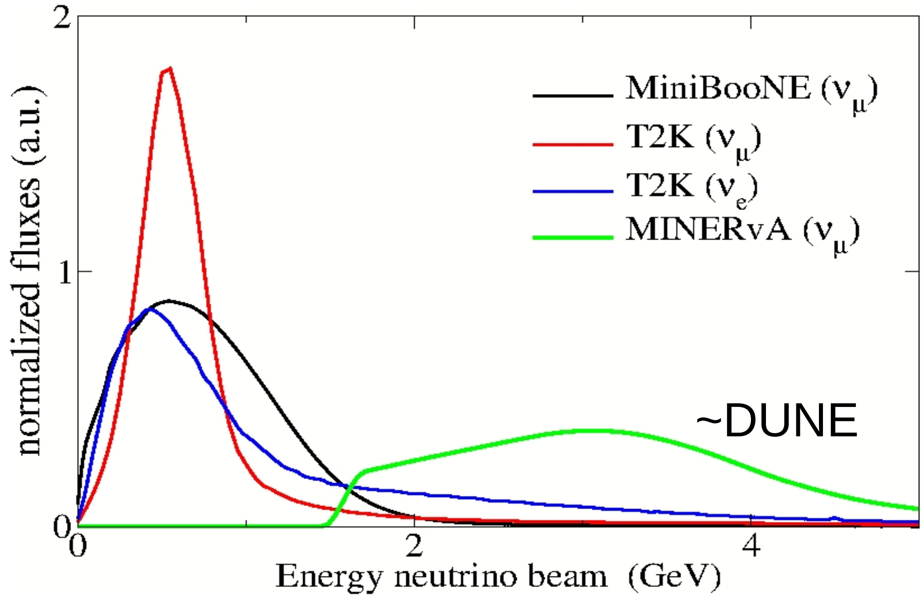
II Interaction model

Single-Pion Production off the nucleon

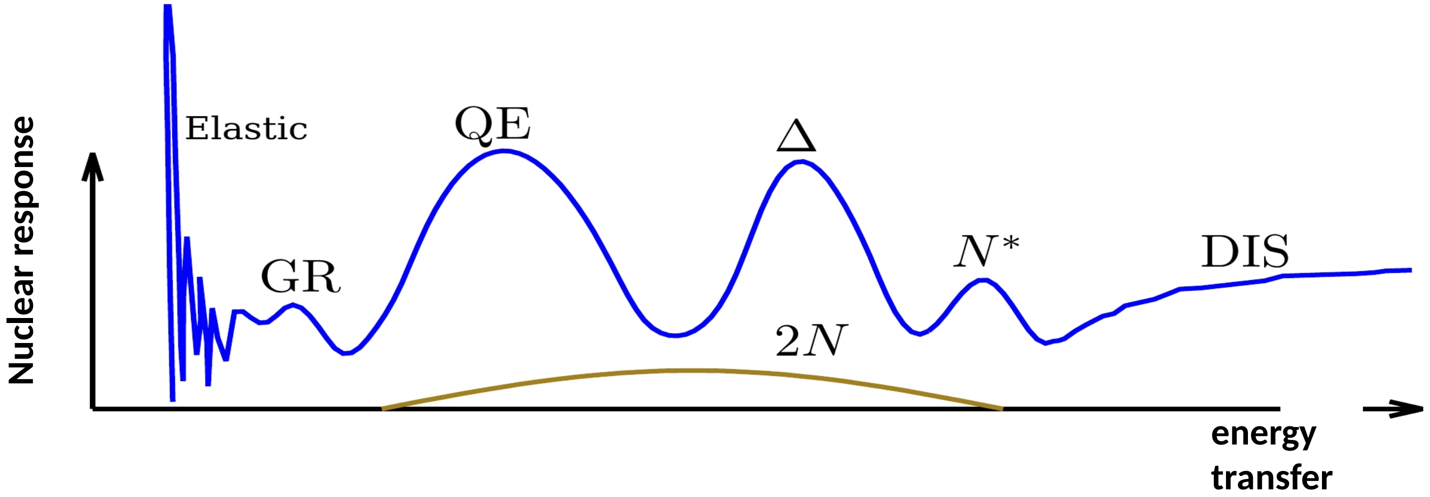


RGJ et al., PRD 95, 113007 (2017)

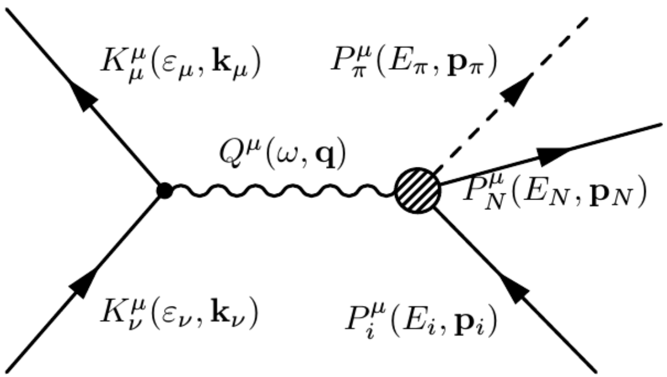
Single pion production



Pion production is more important for DUNE

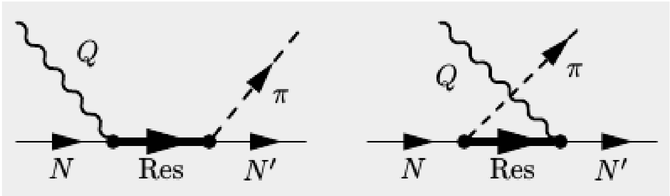


Low-energy model

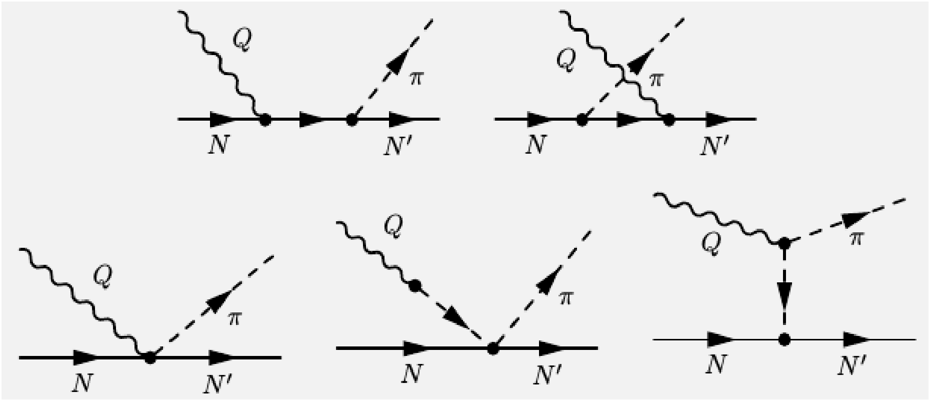


Low-energy model for pion-production on the nucleon:
ChPT background + resonances
Valencia model
(PRD 76 (2007) 033005; PRD 87 (2013) 113009)

Resonances:
P33(1232), D13(1520),
S11(1535), P11(1440)

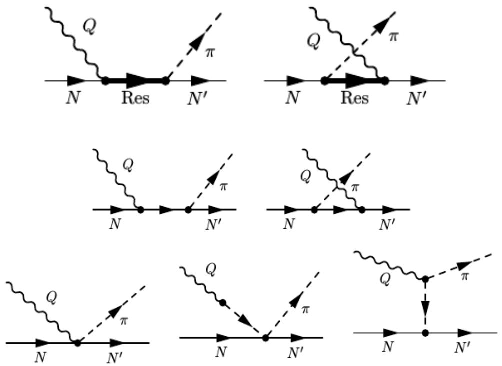


ChPT background:



The Problem

Low-energy model
(resonances + ChPT bg)



Unphysical predictions at large invariant masses.

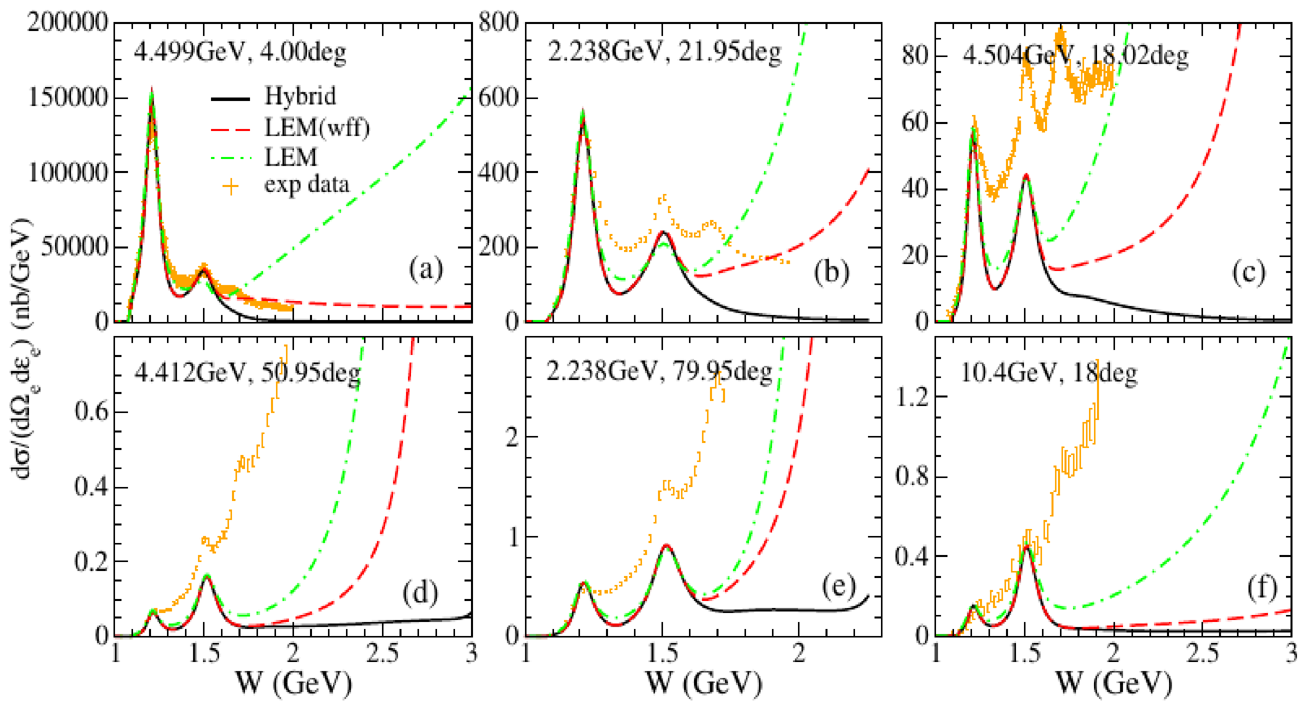
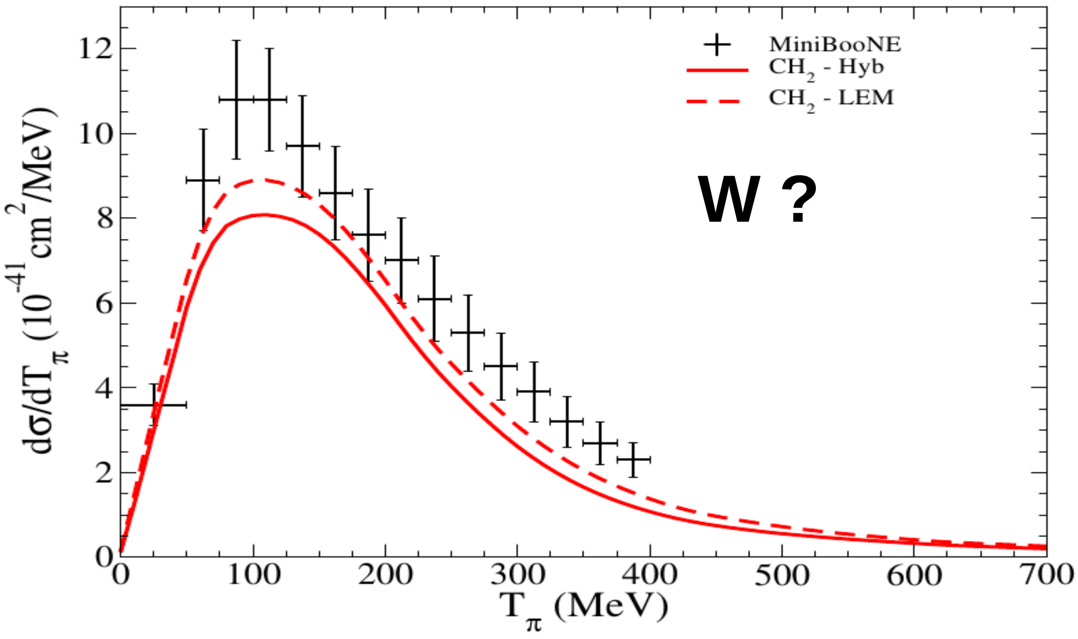


Figure: The model overshoots **inclusive electron-proton** scattering data.

Does it matter for neutrinos?



W values?
+ Fermi motion
+ Flux-folding

Therefore, we need reliable predictions in:

+ the **resonance region** W < 2 GeV,

+ the **high-energy** energy region W > 2 GeV

Regge Theory

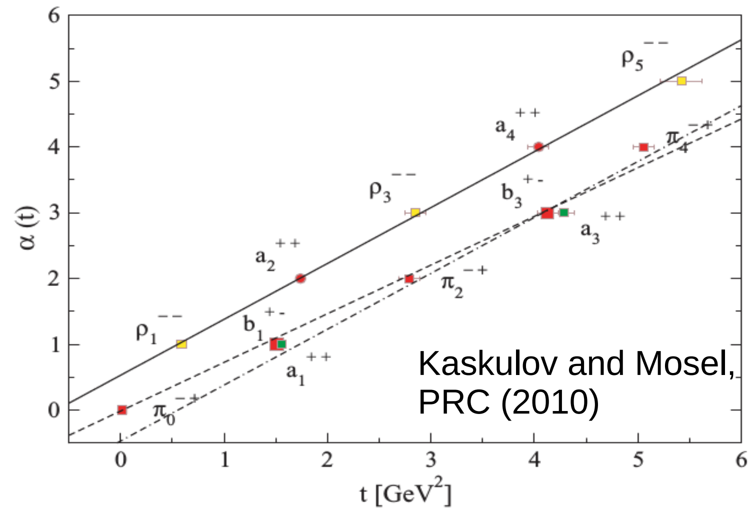
Based on unitarity, causality and crossing symmetry, Regge Theory provides the **high energy ($s \rightarrow \infty$) behavior** of the amplitude:

$$A(s,t) \sim \beta(t)\, s^{\alpha(t)}$$

Regge theory does not predict the **t-dependence** of the amplitude.

For that, one needs a model.

α(t): Families or Regge trajectories

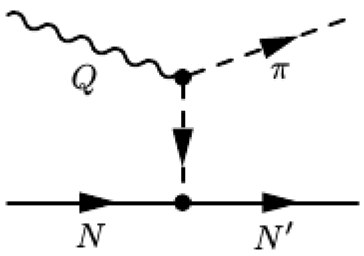


High-energy model

Regge approach for the vector amplitudes.

We use the approach of **Guidal, Laget, and Vanderhaeghen** [NPA627, 645 (1997)], originally developed for pion photoproduction ($Q^2 = 0$):

1) Feynman **meson-exchange diagrams** are reggeized.



$$\frac{1}{t - m_\pi^2}$$

The pion propagator is replace by the Regge trajectory of the pion family

$$\mathcal{P}_\pi(t,s) = -\alpha'_\pi \varphi_\pi(t)\Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$

High-energy model

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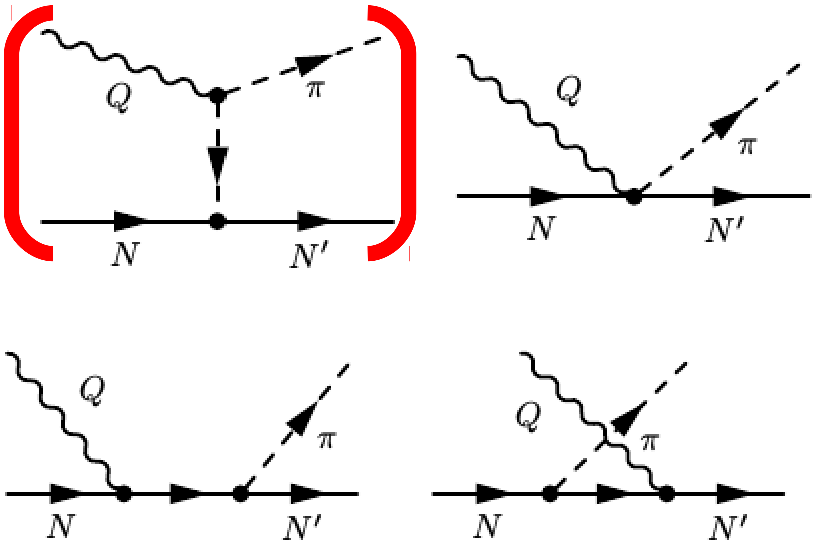
1) Feynman **meson-exchange diagrams** are reggeized.

2) s-channel and u-channel diagrams are included to keep **Conservation of Vector Current**.



$$\frac{1}{t - m_\pi^2}$$

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$$\mathcal{P}_\pi(t,s) = -\alpha'_\pi \varphi_\pi(t)\Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$

High-energy model: N(e, e'π)N' results

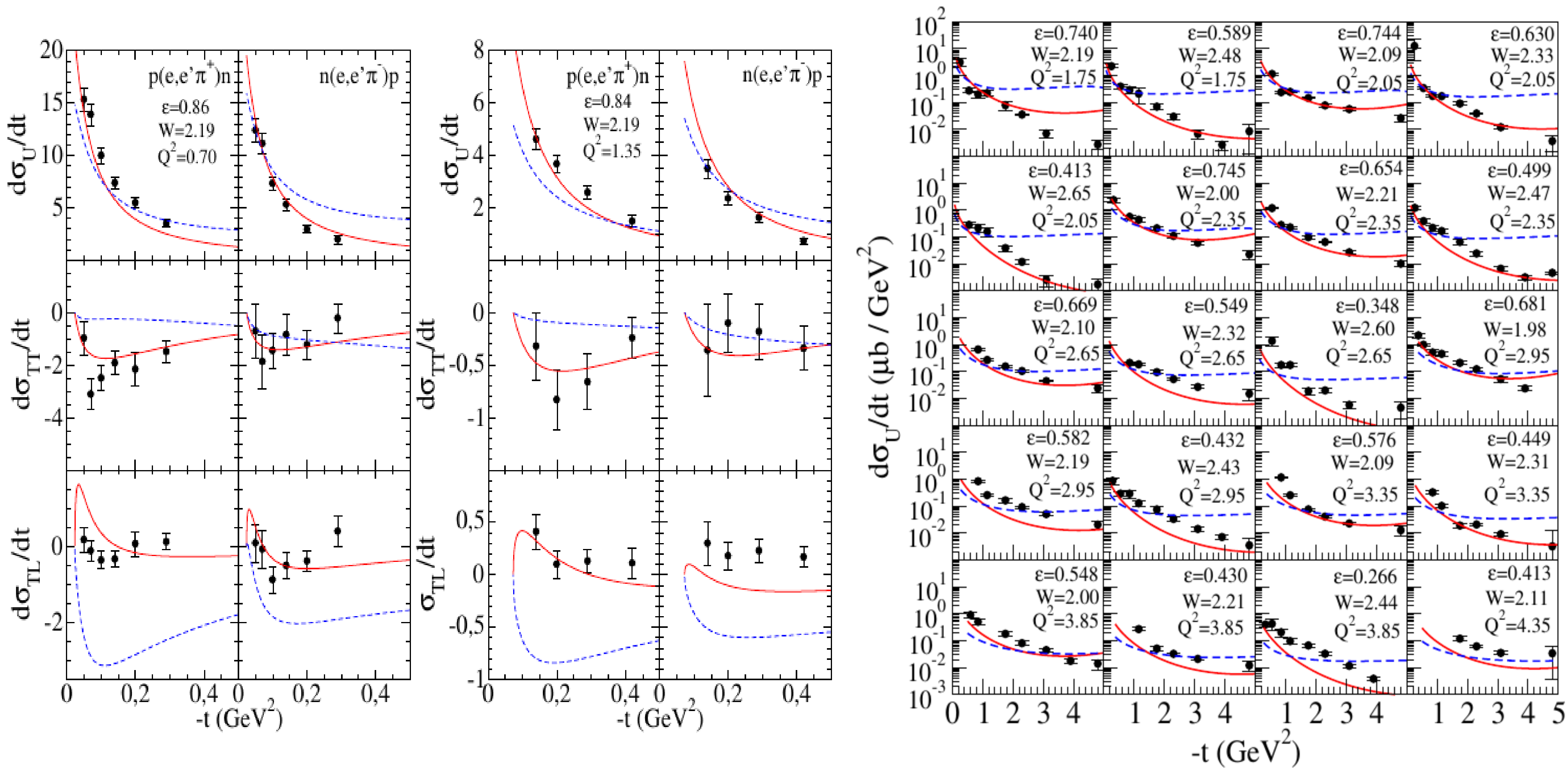


Figure: High-energy model (red lines), low-energy model (blue lines) and **electron-induced single-pion production** data.

High-energy model: results for neutrinos

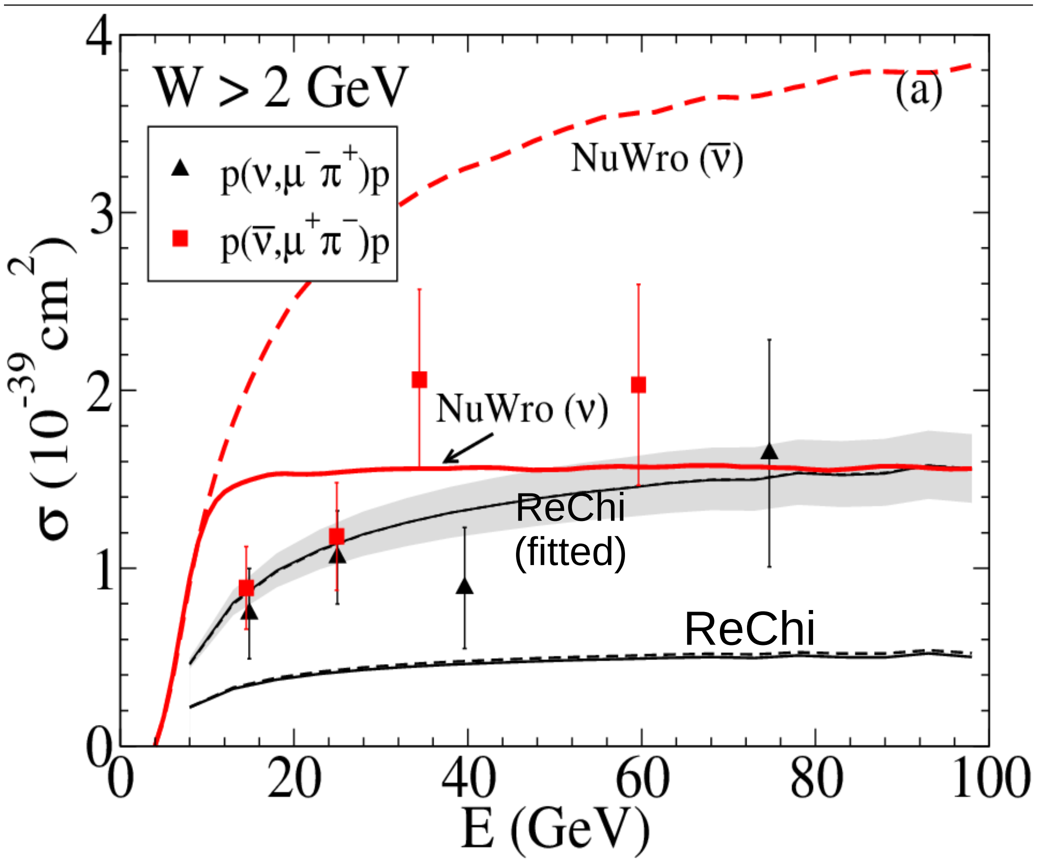


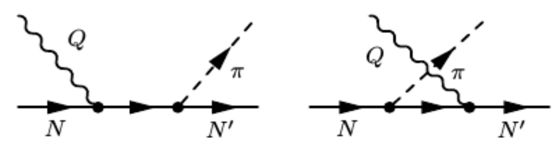
Figure: ReChi model and NuWro predictions are compared with high energy cross section data for neutrino and antineutrino reactions (Note the high energy cut W>2 GeV !!). Data from Allen et al. NPB264, 221 (1986).

