Ab-initio calculations of the electroweak response functions

NUFACT, ‘The 20th International Workshop on Neutrinos from Accelerator’

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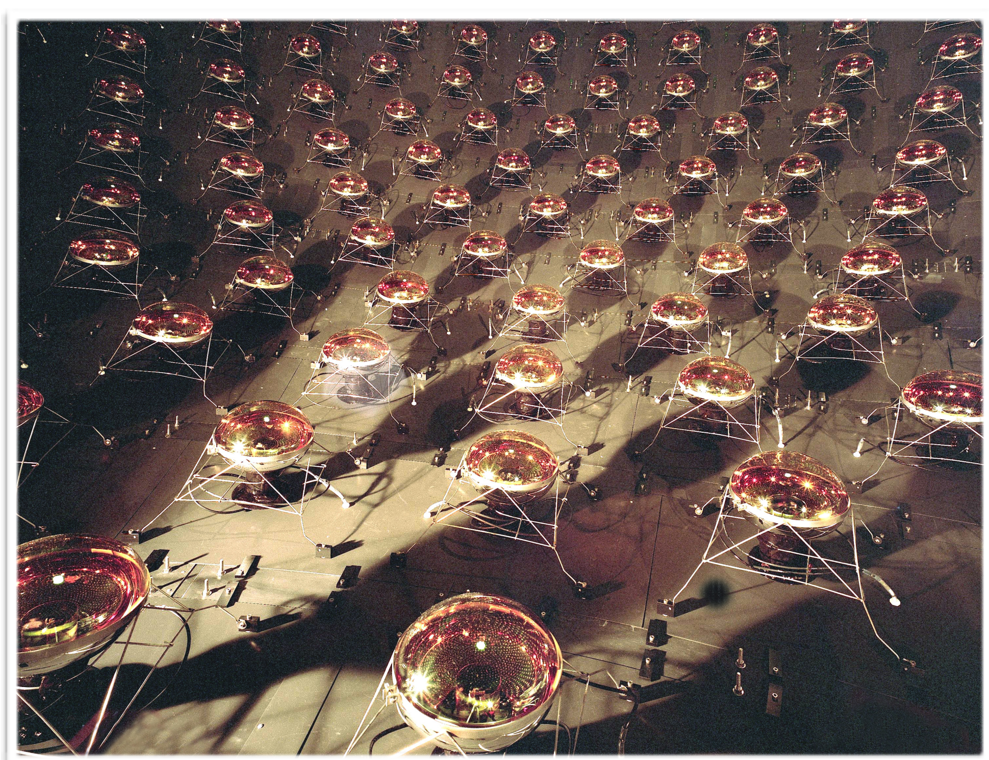
The Physics case

Neutrino-oscillation and $0\nu\beta\beta$ experiments

- Charge-parity (CP) violating phase and the mass hierarchy will be measured
- Determine whether the neutrino is a Majorana or a Dirac particle
- Need for including nuclear dynamics; mean-field models inadequate to describe neutrino-nucleus interaction



Multi-messenger era for nuclear astrophysics

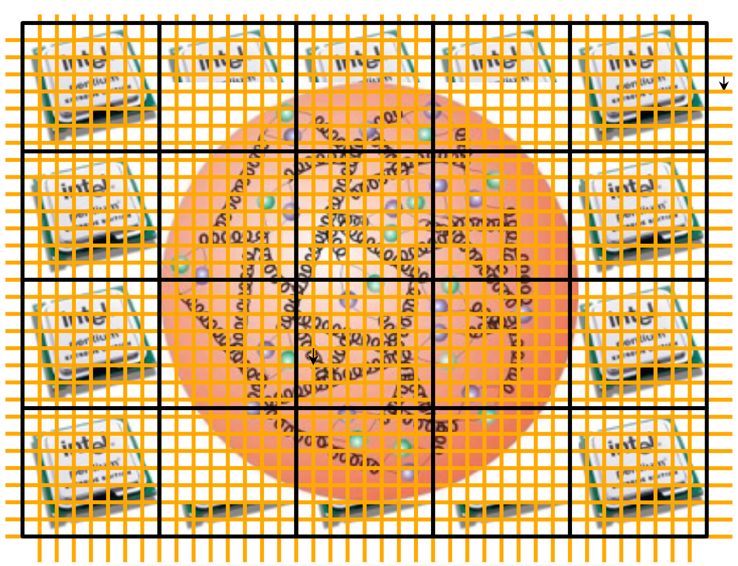
- Gravitational waves have been detected!
- Supernovae neutrinos will be detected by the current and next generation neutrino experiments
- Nuclear dynamics determines the structure and the cooling of neutron stars

The *basic model* of nuclear physics

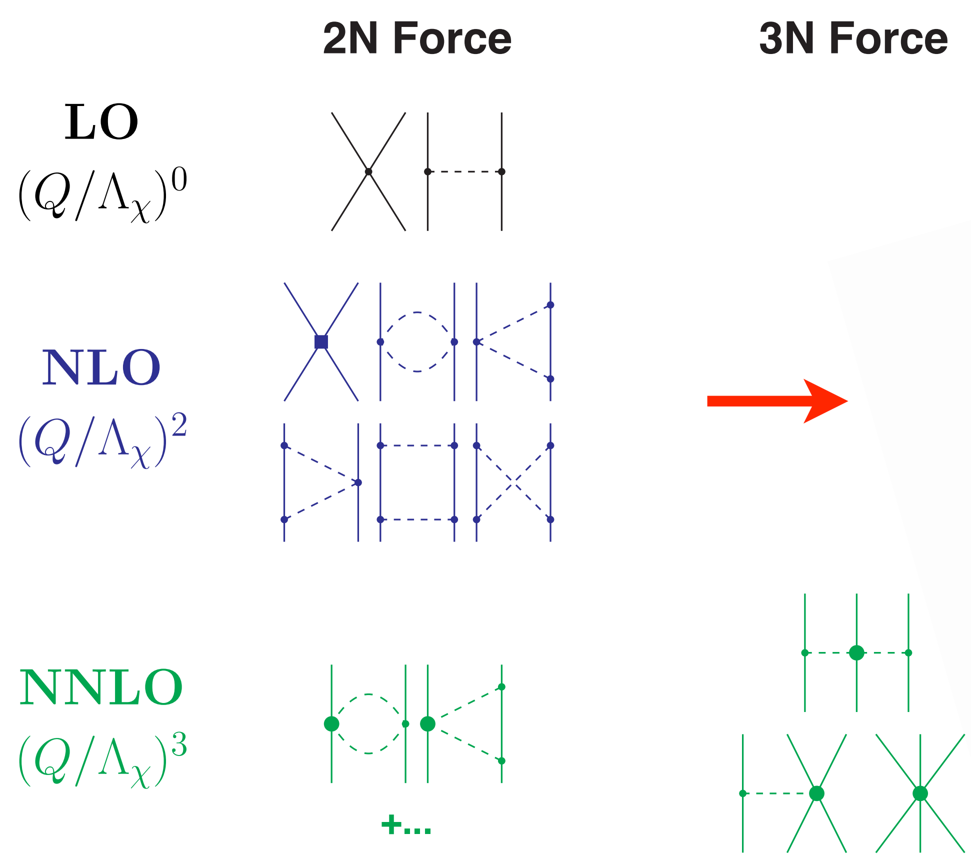
- In the low-energy regime, quark and gluons are confined inside hadrons. Nucleons can treated as point-like particles interacting through the Hamiltonian

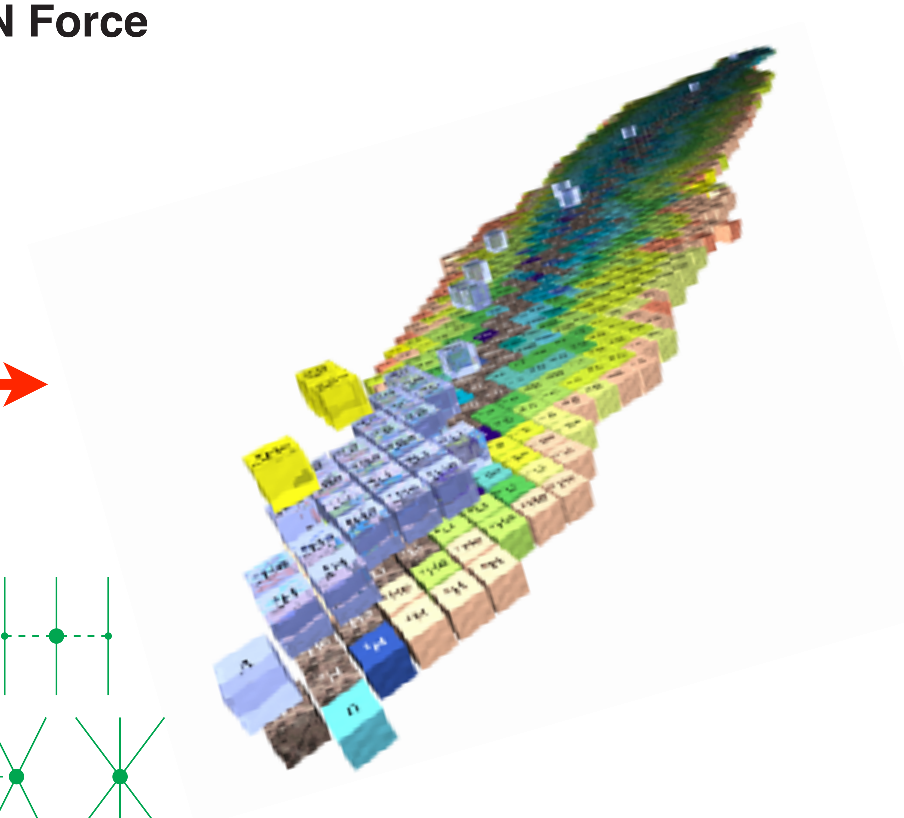
$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

- Effective field theories are the link between QCD and nuclear observables. They exploit the separation between the “hard” (M~nucleon mass) and “soft” (Q ~ exchanged momentum) scales



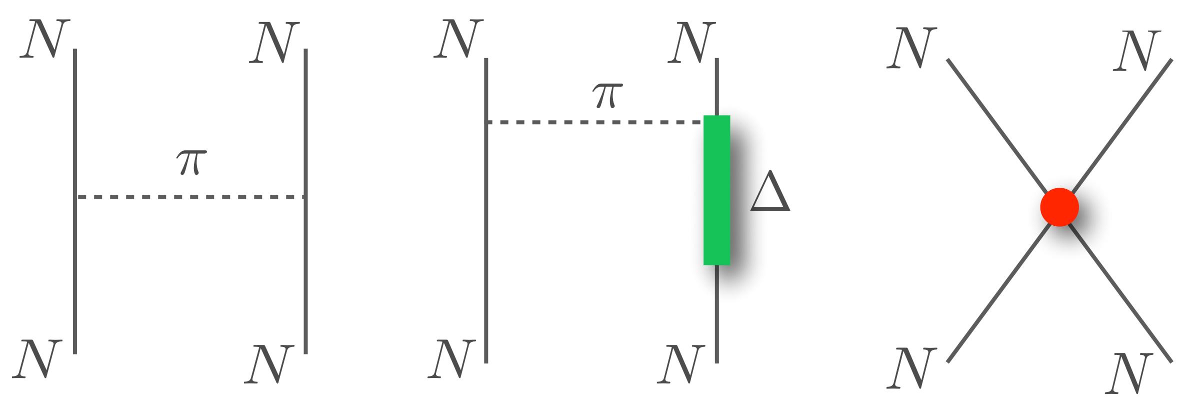
Courtesy of M. Savage



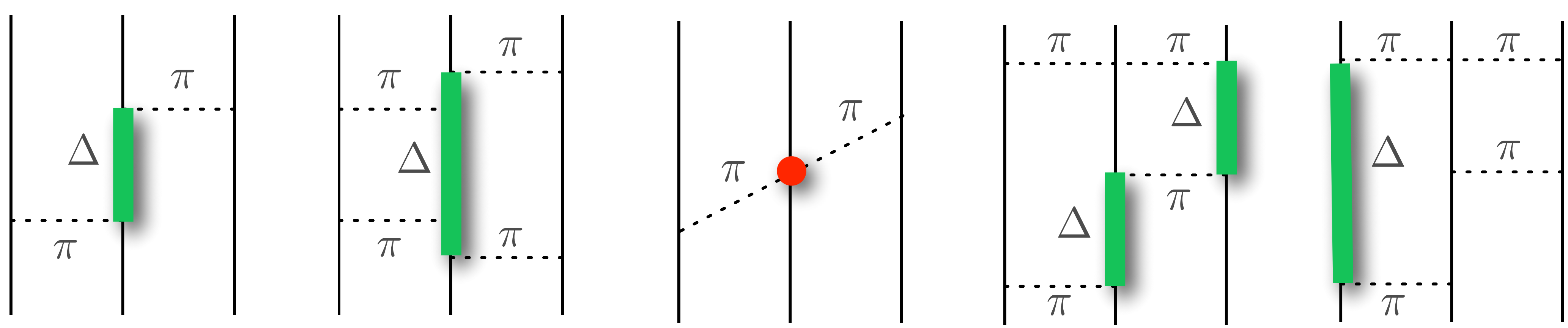


Nuclear (phenomenological) Hamiltonian

The Argonne v_{18} is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database



Three-nucleon interactions effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

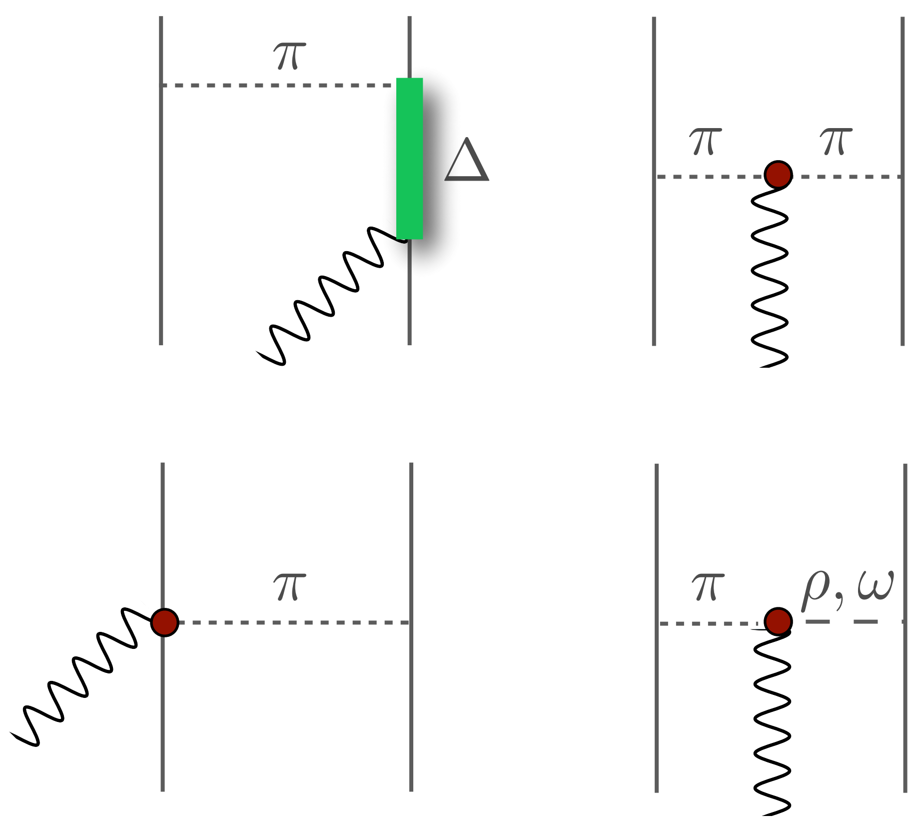


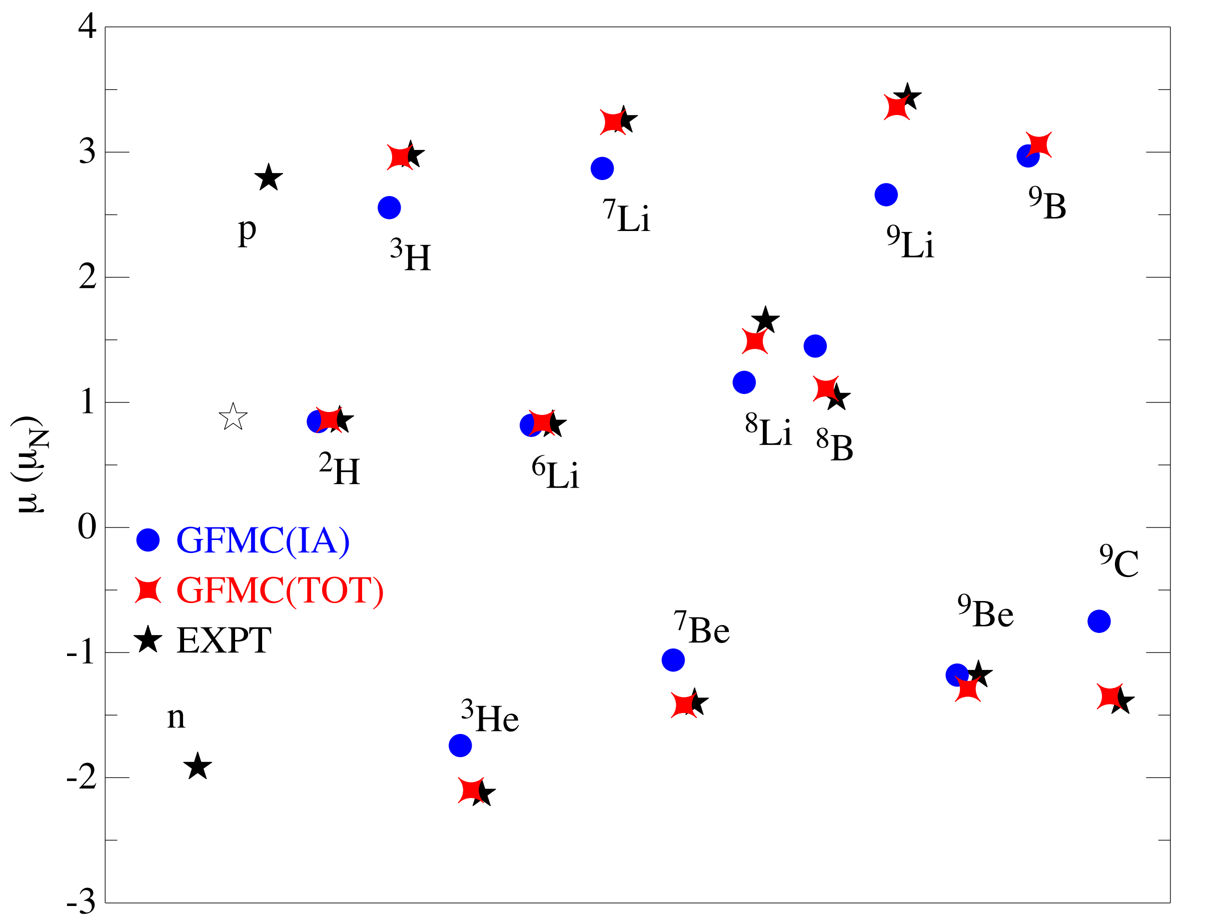
Nuclear electroweak currents

The nuclear electromagnetic current is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.
- They are essential for low-momentum and low-energy transfer transitions.





