Nuclear Theory, Data and Event Generators

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Nuclear Theory and Data

1. Most of the current data consists of "Inclusive" cross sections

2. Theoretical calculations in the "Quasielastic" region consist of

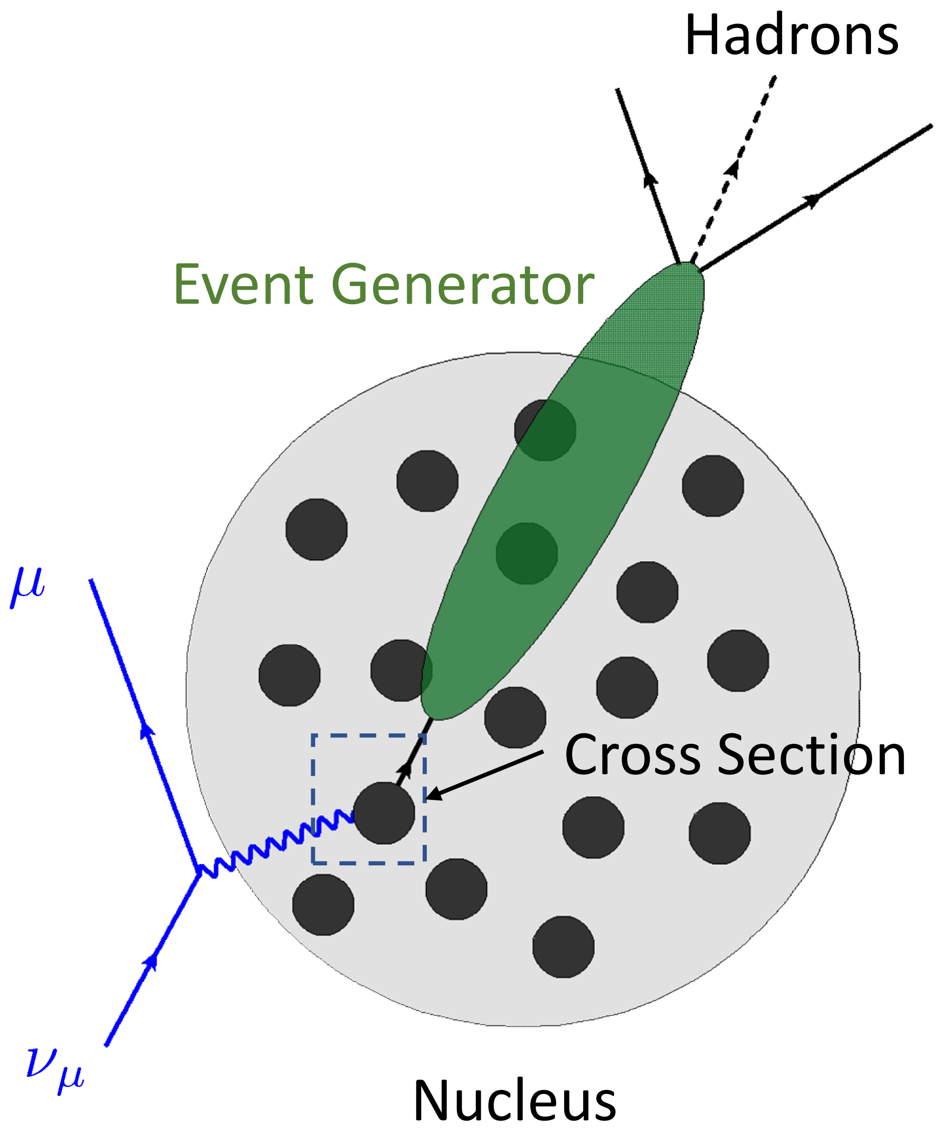
   - Models including quasielastic scattering, meson exchange currents, meson production, etc.

   - The Random Phase Approximation (RPA) plus the above.

   - *Ab Initio* calculations of quasielastic scattering using GFMC.

   - Phenomenological representations of data, SUSAV2.

Nuclear Theory and Event Generators



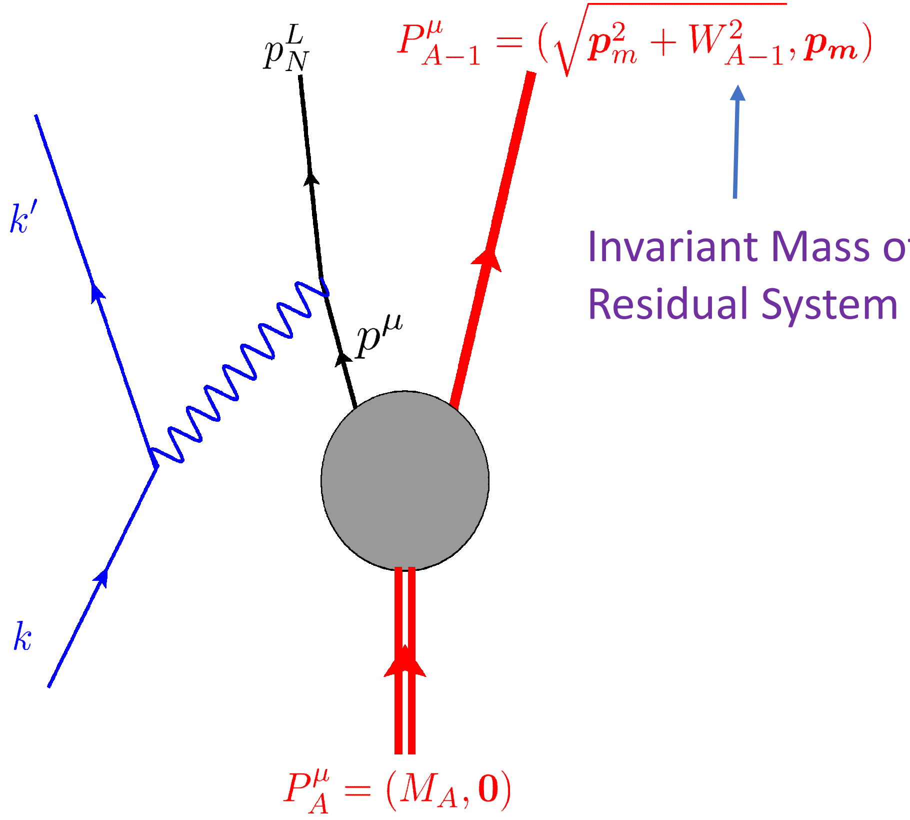
- The theoretical input into event generators consists of calculation of the cross section for an initial scattering event from the ground state of the target nucleus which produces a particle(s) without final state interactions.