NuWro: Based on DIS formalism and PYTHIA for hadronization.

Antineutrino cross section is ~2 the neutrino one:

$$\bar{\nu} + \overbrace{uud}^{p} \to \mu^+ + \overbrace{\bar{u}d}^{\pi^-} + uud\,,$$
$$\nu + uud \to \mu^- + \underbrace{u\bar{d}}_{\pi^+} + uud\,.$$

ReChi model: One free parameter in the boson-nucleon-nucleon vertex



$$G_A[Q^2, s(u)] = g_A \left(1 + \frac{Q^2}{\Lambda_{Apn^*}[s(u)]^2}\right)^{-2}$$

$$\Lambda_{Anp^*}(s) = \Lambda_{Apn} + (\Lambda^A_\infty - \Lambda_{Apn})\left(1 - \frac{M^2}{s}\right)$$

$\Lambda^A_\infty = (7.20 \pm ^{2.09}_{1.32})$ GeV !!!

Hybrid model: results

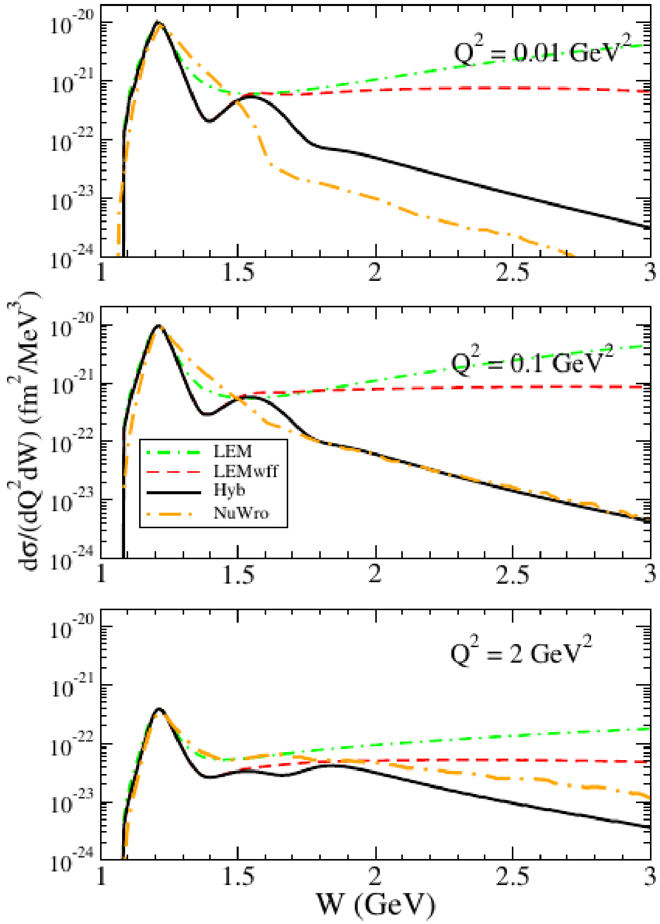
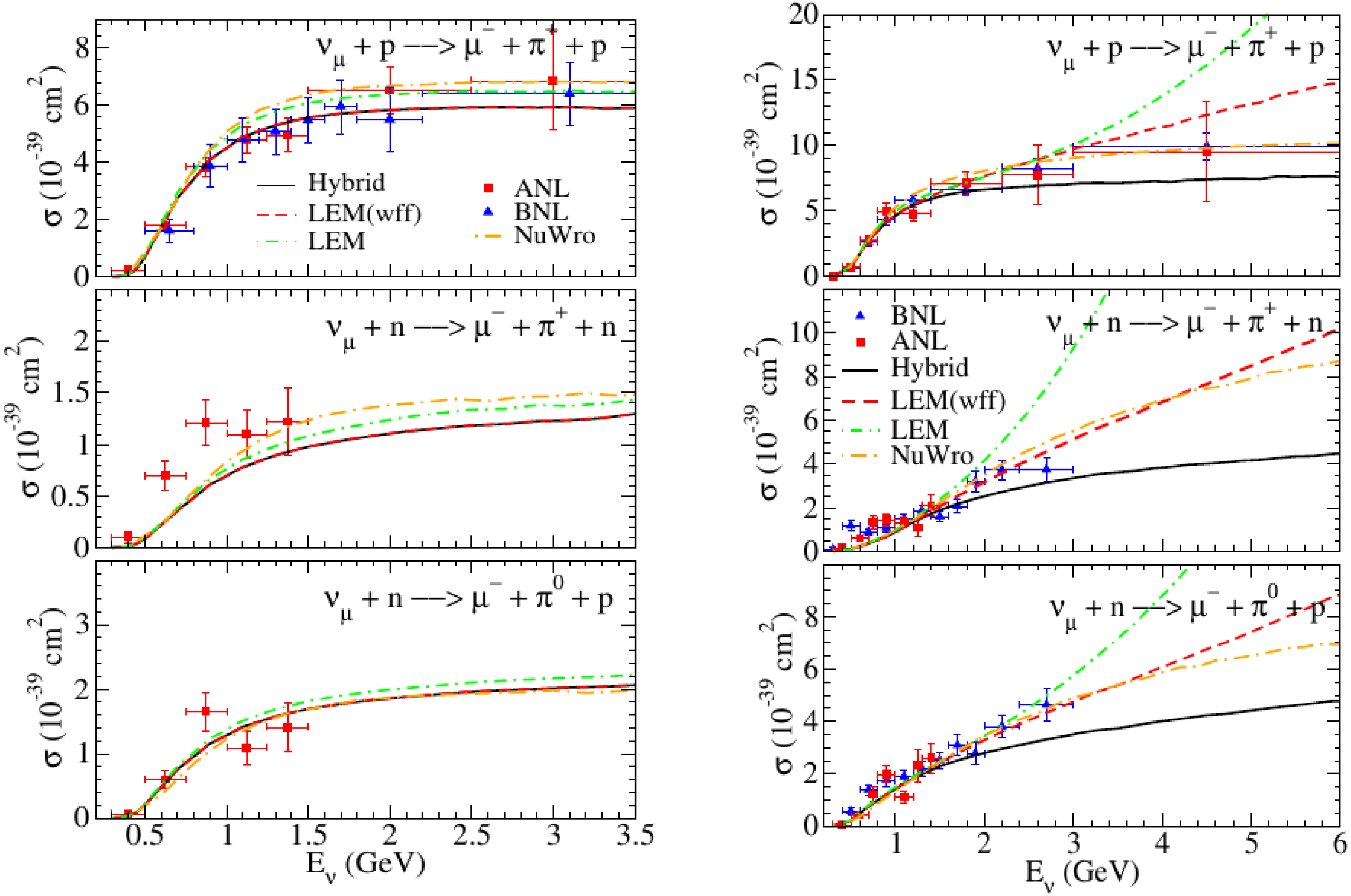


FIG. 21. (Color online) Different model predictions for the differential cross section $d\sigma/(dQ^2dW)$, for the channel $p(\nu_\mu, \mu^-\pi^+)p$. The incoming neutrino energy is fixed to $E_\nu = 10$ GeV.

Hybrid model: results

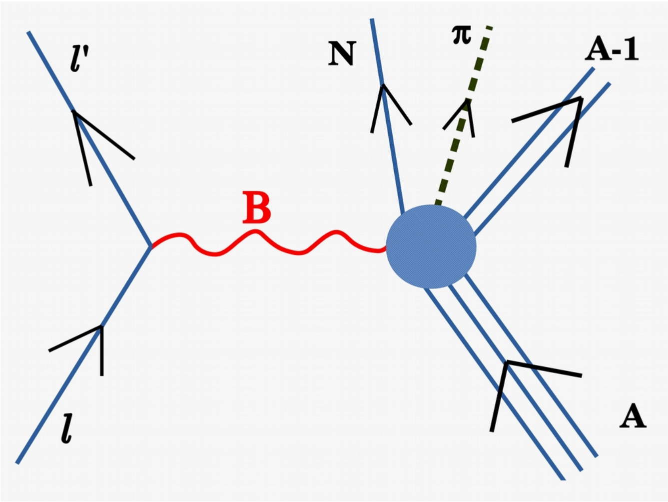
W < 1.4 GeV

No cut in W



III Nuclear effects

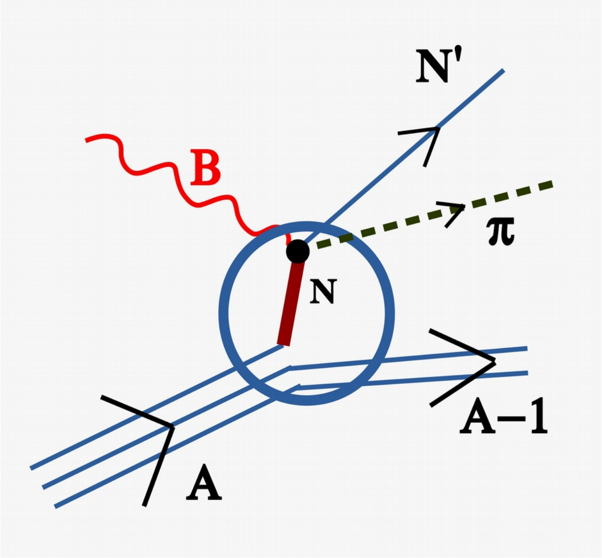
Electroweak one-pion production on nuclei



PRD 97, 013004 (2018), PRD 97, 093008 (2018)

Relativistic mean field model

Relativistic Impulse Approximation



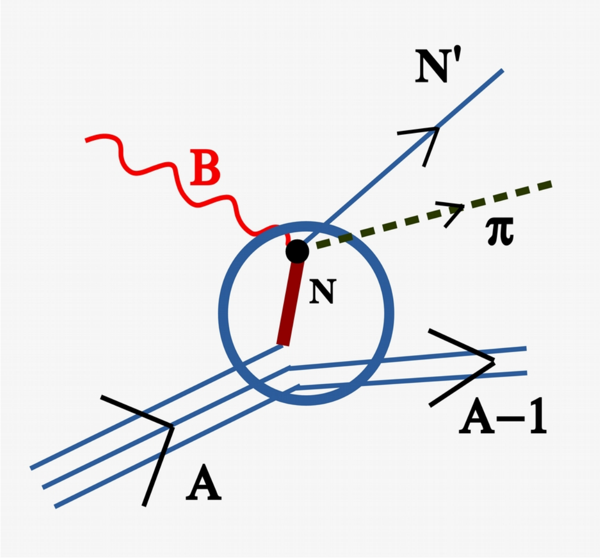
$$J^{\mu}_{had} = \sum_{i}^{A} \int \mathrm{d}\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \phi^*(\mathbf{r}) \hat{\mathcal{O}}^{\mu}_{one\text{-}body}(\mathbf{r})\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

Relativistic mean-field wave functions

$$\frac{d^9\sigma}{d\varepsilon_f d\cos\theta_f d\phi_f\, dE_\pi d\cos\theta_\pi d\phi_\pi\, d\cos\theta_N d\phi_N\, dE_m}$$

Relativistic mean field model

Relativistic Impulse Approximation



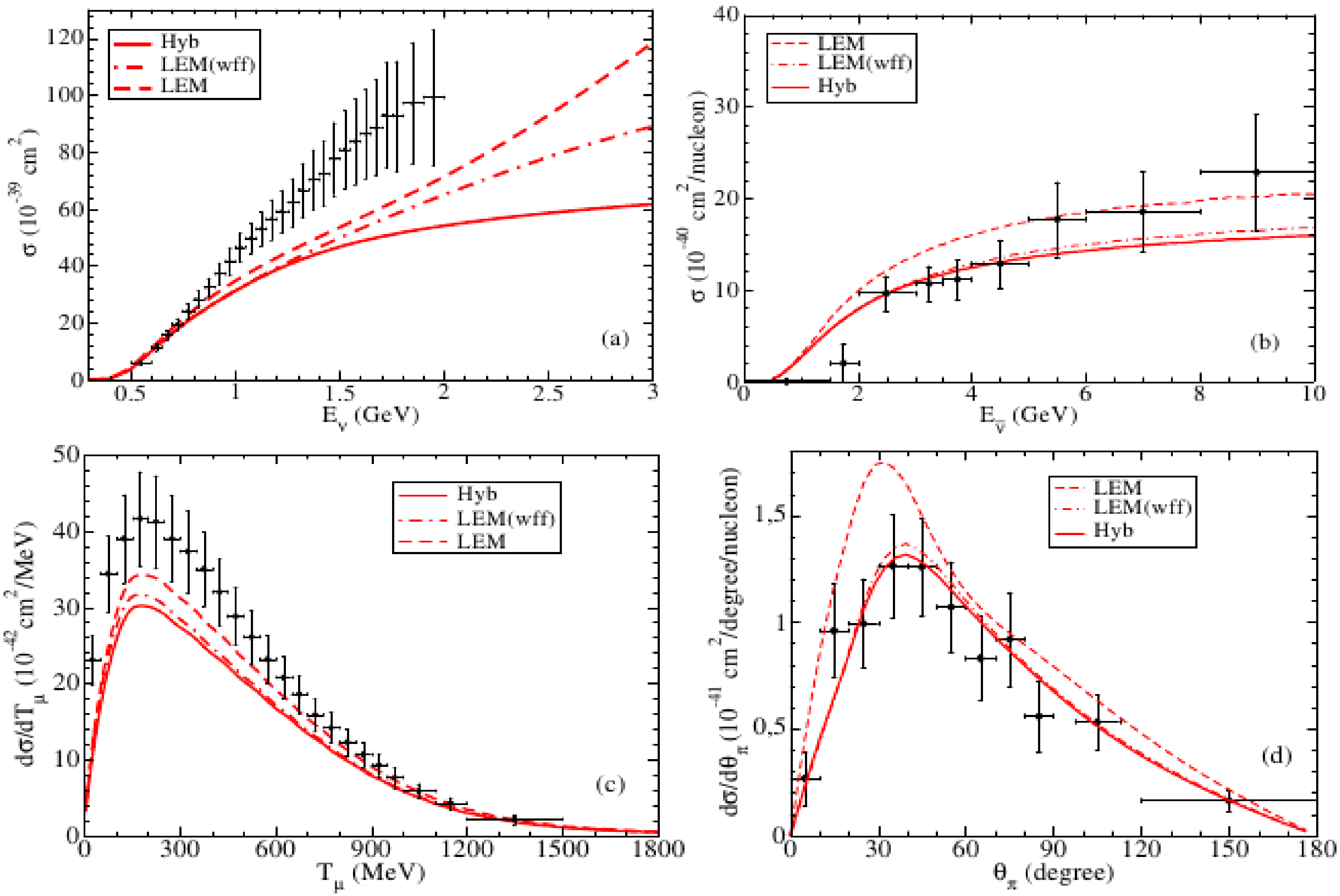
Plane waves (for the moment...)

$$J^{\mu}_{had} = \sum_{i}^{A} \int \mathrm{d}\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \phi^*(\mathbf{r}) \hat{\mathcal{O}}^{\mu}_{one\text{-}body}(\mathbf{r})\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

Relativistic mean-field wave functions

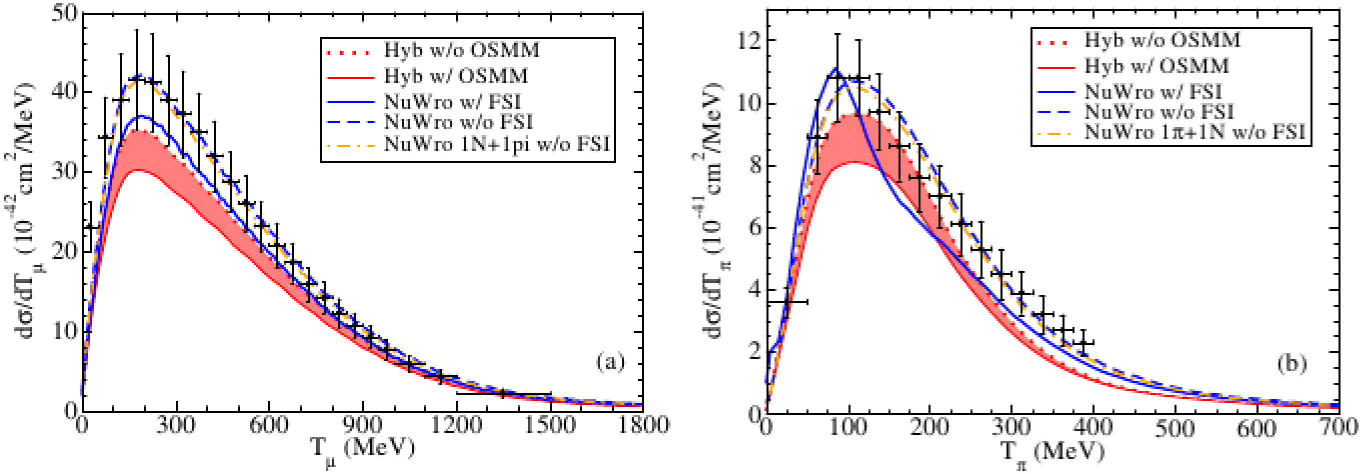
$$\frac{d^9\sigma}{d\varepsilon_f d\cos\theta_f d\phi_f\, dE_\pi d\cos\theta_\pi d\phi_\pi\, d\cos\theta_N d\phi_N\, dE_m}$$

MiniBooNE neutrino CC 1pion+ **MINERvA antineutrino CC 1pion0.**

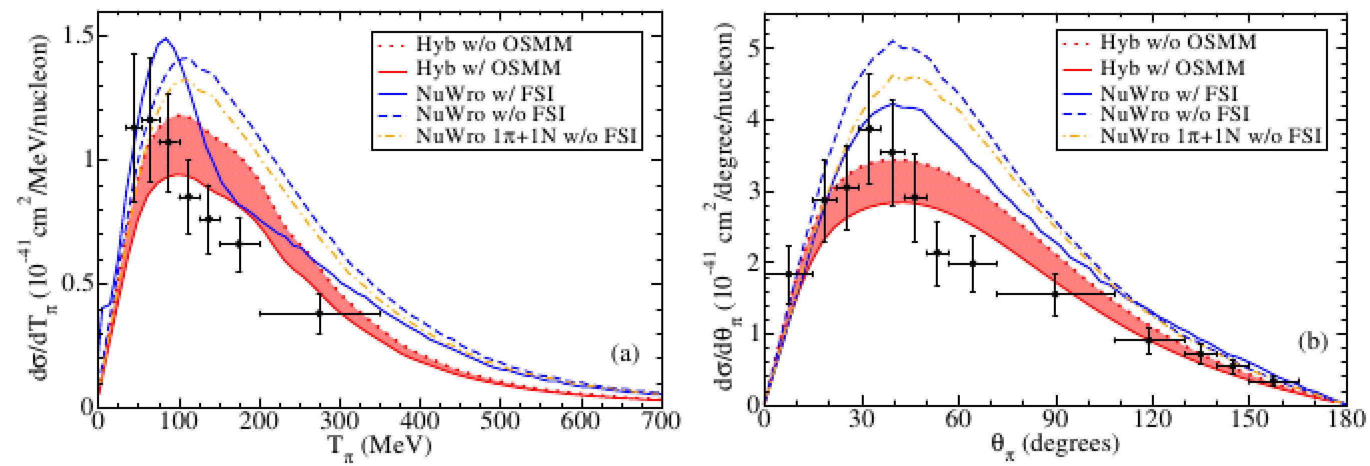


PRD 97, 013004 (2018)

MiniBooNE neutrino CC 1pion+.

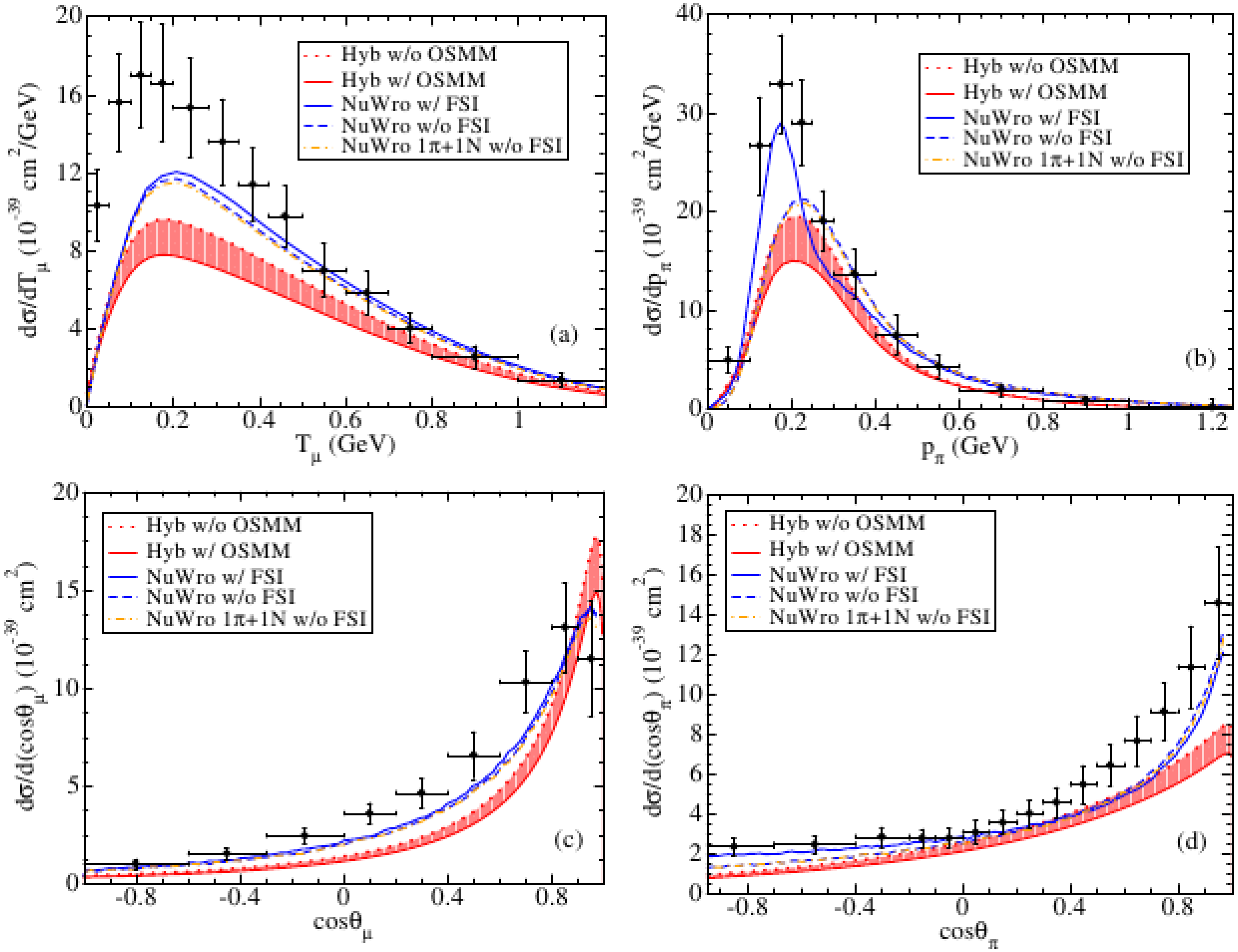


MINERvA neutrino CC 1pion+.