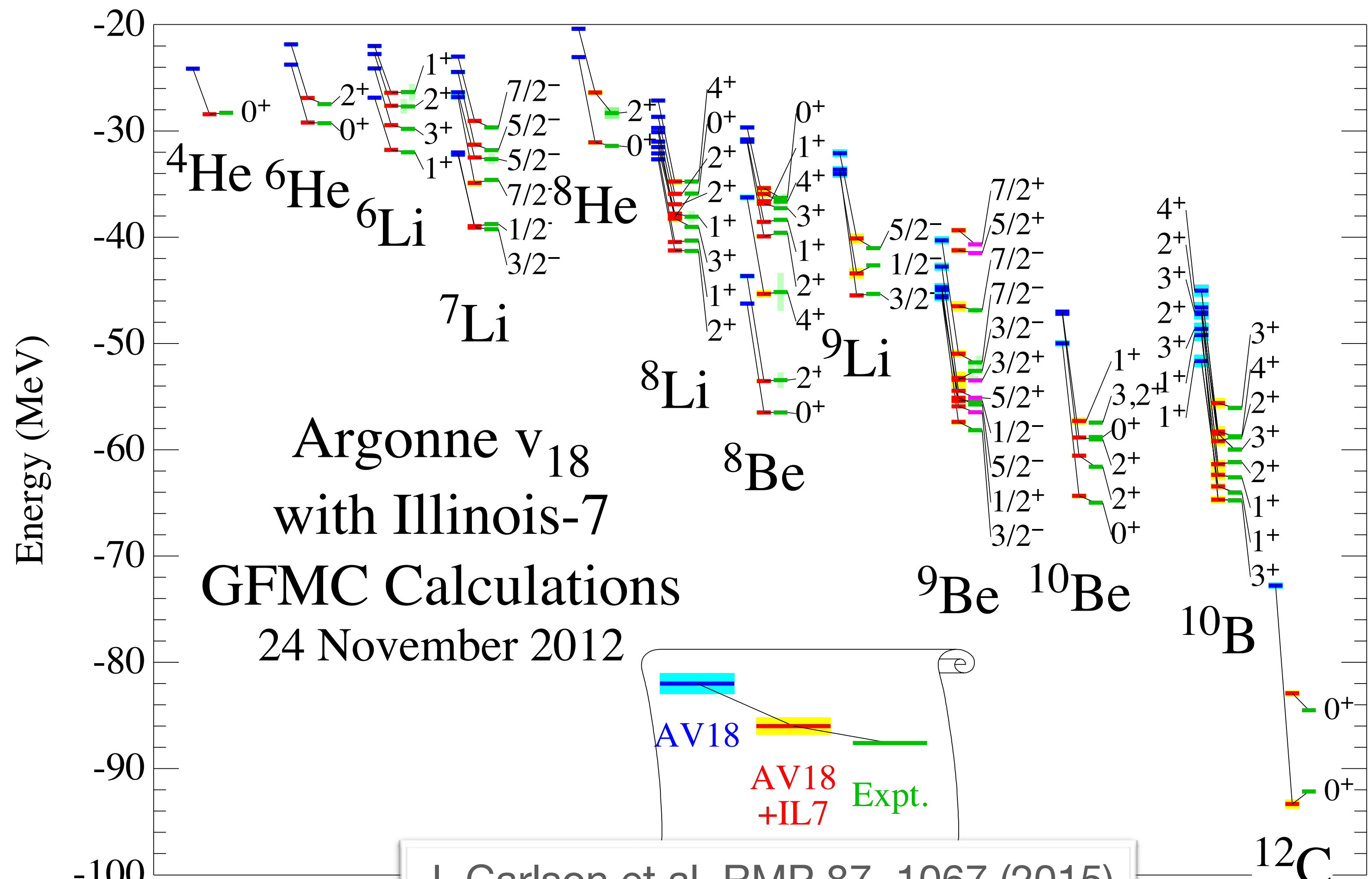
S. Pastore at al., PRC 87, 035503 (2013)

Quantum Monte Carlo

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting trial wave function.

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

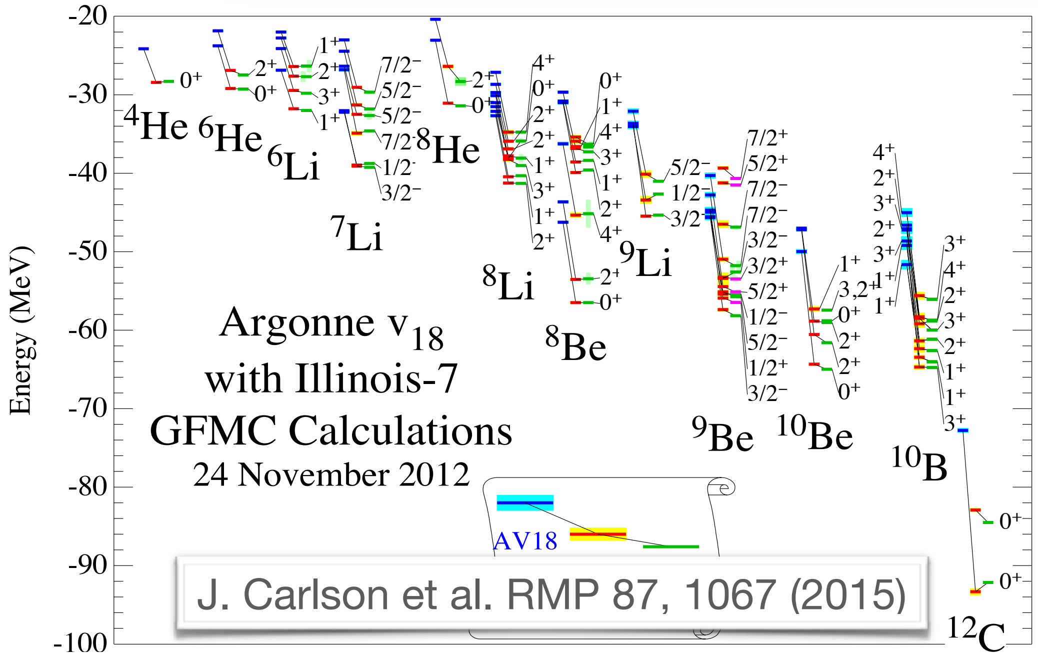
- Suitable to solve of A ≤ 12 nuclei with ~1% accuracy



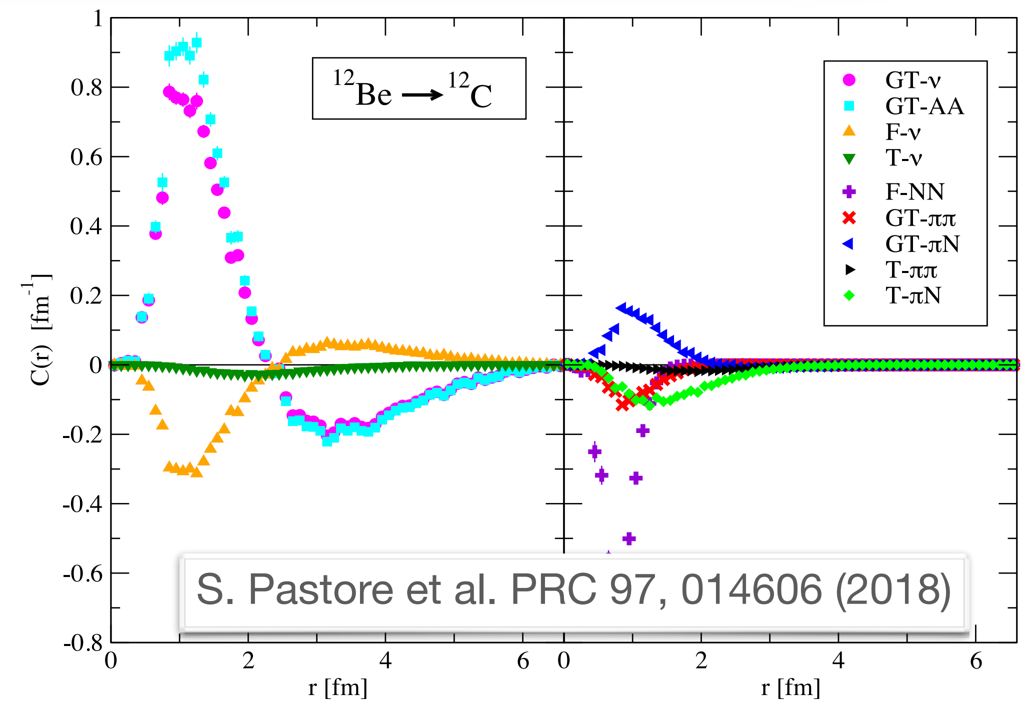
J. Carlson et al. RMP 87, 1067 (2015)

The *basic model* of nuclear Physics

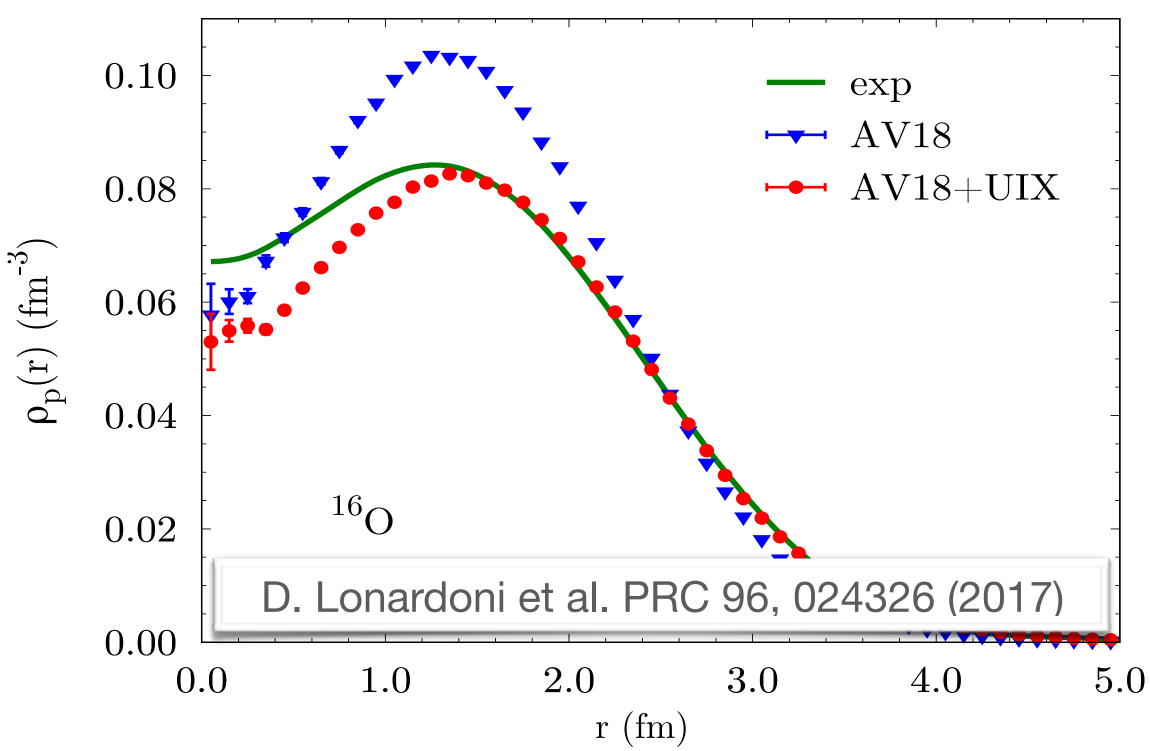
Nuclear effective field theories + Nuclear ab-initio methods



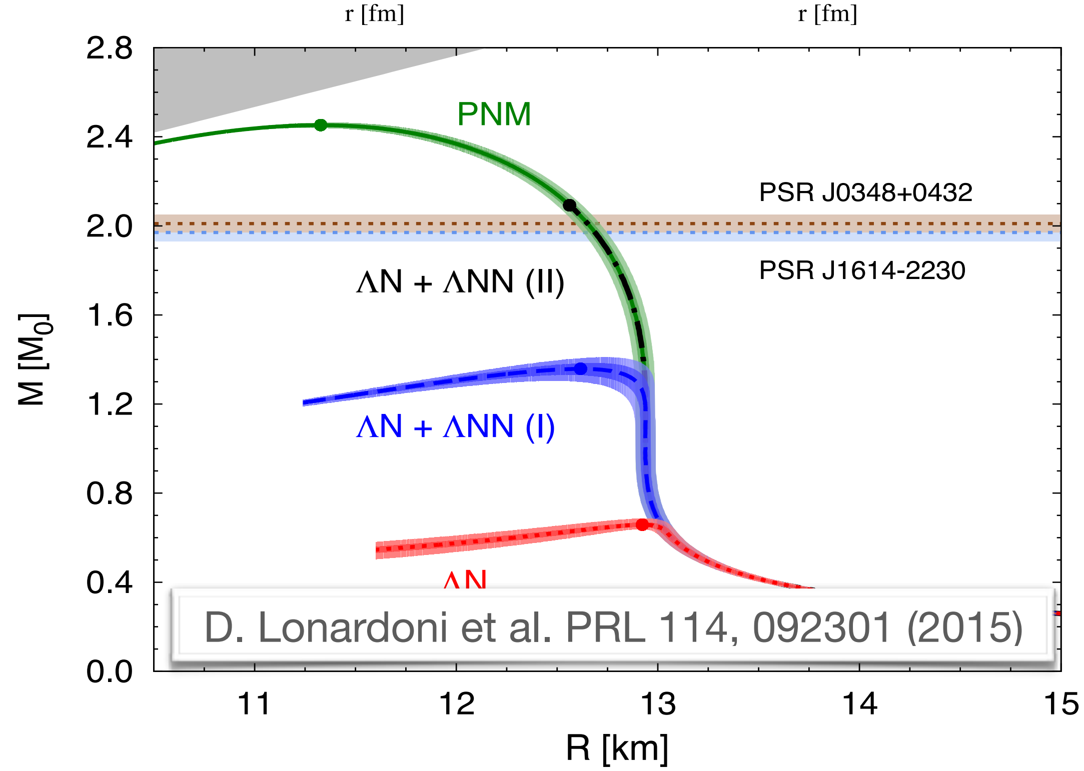
J. Carlson et al. RMP 87, 1067 (2015)



S. Pastore et al. PRC 97, 014606 (2018)



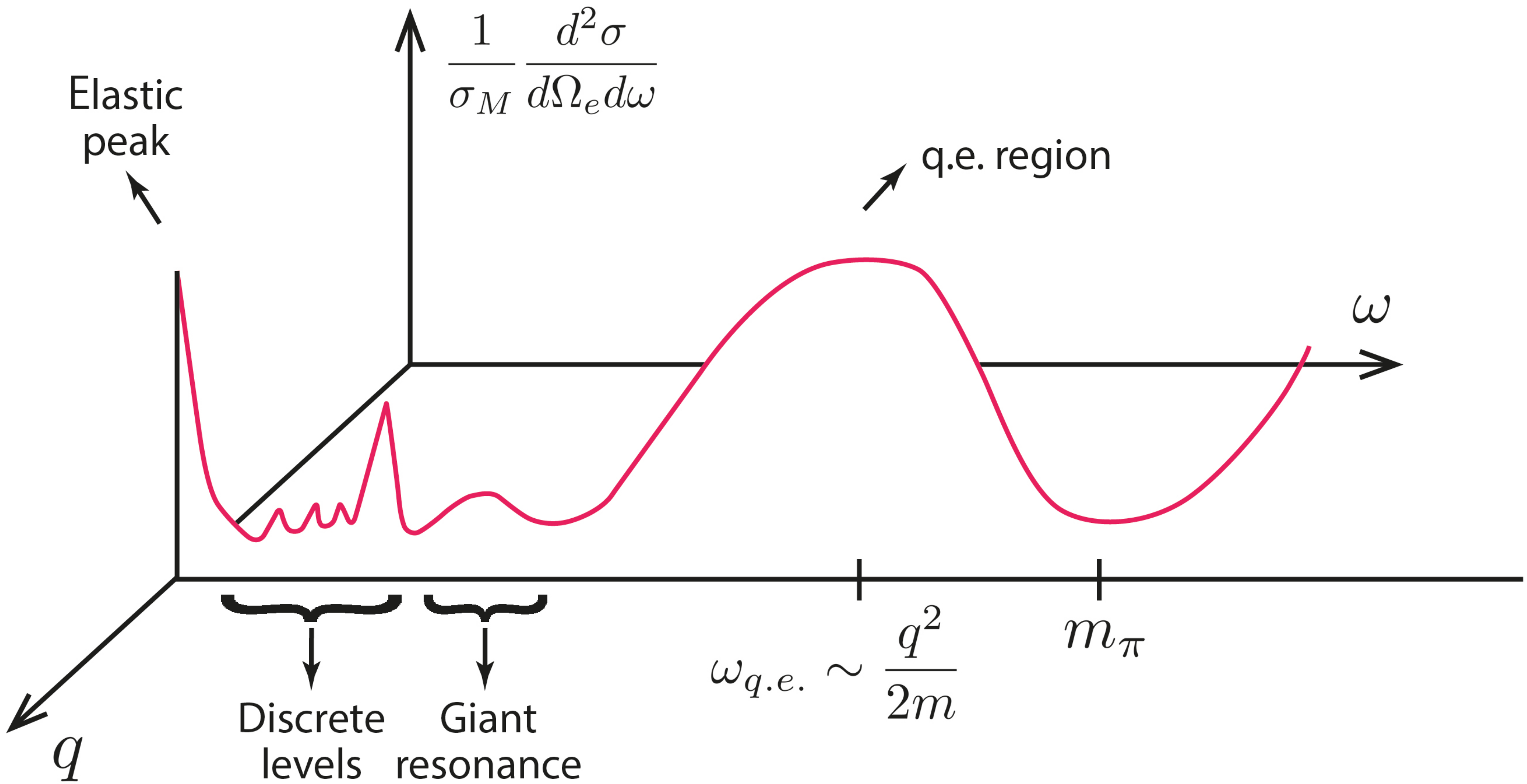
D. Lonardoni et al. PRC 96, 024326 (2017)



D. Lonardoni et al. PRL 114, 092301 (2015)

Lepton-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.