- Final state interactions are produced by the event generator using some prescription for transport of the initial particle(s) through the rest of the nucleus.

- The theoretical input is usually somewhat simplistic.

Factorization Approximation

Spectral Function $$S(p_m, E_m) = \sum_{s_A, s_{A-1}} \langle P_A, s_A | p^\mu; P_{A-1}, s_{A-1} \rangle \langle p^\mu; P_{A-1}, s_{A-1} | P_A, s_A \rangle$$



Excitation energy of the residual system.

$$\mathcal{E} = \sqrt{\boldsymbol{p}_m^2 + W_{A-1}^2} - \sqrt{\boldsymbol{p}_m^2 + M_{A-1}^2} \simeq W_{A-1} - M_{A-1}$$

Lowest Energy of the Residual System

Separation Energy

$$E_s = M_{A-1} + m_N - M_A$$

Missing Energy

$$E_m = \mathcal{E} + E_s$$

Normalization of the Spectral Function

$$n(p_m) = \int_0^\infty dE_m \, S(p_m, E_m)$$

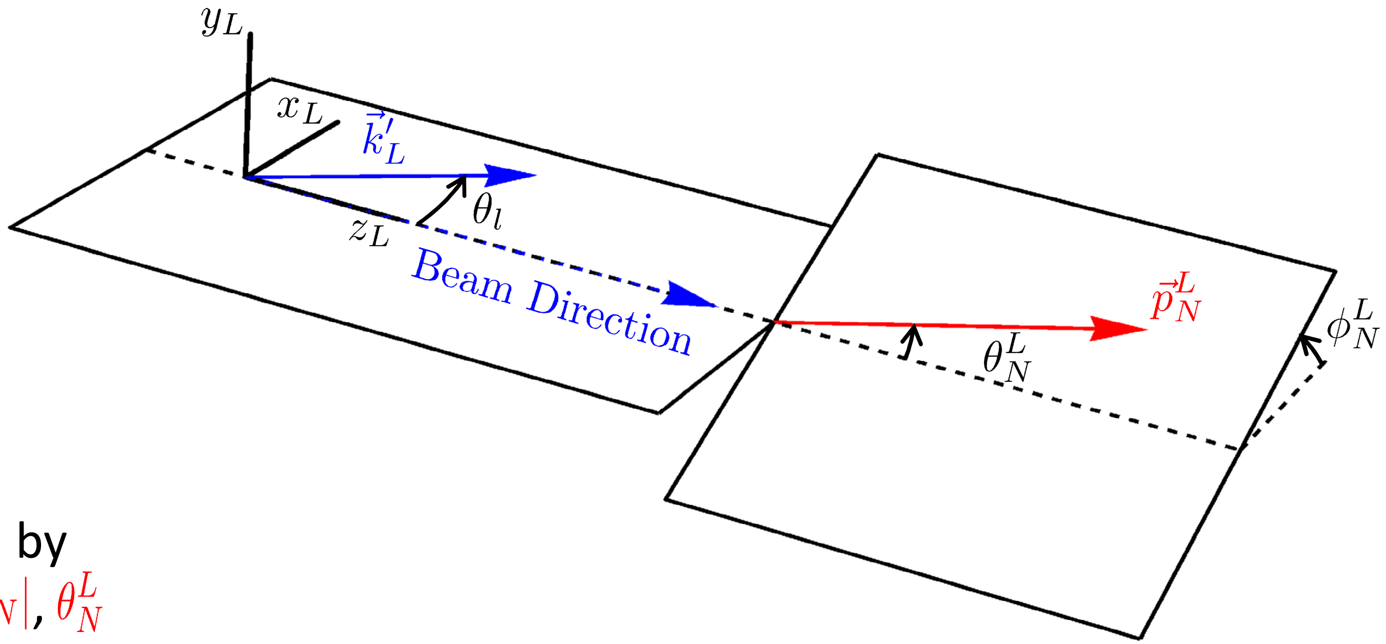
Momentum Density Distribution of the Target Nucleus

$$\mathcal{N} = \frac{1}{(2\pi)^3} \int_0^\infty dp_m \, p_m^2 n(p_m)$$

Number of Active Nucleons in the Target Nucleus

Kinematic Variables in the “Lab” Frame

Since the objective is to determine the incident neutrino energy to study neutrino oscillations and given that the beam direction is known but not the incident momentum, it is best to consider the frame.



$|\vec{k}|$ can be determined by measuring $|\vec{k}'|$, θ_l, $|\vec{p}_N|$, θ_N^L and ϕ_N^L.