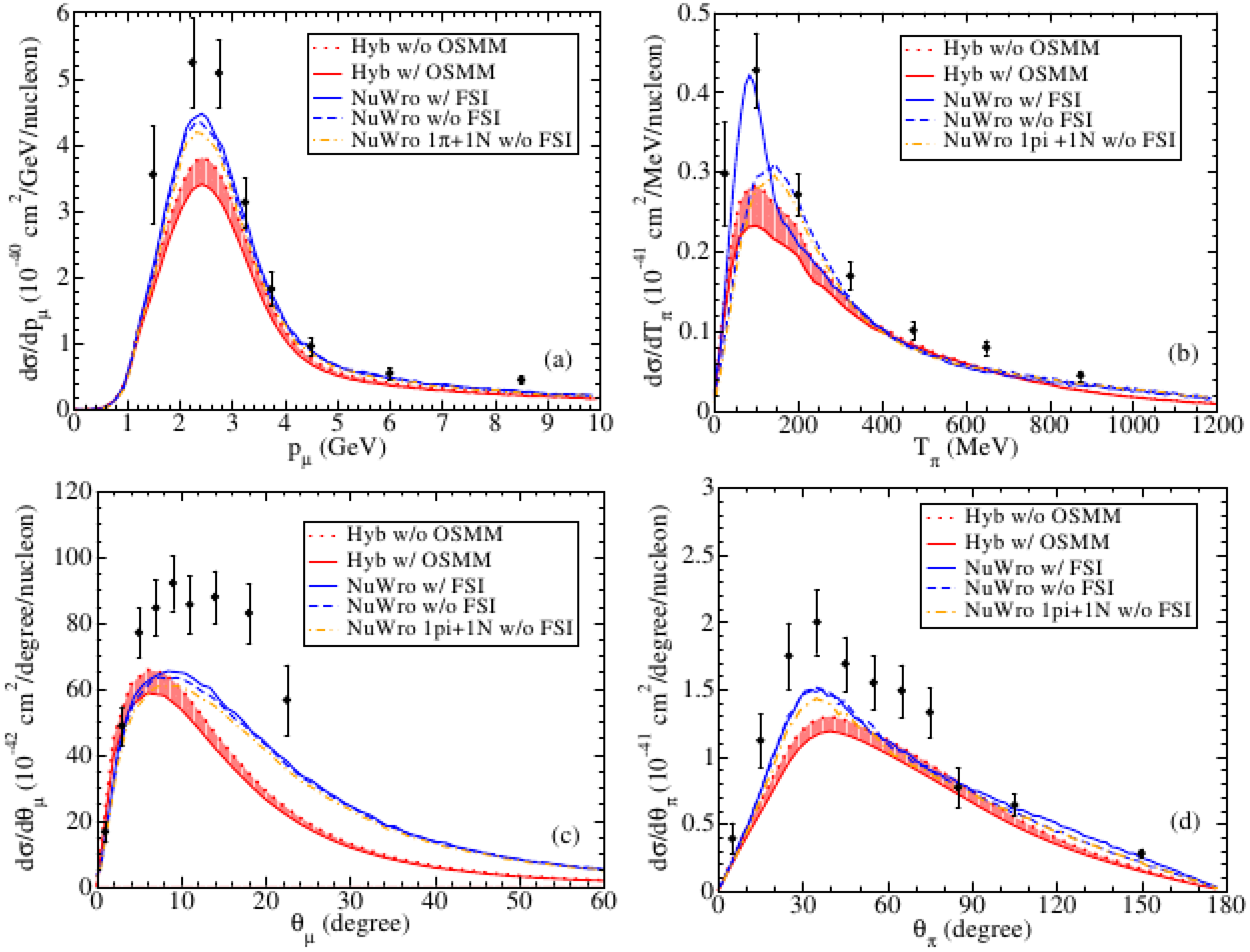
PRD 97, 013004 (2018)

MiniBooNE neutrino CC 1pion0.



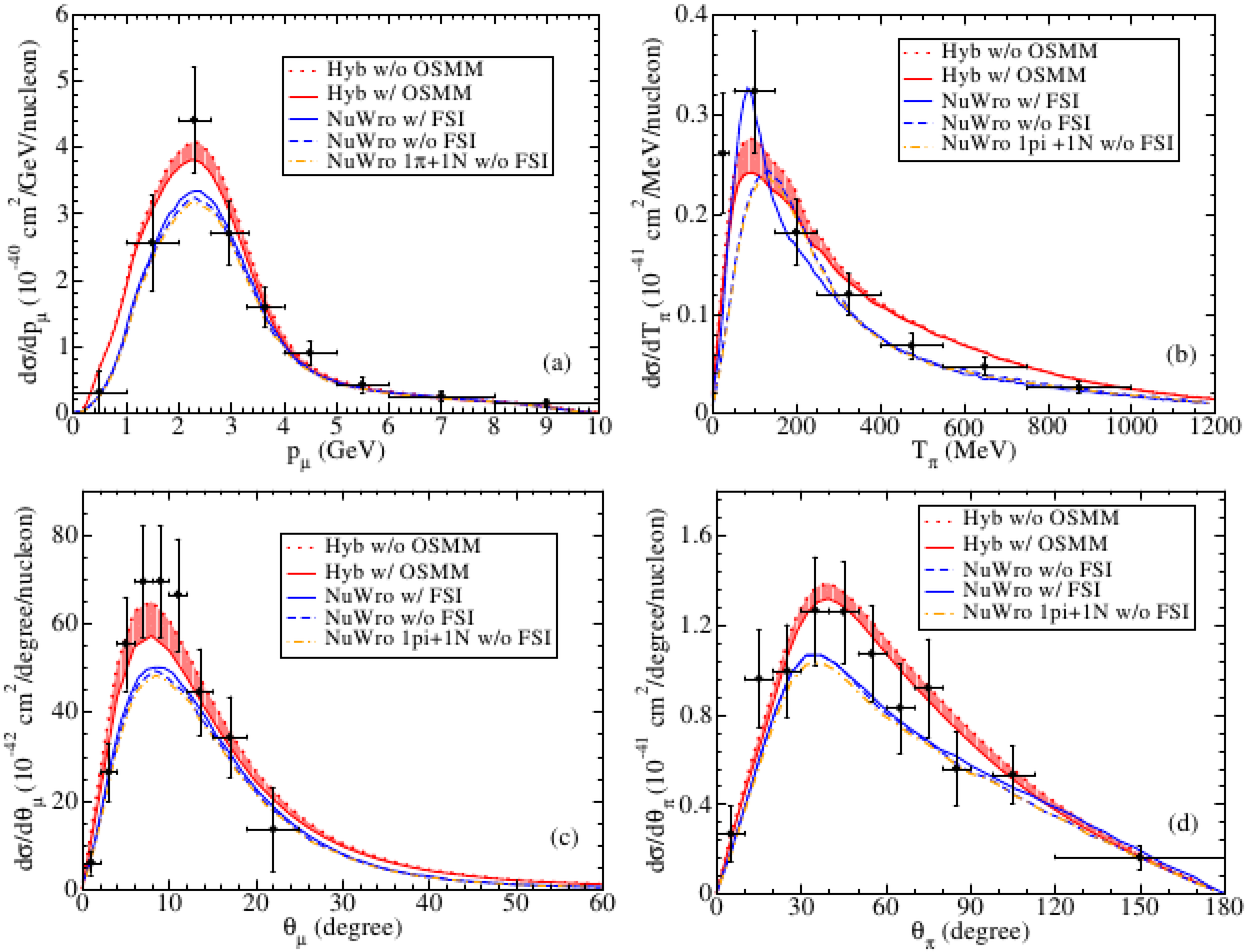
PRD 97, 013004 (2018)

MINERvA neutrino CC 1pion0.



PRD 97, 013004 (2018)

MINERvA antineutrino CC 1pion0.



PRD 97, 013004 (2018)

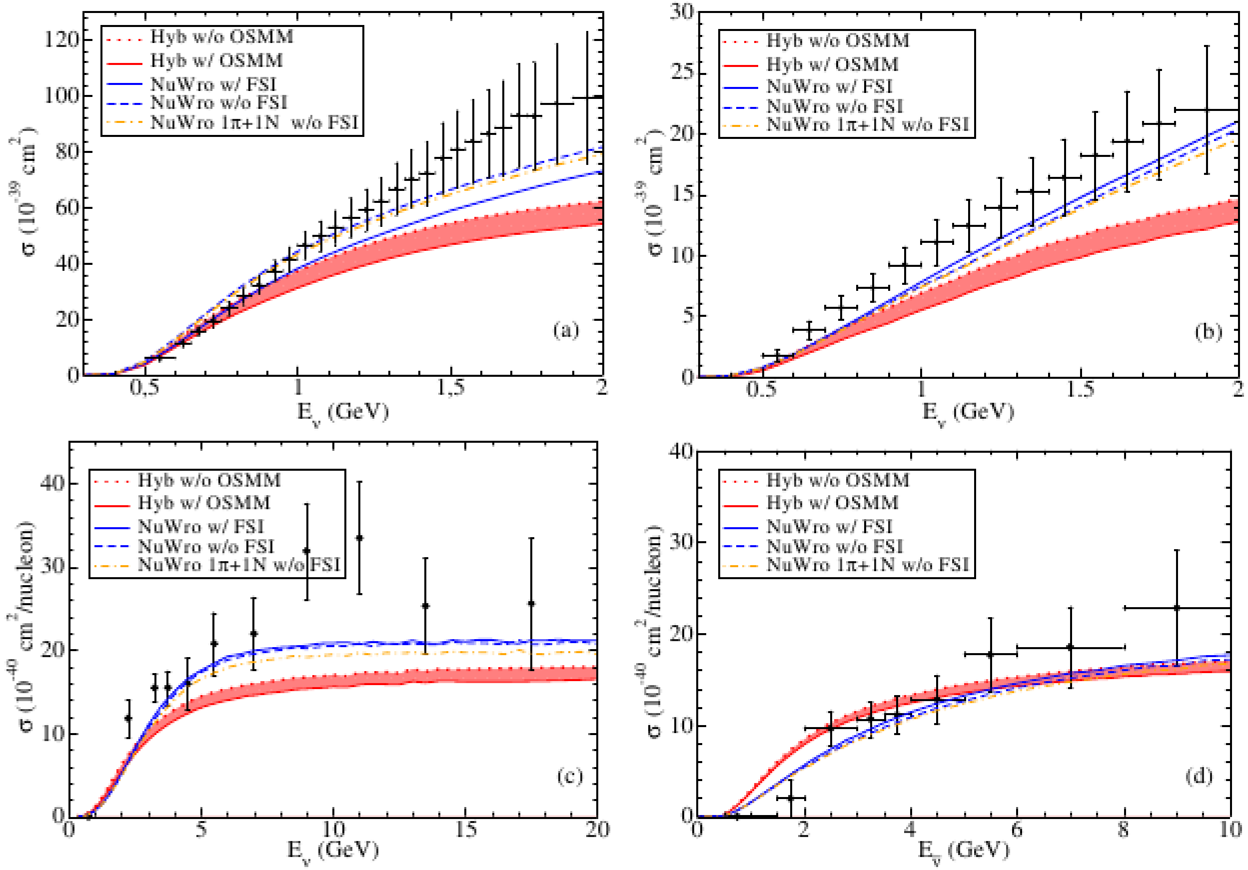


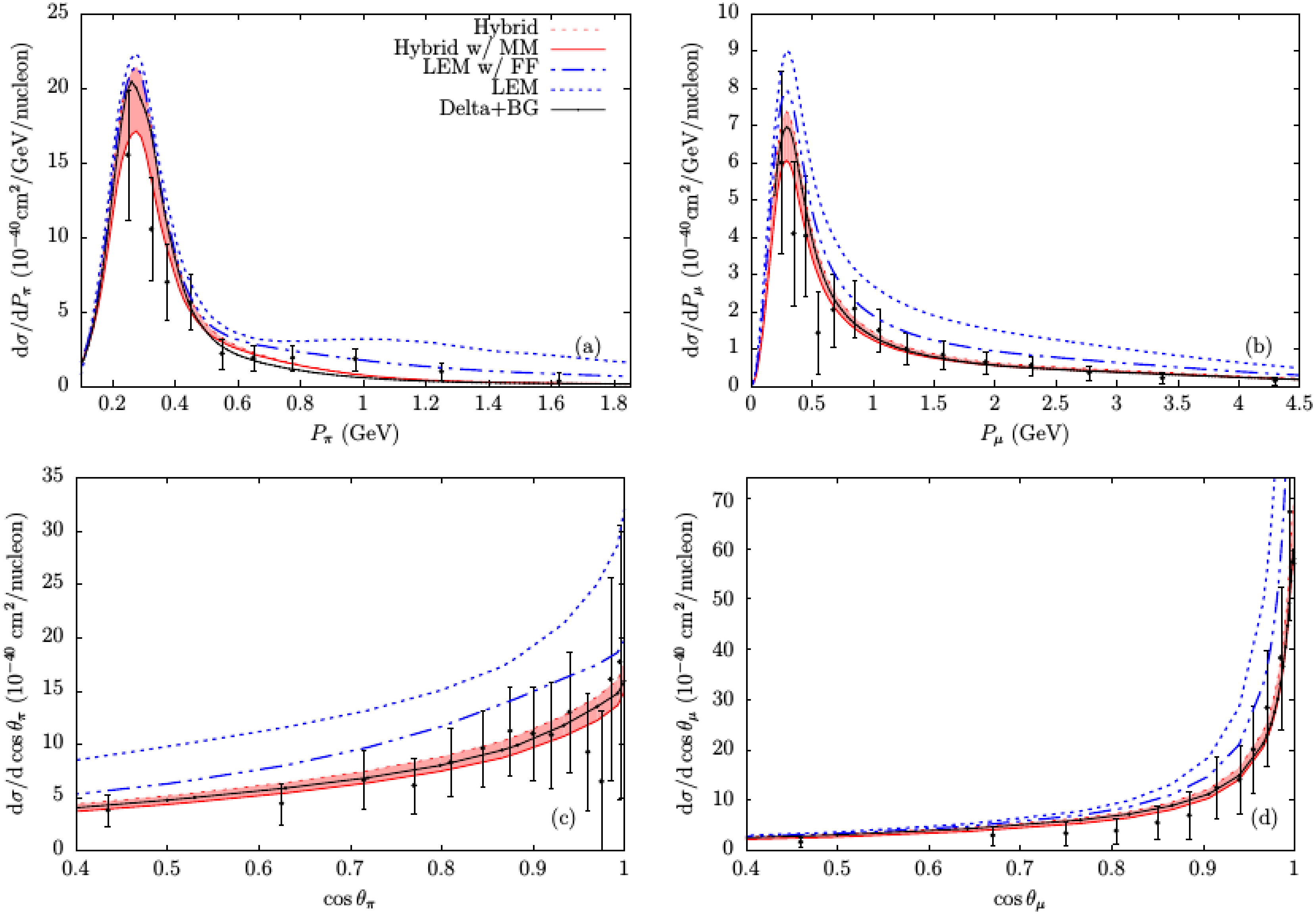
FIG. 10: Total cross section for the reactions (a) MiniBooNE νCC $1\pi^+$ [4], (b) MiniBooNE νCC $1\pi^0$ [62], (c) MINERvA νCC $1\pi^0$ [7], and (d) MINERvA $\bar{\nu}$CC $1\pi^0$ [6]. Labels as in Fig. 5

PRD 97, 013004 (2018)

T2K CC 1pion+

PRD 97, 093008 (2018)

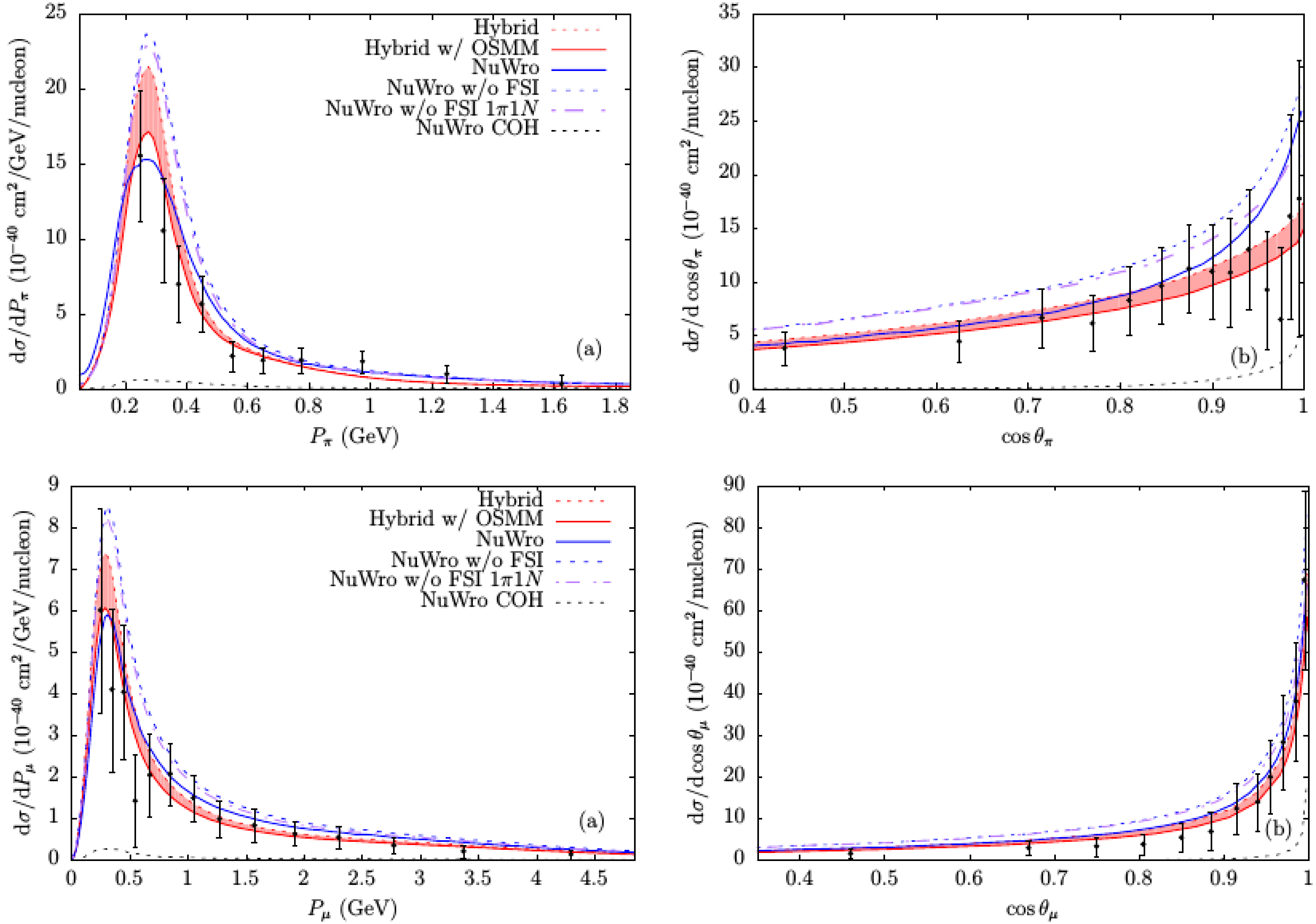
LEM vs Hybrid



T2K CC 1pion+

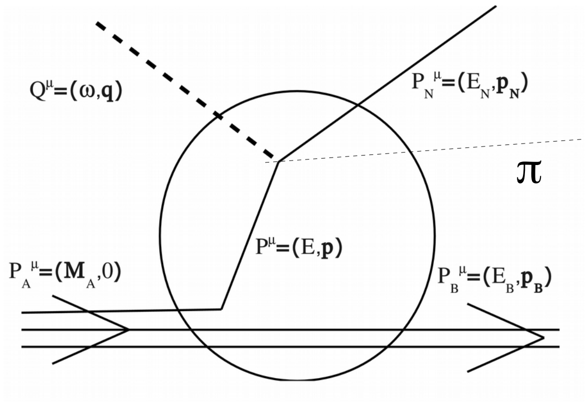
PRD 97, 093008 (2018)

NuWro vs Hybrid: pion FSI effects



What’s next?

So far...



RPWIA: Scattered nucleon wf is described as a Dirac plane wave.

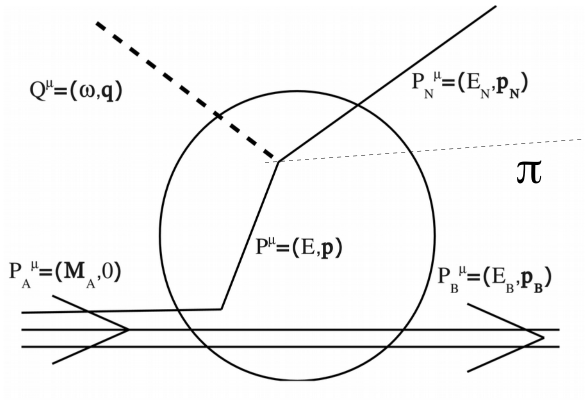
Final Nucleon: Plane wave

Pion:Plane wave

$$J^{\mu}_{had} = \sum_{i}^{A} \int d\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \phi^*(\mathbf{r}) \hat{O}^{\mu}_{one\text{-}body}(\mathbf{r})\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

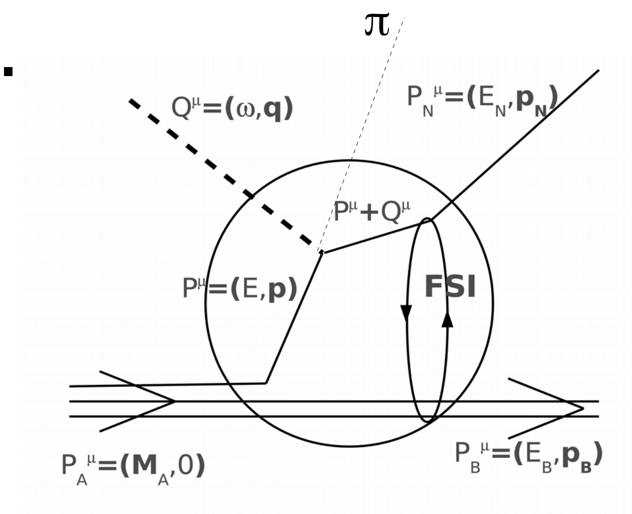
relativistic mean-field wave function

So far...



RPWIA: Scattered nucleon wf is described as a Dirac plane wave.

Now...