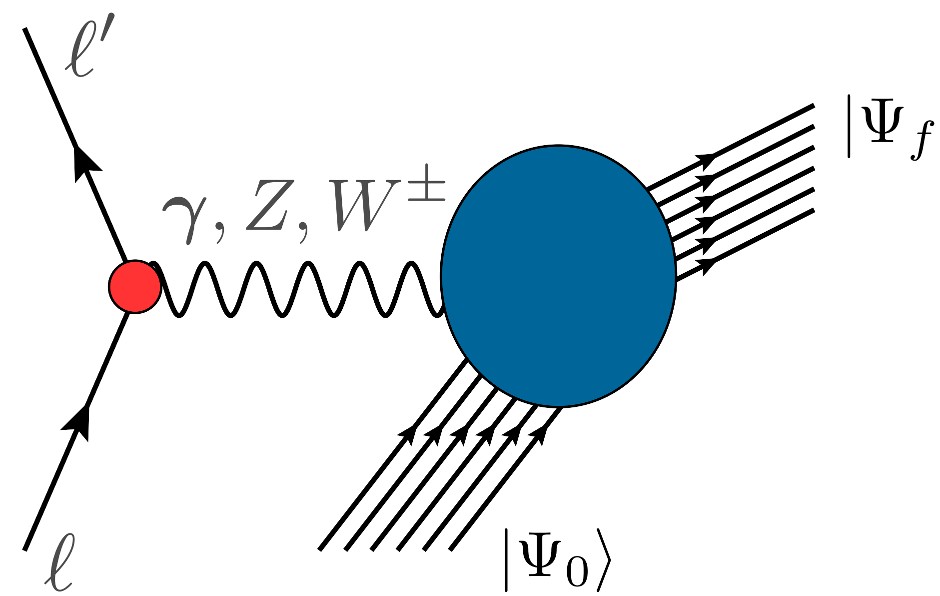


Courtesy of Saori Pastore

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'} d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$

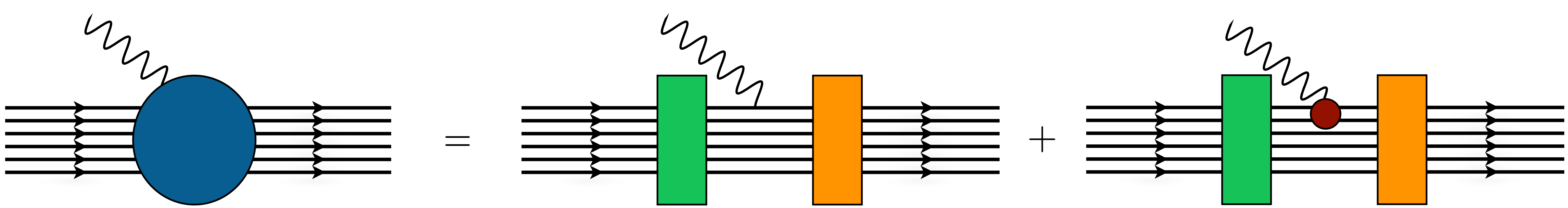


- In the electromagnetic case only the longitudinal and the transverse response functions contribute

- The response functions contain all the information on target structure and dynamics

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

- They account for initial state correlations, final state correlations and two-body currents



Moderate momentum-transfer regime

- At moderate momentum transfer, the inclusive cross section can be written in terms of the response functions

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_\beta(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0)$$

- Both initial and final states are eigenstates of the nuclear Hamiltonian

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle \qquad\qquad H|\Psi_f\rangle = E_f|\Psi_f\rangle$$

- As for the electron scattering on ^{12}C

$$|^{10}\mathrm{Be}, pp\rangle$$

$$|^{12}C^*\rangle, |^{11}\mathrm{B}, p\rangle, \; |^{11}\mathrm{C}, n\rangle, \; |^{10}\mathrm{B}, pn\rangle,$$

- Relativistic corrections are included in the current operators and in the nucleon form factors

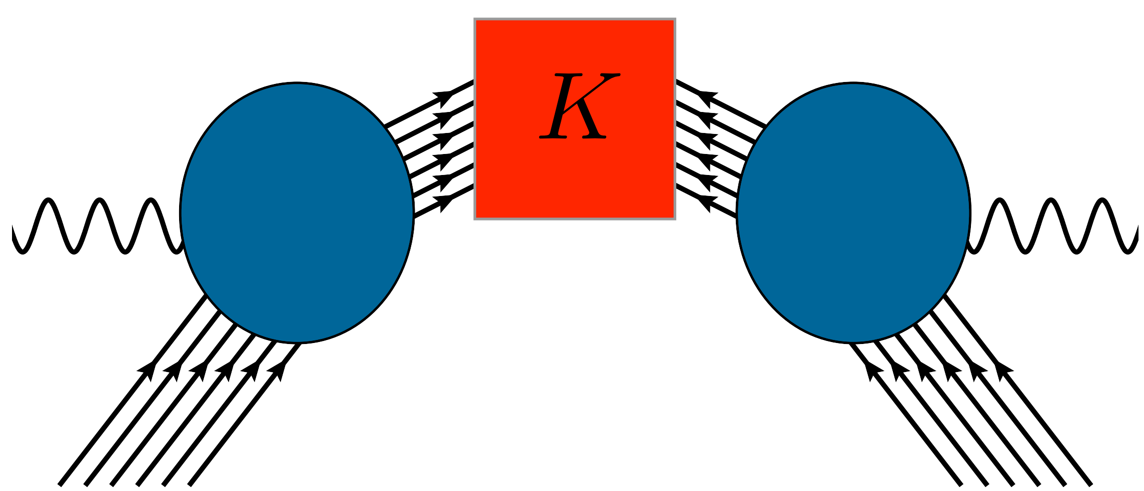
Integral transform techniques

- The integral transform of the response function are generally defined as

$$E_{\alpha\beta}(\sigma, \mathbf{q}) \equiv \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q})$$

- Using the completeness of the final states, they can be expressed in terms of ground-state expectation values

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \Psi_0 \rangle$$

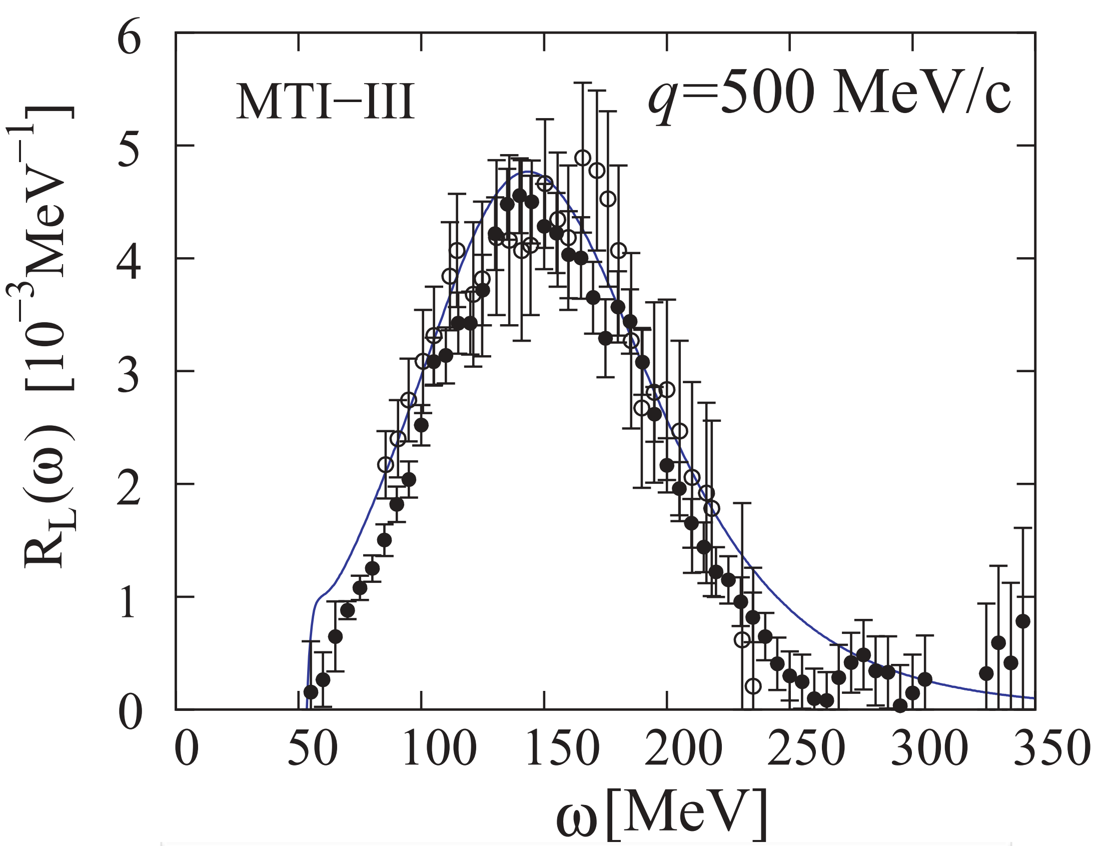


Lorentz integral transform (LIT)

- The Lorentz integral transform

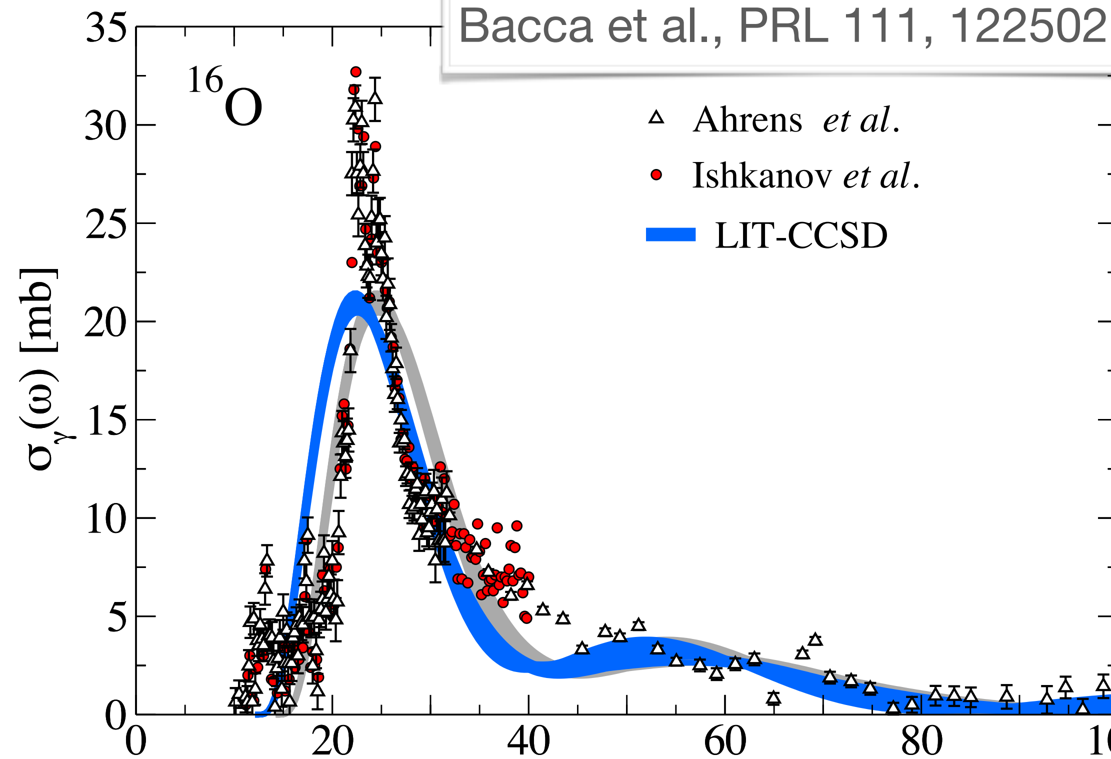
$$K(\sigma,\omega) = \frac{1}{(\omega-\sigma_R)^2+\sigma_I^2}$$

has been successfully exploited in the calculation of electromagnetic and neutral-weak responses

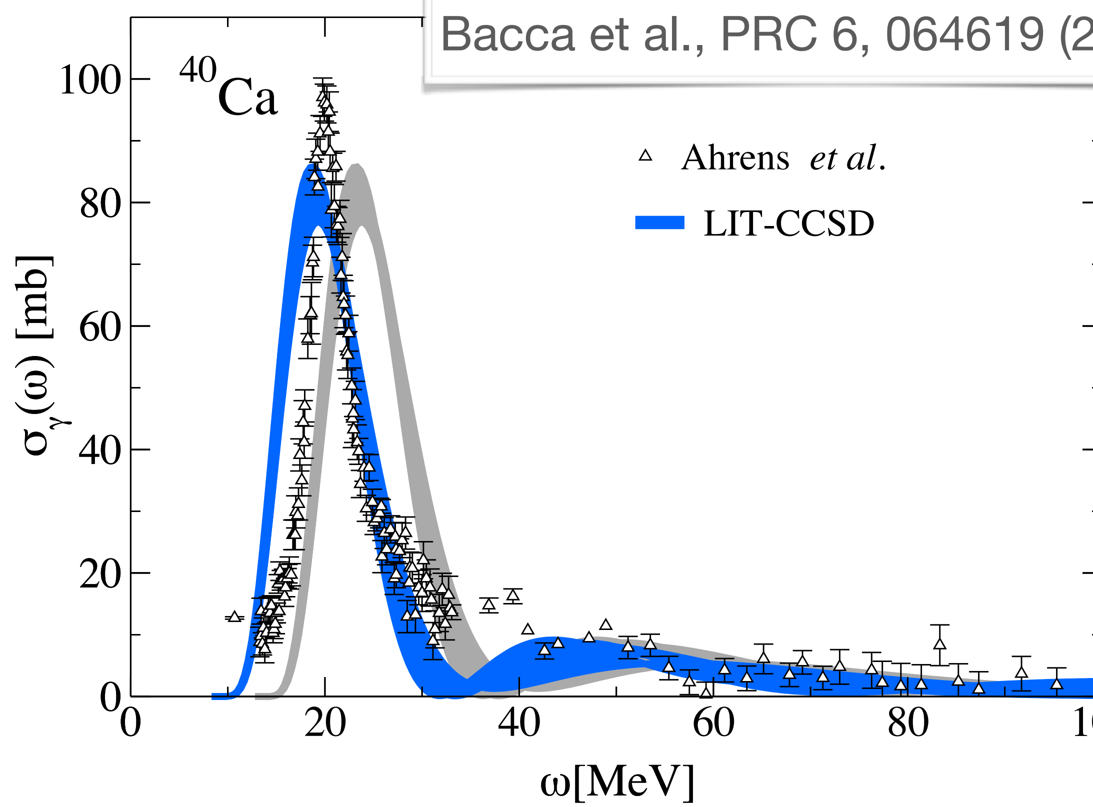


Bacca et al., PRC 76, 014003 (2007)

Bacca et al., PRL 111, 122502 (2013)



Bacca et al., PRC 6, 064619 (2014)