Semi-Inclusive Cross Section

$$\frac{d\sigma}{dk' d\Omega_{k'} dp_N d\Omega_N^L} = \frac{G^2 \cos^2\theta_c m_N k'^2 p_N^2}{8(2\pi)^6 \varepsilon' E_N} \int_0^\infty d\mathcal{E}\, \frac{\varepsilon_0 P(k_0)}{k_0^2} v_0 \widetilde{\mathcal{F}}_\chi^2 S(p_m, E_s + \mathcal{E})$$

where

$$k_0 = \sqrt{(\varepsilon' + \mathcal{E} + E_s + E_N - m_N)^2 - m^2}$$

and

$$p_m = \sqrt{k_0^2 + k'^2 + p_N^2 - 2k_0 k' \cos\theta_l - 2k_0 p_N \cos\theta_N^L + 2k' p_N (\cos\theta_l \cos\theta_N^L + \sin\theta_l \sin\theta_N^L \cos\phi_N^L)}$$

O. Moreno, T. W. Donnelly, J. W. Van Orden and W. P. Ford, PRD **90**, 013014 (2014);
J. W. Van Orden, T. W. Donnelly and O. Moreno, PRD **96**, 113008 (2017).

Reduced, Off-Shell, Single-Nucleon Cross Section

$$
\begin{aligned}
\widetilde{\mathcal{F}}_\chi^2 = & \widehat{V}_{CC}\left(\widetilde{w}_{CC}^{VV(I)} + \widetilde{w}_{CC}^{AA(I)}\right) + \widehat{V}_{CL}\left(\widetilde{w}_{CL}^{VV(I)} + \widetilde{w}_{CL}^{AA(I)}\right) + \widehat{V}_{LL}\left(\widetilde{w}_{LL}^{VV(I)} + \widetilde{w}_{LL}^{AA(I)}\right) \\
& + \widehat{V}_{T}\left(\widetilde{w}_{T}^{VV(I)} + \widetilde{w}_{T}^{AA(I)}\right) + \widehat{V}_{TT}\left(\widetilde{w}_{TT}^{VV(I)} + \widetilde{w}_{TT}^{AA(I)}\right)\cos 2\phi_N \\
& + \widehat{V}_{TC}\left(\widetilde{w}_{TC}^{VV(I)} + \widetilde{w}_{TC}^{AA(I)}\right)\cos\phi_N + \widehat{V}_{TL}\left(\widetilde{w}_{TL}^{VV(I)} + \widetilde{w}_{TL}^{AA(I)}\right)\cos\phi_N \\
& + \chi\left[\widehat{V}_{T'}\widetilde{w}_{T'}^{VA(I)} + \widehat{V}_{TC'}\widetilde{w}_{TC'}^{VA(I)}\cos\phi_N + \widehat{V}_{TL'}\widetilde{w}_{TL'}^{VA(I)}\cos\phi_N\right]
\end{aligned}
$$

CCν Inclusive Cross Section

$$\frac{d\sigma}{dk' d\Omega_{k'}} = \frac{G^2 \cos^2 \theta_c m_N {k'}^2}{8(2\pi)^5 \varepsilon'} \int_0^\infty dk \frac{P(k)}{qk} \int_0^\infty d\mathcal{E} \int_0^\infty dp_m p_m v_0 \widetilde{\mathcal{F}}_\chi^2 S(p_m, E_s + \mathcal{E})$$
$$\times \theta(p_m^+ - p_m)\theta(p_m - p_m^-)$$

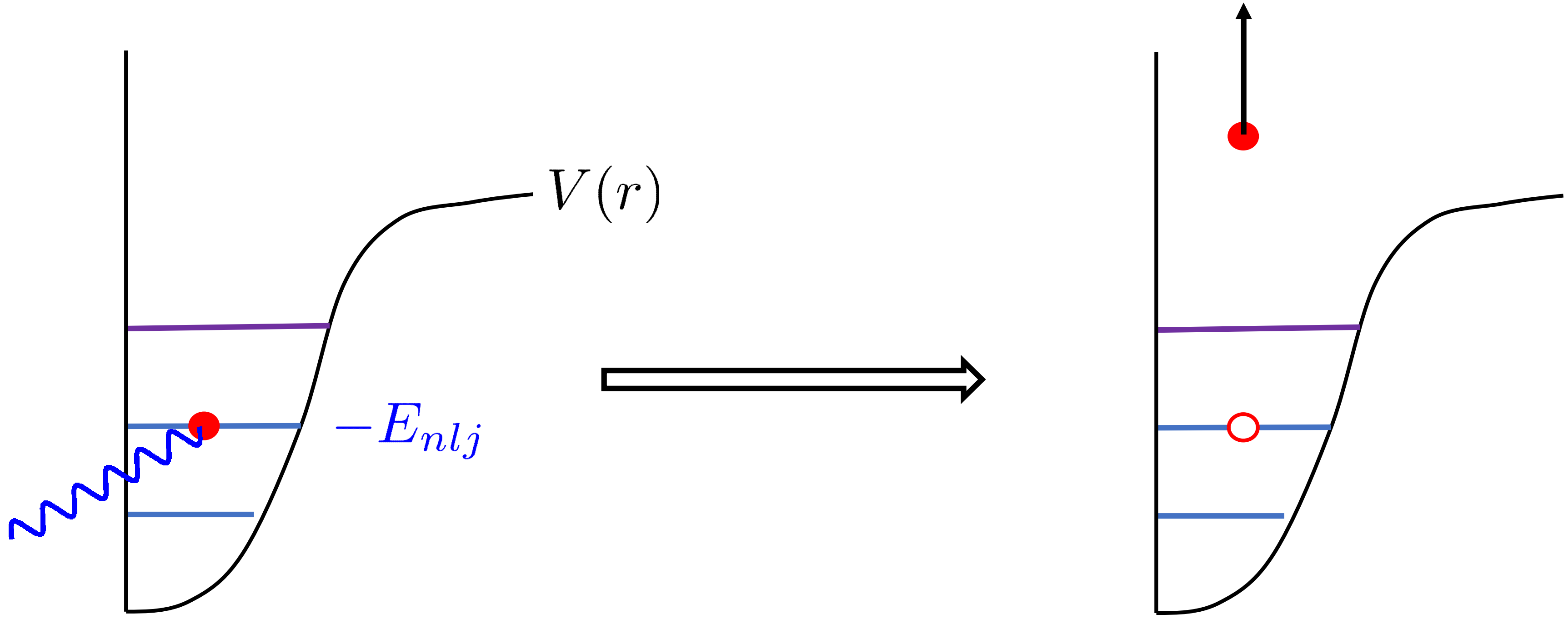
where

$$p_m^+ = \sqrt{(\omega - E_s - \mathcal{E})(\omega - E_s - \mathcal{E} + 2m_N)} + q$$