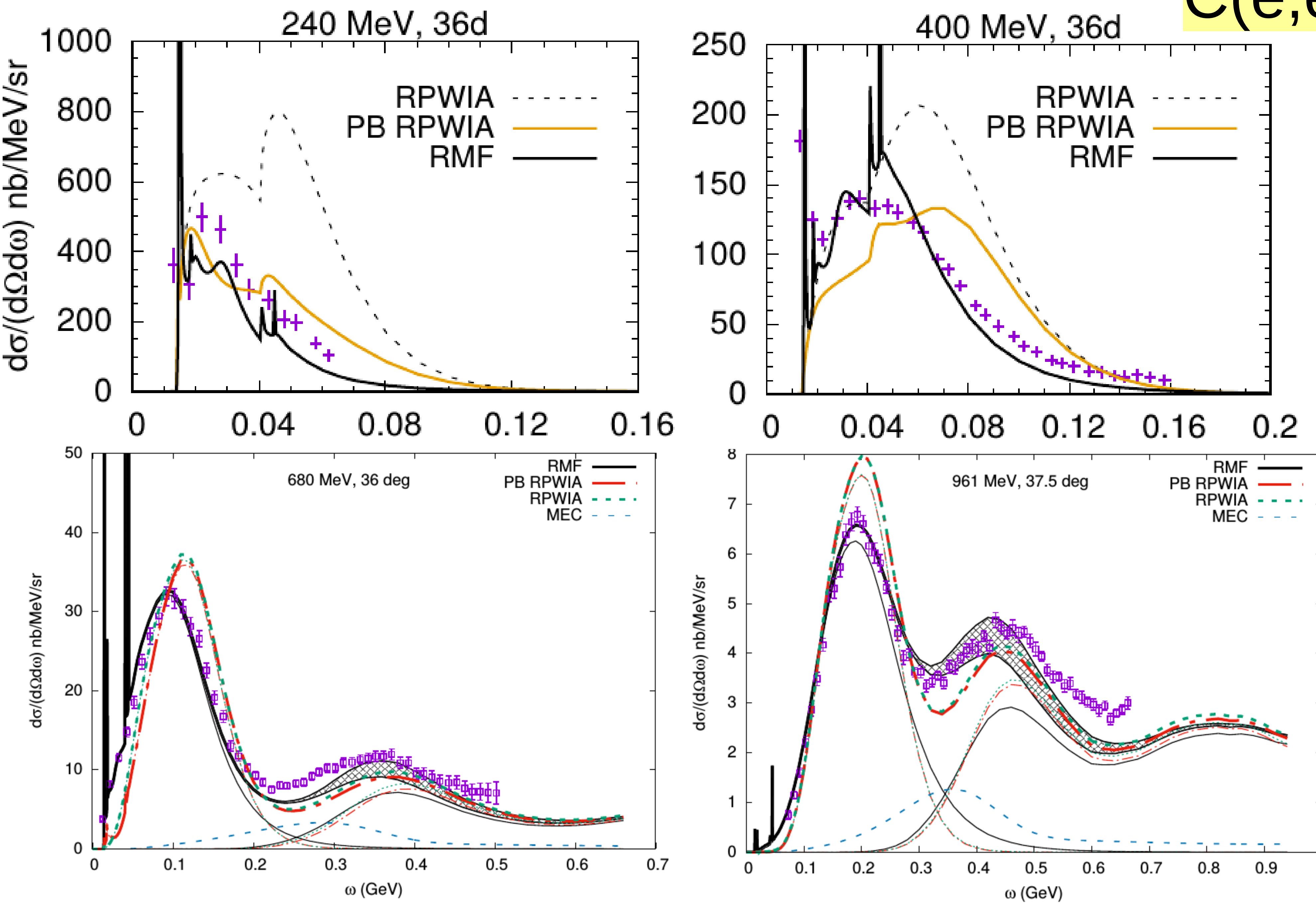
RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

Pion:Plane wave

$$J^\mu_{had} = \sum_i^A \int d\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \phi^*(\mathbf{r}) \hat{O}^\mu_{one\text{-}body}(\mathbf{r})\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

relativistic mean-field wave functions

$^{12}C(e,e')$



240 MeV, 36d
400 MeV, 36d
RPWIA
PB RPWIA
RMF
dσ/(dΩdω) nb/MeV/sr
680 MeV, 36 deg
961 MeV, 37.5 deg
MEC
ω (GeV)

Orthogonality: Pauli blocking

Partial wave expansion of a relativistic plane wave:

$$\Psi_{PW}(\mathbf{r},\mathbf{p},m_s) \;=\; 4\pi\sqrt{\frac{E+M}{2EV}}\sum_{\kappa=-\infty}^{+\infty}\sum_{m_j=-j}^{+j} i^{\ell}\langle \ell(m_j-m_s),\frac{1}{2}m_s|jm_j\rangle[Y_{\ell}^{m_\ell}(\Omega_{\mathbf{p}})]^{*}$$

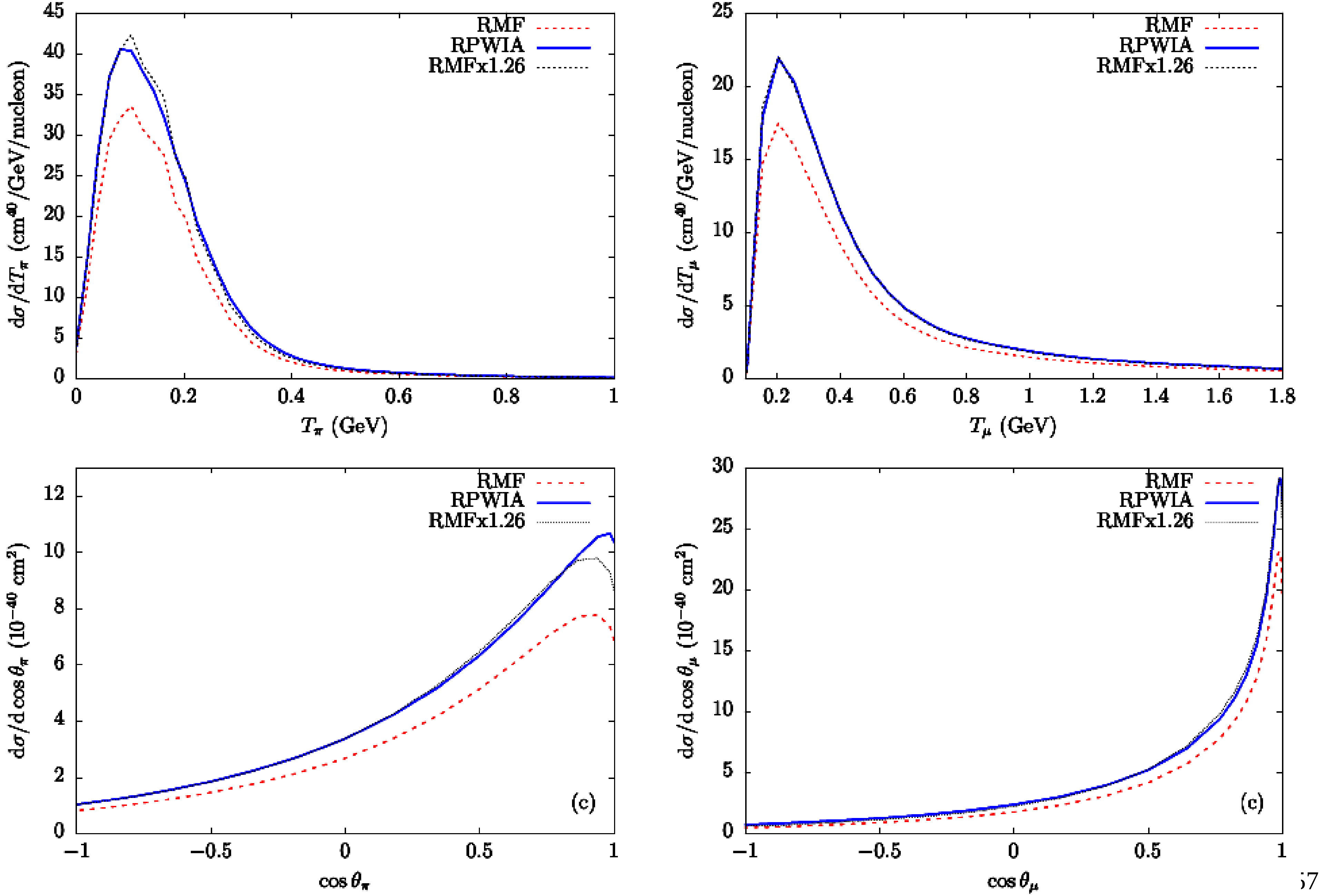
$$\times\;\begin{pmatrix} j_\ell(pr)\phi_\kappa^{m_j}(\Omega_{\mathbf{r}}) \\ i\frac{|\kappa|}{\kappa}j_{\bar{\ell}}(pr)\phi_{-\kappa}^{m_j}(\Omega_{\mathbf{r}}) \end{pmatrix}.$$

The **orthogonalized** final nucleon wave function (Pauli blocked) is built by subtracting to the plane wave the partial waves that overlap with the initial state nucleus:

$$|\Psi^{s_N}(\mathbf{p}_N)\rangle = |\psi_{pw}^{s_N}(\mathbf{p}_N)\rangle - \sum_{\kappa,m_j}[C_\kappa^{m_j,s_N}(\mathbf{p}_N)]^{\dagger}\,|\psi_\kappa^{m_j}\rangle$$

$$C_\kappa^{m_j,s_N}(\mathbf{p}_N) \equiv \langle\psi_{pw}^{s_N}(\mathbf{p}_N)|\psi_\kappa^{m_j}\rangle$$

PRELIMINARY RESULTS
CC 1pion+ (folded with T2K flux, only one shell)



PRELIMINARY RESULTS
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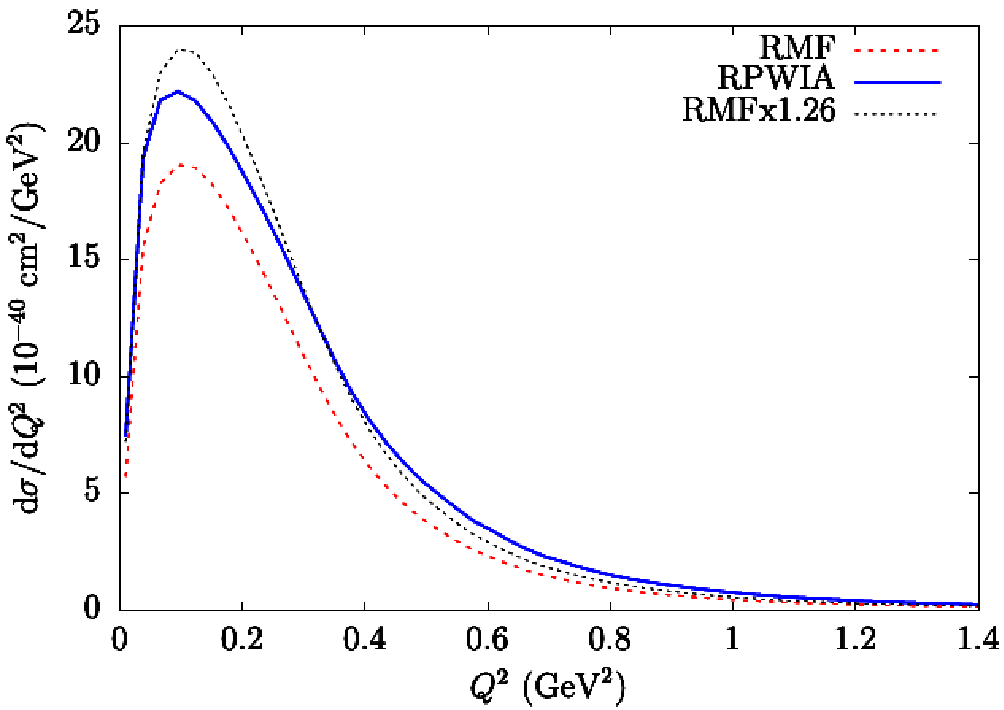

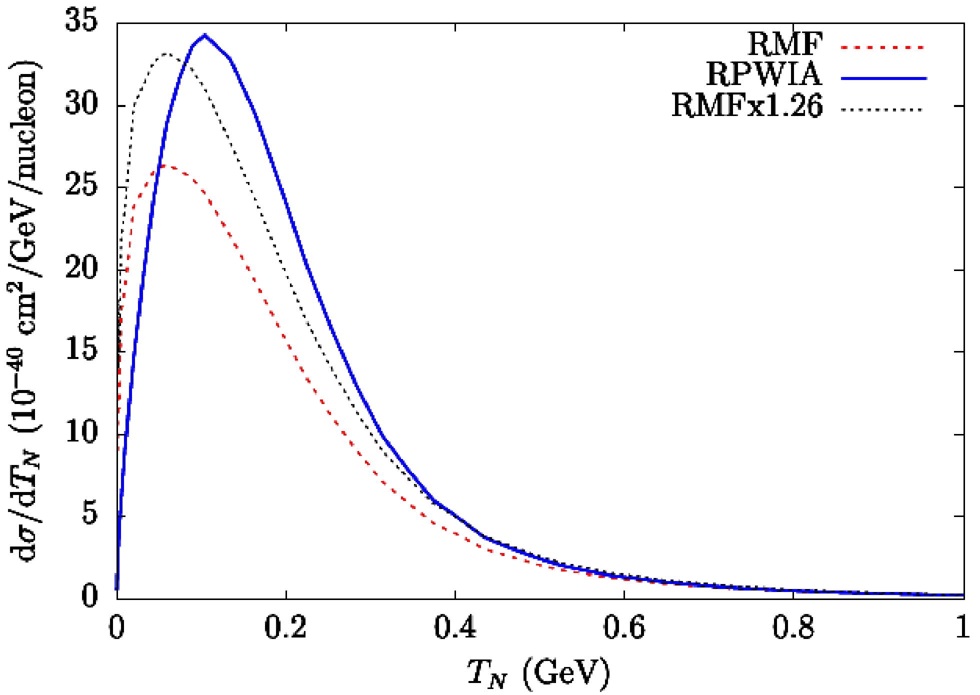

PRELIMINARY RESULTS
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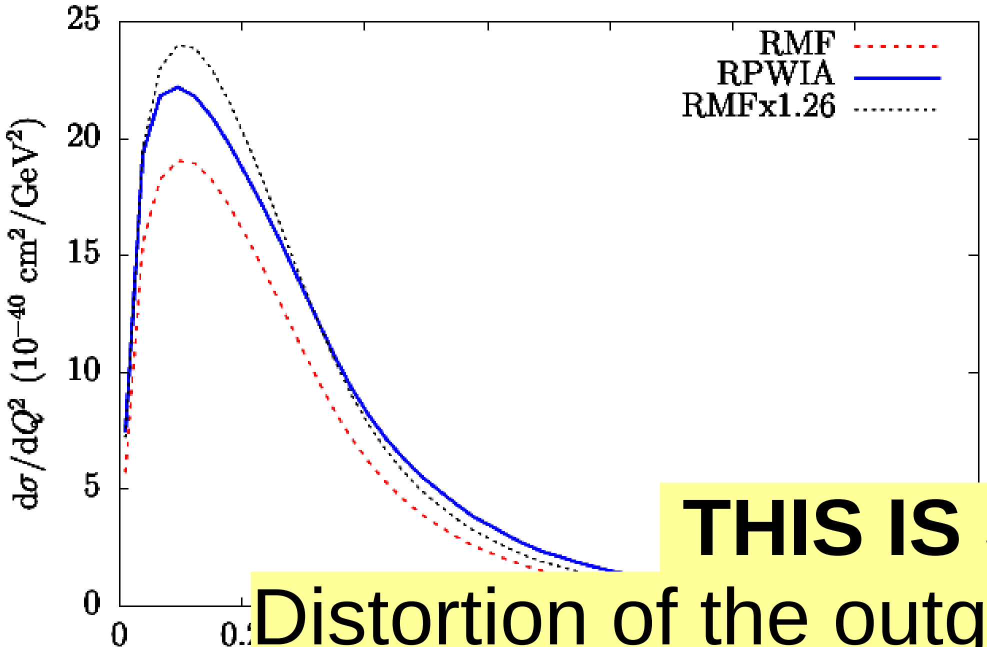
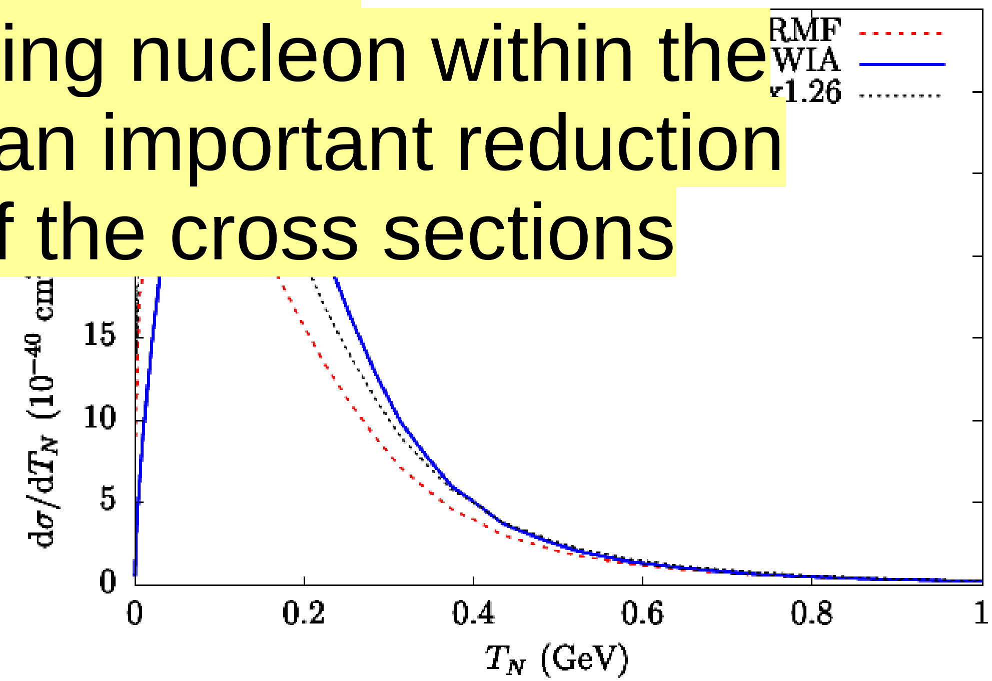


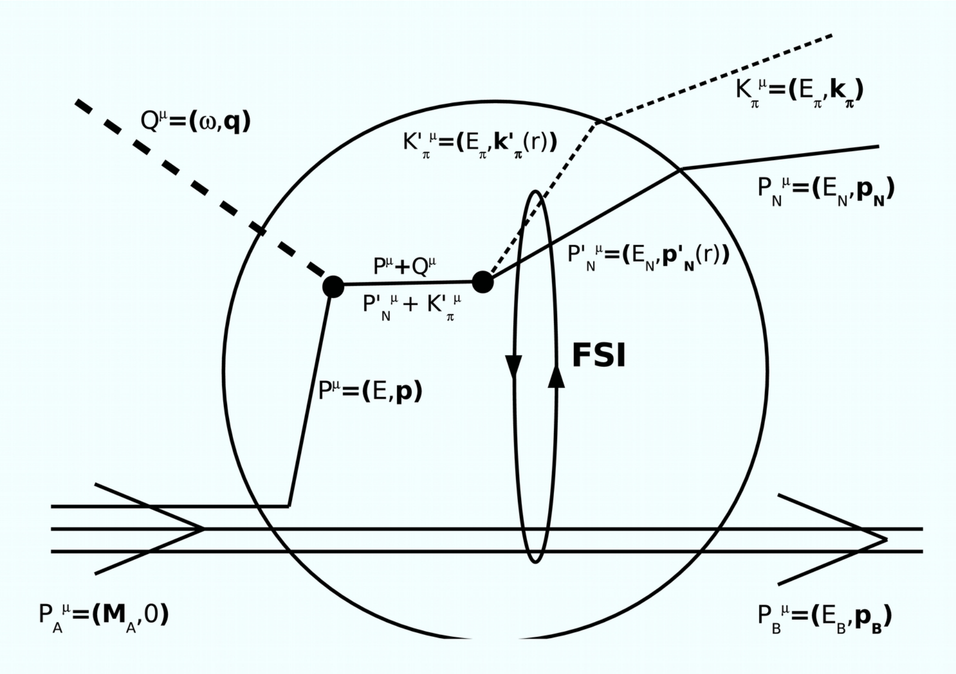
THIS IS SERIOUS !

Distortion of the outgoing nucleon within the RMF model produces an important reduction and redistribution of the cross sections



What’s next?

What's next?



... distortion and absorption of the pion

Conclusions

- ✔ Simple analysis of the **kinematics of the problem** tell us the minimum set of independent variables that is needed to describe the scattering process.

  ➔ A model that depends on less variables is missing something. Is it important?

- ✔ Distortion of the outgoing nucleon within the RMF model produces an important reduction and redistribution of the cross sections: **THIS IS A SERIOUS ISSUE!**

- ✔ Is it possible to implement (complex) microscopic models with full kinematics in the MC event generators?

  ➔ Yes, it is possible.
  ➔ Is it worthy? Are you interested?
  ➔ Let's talk about it.

Collaborators

JM Udías (Madrid)

Alexis Nikolakopoulos (Ghent)

Kajetan Niewczas (Ghent)

Natalie Jachowicz (Ghent)

Nils Van Dessel (Ghent)

Jannes Nys (Ghent)

Tom Van Cuyck (Ghent)

Vishvas Pandey (Virginia Tech)

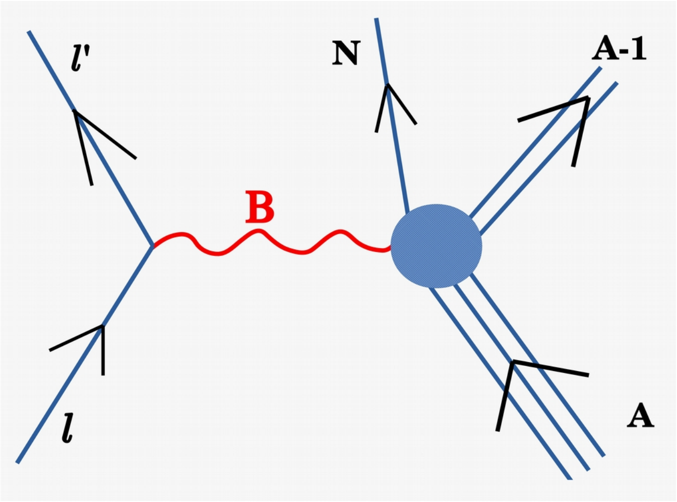
TW Donnelly (MIT)

The end...