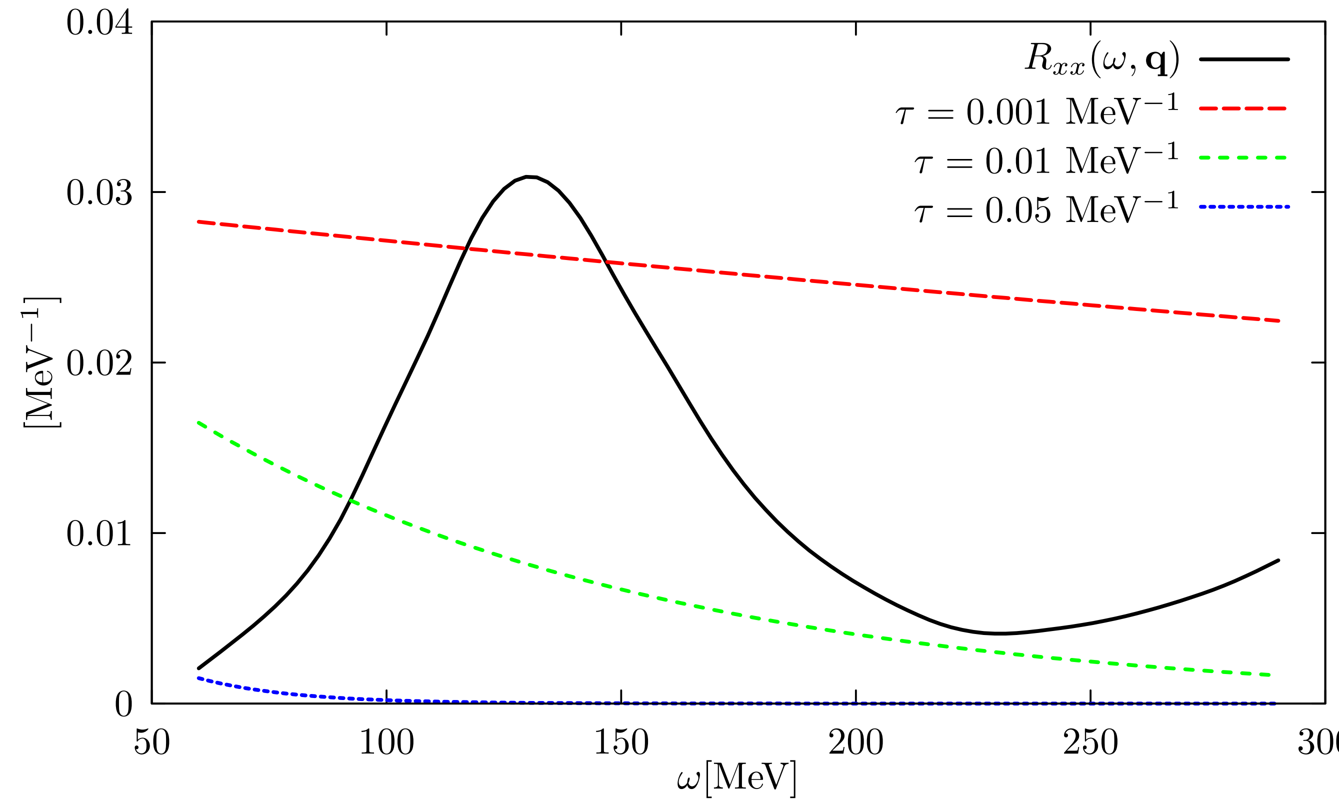


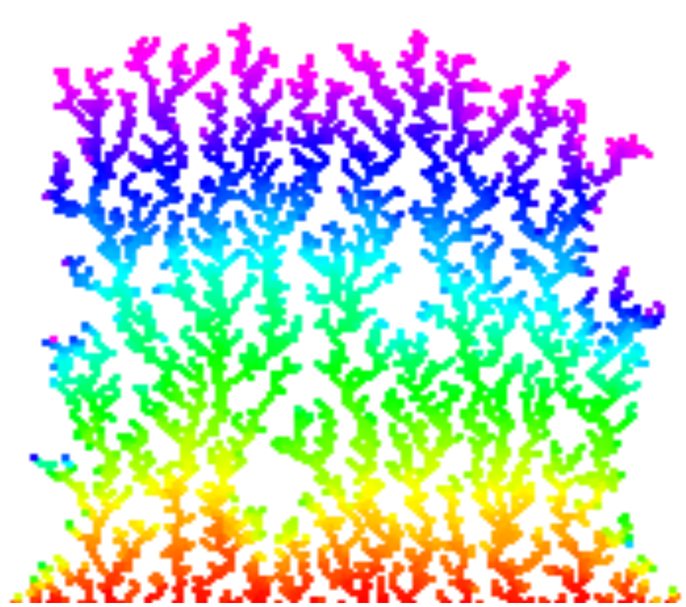
Euclidean response function

Valuable information on the energy dependence of the response functions can be inferred from their Laplace transforms

$$E_{\alpha\beta}(\tau,\mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega,\mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed





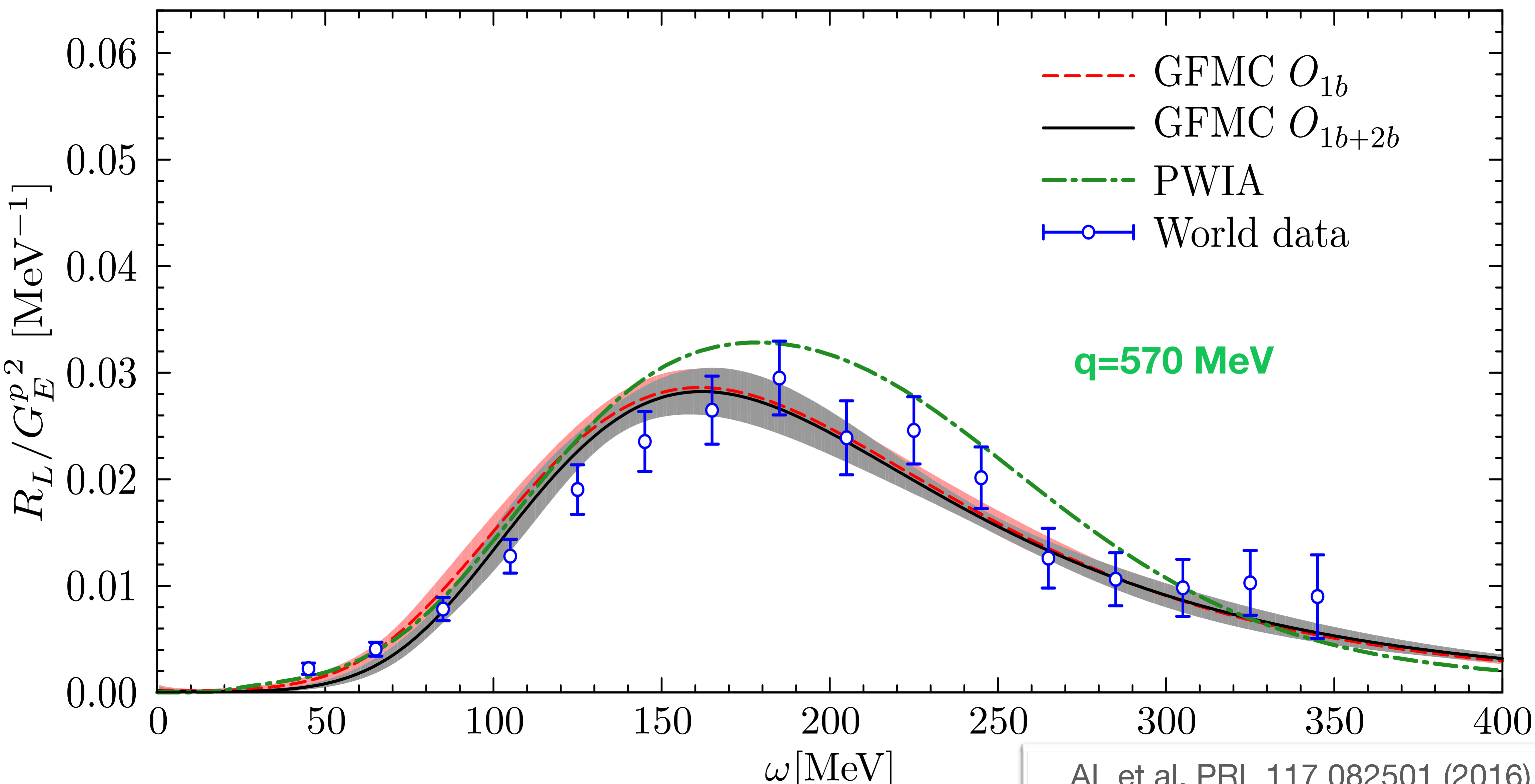
The system is first heated up by the transition operator. Its cooling determines the Euclidean response of the system

$$E_{\alpha\beta}(\tau,\mathbf{q}) = \langle\Psi_0|J^\dagger_\alpha(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$$

Same technique used in Lattice QCD, condensed matter physics…

^{12}C electromagnetic response

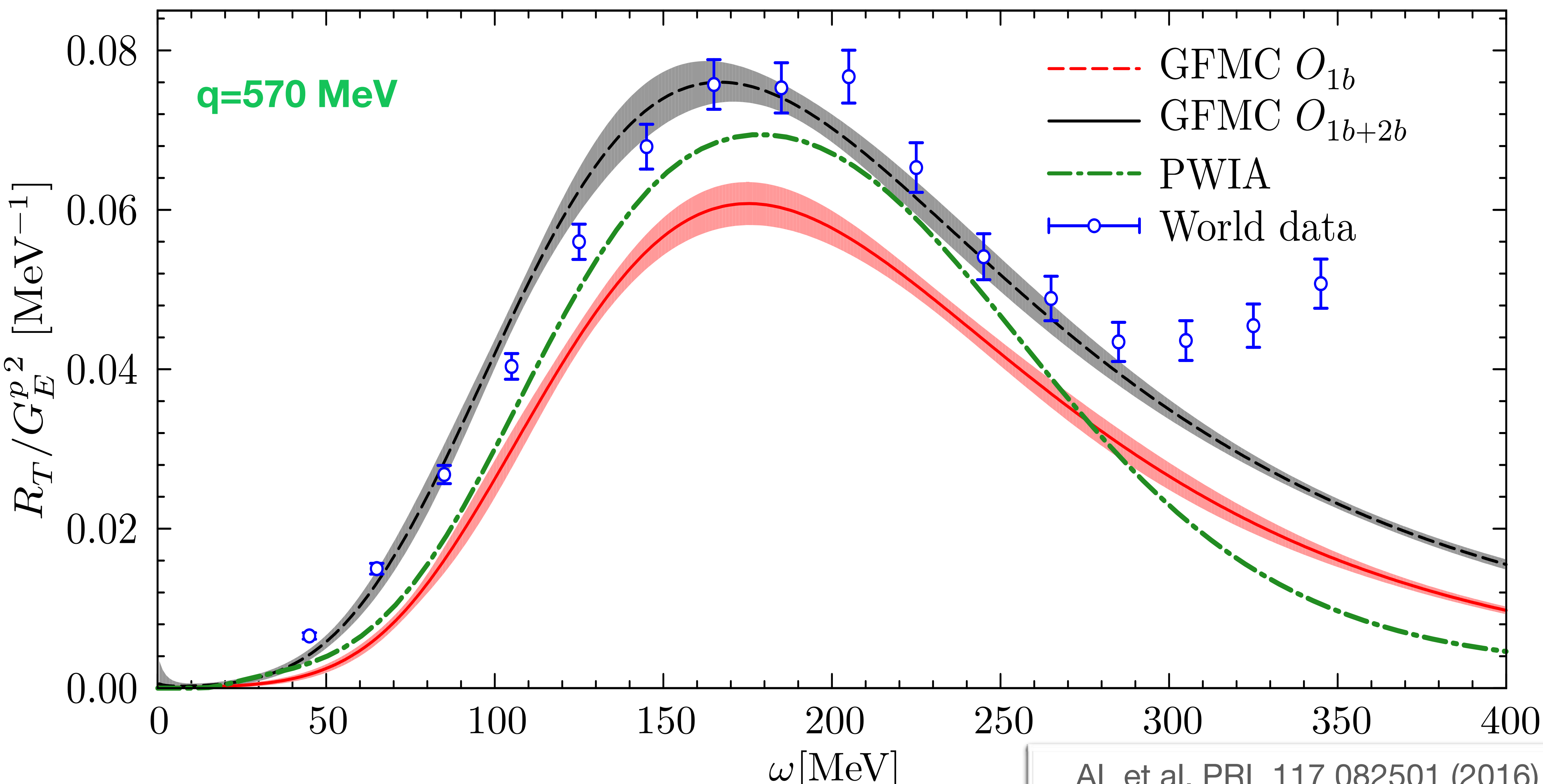
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data. Small contribution from two-body currents.



AL et al. PRL 117 082501 (2016)

^{12}C electromagnetic response

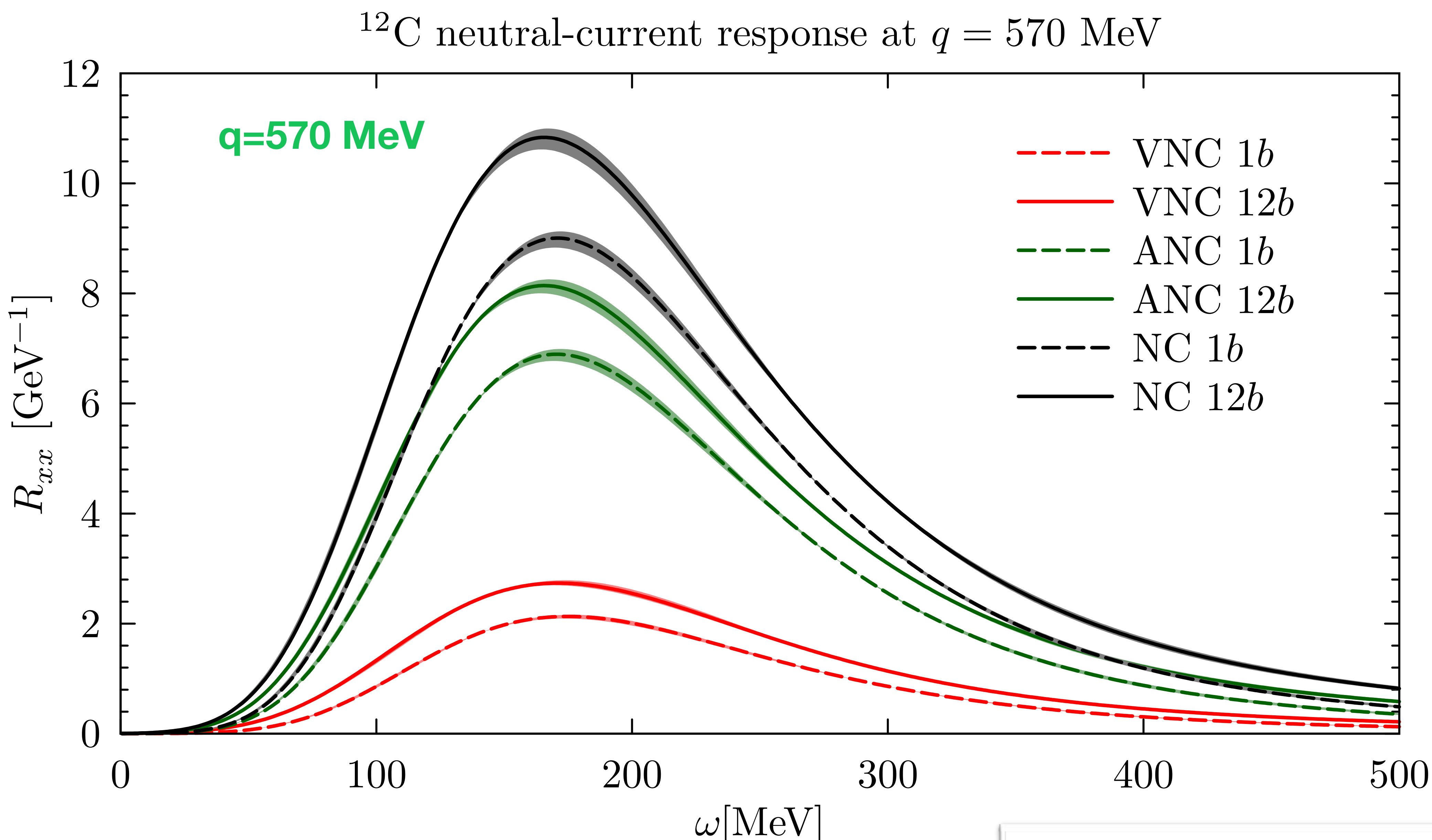
- We inverted the electromagnetic Euclidean response of ^{12}C
- Very good agreement with the experimental data once two-body currents are accounted for



AL et al. PRL 117 082501 (2016)

^{12}C neutral-current response

- We were recently able to invert the neutral-current Euclidean responses of ^{12}C



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current response

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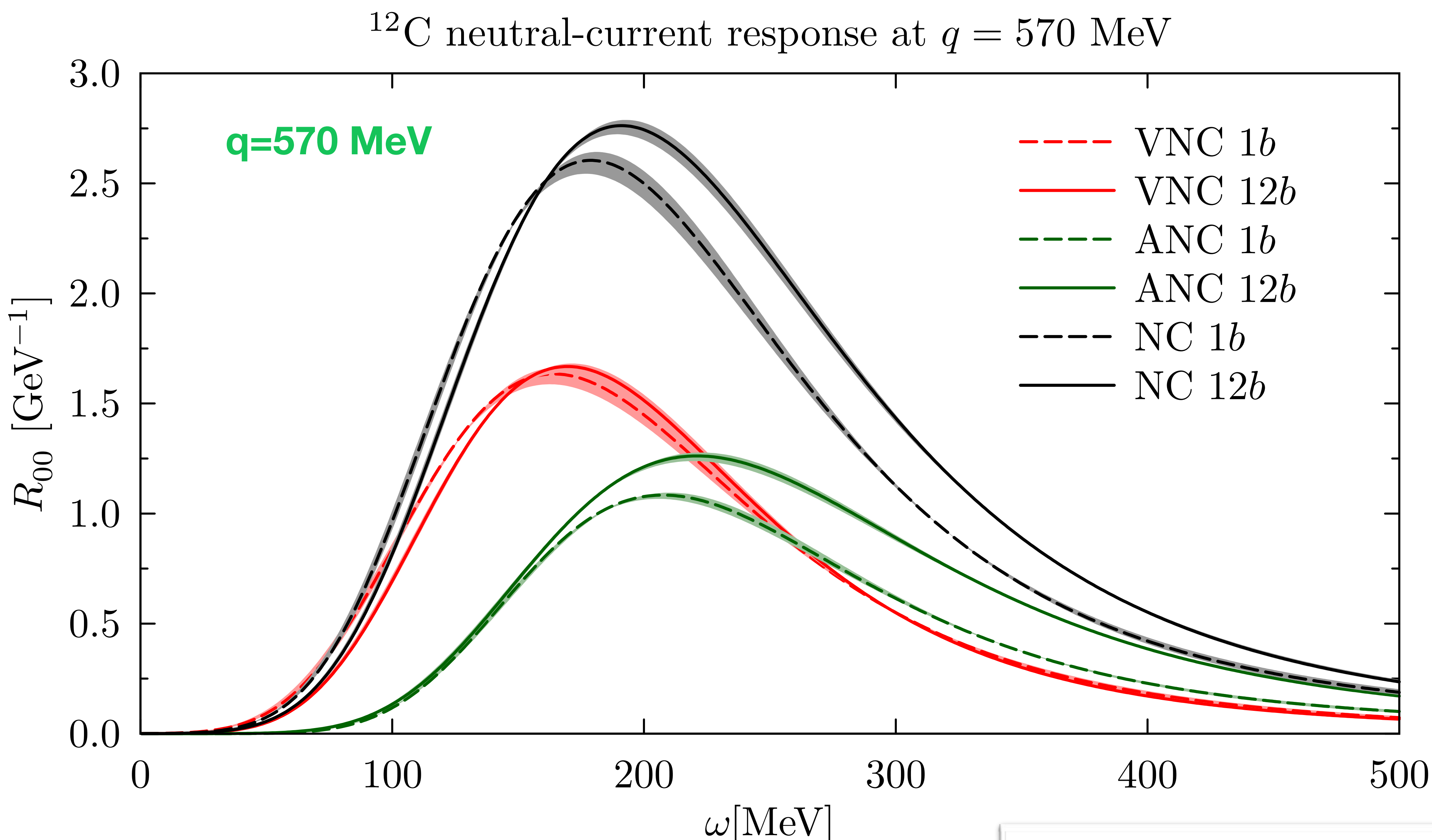