$$p_m^- = \left|\sqrt{(\omega - E_s - \mathcal{E})(\omega - E_s - \mathcal{E} + 2m_N)} - q\right|$$

Spectral Functions

Independent Particle Shell Model



$$S_{SM}(p_m, E_m) = \sum_{n,j,l} 2(2j+1) n_{nlj}(p_m) \delta(E_m - E_{nlj})$$

Relativistic Fermi Gas

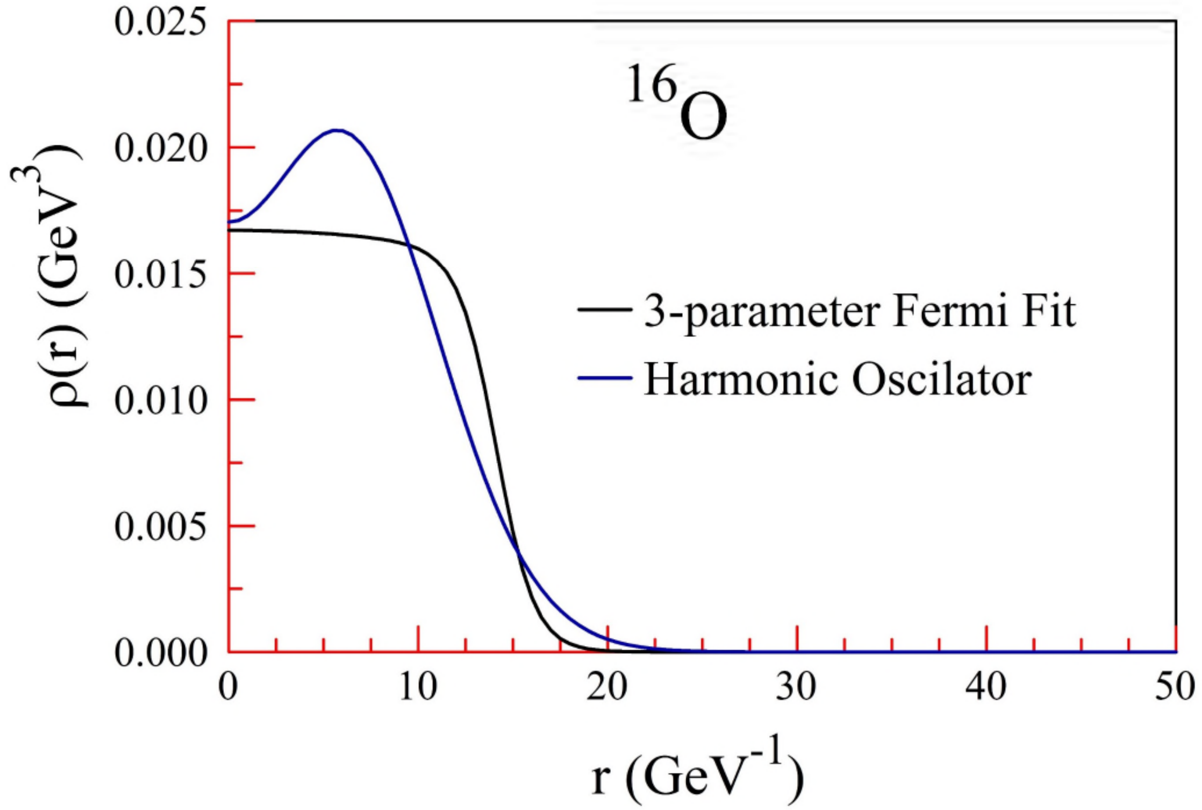
$$S_{RFG}(p_m, \mathcal{E}+E_s, k_F) = \frac{3(2\pi)^3\mathcal{N}}{k_F^3}\delta\left(\mathcal{E} - \sqrt{k_F^2 + m_N^2} + \sqrt{p_m^2 + m_N^2}\right)\theta(k_F - p_m)$$