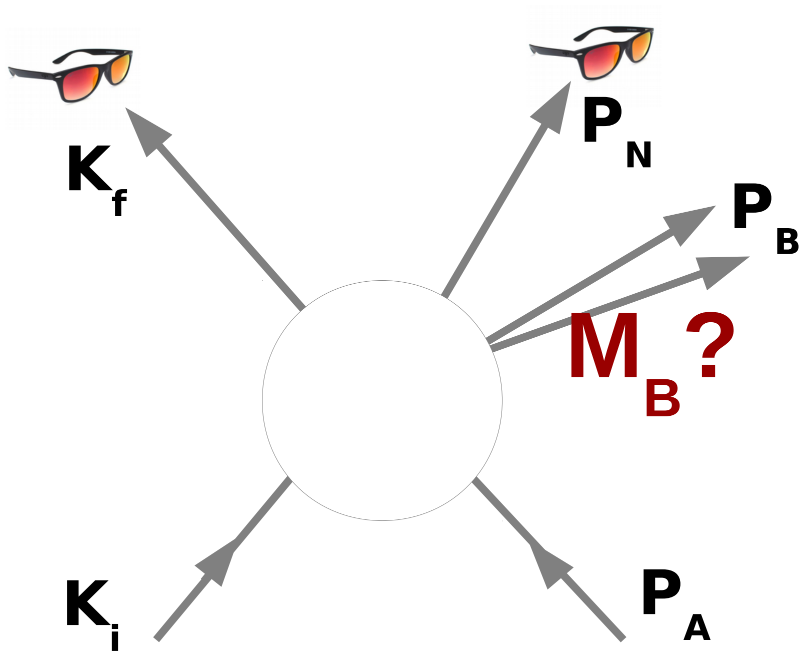
Thanks for your attention

Backup slides

Quasielastic scattering

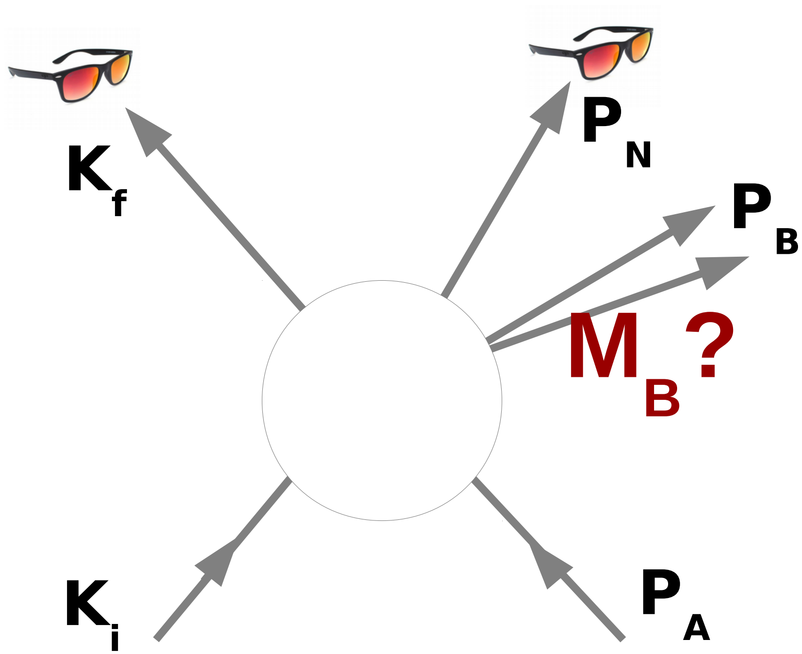


One hadron is detected in coincidence with the scattered lepton, e.g., **quasielastic scattering.**



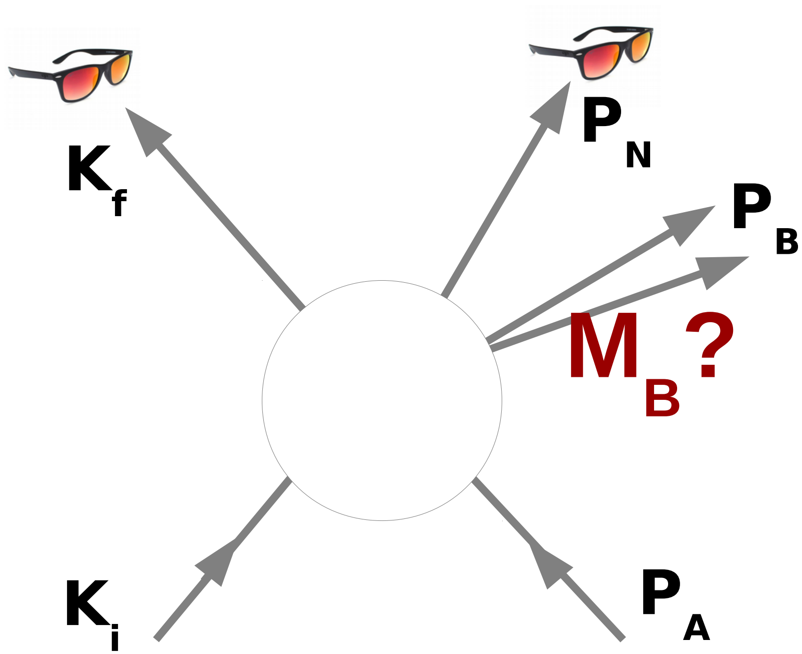
5 x 4-momentum → 20 variables

One hadron is detected in coincidence with the scattered lepton, e.g., **quasielastic scattering.**



5 x 4-momentum	→	**20 variables**
1 x 4-mom. conserv.	→	**-4 constraints**
4 x ($E^2=M^2+p^2$)	→	**-4 constraints**

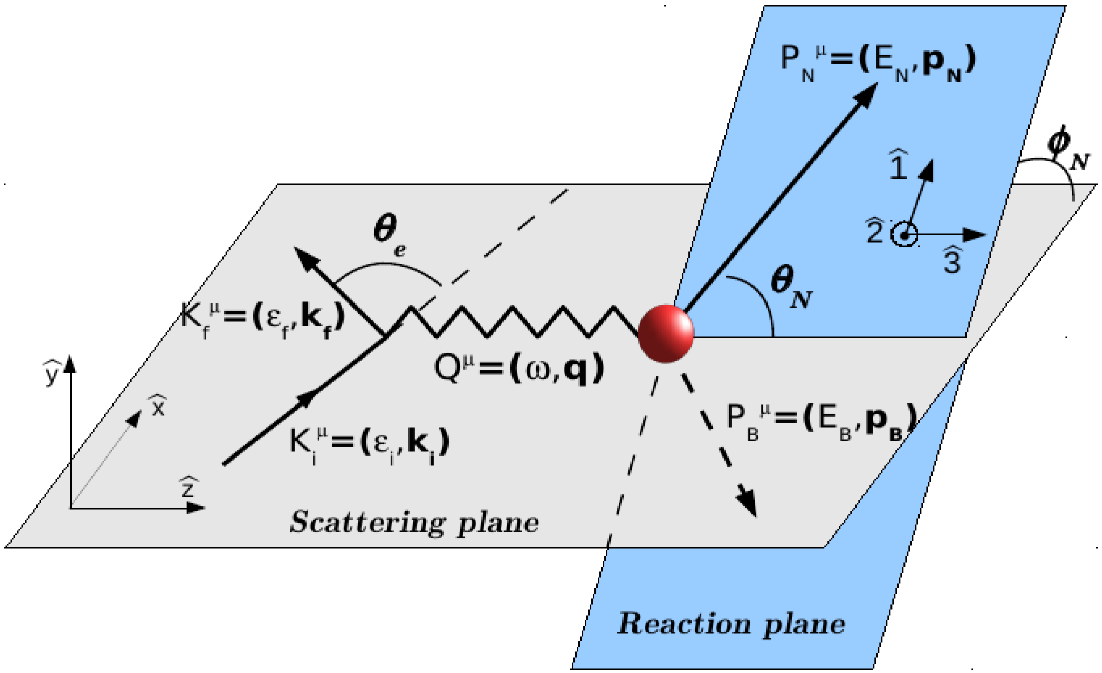
One hadron is detected in coincidence with the scattered lepton, e.g., **quasielastic scattering.**



5 x 4-momentum	→	**20 variables**
1 x 4-mom. conserv.	→	**-4 constraints**
4 x ($E^2=M^2+p^2$)	→	**-4 constraints**
3-mom. of the beam	→	**-3 known**
3-mom. of the target	→	**-3 known**

Independent variables left → 6

$$\frac{d^6\sigma}{d\varepsilon_f d\Omega_f \, dE_N d\Omega_N}$$



In this reference frame (**q** // **z**):

1) The cross section can be decomposed in **response functions** (Rosenbluth decomposition).

2) **Leptonic and hadronic variables are not mixed.**

3) The ϕ_N **dependence factorizes** in terms of sinos and cosinos.

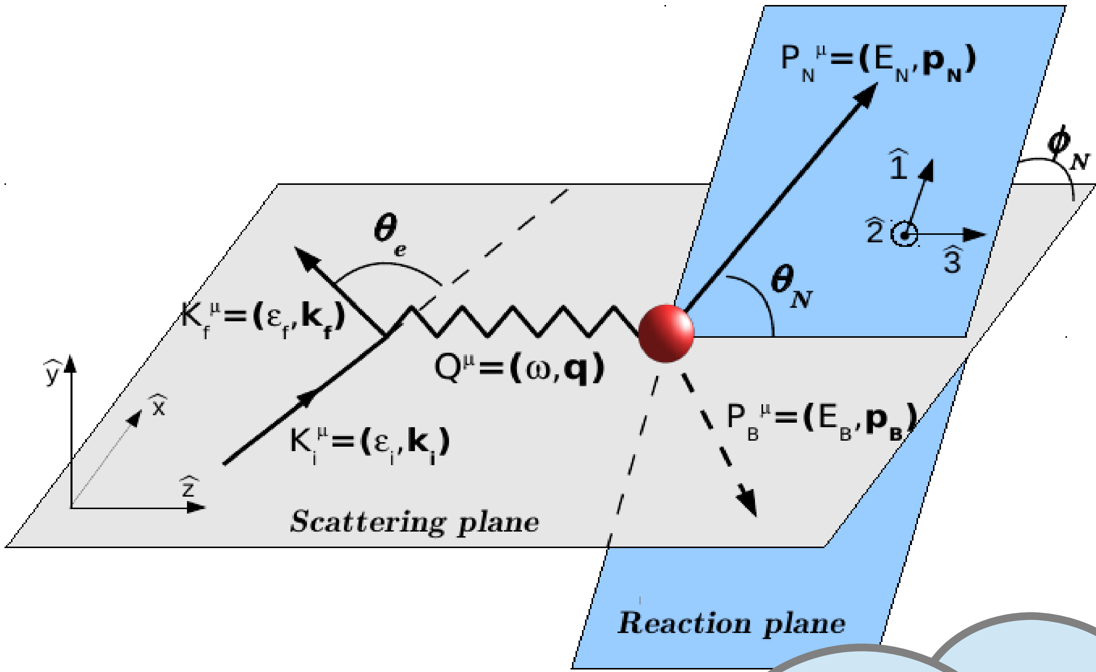
$$\frac{d^6\sigma}{d\varepsilon_f d\Omega_f\, dE_N d\Omega_N} \propto$$

$$(v_L R_L + v_T R_T + v_{TL} R_{TL} \cos\phi_N + v_{TT} R_{TT} \cos 2\phi_N)$$

v's and R's depend on the independent variables (I included the explicit dependence on the incoming energy)

$$v_K = v_K(\varepsilon_i, q, \omega)$$

$$R_K = R_K(q, \omega, \theta_N, E_m)$$



In this reference frame (**q** // **z**):

1) The cross section can be decomposed in **response functions** (Rosenbluth decomposition).

2) **Leptonic and hadronic variables are not mixed.**

3) The ϕ_N **dependence** … terms of sinos and …sinos.

4 indep. variables.
If you have less...
better think about it!

$$\frac{d^6\sigma}{d\varepsilon_f d\Omega_f\, dE_N d\Omega_N \ldots}$$

$$(v_L R_L + v_T R_T \ldots {}_{TT} R_{TT} \cos 2\phi_N)$$

v's and R's depend on the independent variables (I included the explicit dependence on the incoming energy)

$$v_K = v_K(\varepsilon_i, q, \omega)$$

$$R_K = R_K(q, \omega, \theta_N, E_m)$$

$$\frac{d^6\sigma}{d\varepsilon_f d\Omega_f \, dE_m dp_m \, d\phi_N} \propto \ell_{\mu\nu} H^{\mu\nu}$$

$$\ell_{\mu\nu} = \overline{\sum} (j_\mu)^* j_\nu$$

$$H^{\mu\nu} = \overline{\sum} (J^\mu)^* J^\nu$$

$$j_\mu = j_\mu(\varepsilon_i, q, \omega),$$

$$J^\mu = J^\mu(q, \omega, \theta_N, \phi_N, E_m)$$

If **q** is not along **z**, then hadronic-leptonic variables are mixed. The neutrino energy appears in the hadronic current:

$$J^\mu = J^\mu(\varepsilon_i, q, \omega, \theta_N, \phi_N, E_m)$$

$$\frac{d^6\sigma}{d\ldots} \propto \ell_{\mu\nu} H^{\mu\nu}$$

5 indep. variables.
If you have less...
better think about it!

$$\ldots j_\nu$$

$$\ldots \sum (J^\mu)^* J^\nu$$

$$j_\mu = j_\mu(\varepsilon_i, q, \omega),$$

$$J^\mu = \boxed{J^\mu(q, \omega, \theta_N, \phi_N, E_m)}$$

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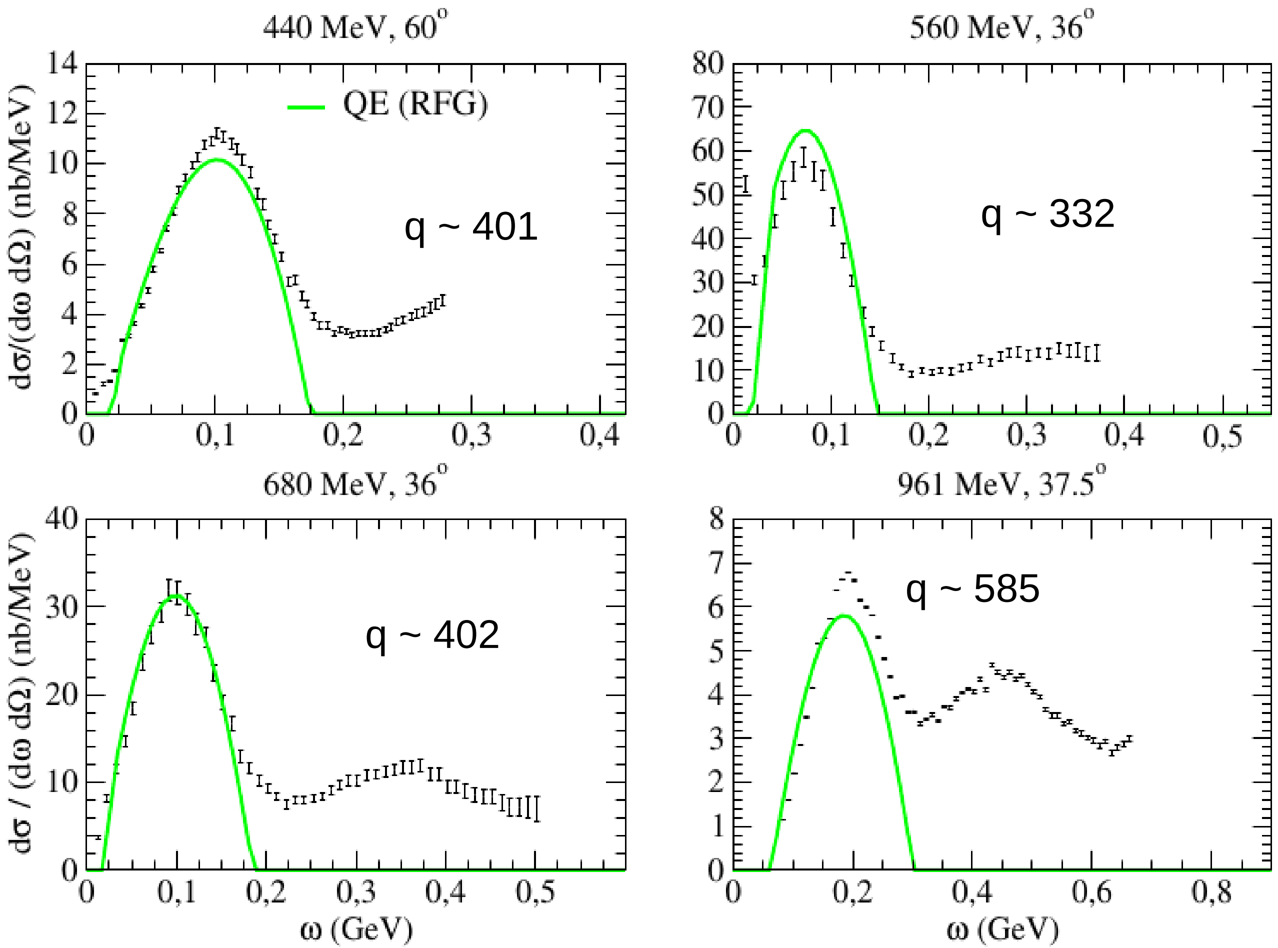
Mean-field vs plane waves

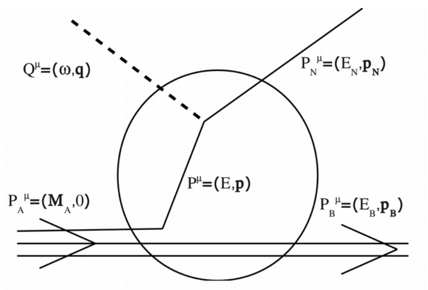
Intermediate energies (typical QE regime)



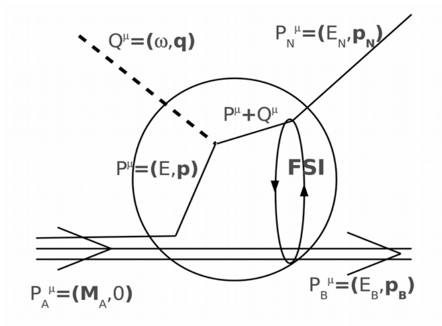
Mean-field vs plane waves

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

$$J^{\mu}_{had} = \sum_i^A \int d\mathbf{r}\,\overline{\Psi}_F(\mathbf{r})\,\hat{O}^{\mu}_{one\text{-}body}\,\Psi_B(\mathbf{r})\,e^{i\mathbf{q}\cdot\mathbf{r}}$$

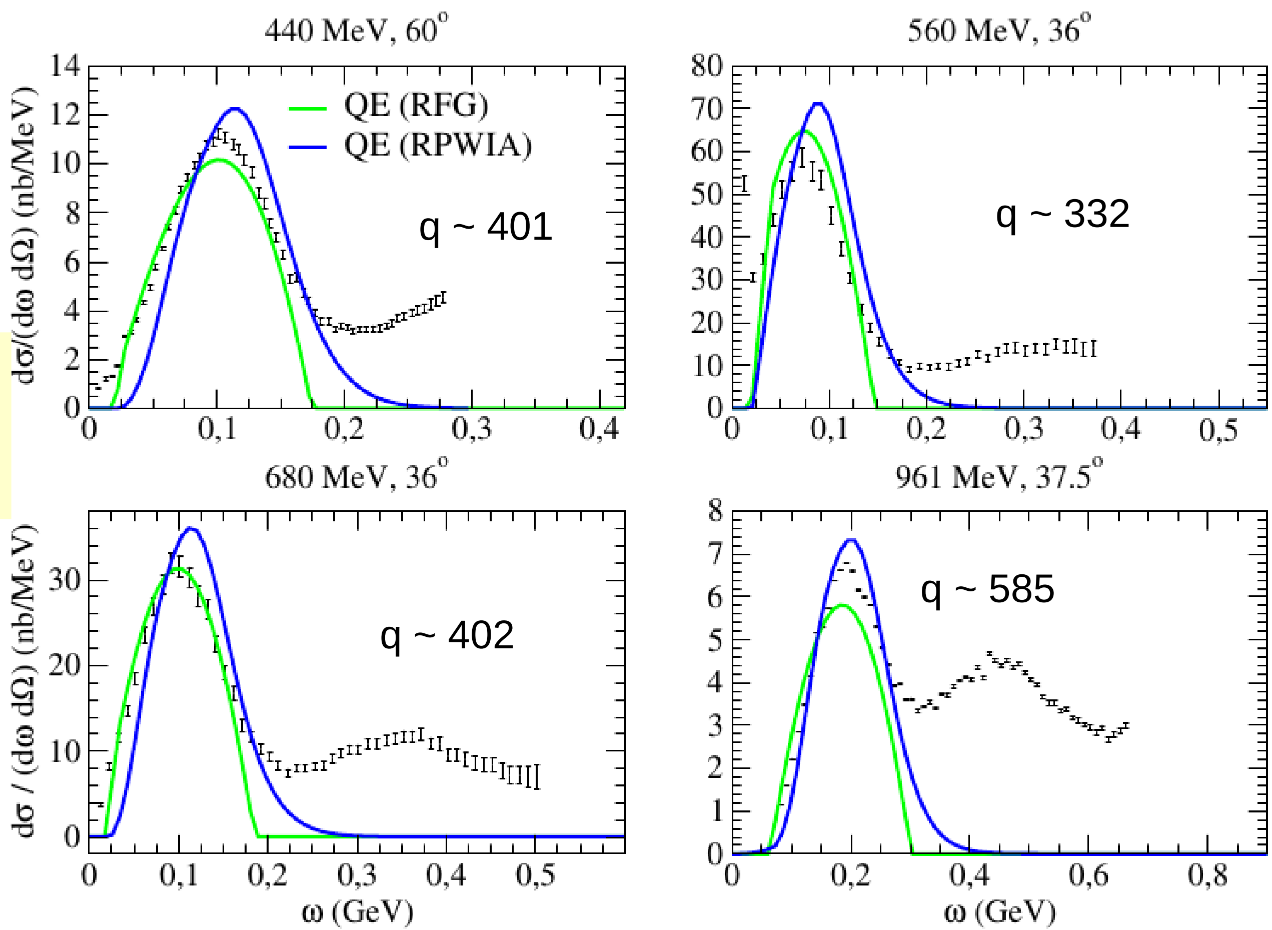


RPWIA: Scattered nucleon wf is described as a Dirac plane wave.