AL et al. PRC 97 022502 (2018)

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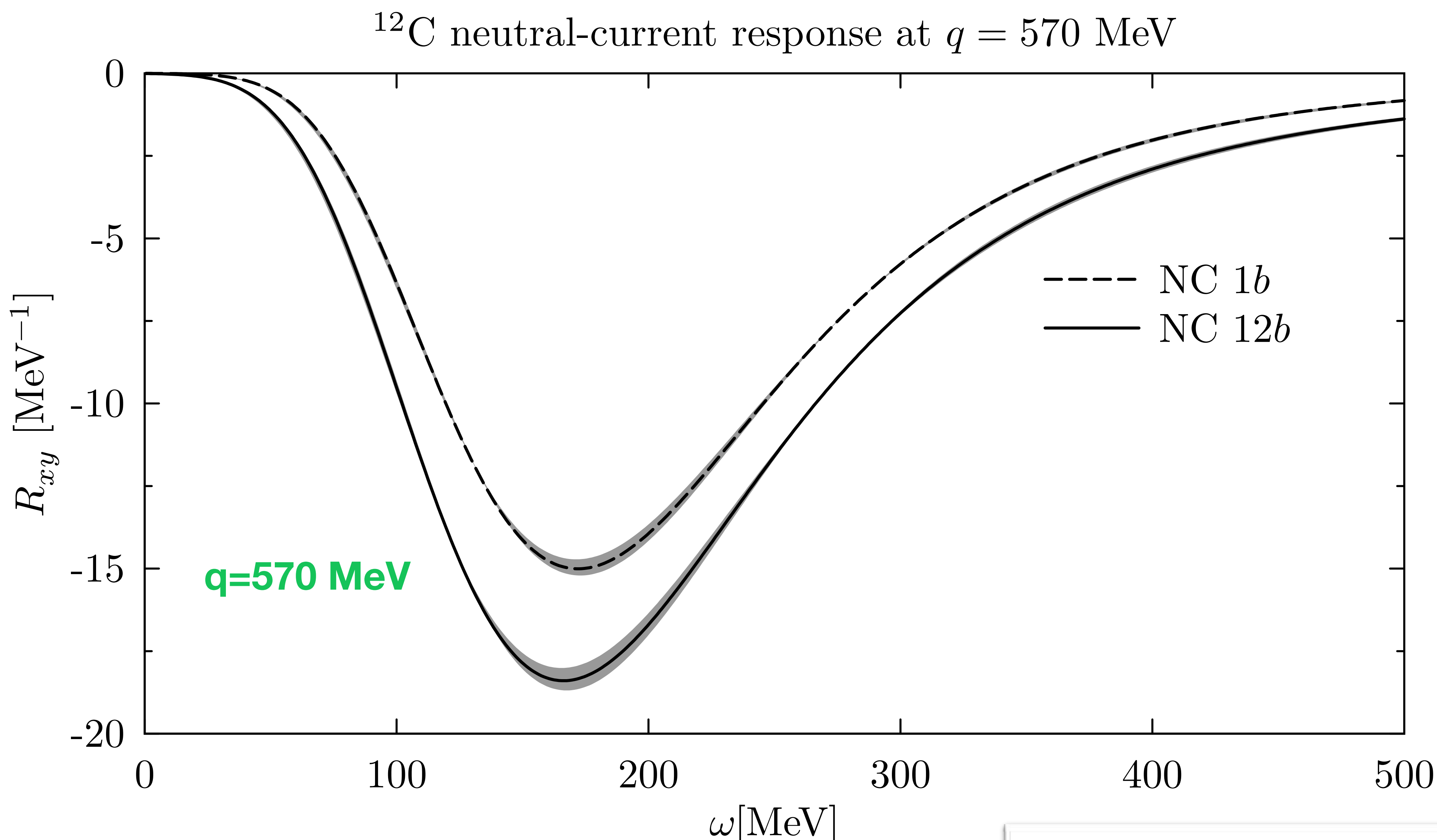
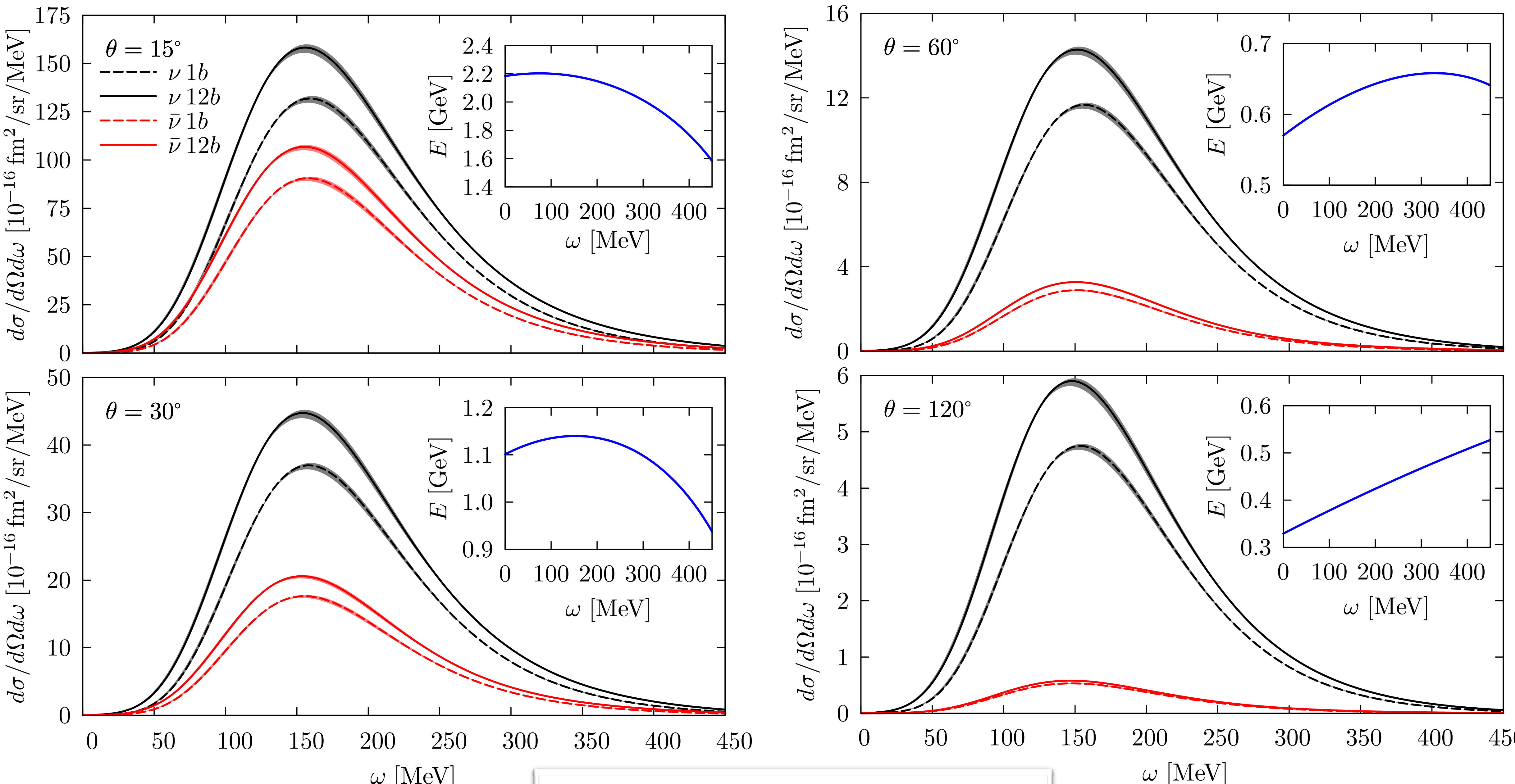


AL et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

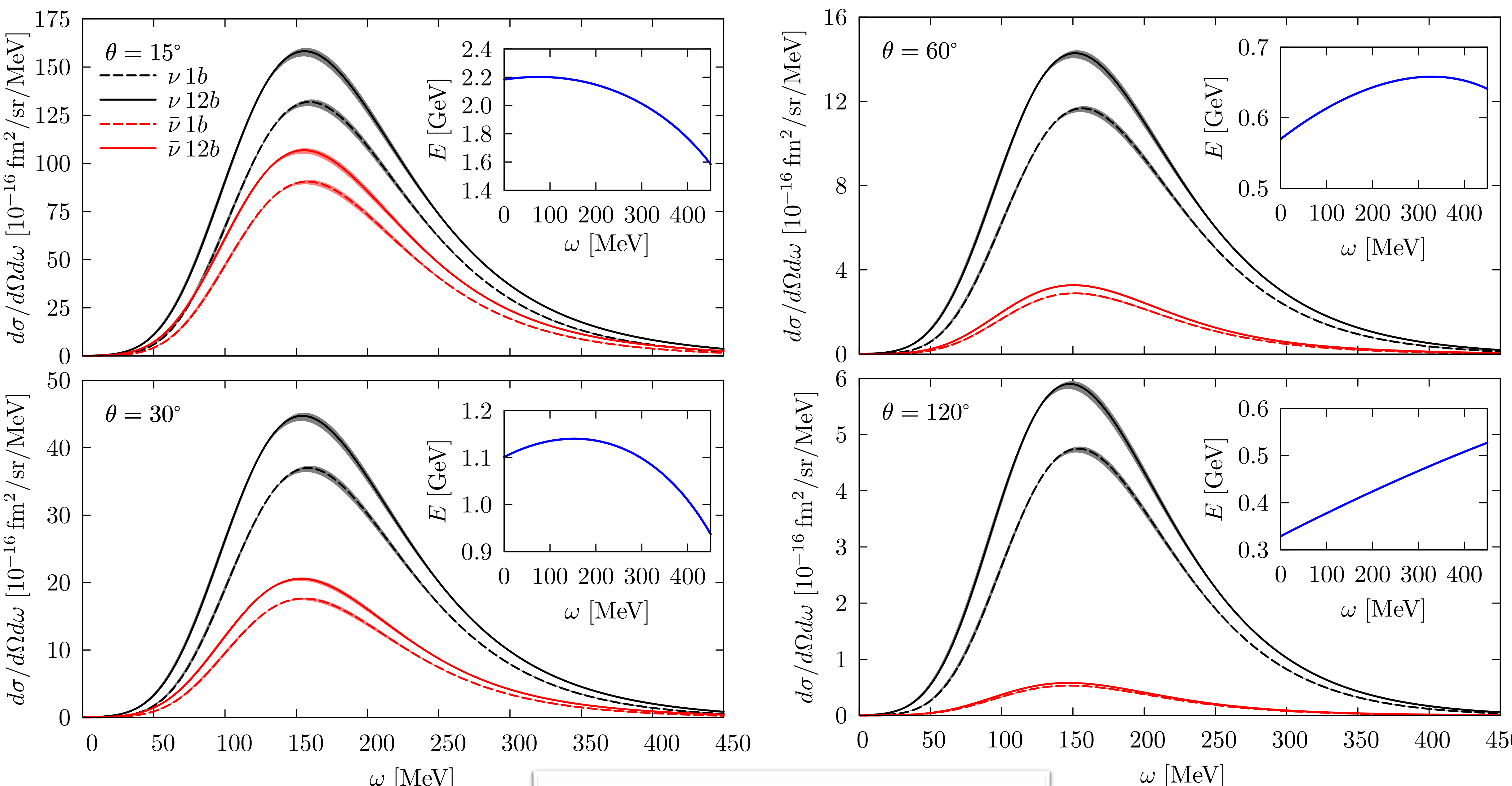
- We computed the neutrino and anti-neutrino differential cross sections for a fixed value of the three-momentum transfer as function of the energy transfer for a number of scattering angles



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

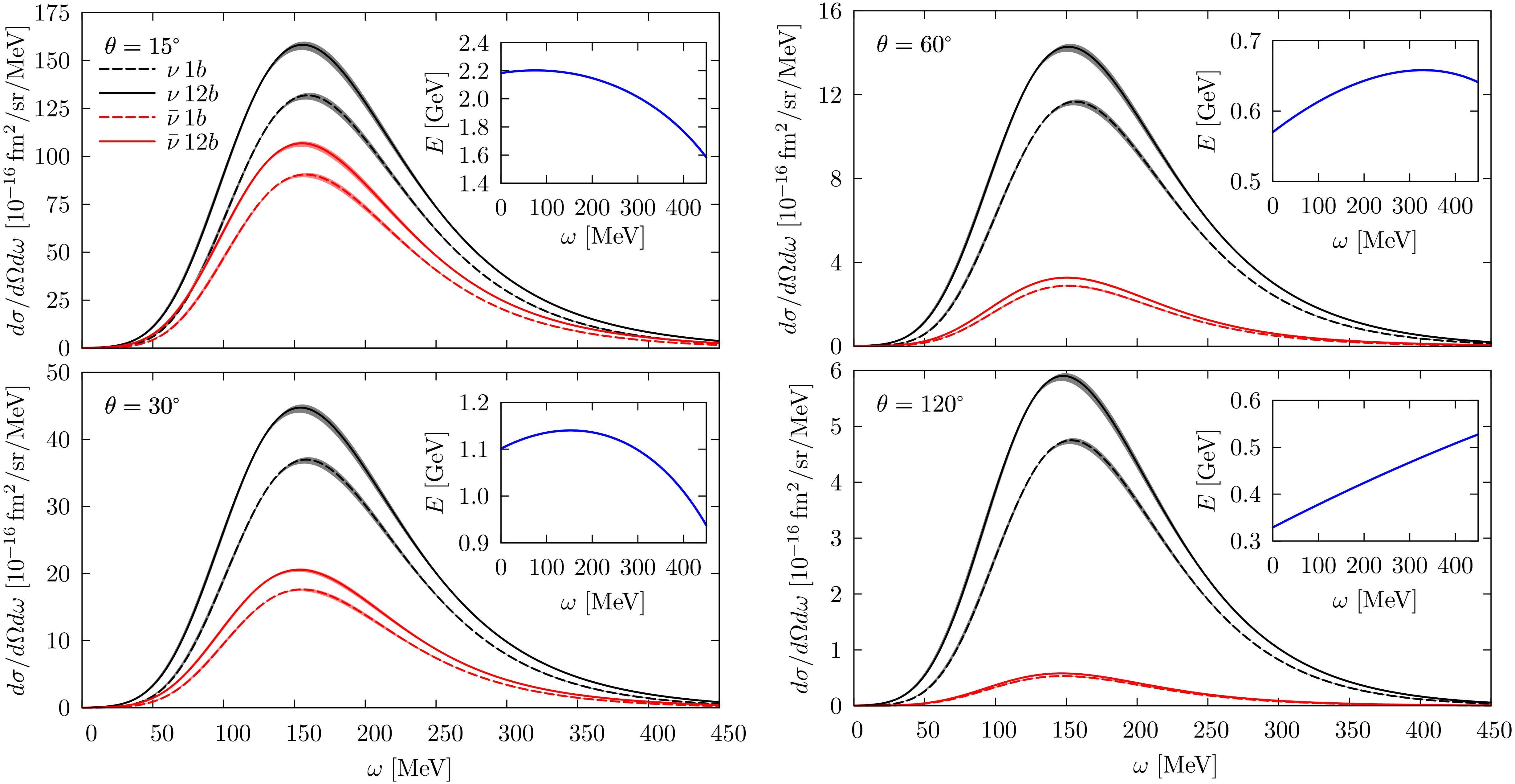
- The anti-neutrino cross section decreases rapidly relative to the neutrino cross section as the scattering angle changes from the forward to the backward hemisphere



AL et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

- For this same reason, two-body current contributions are smaller for the antineutrino than for the neutrino cross section



Relativistic effects in a correlated system

- Non relativistic approaches are limited to moderate momentum transfers

- In a generic reference frame the longitudinal response reads

$$R_L^{fr} = \sum_f \left| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \right|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