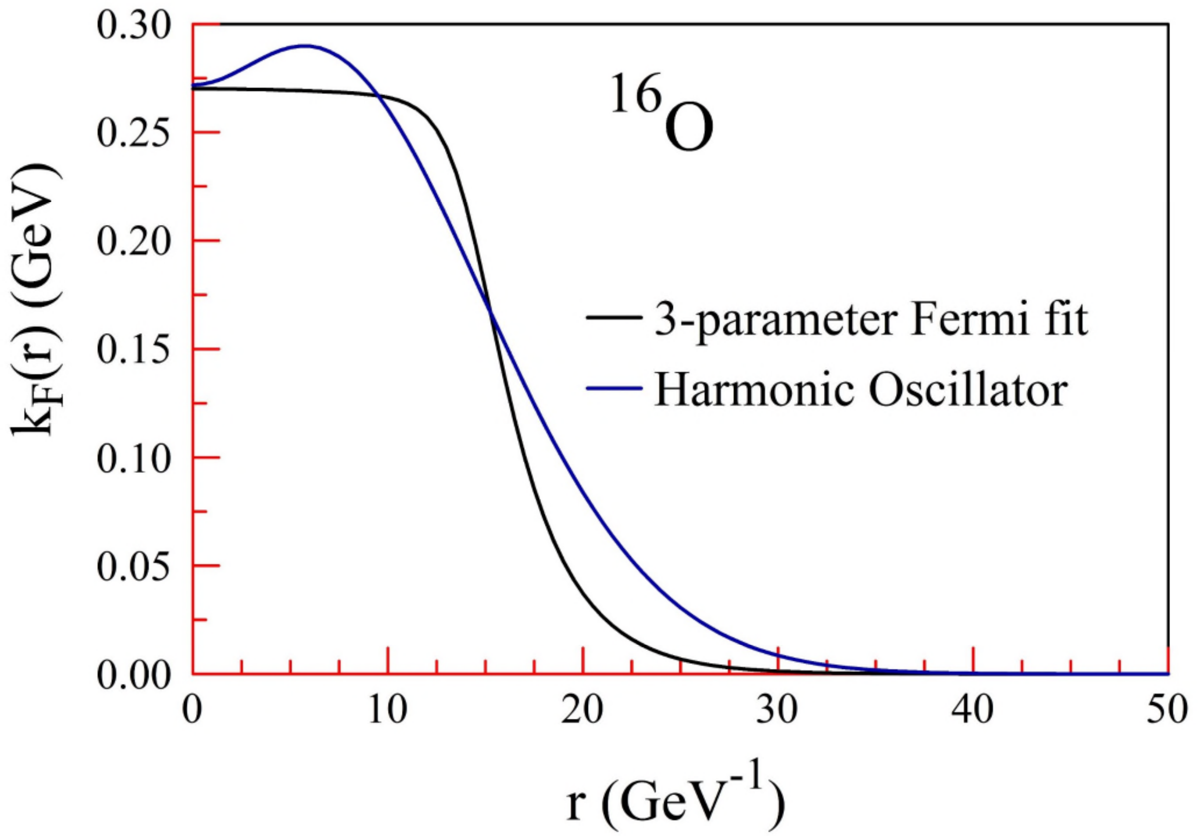
Local Density Approximation

$$4\pi \int_0^\infty drr^2\rho(r) = \mathcal{N} \qquad \rho(r) = \frac{2k_F^3(r)}{3\pi^2} \qquad k_F(r) = \left(\frac{3\pi^2\rho(r)}{2}\right)^{\frac{1}{3}}$$

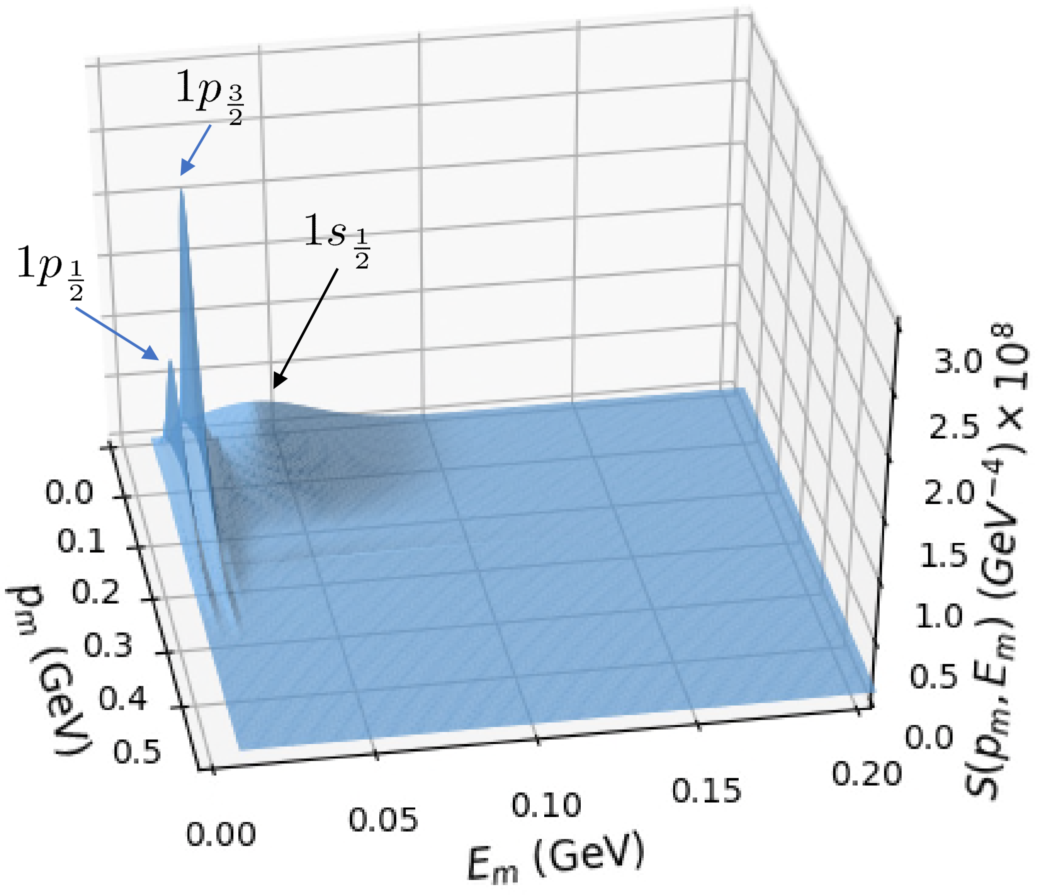
$$\begin{aligned} S_{LD}(p_m, \mathcal{E}+E_s) &= 4\pi \int_0^\infty drr^2\rho(r)S_{RFG}(p_m, \mathcal{E}+E_s, k_F(r)) \\ &= (8\pi)^2 \int_0^\infty drr^2\delta\left(\mathcal{E} - \sqrt{k_F^2(r) + m_N^2} + \sqrt{p_m^2 + m_N^2}\right)\theta(k_F(r) - p_m) \end{aligned}$$