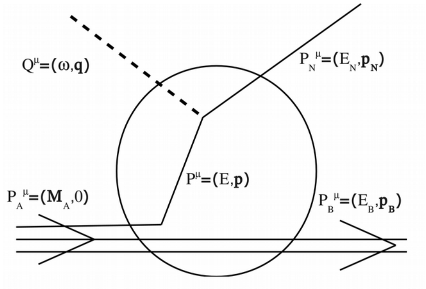


RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

Intermediate energies (typical QE regime)



Mean-field vs plane waves



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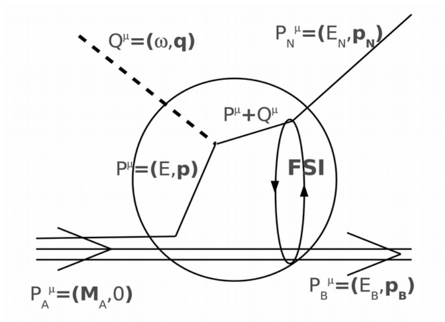
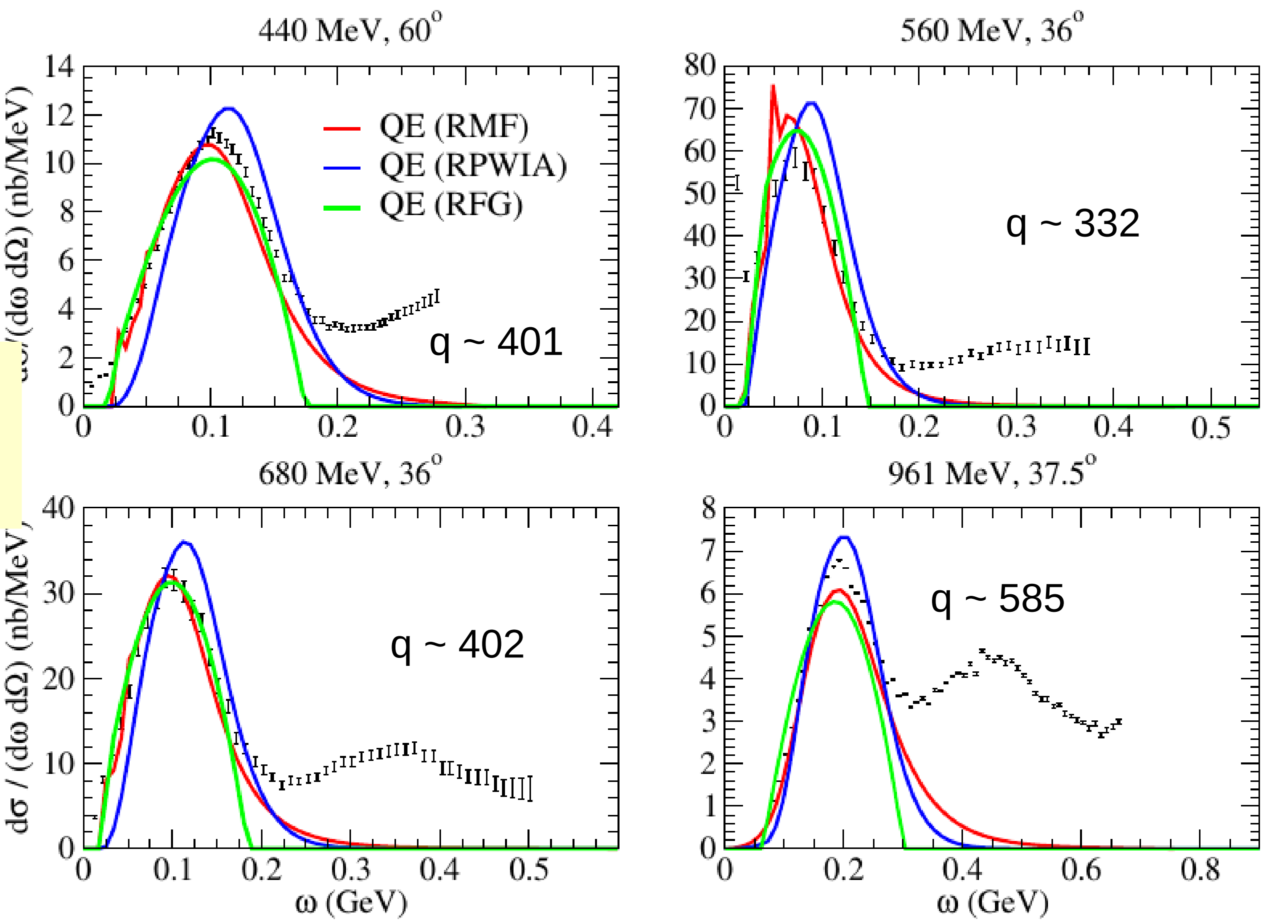


RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+V(r)+\beta(M+S(r)]\Psi_i(\boldsymbol{r})=E_i\Psi_i(\boldsymbol{r})$$

$$J^\mu_{had}=\sum_i^A\int d\mathbf{r}\,\overline{\Psi}_F(\mathbf{r})\,\hat{\mathcal{O}}^\mu_{one\text{-}body}\,\Psi_B(\mathbf{r})\,e^{i\mathbf{q}\cdot\mathbf{r}}$$

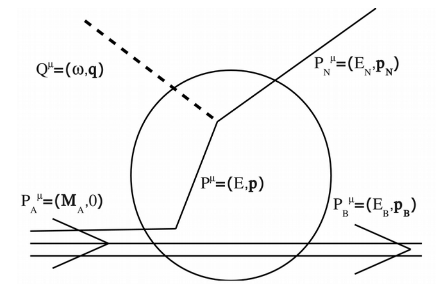
Intermediate energies (typical QE regime)



Mean-field vs plane waves

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

$$J^{\mu}_{had} = \sum_{i}^{A} \int d\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \hat{\mathcal{O}}^{\mu}_{one\text{-}body}\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$



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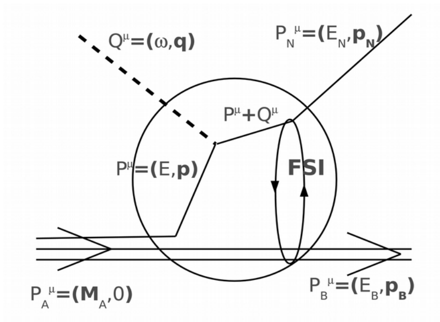
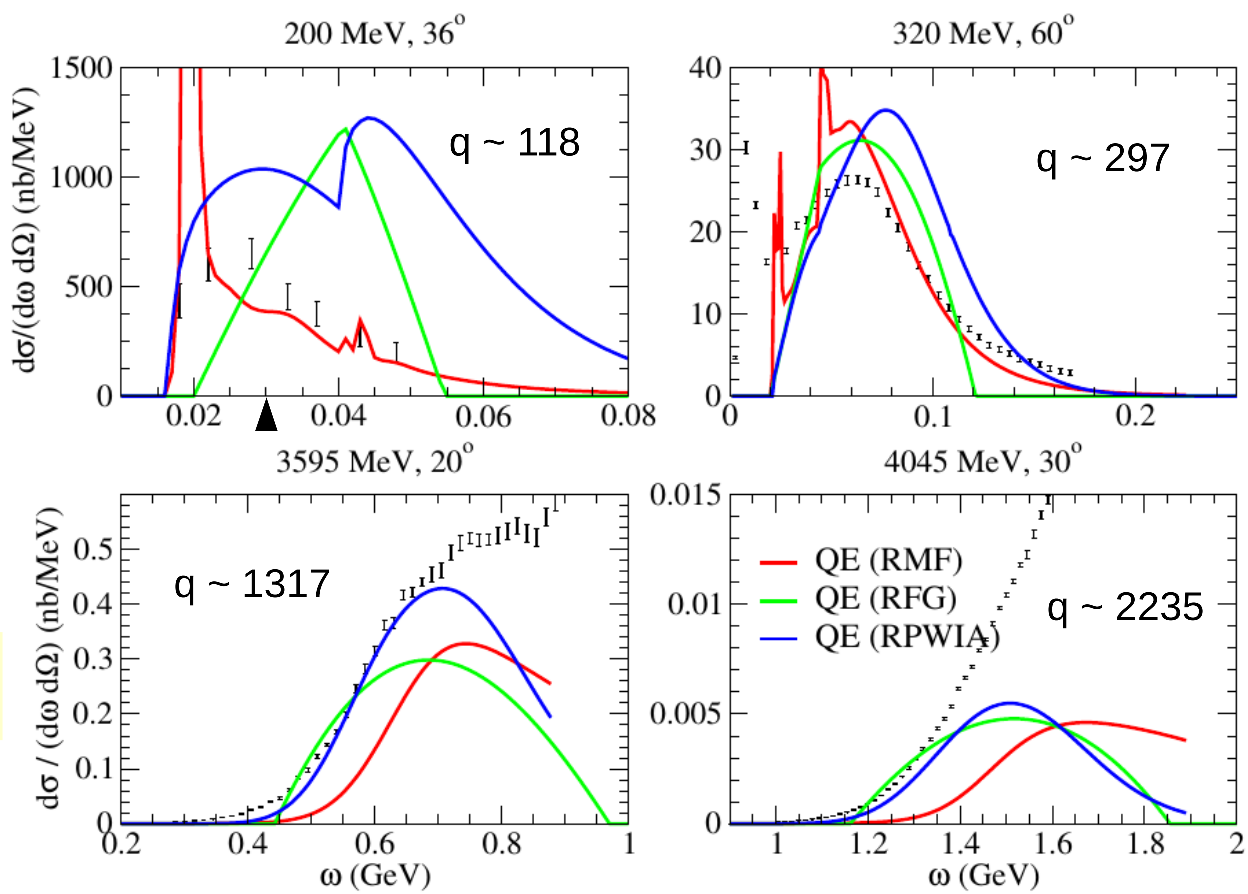


RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

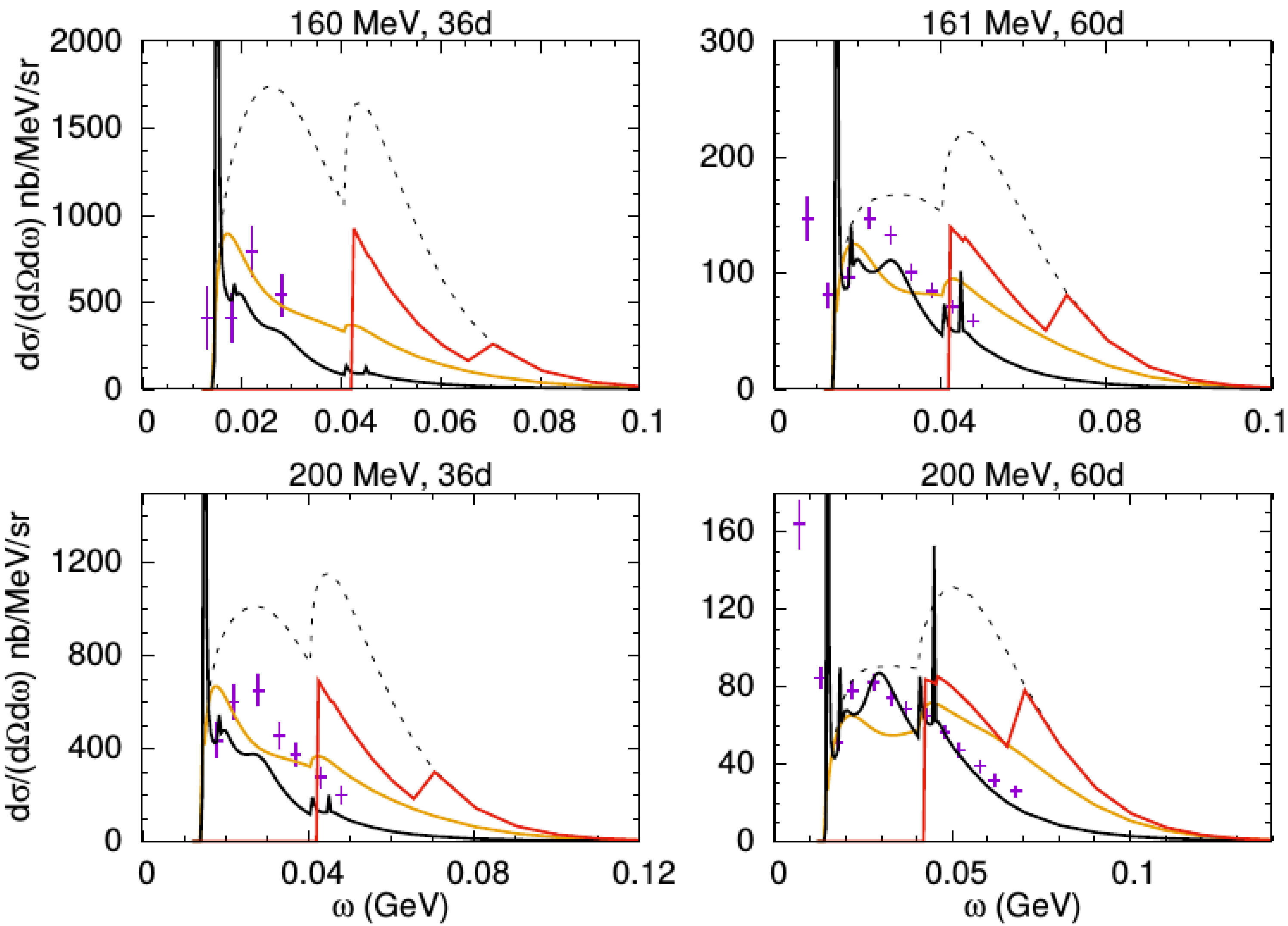
Low energies

High energies



$^{12}C(e,e')$

Quasielastic: RPWIA, Pauli blocked RPWIA and RMF



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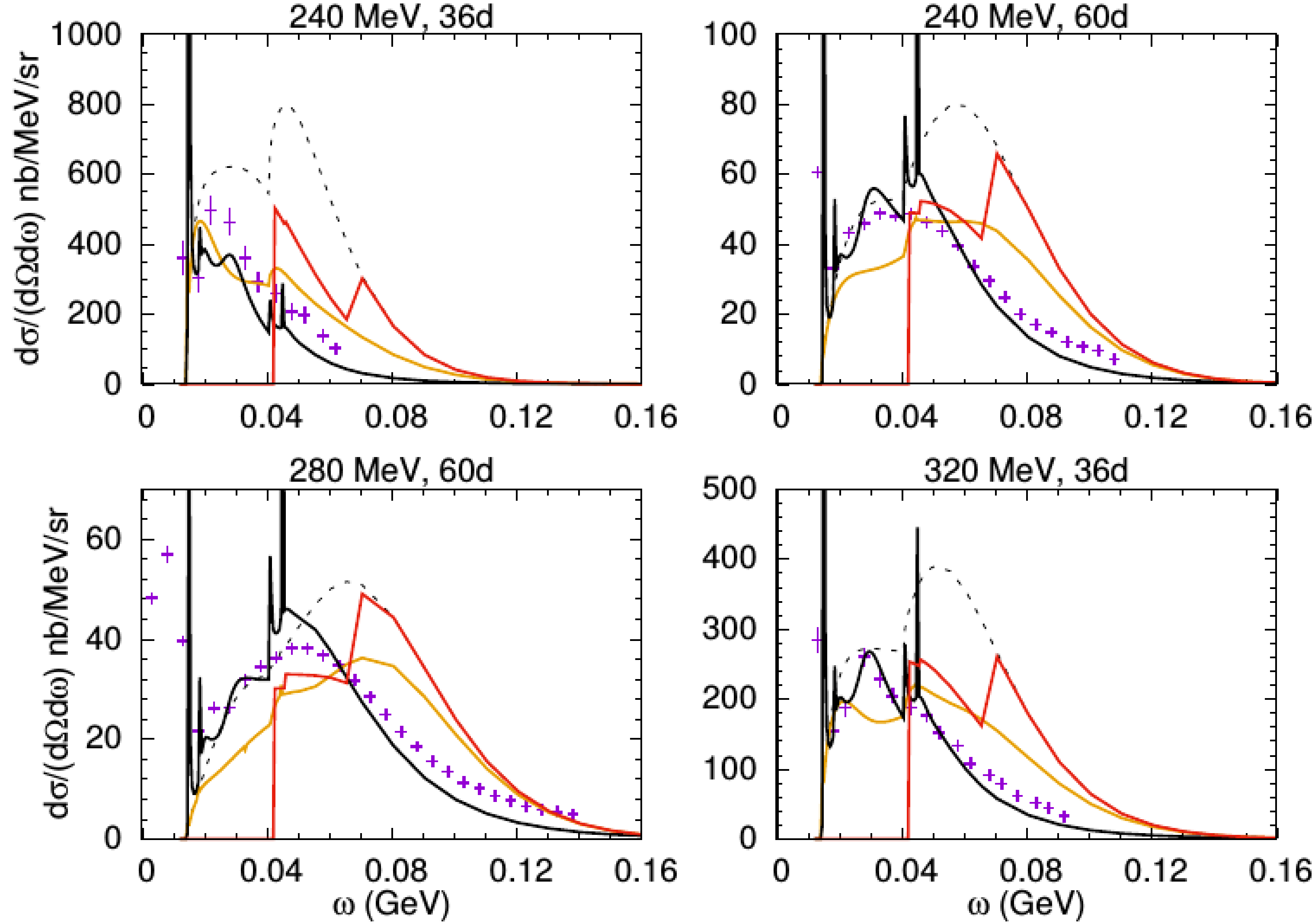

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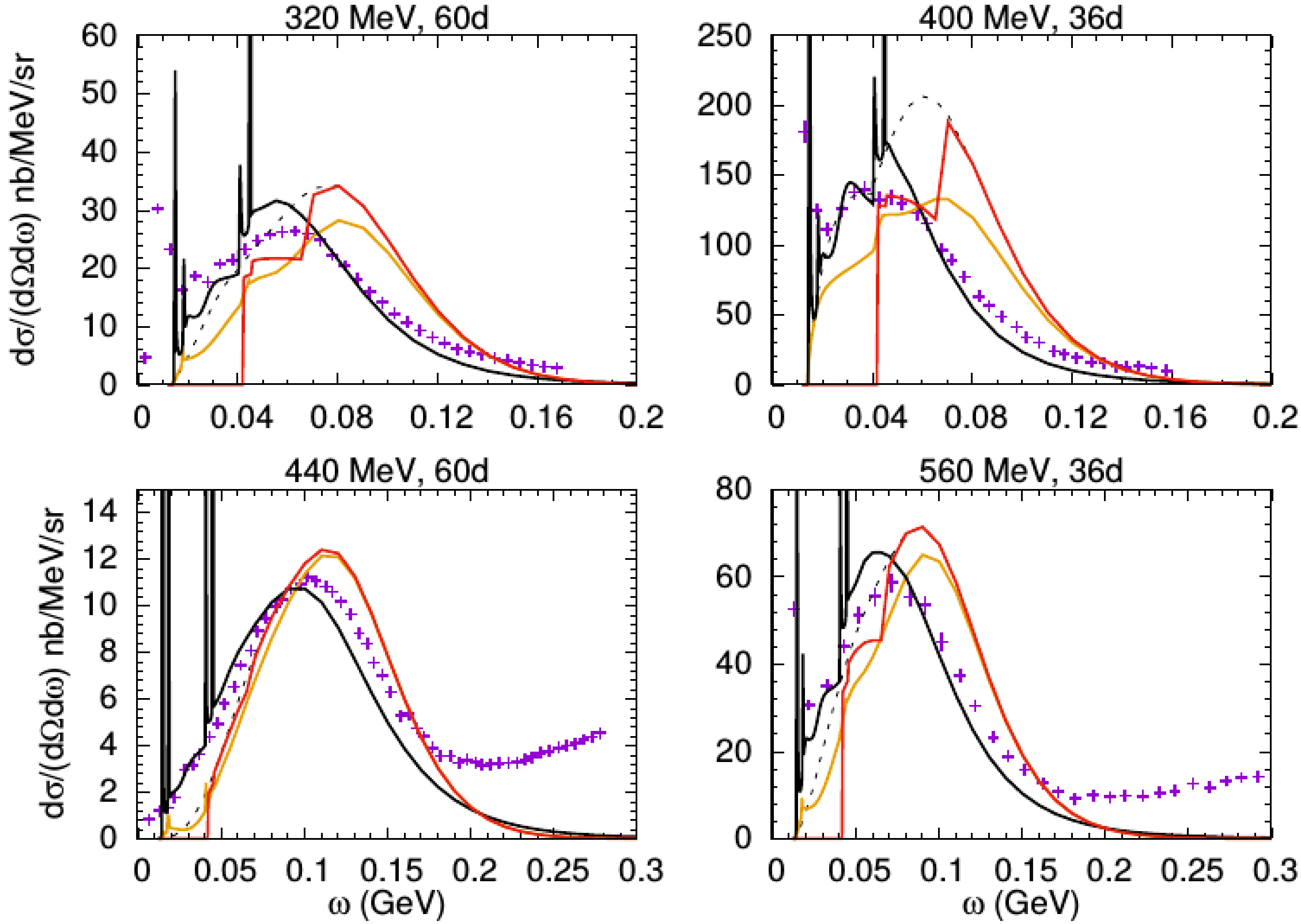

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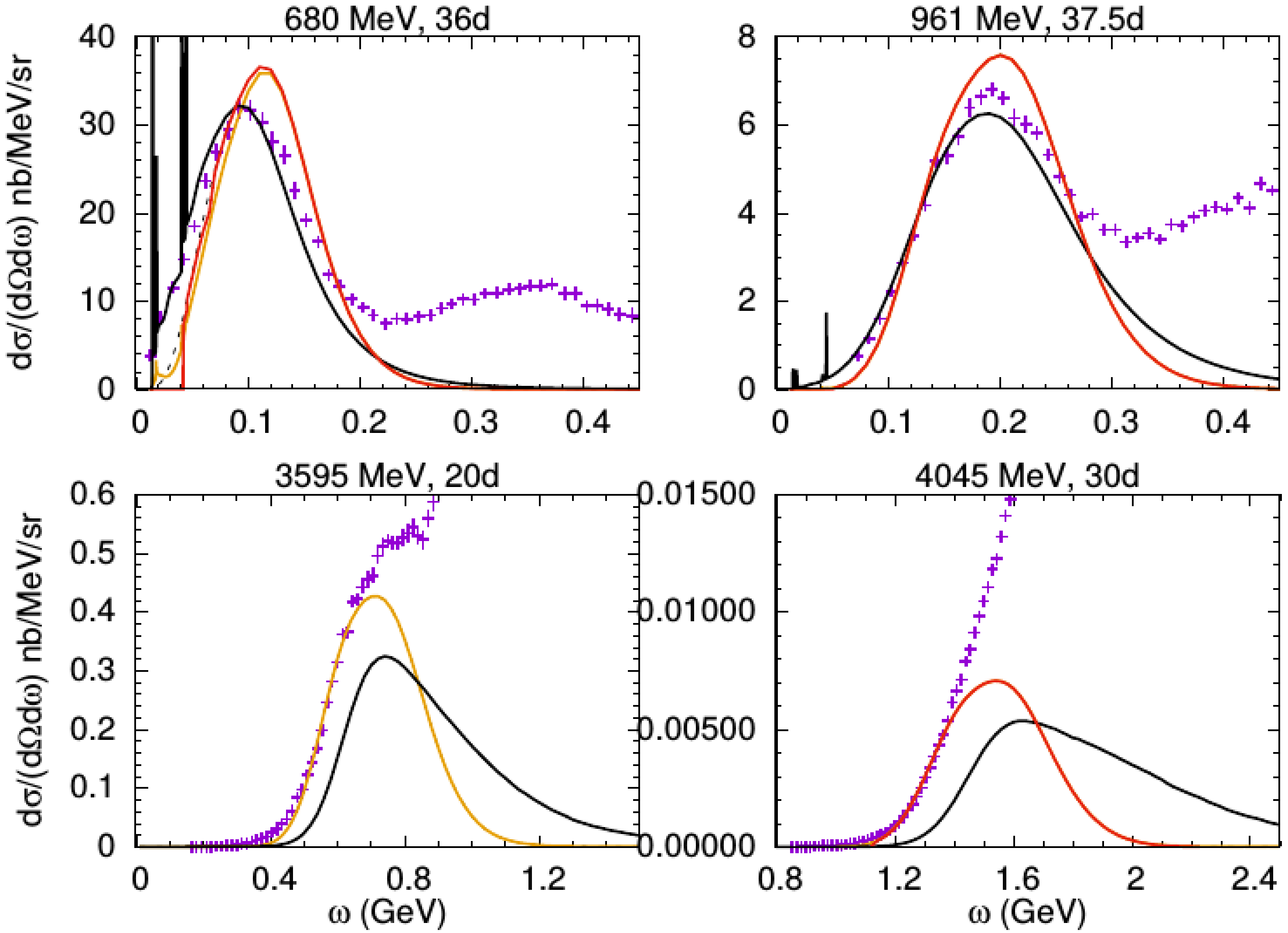


Relativistic mean-field model

RMF model provides a microscopic description of the ground state of finite nuclei which is consistent with Quantum Mechanic, Special Relativity and symmetries of strong interaction.