- The response in the LAB frame is given by the Lorentz transform

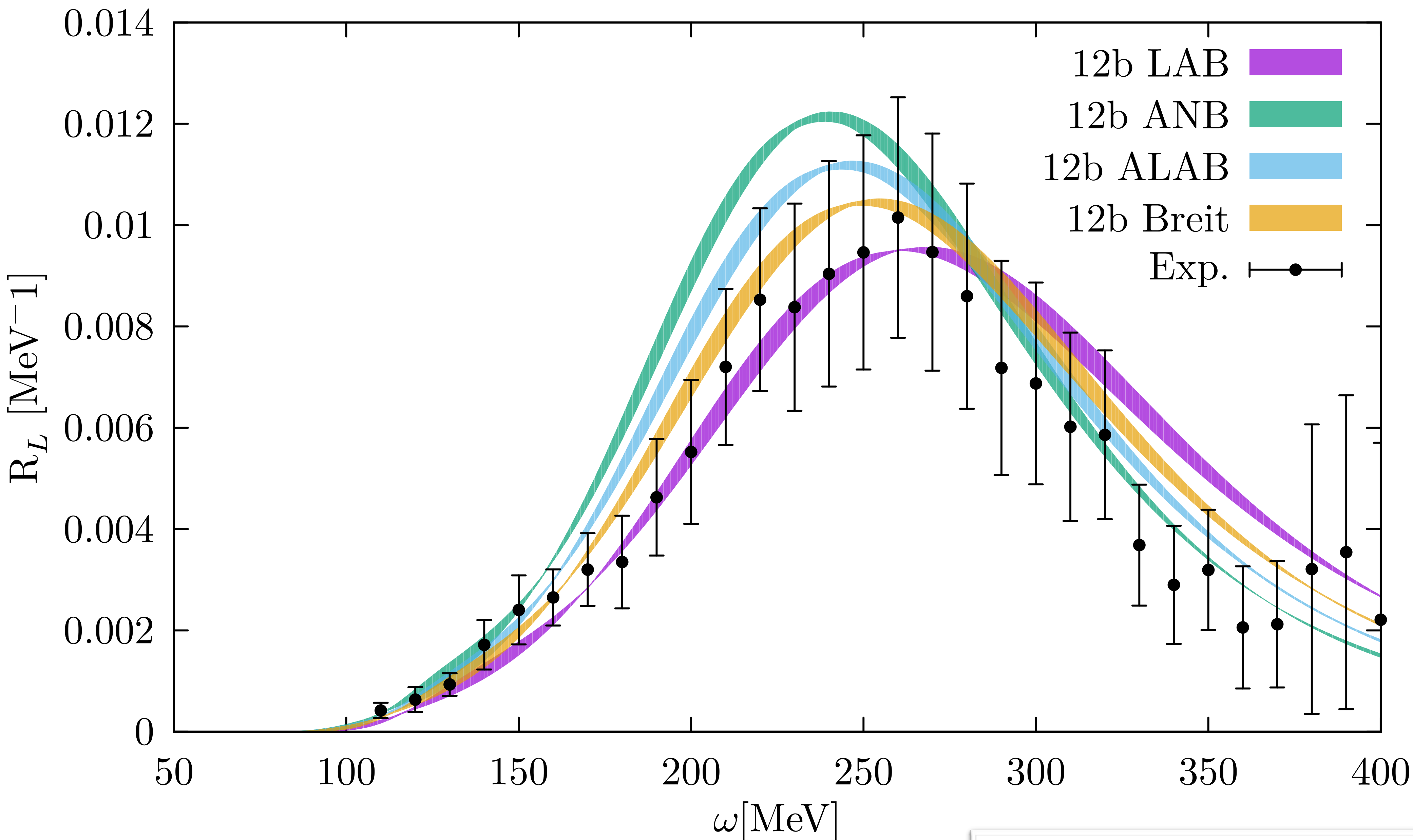
$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega),\ \omega^{fr} = \gamma(\omega - \beta q),\ P_i^{fr} = -\beta\gamma M_0,\ E_i^{fr} = \gamma M_0$$

Relativistic effects in a correlated system

- The ^{4}He longitudinal response at q=700 MeV **strongly** depends on the original reference frame



N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system

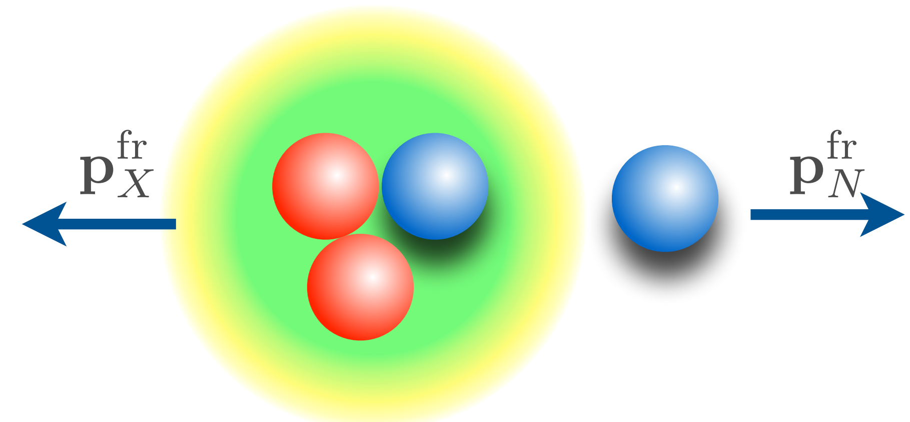
- To determine the relativistic corrections, we consider a two-body breakup model

$$\mathbf{p}^{\rm fr} = \mu(\frac{\mathbf{p}_N^{\rm fr}}{m} - \frac{\mathbf{p}_X^{\rm fr}}{M_X})$$

$$\mathbf{P}_f^{\rm fr} = \mathbf{p}_N^{\rm fr} + \mathbf{p}_X^{\rm fr}$$

$$\mu = \frac{mM_X}{m+M_X}$$

$$M_X = (A-1)m + \epsilon_0^{A-1}$$



- The relative momentum is derived in a relativistic fashion

$$\omega^{\rm fr} = E_f^{\rm fr} - E_i^{\rm fr}$$

$$E_f^{\rm fr} = \sqrt{m^2 + (\mathbf{p}^{\rm fr} + (\mu/M_{A-1})\mathbf{P}_f^{\rm fr})^2} + \sqrt{M_{A-1}^2 + (\mathbf{p}^{\rm fr} - (\mu/m)\mathbf{P}_f^{\rm fr})^2}$$

- And it is used as input in the non relativistic kinetic energy

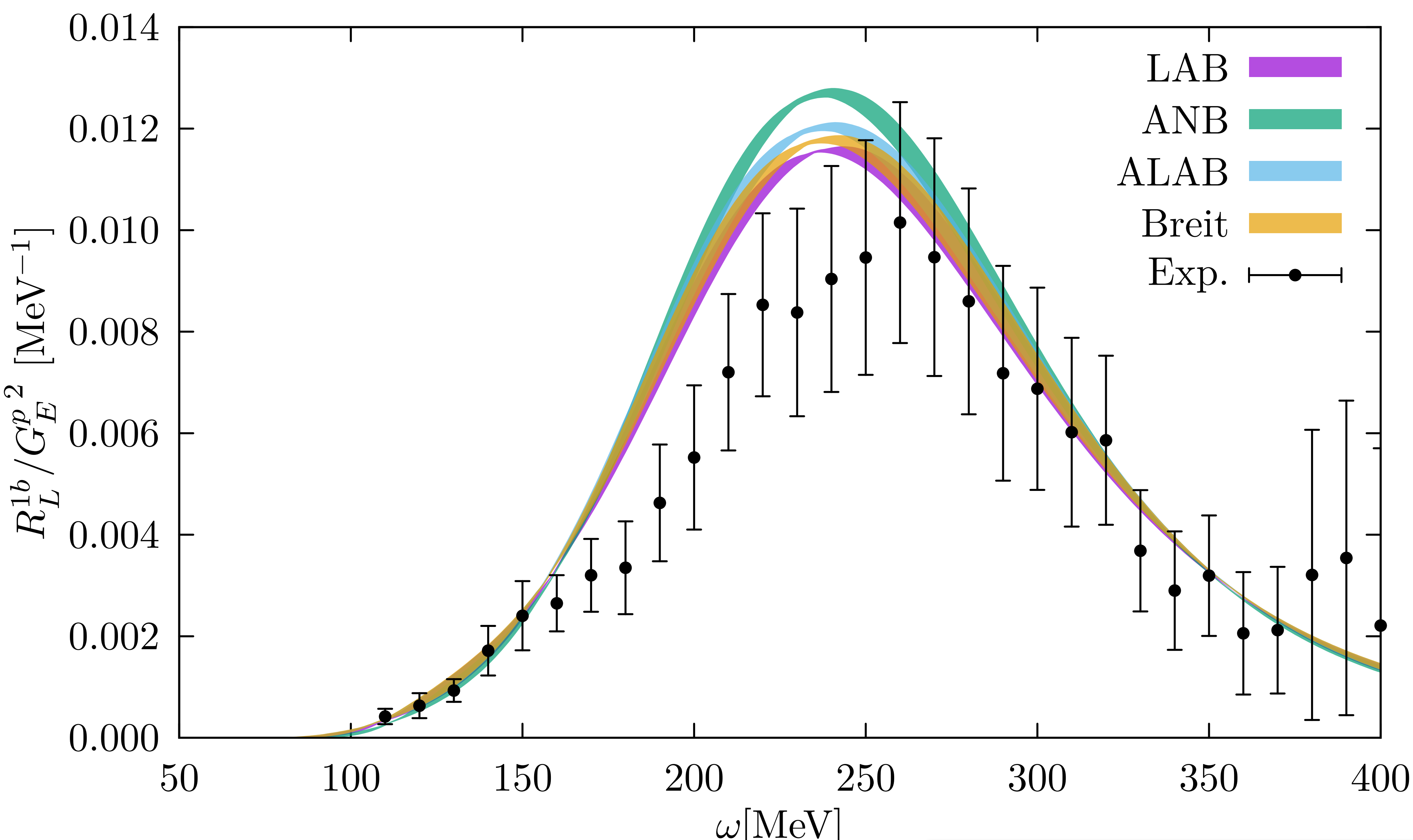
$$\epsilon_f = \frac{p_f^2}{2\mu} + \epsilon_0^{A-1}$$

- The energy-conserving delta function reads

$$\delta(\omega^{fr} - E_f^{\rm fr}(\epsilon_f) + E_0^{fr}) = \left(\frac{\partial E_f^{\rm fr}(\epsilon_f)}{\partial \epsilon_f^{\rm fr}}\right)^{-1} \delta\left(\epsilon_f - \frac{p_f^2(\omega^{\rm fr}, |\mathbf{q}^{\rm fr})}{2\mu} - \epsilon_0^{A-1}\right)$$

Relativistic effects in a correlated system

- The ^{4}He longitudinal response at q=700 MeV **mildly** depends on the original reference frame



N. Rocco et al. PRC 97 055501(2018)

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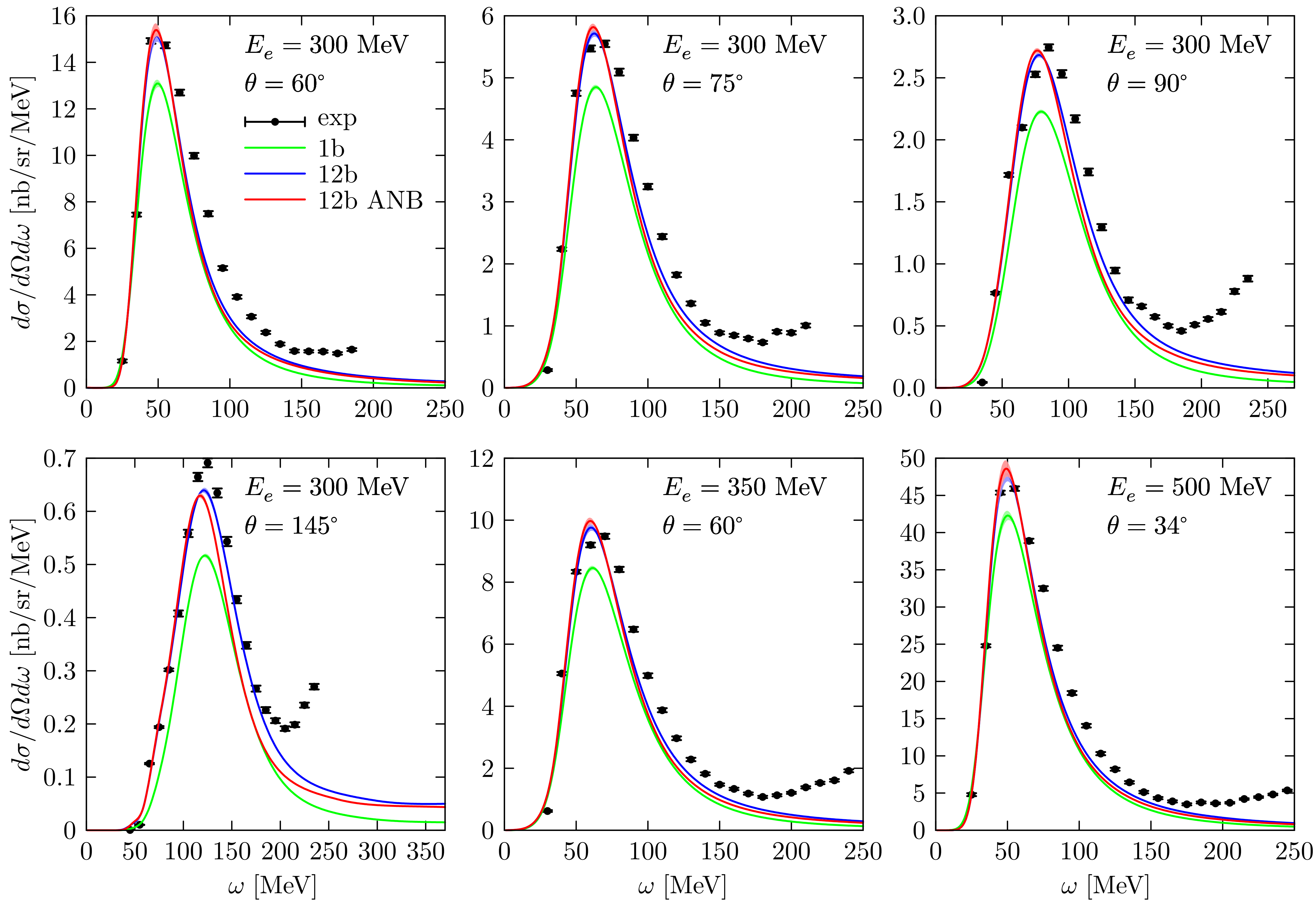
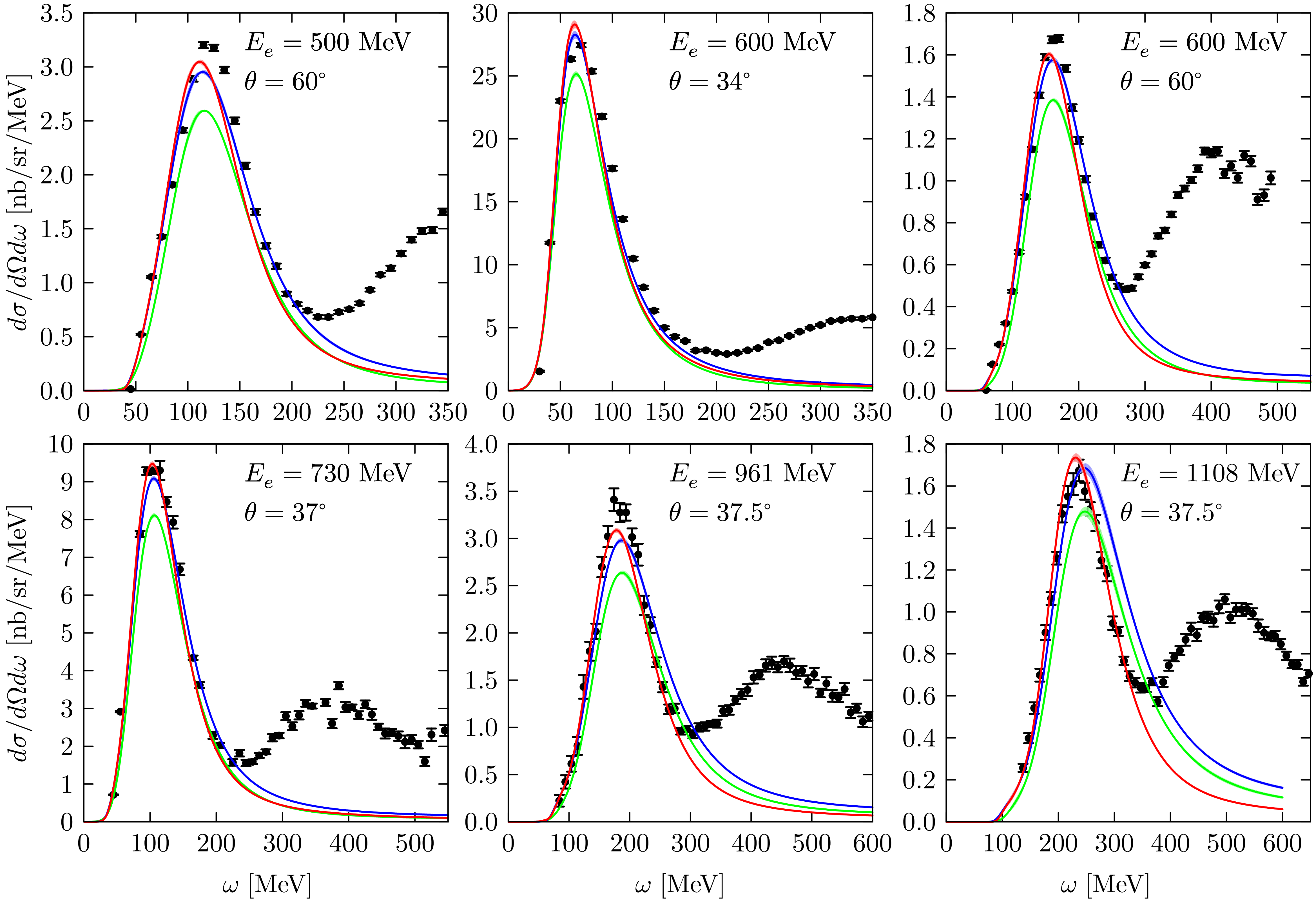
Relativistic effects in a correlated system



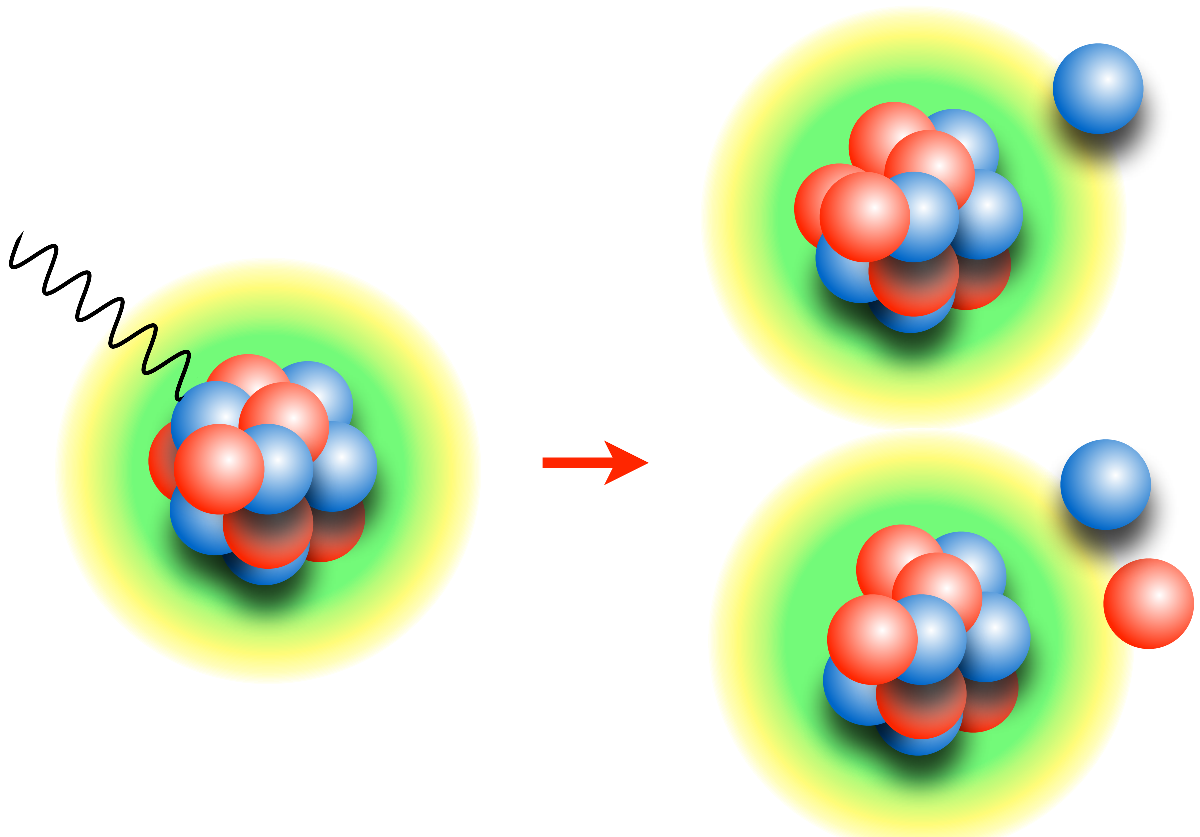
N. Rocco et al. PRC 97 055501(2018)

Relativistic effects in a correlated system



Fully-relativistic regime

- At (very) large momentum transfer, scattering off a nuclear target reduces to the sum of scattering processes involving bound nucleons → short-range correlations.



$$|\Psi_f\rangle \simeq |p_1\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$|\Psi_f\rangle \simeq |p_1, p_2\rangle \otimes |\Psi_f\rangle_{A-2}$$

- Relativistic effects and resonance production-mechanisms play a major role and need to be accounted for along with nuclear correlations.