Nuclear Densities and Local Fermi momenta for ^{16}O

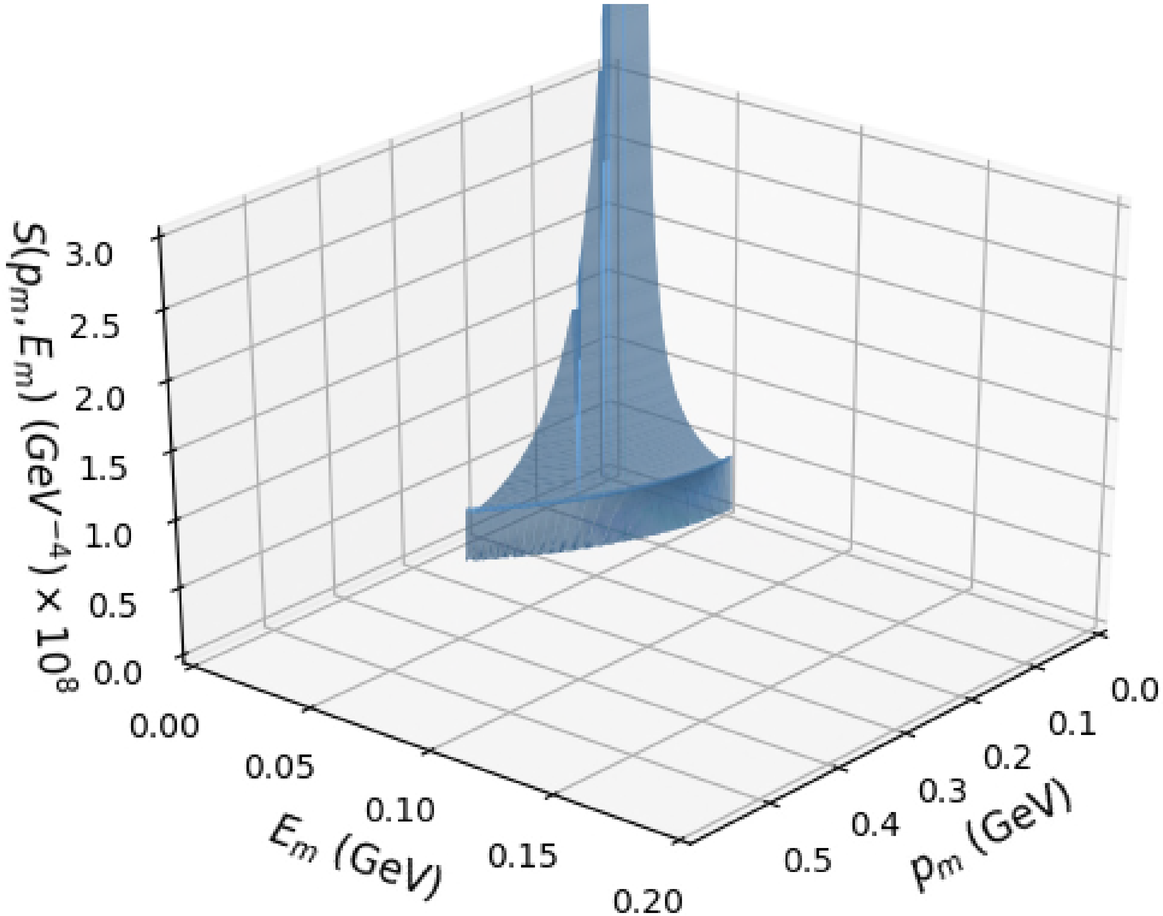




Spectral Function*



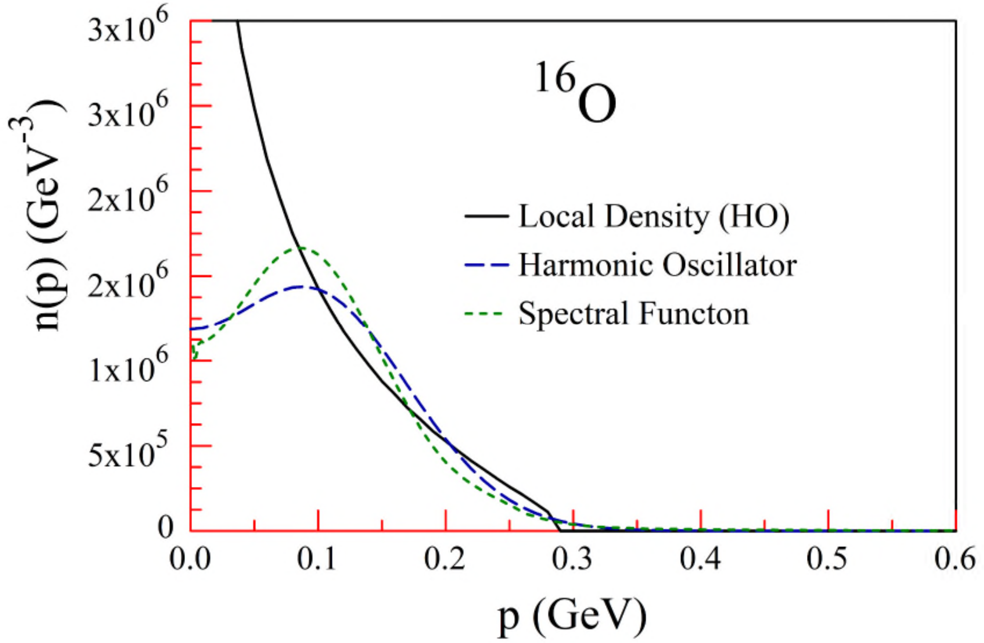
Local Density Spectral Function 3-parameter Fermi Fit