Extension of the original σ-ω Walecka model (Ann. Phys.83,491 (1974)).

The starting point is a Lorentz covariant Lagrangian density

$$
\begin{aligned}
\mathcal{L} &= \overline{\Psi}\left(i\gamma_\mu\partial^\mu - M\right)\Psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) \\
&- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&- g_\sigma\overline{\Psi}\sigma\Psi - g_\omega\overline{\Psi}\gamma_\mu\omega^\mu\Psi - g_\rho\overline{\Psi}\gamma_\mu\boldsymbol{\tau}\boldsymbol{\rho}^\mu\Psi - g_e\frac{1+\tau_3}{2}\overline{\Psi}\gamma_\mu A^\mu\Psi\,.
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega^{\mu\nu} &= \partial^\mu\omega^\nu - \partial^\nu\omega^\mu\,, \\
\boldsymbol{R}^{\mu\nu} &= \partial^\mu\boldsymbol{\rho}^\nu - \partial^\nu\boldsymbol{\rho}^\mu\,, \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu\,. \\
U(\sigma) &= \tfrac{1}{3}g_2\sigma^3 + \tfrac{1}{4}g_3\sigma^4
\end{aligned}
$$

Main approximations:

1) Mean-field approximation:

$\omega_\mu \rightarrow \langle\omega_\mu\rangle$ $\quad \sigma \rightarrow \langle\sigma\rangle$ $\quad \boldsymbol{\rho}_\mu \rightarrow \langle\boldsymbol{\rho}_\mu\rangle$

2) Static limit:

$\partial^0\omega_0 = \partial^0\boldsymbol{\rho}_0 = \partial^0\sigma = 0$ $\quad \omega_\mu = \delta_{\mu 0}\omega_0\,, \quad \boldsymbol{\rho}_\mu = \delta_{\mu 0}\boldsymbol{\rho}_0$

3) Spherical symmetry for finite nuclei:

$\omega_0 = \omega_0(r)$ $\quad \boldsymbol{\rho}_0 = \boldsymbol{\rho}_0(r)$ $\quad \sigma = \sigma(r)$

Relativistic mean-field model

Dirac equation for nucleons (eq. of motion for the barionic fields):

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

where the scalar (S) and vector (V) potential are given by:

$$\begin{aligned} S(r) &= g_\sigma\sigma(r)\,, \\ V(r) &= g_\omega\omega^0(r) + g_\rho\tau_3\rho_3^0(r) + e\frac{1+\tau_3}{2}A^0(r) \end{aligned}$$

Eqs. of motion for the mesons and the photon:

$$\begin{aligned} \left[-\nabla^2 + m_\sigma^2\right]\sigma(r) &= -g_\sigma\rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r)\,, \\ \left[-\nabla^2 + m_\omega^2\right]\omega^0(r) &= -g_\omega\rho_B(r)\,, \\ \left[-\nabla^2 + m_\rho^2\right]\rho_3^0(r) &= -g_\rho\rho_\rho(r)\,, \\ -\nabla^2 A^0 &= e\rho_c\,, \end{aligned}$$

Current densities

$$\begin{aligned} \rho_s(r) &= \sum_i^A \overline{\Psi}_i(\boldsymbol{r})\Psi_i(\boldsymbol{r})\,, \\ \rho_B(r) &= \sum_i^A \Psi_i^\dagger(\boldsymbol{r})\Psi_i(\boldsymbol{r})\,, \\ \rho_\rho(r) &= \sum_i^A \Psi_i^\dagger(\boldsymbol{r})\tau_3\Psi_i(\boldsymbol{r}) \\ \rho_c(r) &= \sum_i^A \Psi_i^\dagger(\boldsymbol{r})\frac{1+\tau_3}{2}\Psi_i(\boldsymbol{r}) \end{aligned}$$

Solution of the couple equations for the fields in a self-consistent way.

Relativistic mean-field model

In general, the parameters are fit to reproduce some general properties of some closed shell spherical nuclei and nuclear matter.

Parameters for the NLSH model (fitted to the mean charge radius, binding energy and neutron radius of the **^{16}O, ^{40}Ca, ^{90}Zr, ^{116}Sr, ^{124}Sn** and **^{208}Pb.**

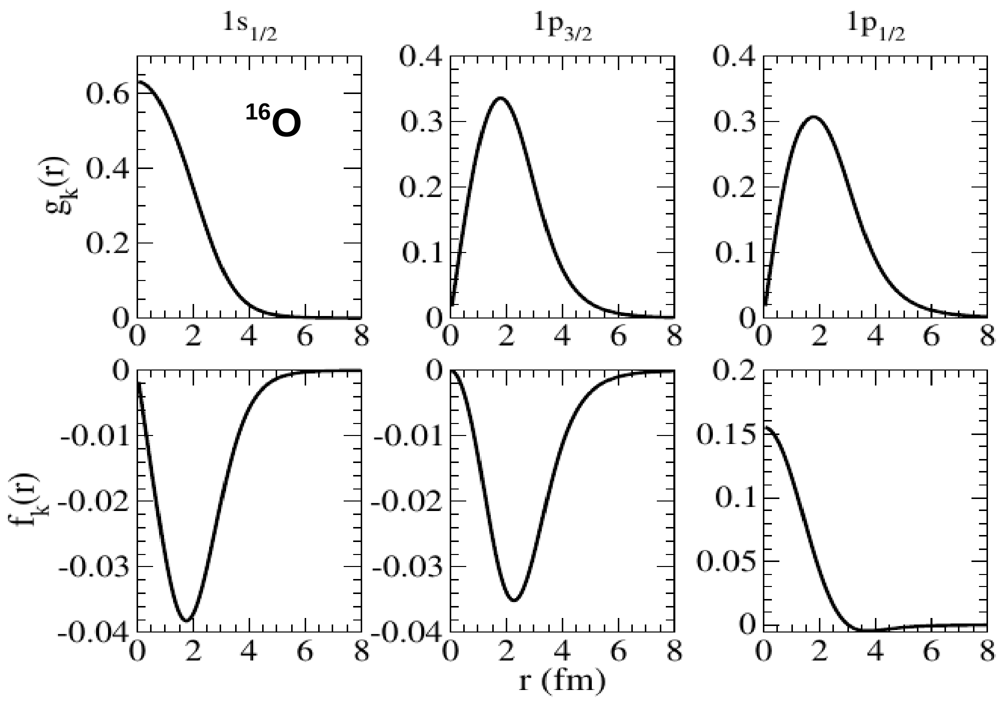
M_N	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2	g_3
939.0	526.059	783.0	763.0	10.444	12.945	4.3830	-6.9099	-15.8337

6 free parameters

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

$$\Psi_k^{m_j}(\boldsymbol{r}) = \begin{pmatrix} g_k(r)\varphi_k^{m_j}(\Omega_r) \\ i f_k(r)\varphi_{-k}^{m_j}(\Omega_r) \end{pmatrix},$$

$$\varphi_k^{m_j}(\Omega_r) = \sum_{m_\ell s}\langle \ell m_\ell \frac{1}{2} s | j m_j\rangle Y_\ell^{m_\ell}(\Omega_r)\chi^s$$



Back slides: isospin coefficients and resonances parameters

Channel	ΔP	$C\Delta P$	NP	CNP	Others
$p \to \pi^+ + p$	$\sqrt{3/2}$	$\sqrt{1/6}$	0	1	1
$n \to \pi^0 + p$	$-\sqrt{1/3}$	$\sqrt{1/3}$	$\sqrt{1/2}$	$-\sqrt{1/2}$	$-\sqrt{2}$
$n \to \pi^+ + n$	$\sqrt{1/6}$	$\sqrt{3/2}$	1	0	-1
$n \to \pi^- + n$	$\sqrt{3/2}$	$\sqrt{1/6}$	0	1	1
$p \to \pi^0 + n$	$\sqrt{1/3}$	$-\sqrt{1/3}$	$-\sqrt{1/2}$	$\sqrt{1/2}$	$\sqrt{2}$
$p \to \pi^- + p$	$\sqrt{1/6}$	$\sqrt{3/2}$	1	0	-1

Table: Isospin coefficients for the CC reaction.

Channel	ΔP	$C\Delta P$	NP	CNP	Others
$p \to \pi^0 + p$	$\sqrt{1/3}$	$\sqrt{1/3}$	$\sqrt{1/2}$	$\sqrt{1/2}$	0
$p \to \pi^+ + n$	$-\sqrt{1/6}$	$\sqrt{1/6}$	1	1	-1
$n \to \pi^- + p$	$\sqrt{1/6}$	$-\sqrt{1/6}$	1	1	1
$n \to \pi^0 + n$	$\sqrt{1/3}$	$\sqrt{1/3}$	$-\sqrt{1/2}$	$-\sqrt{1/2}$	0

Table: Isospin coefficients for the neutral current (EM and WNC) reactions.

	I	S	P	M_R	πN-br	$\Gamma^{exp}_{\text{width}}$	$f_{\pi N R}$
P_{33}	3/2	3/2	+	1232	100%	120	2.18
D_{13}	1/2	3/2	−	1515	60%	115	1.62
P_{11}	1/2	1/2	+	1430	65%	350	0.391
S_{11}	1/2	1/2	−	1535	45%	150	0.16

Table: quantum numbers and other parameters of the nucleon resonances.

Medium modifications of the Delta

Delta propagator:

$$S_{\Delta,\alpha\beta} = \frac{-(\not{K}_\Delta + M_\Delta)}{K_\Delta^2 - M_N^2 + iM_\Delta \Gamma_{\text{width}}} \left(g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3M_\Delta^2} K_{\Delta,\alpha}K_{\Delta,\beta} - \frac{2}{3M_\Delta}(\gamma_\alpha K_{\Delta,\beta} - K_{\Delta,\alpha}\gamma_\beta) \right)$$

with the energy dependent Delta width:

$$\Gamma_{\text{width}}(W) = \frac{1}{12\pi}\frac{(f_{\pi N\Delta})^2}{m_\pi^2 W}(p_{\pi,cm})^3(M + E_{N,cm})$$

$$\Gamma^{\text{free}}_{\text{width}} \longrightarrow \Gamma^{\text{in-medium}}_{\text{width}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_\Delta)\,, \quad M^{\text{free}}_\Delta \longrightarrow M^{\text{in-medium}}_\Delta = M^{\text{free}}_\Delta + \Re(\Sigma_\Delta)\,.$$

+ Γ_{Pauli}: some nucleons from Δ-decay are Pauli blocked (the Δ-decay width decreases).

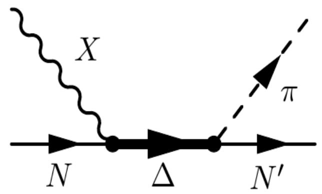
+ The parametrization of $\Im(\Sigma_\Delta)$ and $\Re(\Sigma_\Delta)$ is given in terms of the nuclear density ρ:

$$\begin{aligned} -\Im(\Sigma_\Delta) &= C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma\,, \\ \Re(\Sigma_\Delta) &= 40\text{ MeV}\,(\rho/\rho_0)\,. \end{aligned}$$

We modify the free $\Delta\pi N$-decay constant ($f_{\Delta\pi N}$) to take into account the E-dependent medium modification of the Δ width:

$$f^{\text{in-medium}}_{\Delta\pi N}(W) = f_{\Delta\pi N}\sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE}\,(\rho/\rho_0)^\alpha}{\Gamma^{\text{free}}_{\text{width}}}}$$

Medium modifications of the Delta



$$-\Im(\Sigma_\Delta) = C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma$$

Each contribution corresponds to a different process:

- QE $\Longrightarrow \Delta N \to \pi NN$ (still one pion in the final state)
- A2 $\Longrightarrow \Delta N \to NN$ (no pions in the final state)
- A3 $\Longrightarrow \Delta NN \to NNN$ (no pions in the final state)

We modify the free Delta decay constant to take into account the E-dependent medium modification of the Delta-width

$$\Gamma^\alpha_{\Delta\pi N} = \frac{f_{\pi N\Delta}}{m_\pi} P^\alpha_\pi$$

$$f^{\text{in-medium}}_{\Delta\pi N}(W) = f_{\Delta\pi N}\sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE}\,(\rho/\rho_0)^\alpha}{\Gamma^{\text{free}}_{\text{width}}}}$$

References: [*] E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987).

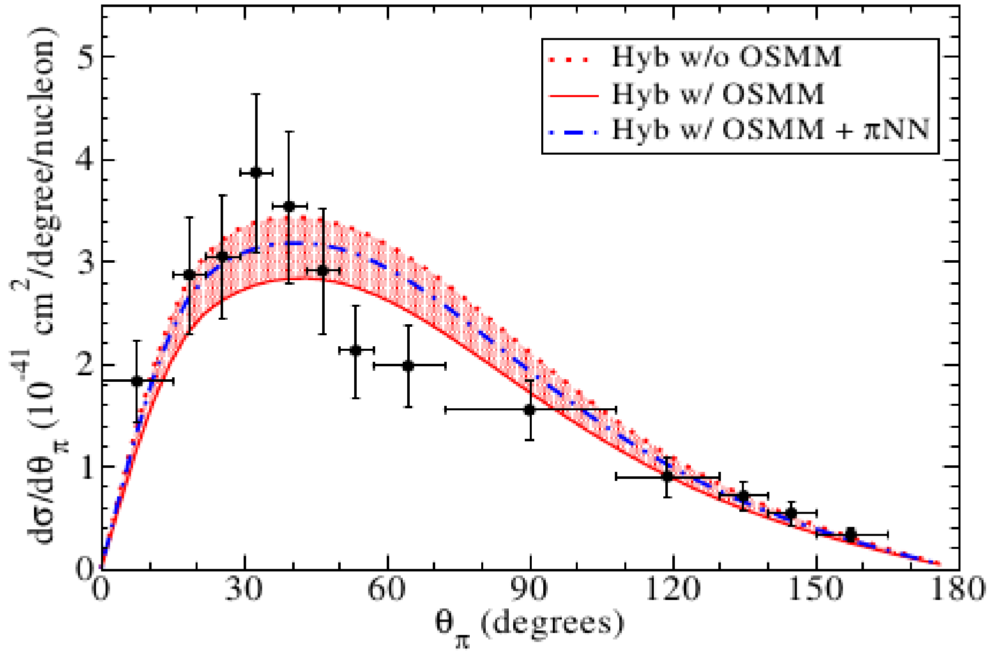


FIG. 3: MINERvA ν-induced $1\pi^+$ production sample [5] compared with RPWIA predictions. Solid (dotted) line is the result with (without) medium modification of the Delta width. The dash-dotted line is the result with OSMM when the contribution from the $\Delta N \to \pi NN$ channel is added to the cross section. The results were computed with the Hybrid model (see Sec IV B).

PRD 97, 013004 (2018)

Interferences

$$J^\nu = \langle J^\nu_{\Delta P}\rangle + \langle J^\nu_{C\Delta P}\rangle + \langle J^\nu_{CT,V}\rangle + \langle J^\nu_{CT,A}\rangle + \langle J^\nu_{NP}\rangle + \langle J^\nu_{CNP}\rangle + \langle J^\nu_{PF}\rangle + \langle J^\nu_{PP}\rangle$$

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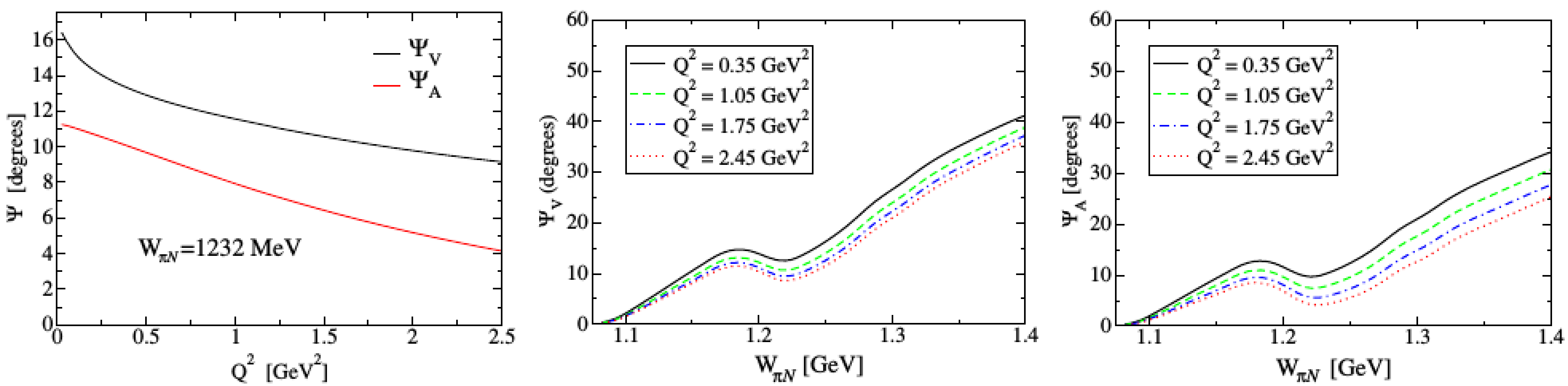
Watson's theorem and the $N\Delta(1232)$ axial transition

L. Alvarez-Ruso,[1] E. Hernández,[2] J. Nieves,[1] and M. J. Vicente Vacas[3]

We present a new determination of the $N\Delta$ axial form factors from neutrino induced pion production data. For this purpose, the model of Hernandez *et al.* [Phys. Rev. D 76, 033005 (2007)] is improved by partially restoring unitarity. This is accomplished by imposing Watson's theorem on the dominant vector and axial multipoles. As a consequence, a larger $C_5^A(0)$, in good agreement with the prediction from the off-diagonal Goldberger-Treiman relation, is now obtained.



High-energy model

“Reggeizing” the ChPT background:

$$\mathcal{O}^{\mu}_{ReChi,V} = \mathcal{O}^{\mu}_{ChPT,V}\, \mathcal{P}_{\pi}(t,s)(t-m_{\pi}^{2})$$

high-energy model: ReChi (from Reggeized ChPT background)

low-energy model (only the ChPT background)

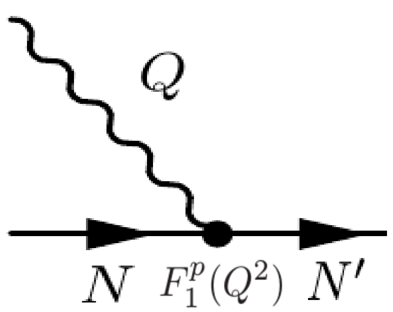
The pion propagator is replaced by the **Regge propagator** of the pion trajectory

$$\mathcal{O}^{\mu}_{ReChi,A} = \mathcal{O}^{\mu}_{ChPT,A}\, \mathcal{P}_{\rho}(t,s)(t-m_{\rho}^{2})$$

High-energy model

Regge approach for the vector amplitudes.

We use the approach proposed by **Kaskulov and Mosel** [PRC81, 045202 (2010)] to extends GLV to the case of pion electroproduction ($Q^2 \neq 0$).



The nucleon N' may be highly off its mass shell.
Therefore, instead of using the on shell form factor $F_1^p(Q^2)$.
We use a form factor that accounts for the off shell character of the nucleon [**Vrancx and Ryckebusch**, PRC89, 025203 (2014)]:

$$F_1^p(Q^2, s) = \left(1 + \frac{Q^2}{\Lambda_{\gamma pp^*}(s)^2}\right)^{-2}$$

$$\Lambda_{\gamma pp^*}(s) = \Lambda_{\gamma pp} + (\Lambda_\infty - \Lambda_{\gamma pp})\left(1 - \frac{M^2}{s}\right)$$

$$\Lambda_\infty = 2.194 \text{ GeV}$$

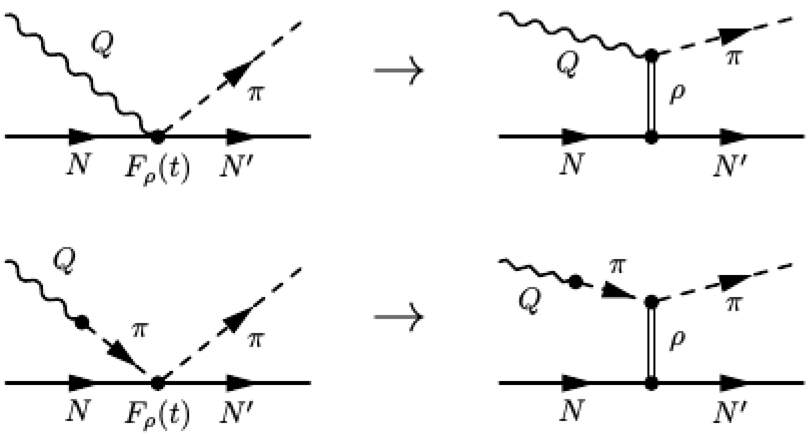
In the (on shell) limit the Dirac form factor is recovered.

High-energy model

Regge approach for the axial amplitudes.

We need meson exchange diagrams to apply the reggeization procedure of the current.

Effective rho-exchange diagrams. This allows us to consider the rho-exchange as the main Regge trajectory in the axial current.



$$\mathcal{O}^{\mu}_{CT\rho} = i\mathcal{I}\,\frac{m_\rho^2}{m_\rho^2 - t} F_{A\rho\pi}(Q^2)\frac{1}{\sqrt{2}f_\pi} \times \left(\gamma^\mu + i\frac{\kappa_\rho}{2M}\sigma^{\mu\nu}K_{t,\nu}\right).$$

We consider $\kappa_\rho = 0$ so that the low-energy model amplitude is recovered.

The propagator of the rho is replaced by the Regge trajectory of the **rho family**:

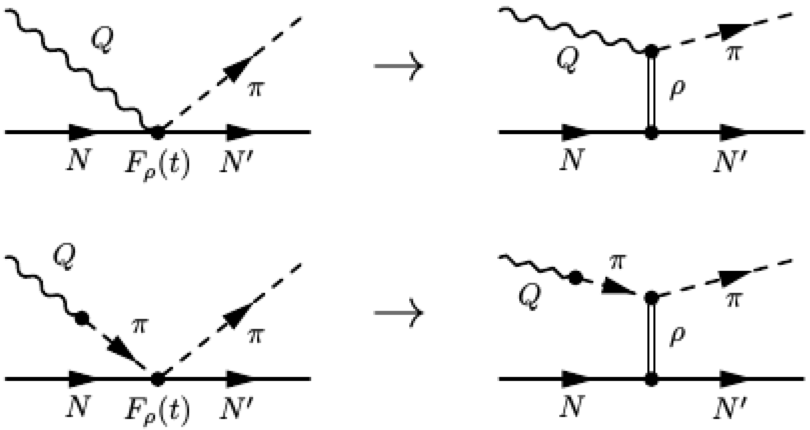
$$\mathcal{P}_\rho(t,s) = -\alpha'_\rho \varphi_\rho(t)\Gamma[1-\alpha_\rho(t)](\alpha'_\rho s)^{\alpha_\rho(t)-1}$$

High-energy model

Regge approach for the axial amplitudes.