- The approach based on realistic spectral functions has recently been extended to include two-body currents in electron and neutrino scattering → **N. Rocco**'s talk on Thursday

(Intermediate) Conclusions

- ^{12}C electromagnetic responses are in good agreement with experiments.
- Two-body current contributions enhance the longitudinal and transverse axial responses
- Quantum Monte Carlo is suitable to compute cross-sections, not only responses
- Charged-current responses calculations underway

Disclaimer

- The continuity equation only constraints the longitudinal components of the current
- The transverse component and the axial terms are phenomenological (the coupling constant is fitted on the tritium beta-decay)
- Two- and three- body forces not consistent

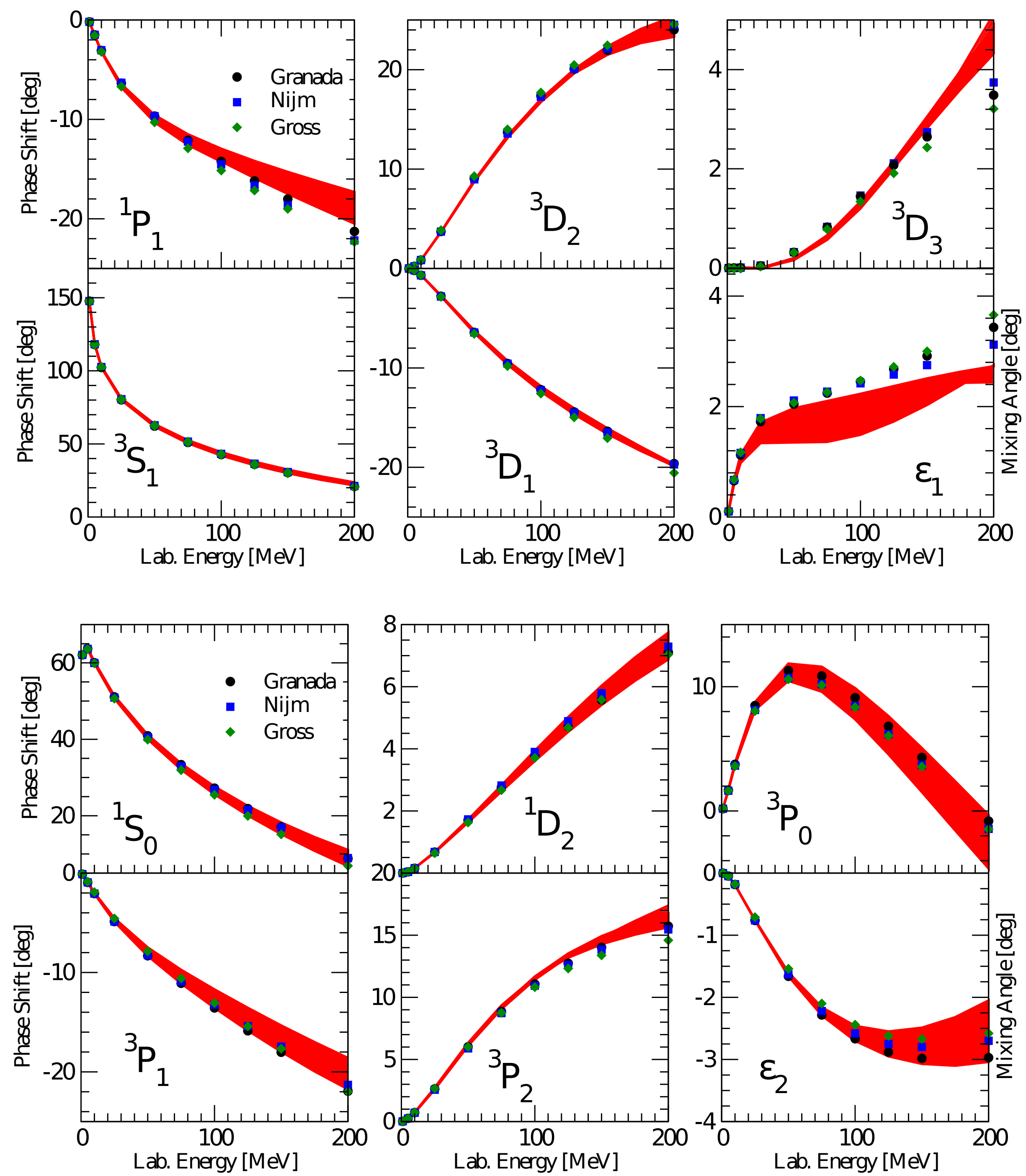
The theoretical error arising from modeling the nuclear dynamics cannot be properly assessed

Δ-full local chiral potential

We have complemented the historical “Argonne” approach by considering a local chiral Δ -full potential giving an excellent fit to the NN scattering data that can be readily used in QMC.

- Closer connection with QCD
- Consistent MEC available
- Theoretical uncertainty estimation

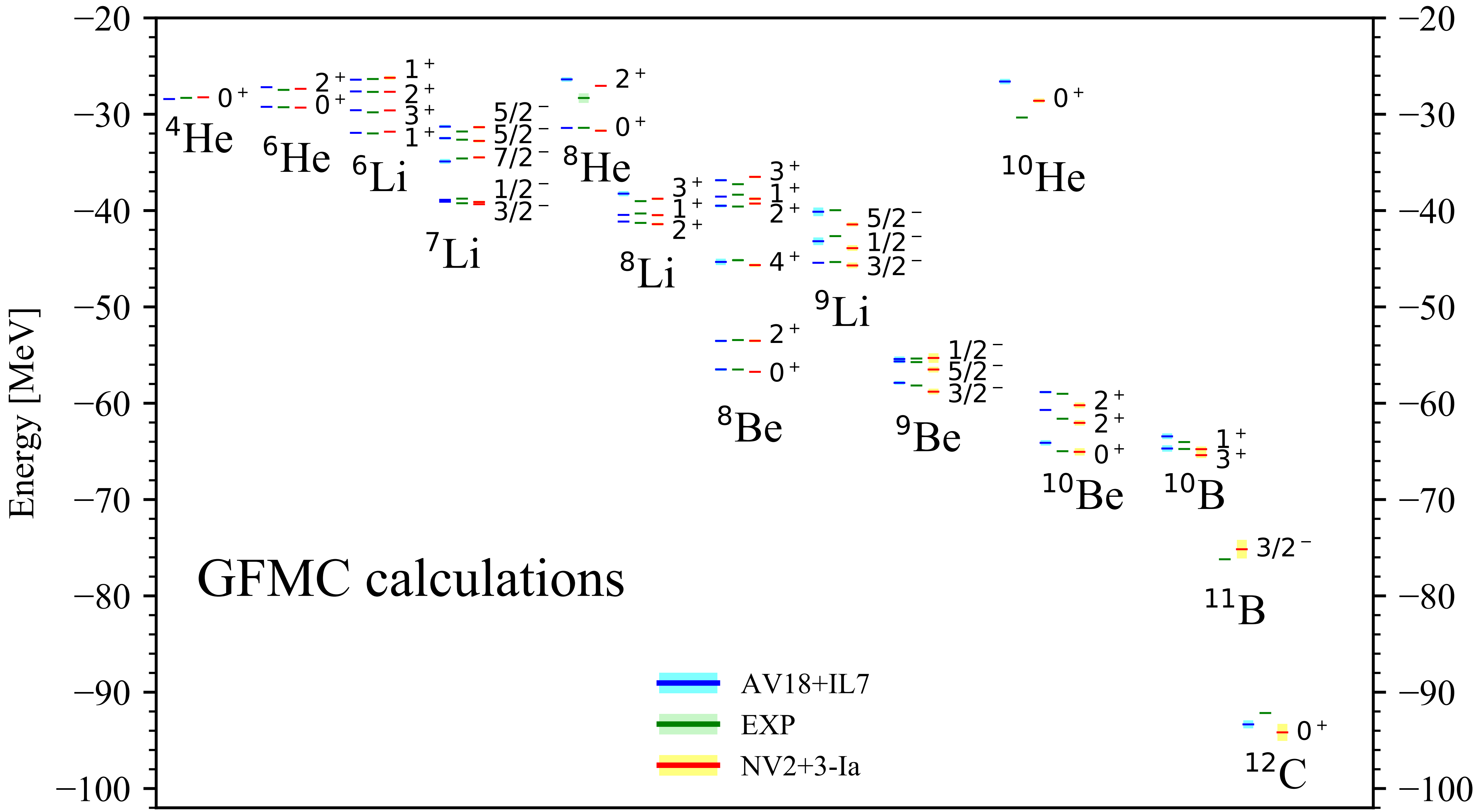
model	order	E_{Lab} (MeV)	N_{pp+np}	χ^2/datum
b	LO	0–125	2558	59.88
b	NLO	0–125	2648	2.18
b	N2LO	0–125	2641	2.32
b	N3LO	0–125	2665	1.07
a	N3LO	0–125	2668	1.05
c	N3LO	0–125	2666	1.11
$\widetilde{a}$	N3LO	0–200	3698	1.37
$\widetilde{b}$	N3LO	0–200	3695	1.37
$\widetilde{c}$	N3LO	0–200	3693	1.40



M. Piarulli et al. PRC 94 (2016) 054007

Δ-full local chiral potential

The experimental A≤12 ground- and excited state energies are very well reproduced by the local Δ-full NN+NNN chiral interaction

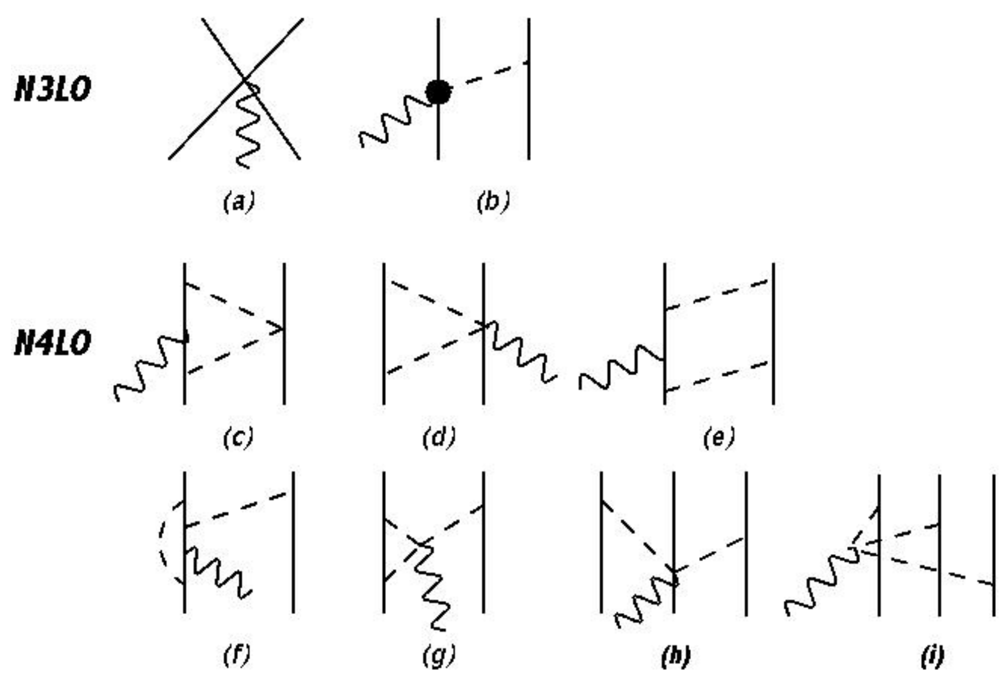


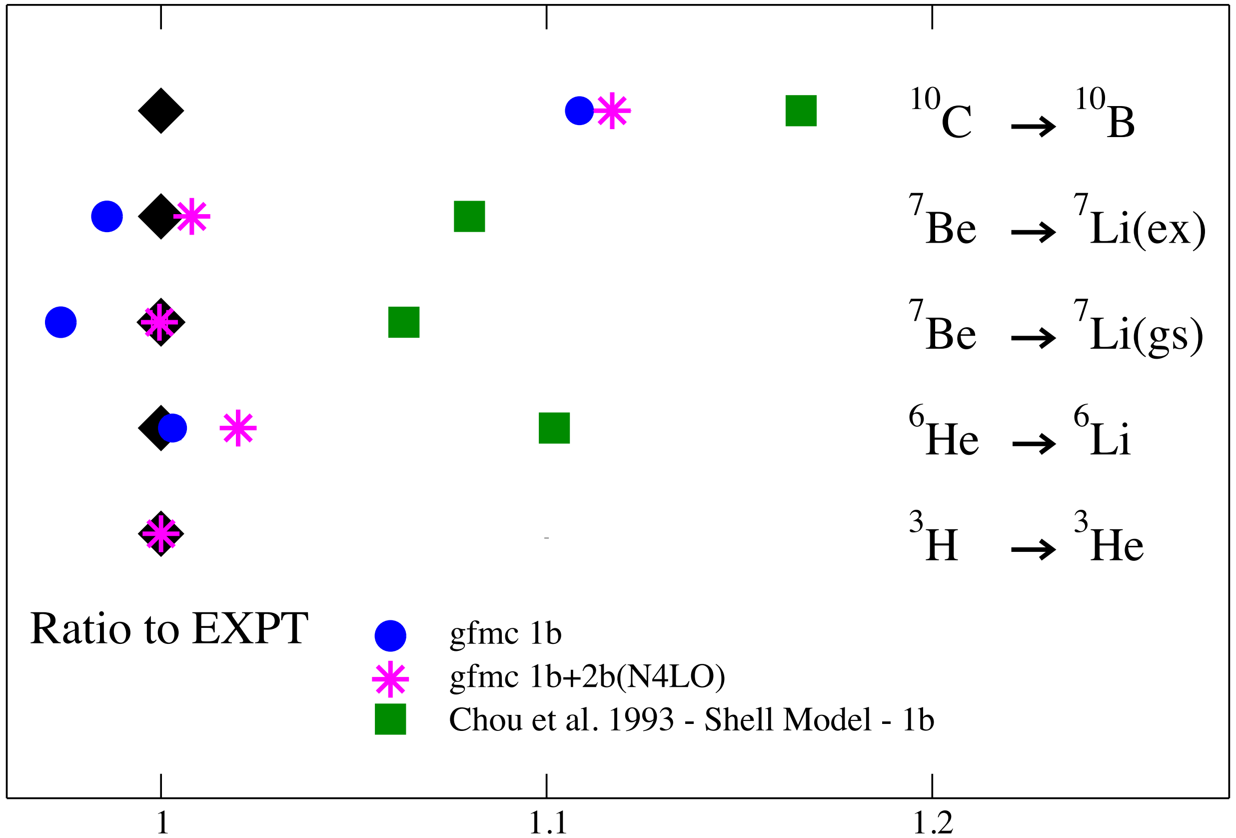
M. Piarulli, et al. PRL 120 (2018) 052503

Chiral-EFT currents

- Chiral currents consistent with the Δ-full local chiral potential are being developed

- Mixed-approach calculations indicate a slight enhancement of the decay rates from MEC