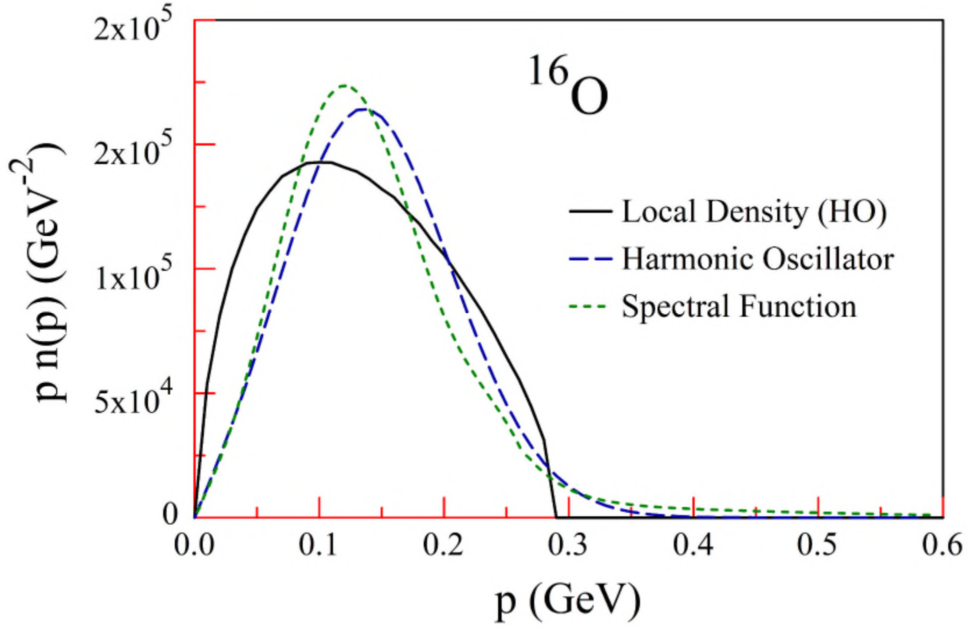


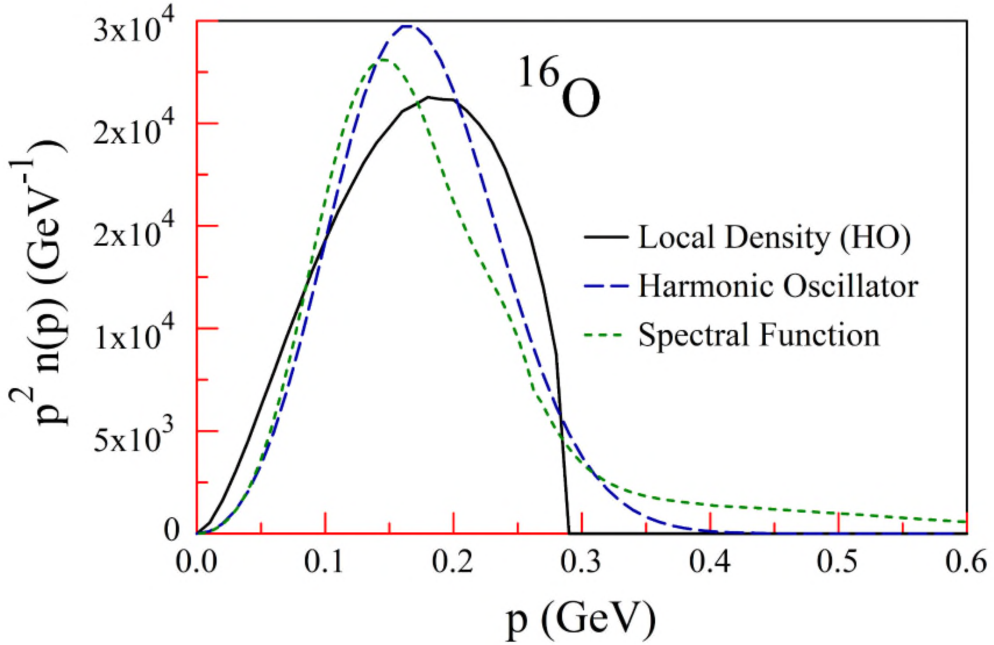
* O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl. Phys. A579, 493 (1994).
O. Benhar, N. Farina, H. Nakamura, M. Sakuda, and R.Seki, Phys. Rev. D 72, 053005 (2005).