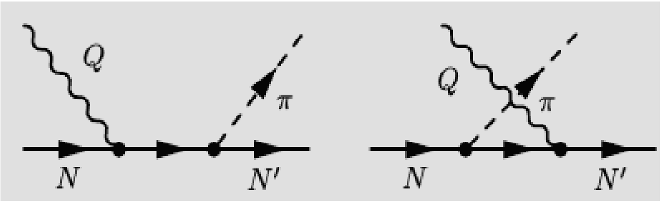
We need meson exchange diagrams to apply the reggeization procedure of the current.

Effective rho-exchange diagrams. This allows us to consider the rho-exchange as the main Regge trajectory in the axial current.



$$\mathcal{O}^{\mu}_{CT\rho} = i\mathcal{I}\frac{m_\rho^2}{m_\rho^2 - t} F_{A\rho\pi}(Q^2)\frac{1}{\sqrt{2}f_\pi} \times \left(\gamma^\mu + i\frac{\kappa_\rho}{2M}\sigma^{\mu\nu}K_{t,\nu}\right) .$$

We consider $\kappa_\rho = 0$ so that the low-energy model amplitude is recovered.

\+  ?

High-energy model: results

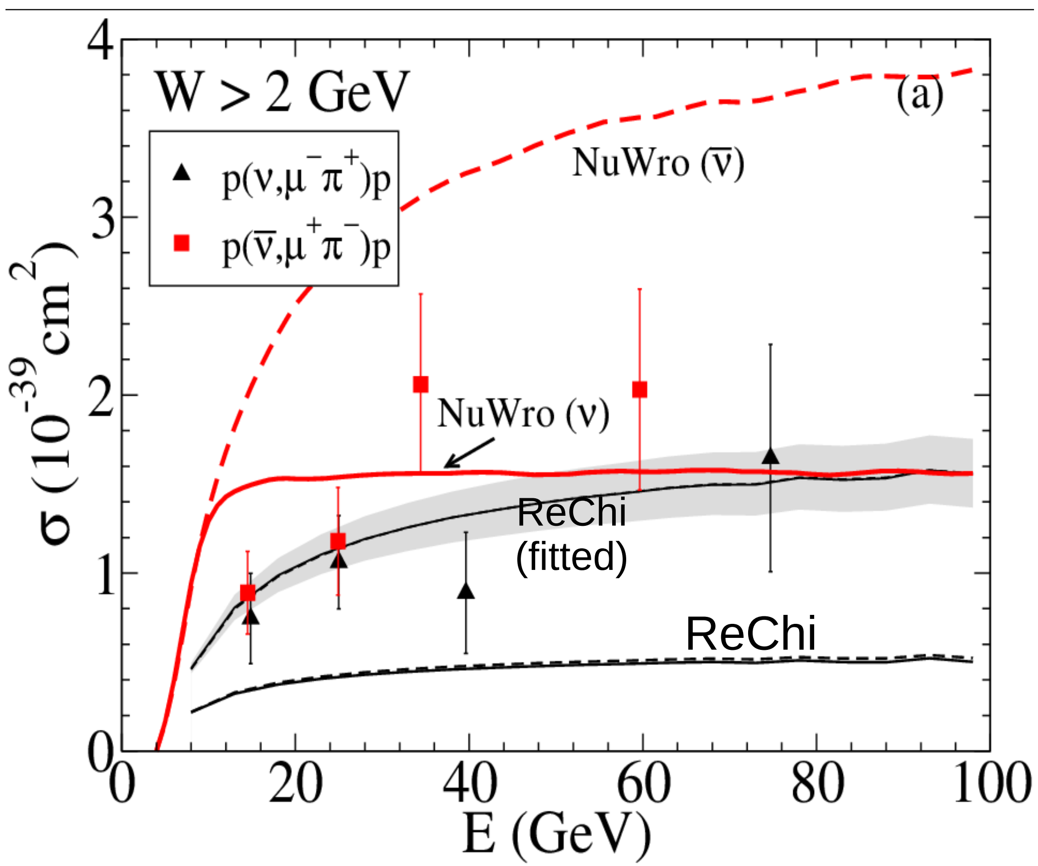


Figure: ReChi model and NuWro predictions are compared with high energy cross section data for neutrino and antineutrino reactions (Note the high energy cut W>2 GeV !!). Data from Allen et al. NPB264, 221 (1986).

NuWro: Based on DIS formalism and PYTHIA for hadronization.

Antineutrino cross section is ~2 the neutrino one:

$$\bar{\nu} + \overbrace{uud}^{p} \to \mu^{+} + \overbrace{\bar{u}d}^{\pi^{-}} + uud\,,$$
$$\nu + uud \to \mu^{-} + \underbrace{u\bar{d}}_{\pi^{+}} + uud\,.$$

High-energy model: results

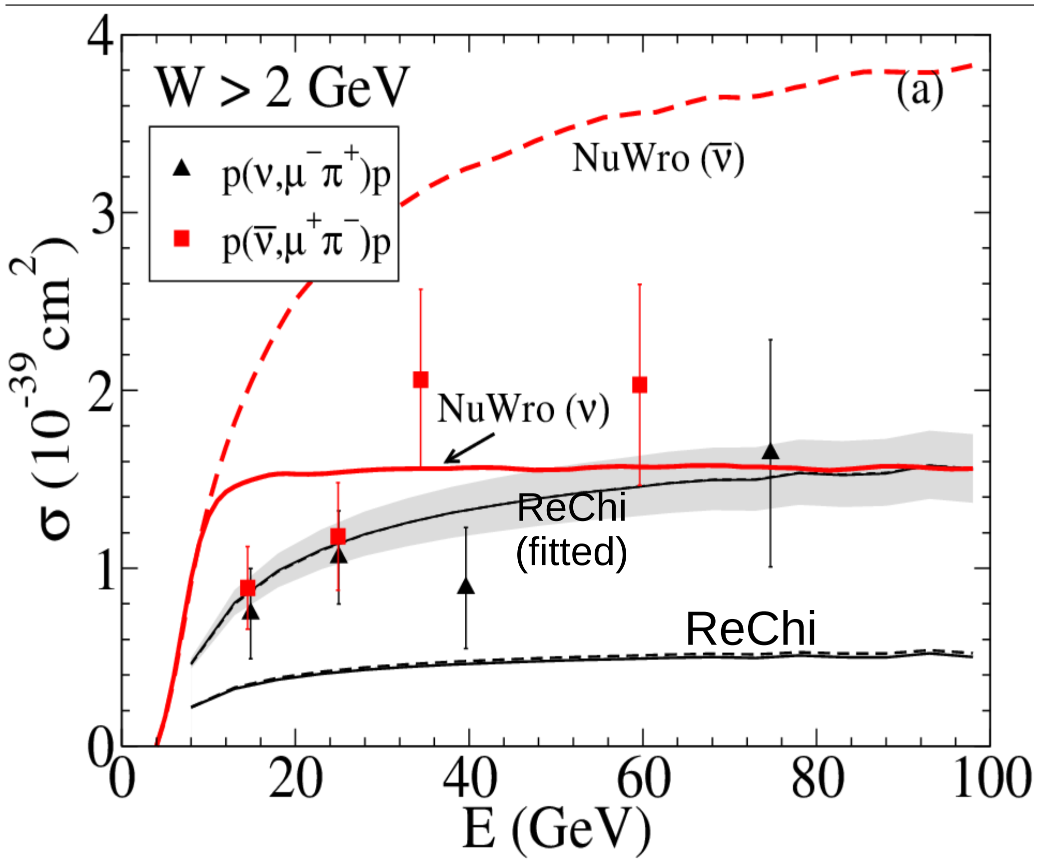
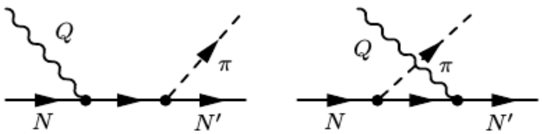


Figure: ReChi model and NuWro predictions are compared with high energy cross section data for neutrino and antineutrino reactions (Note the high energy cut W>2 GeV !!). Data from Allen et al. NPB264, 221 (1986).

ReChi model: One free parameter in the boson-nucleon-nucleon vertex



$$G_A[Q^2, s(u)] = g_A \left(1 + \frac{Q^2}{\Lambda_{Apn^*}[s(u)]^2}\right)^{-2}$$

$$\Lambda_{Anp^*}(s) = \Lambda_{Apn} + (\Lambda^A_\infty - \Lambda_{Apn})\left(1 - \frac{M^2}{s}\right)$$

$\Lambda^A_\infty = (7.20 \pm {}^{2.09}_{1.32})$ GeV !!!

NuWro: Based on DIS formalism and PYTHIA for hadronization.

Antineutrino cross section is ~2 the neutrino one:

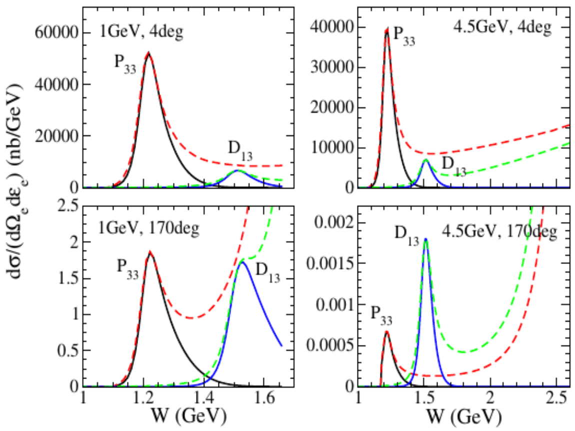
$$\bar{\nu} + \overbrace{uud}^{p} \to \mu^+ + \overbrace{\bar{u}d}^{\pi^-} + uud\,,$$
$$\nu + uud \to \mu^- + \underbrace{u\bar{d}}_{\pi^+} + uud\,.$$

Hybrid model

1) Regularizing the behavior of resonances (u- and s-channel contributions): we multiply the resonance amplitude by a dipole-Gaussian form factor

$$F(s,u) = F(s) + F(u) - F(s)F(u)$$

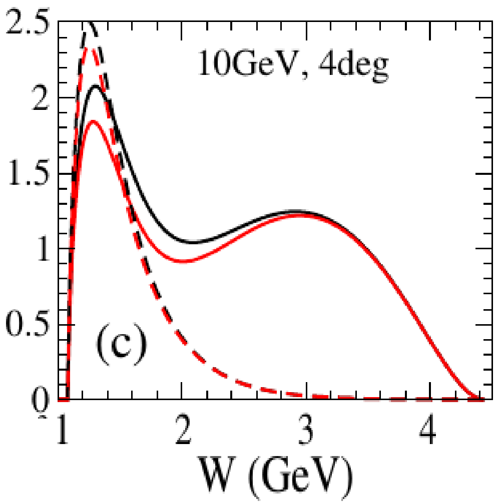
$$F(s) = \exp\left(\frac{-(s-M_R^2)^2}{\lambda_R^4}\right)\ \frac{\lambda_R^4}{(s-M_R^2)^2+\lambda_R^4}$$



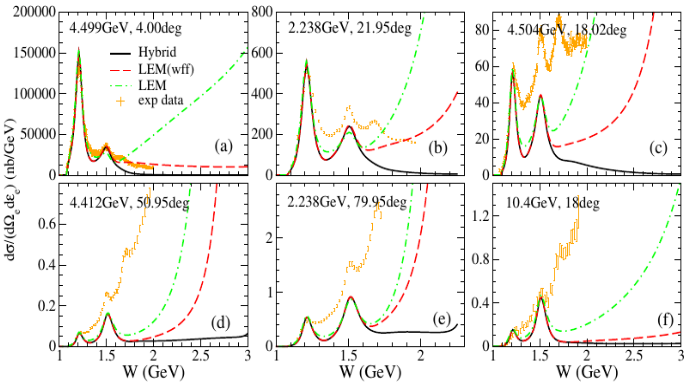
2) Gradually replacing the ChPT background by the High-energy (ReChi) model: we use a phenomenological transition function

$$\widetilde{\mathcal{O}} = \cos^2\phi(W)\mathcal{O}_{ChPT} + \sin^2\phi(W)\mathcal{O}_{ReChi}$$

$$\phi(W) = \frac{\pi}{2}\left(1 - \frac{1}{1+\exp\left[\frac{W-W_0}{L}\right]}\right), \qquad W_0 = 1.7\ \text{GeV}, \quad L = 100\ \text{MeV}$$

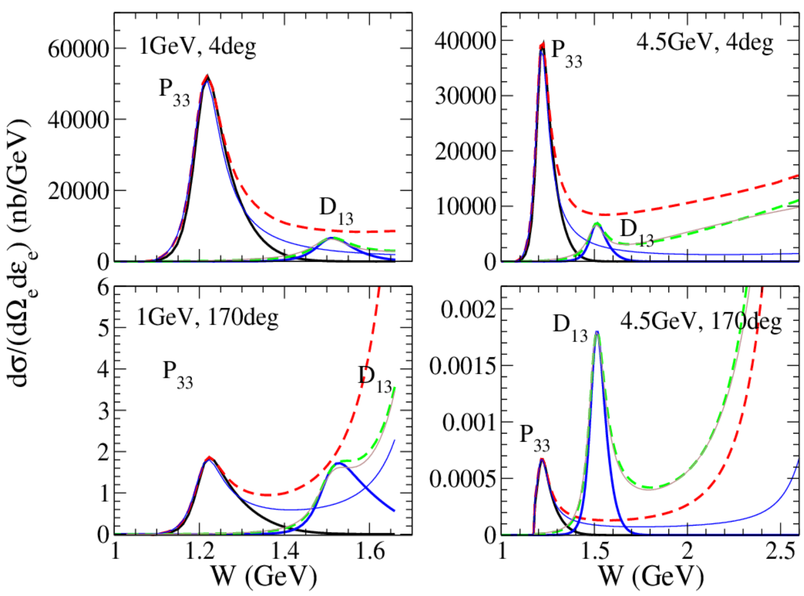


The Problem

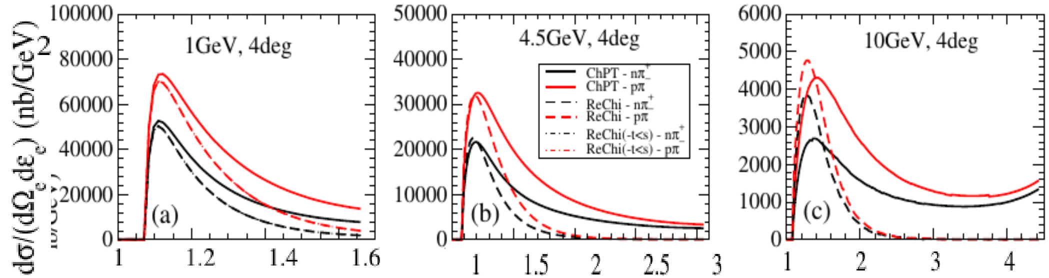


The pathologies come from the

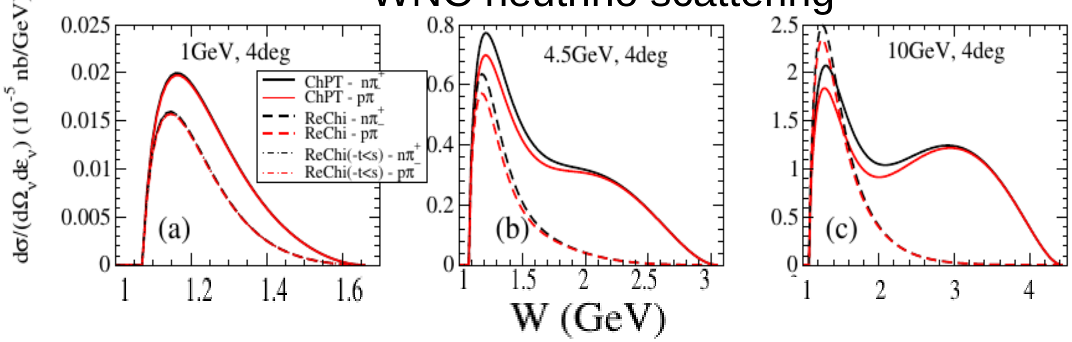
resonances and **background terms**



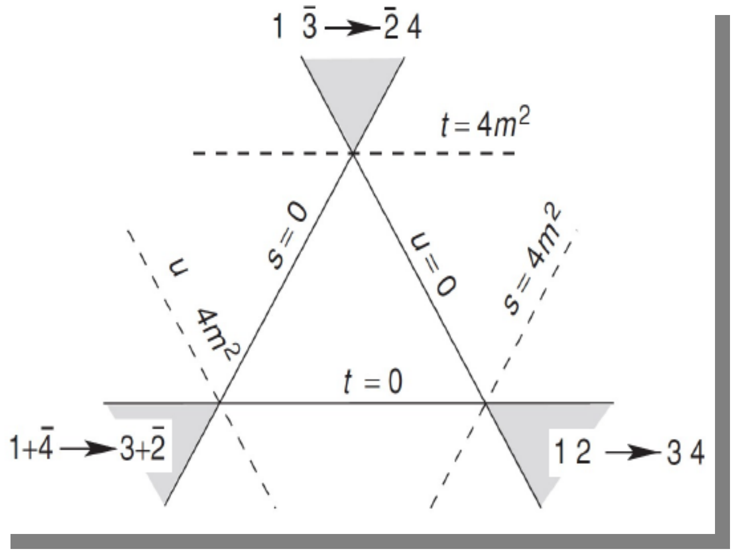
Electron scattering

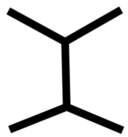


WNC neutrino scattering



Why does this happen?



Cross channels: 

$$\mathcal{A}(t,s) = \sum_{\ell} (2\ell+1)\, A_\ell(t)\, P_\ell(z_t)$$

$$P_\ell(z_t) \xrightarrow{s\to\infty} (2s)^\ell$$

$$z_t \equiv \cos\theta_t = 1 + \frac{2s}{t-4m^2}$$

Direct channels: 

$$\mathcal{A}(s,t) = \sum_{\ell} (2\ell+1)\, A_\ell(s)\, P_\ell(z_s)$$

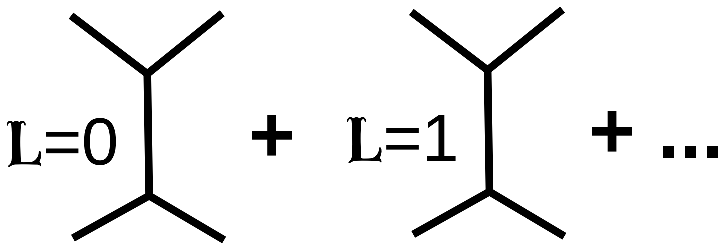
$$z_s \equiv \cos\theta_s = 1 + \frac{2t}{s-4m^2}$$

$$A_\ell(s) \sim \left(\frac{s-4m^2}{2}\right)^\ell$$

Behavior at threshold (barrier factor).
Feynman diagrams provide the right behavior at threshold but not at high s

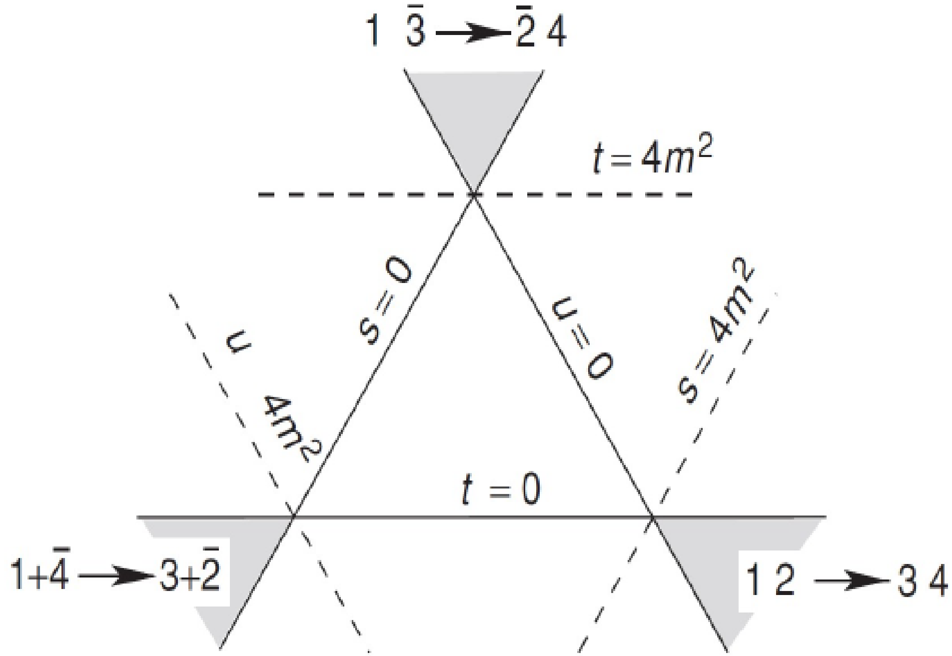
Regge Theory

$$z_t \equiv \cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$



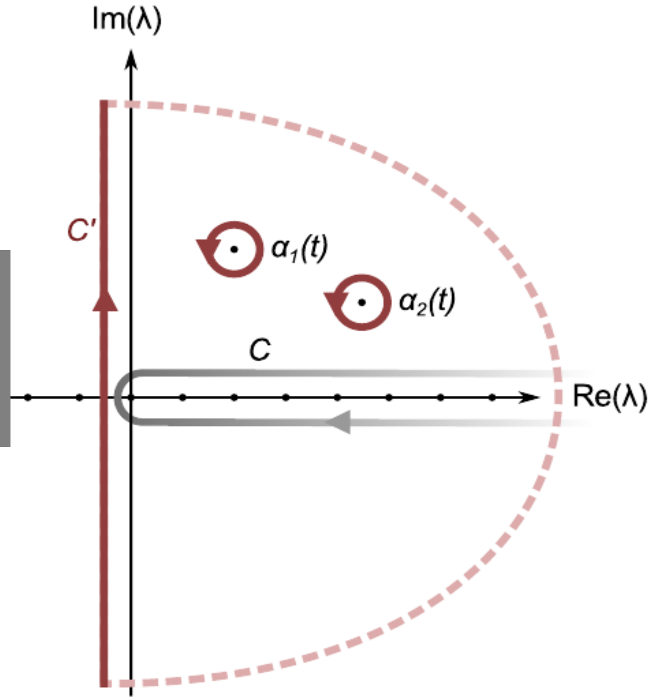
$$\frac{\lambda^2}{m^2 - t}$$

$$P_\ell(z_t) \xrightarrow{s\to\infty} (2s)^\ell$$



Regge Theory

$$\mathcal{M}(s,t) = -\frac{1}{2i}\oint_{C_1} d\lambda \frac{(2\lambda+1)\,\mathcal{M}_\lambda(t)\,P_\lambda(-\cos\theta_t)}{\sin(\pi\lambda)}$$



$$\mathcal{M}^{\zeta}_{\text{Regge}}(s,t) = C\sum_i \left(\frac{s}{s_0}\right)^{\alpha_i^\zeta(t)} \frac{\beta_i^\zeta(t)}{\sin\left(\pi\alpha_i^\zeta(t)\right)} \frac{1+\zeta e^{-i\pi\alpha_i^\zeta(t)}}{2} \frac{1}{\Gamma\left(\alpha_i^\zeta(t)+1\right)}.$$

Regge Theory

Based on unitarity, causality and crossing symmetry, Regge Theory predicts the following **high energy ($s \to \infty$) behavior** for the invariant amplitude:

$$A(s,t) \sim \beta(t)\, s^{\alpha(t)}$$

Regge theory does not predict the **t-dependence** of the amplitude.

For that, one needs a model.

$\alpha(t)$: Families or Regge trajectories