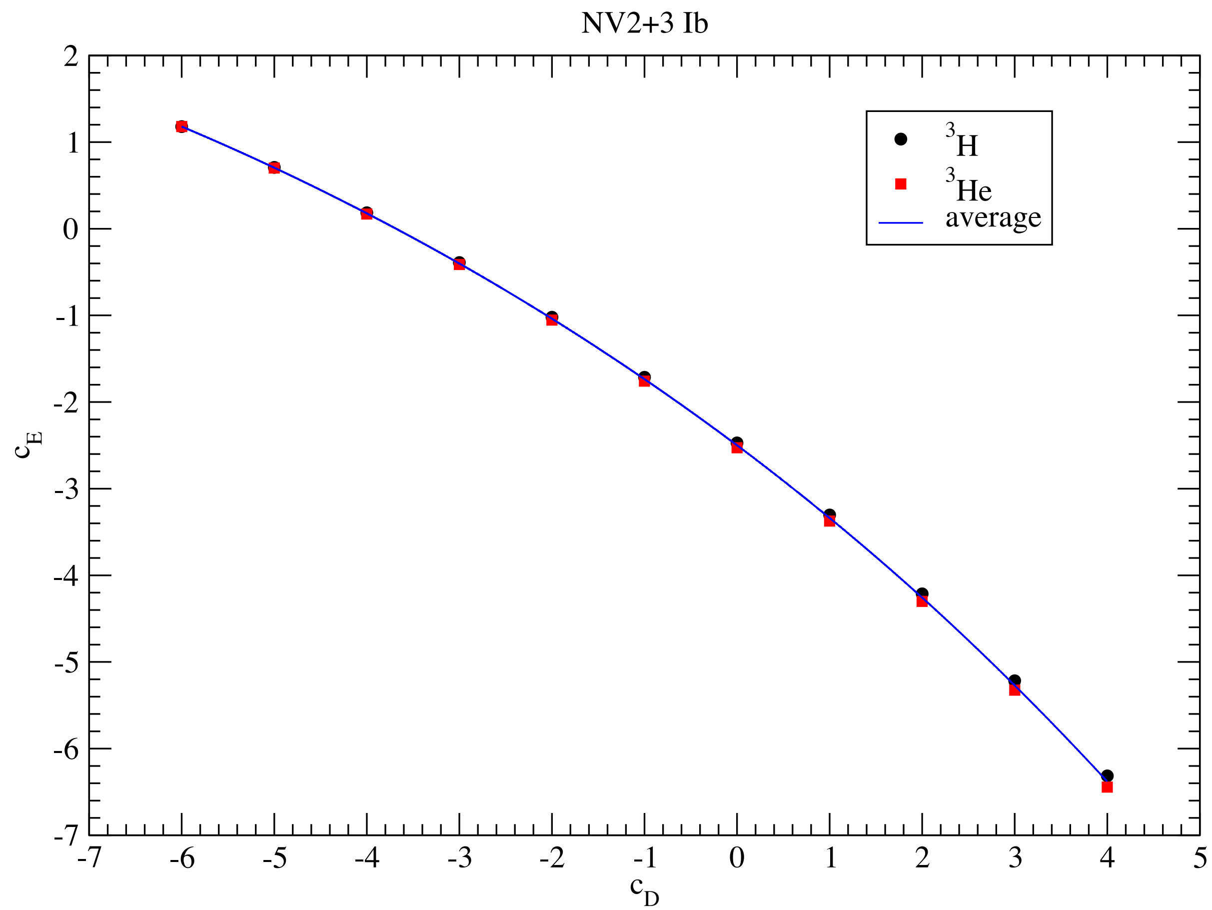




S. Pastore et al. PRC 97, 022501 (2018)

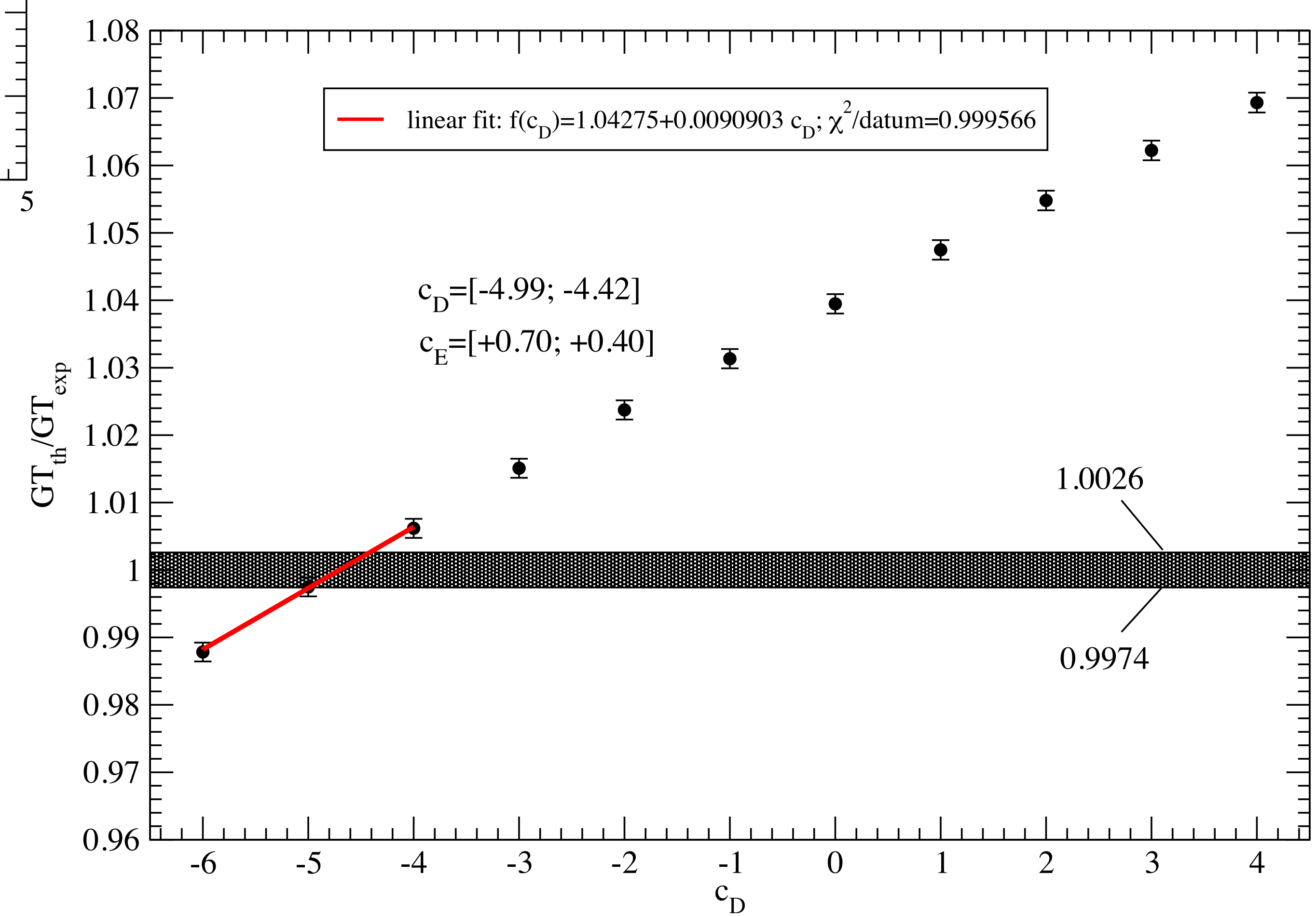
Nuclear spectra and decays

A. Baroni et al. arXiv:1806.10245



- Triton and ^{3}He average binding energies provide a correlation line between c_D and c_E

- Triton decay is most sensitive to c_D



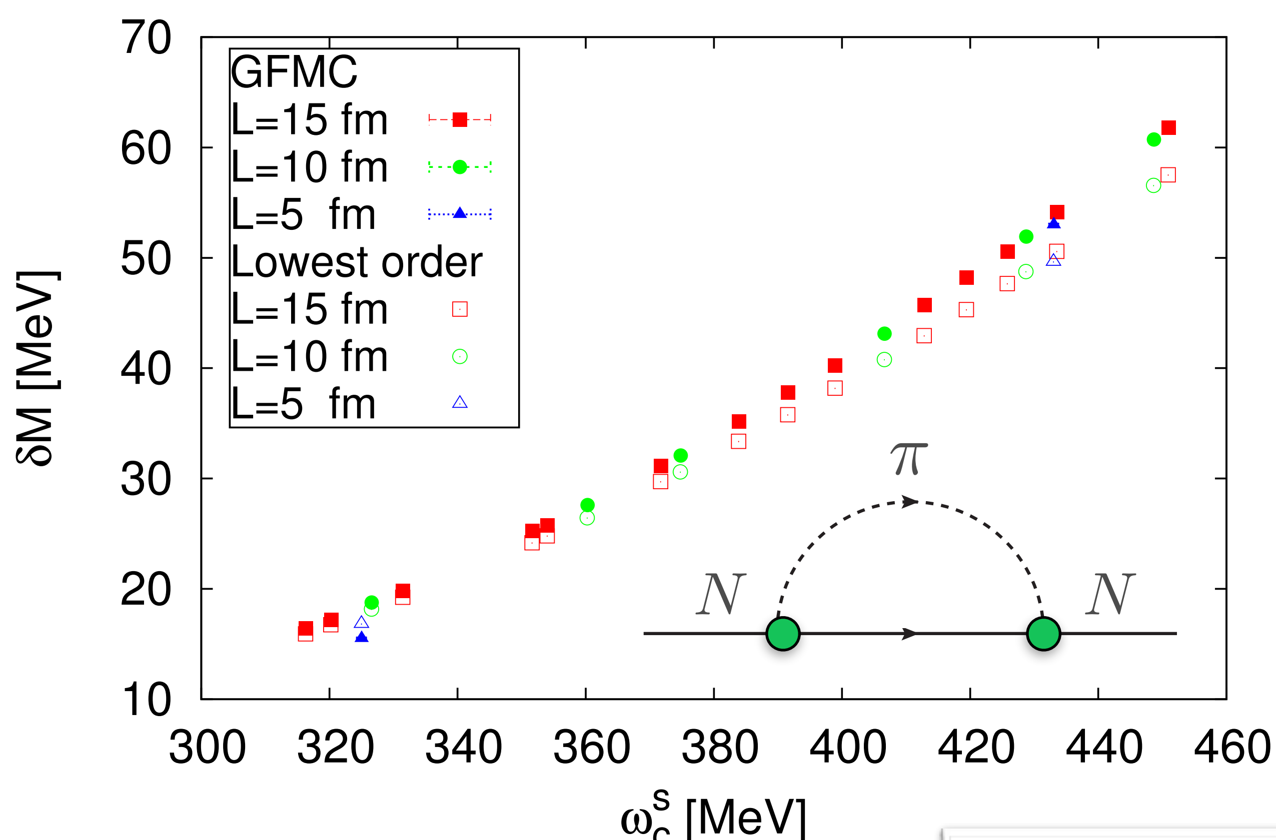
Explicit-pion QMC

The non-relativistic wave functions found solving the many-body Schrödinger equations describe the quantum-mechanical amplitudes of the nucleonic degrees of freedom.

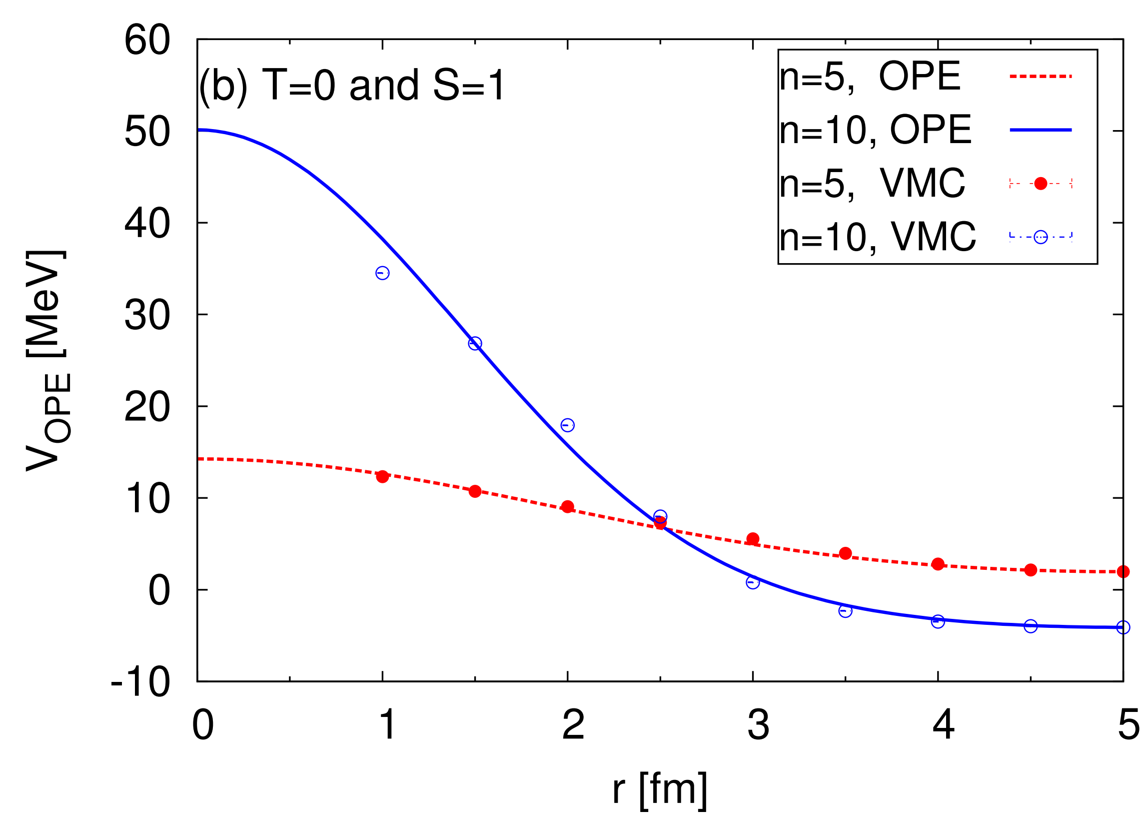
The relativistic pions and the nucleons are **both** explicitly included in the quantum-mechanical states of the system

$$\langle \mathbf{R}, S | \Psi \rangle \longrightarrow \langle \mathbf{R}, S, \boldsymbol{\pi}_k | \Psi \rangle \qquad \boldsymbol{\pi}(\mathbf{x}) = \frac{1}{\sqrt{L^3}} \sum_{\mathbf{k}} \boldsymbol{\pi}_k e^{i\mathbf{k}\cdot\mathbf{x}}$$

The **nucleon-mass renormalization** is consistent with quantum-field theory



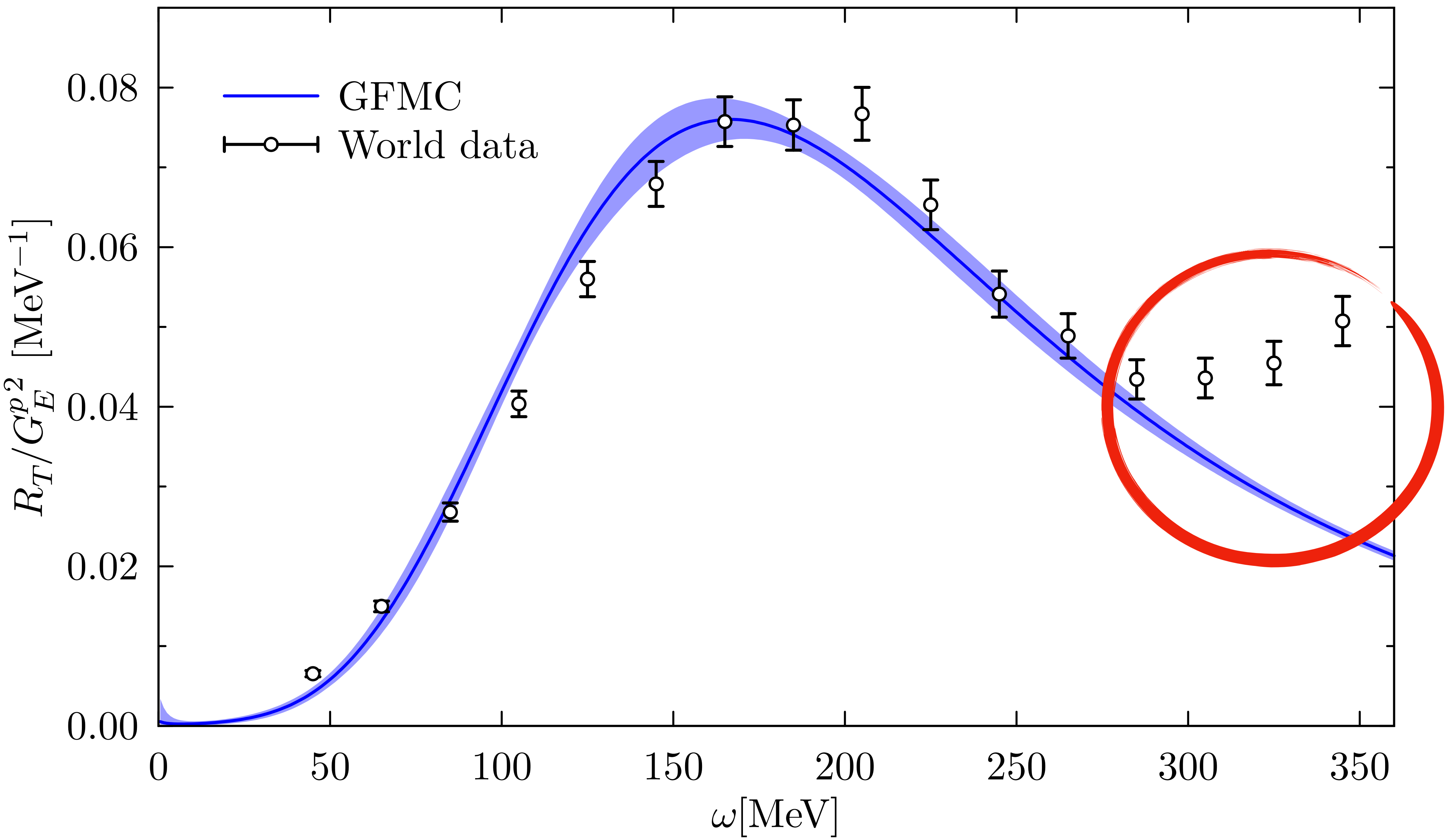
The interaction between two static nucleons reduces to one-pion exchange at large distance



L. Madeira, et al. arXiv:1803.10725

Explicit-pion QMC

Our goal (a long way ahead) is to perform reliable predictions for **pion production** in electron- and neutrino-nucleus scattering



Thank you