Momentum Distributions

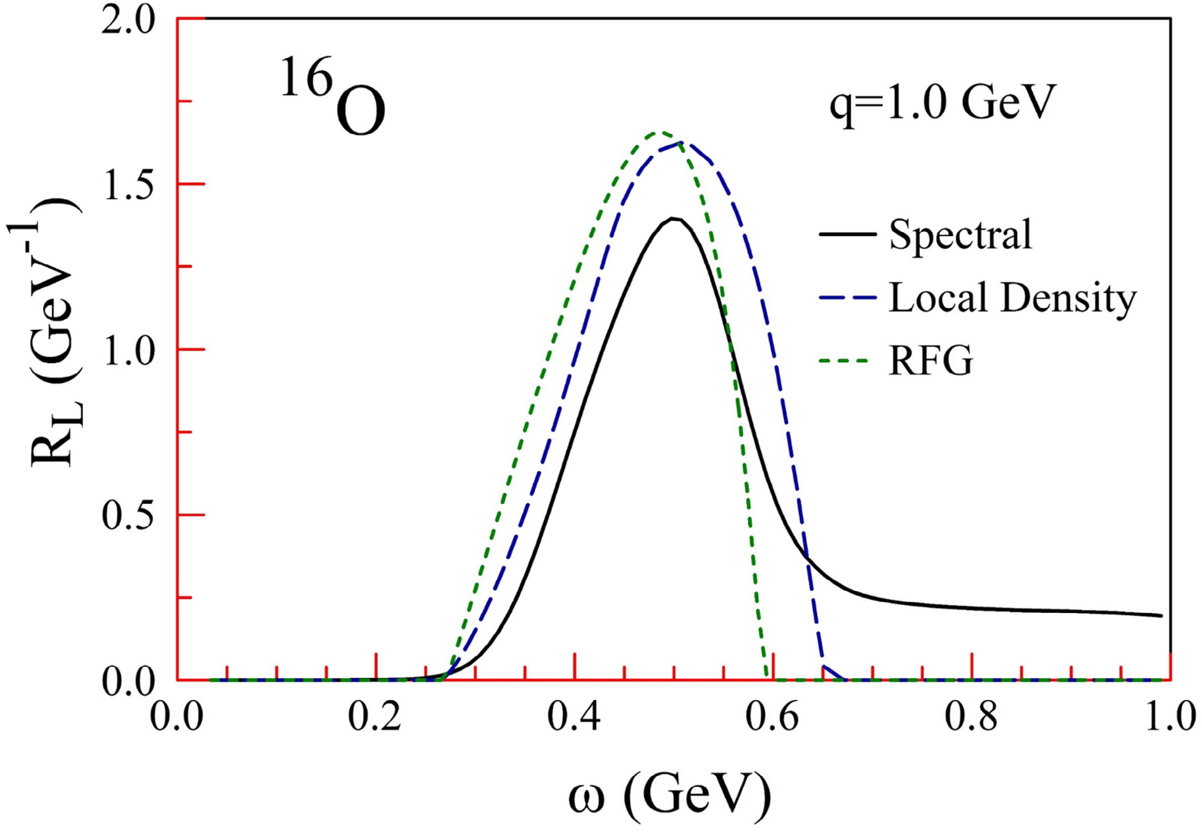
$$n(p) = \int_0^\infty dE\, S(p, E)$$

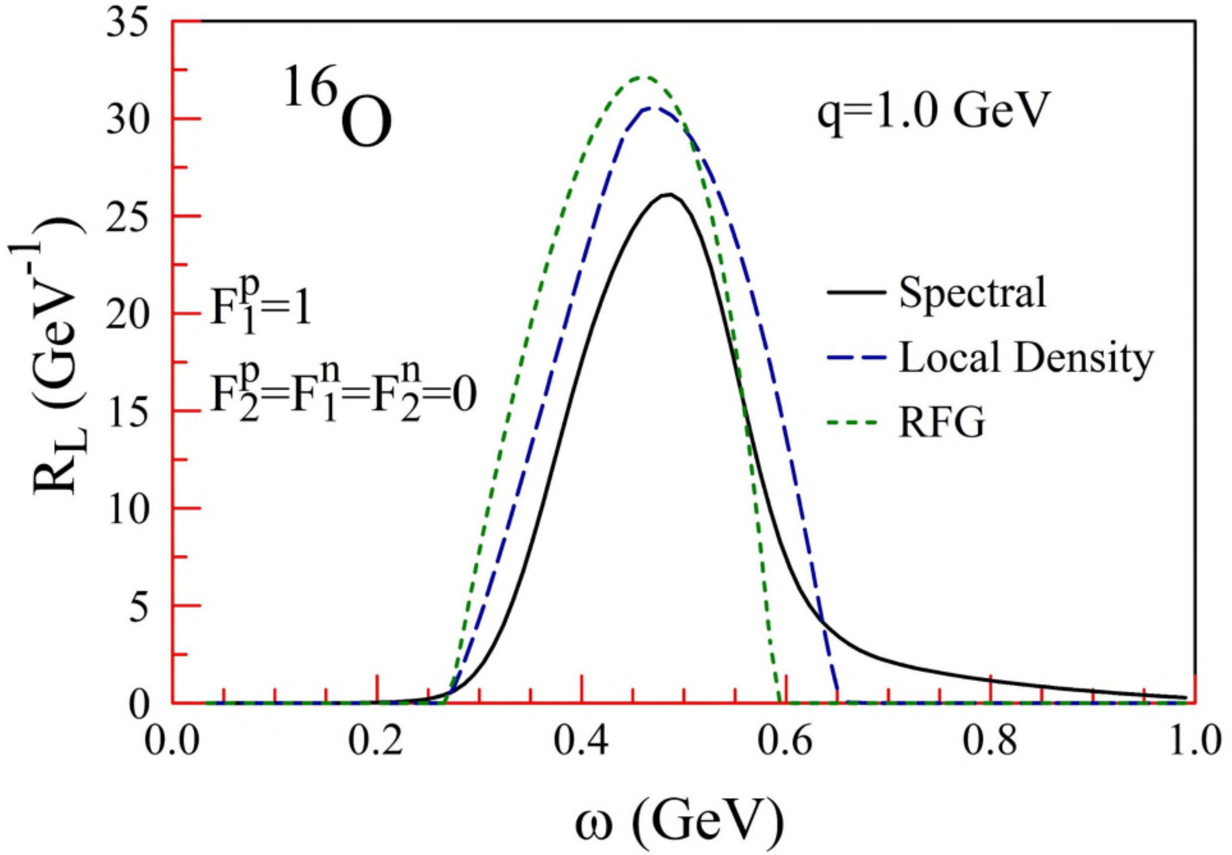






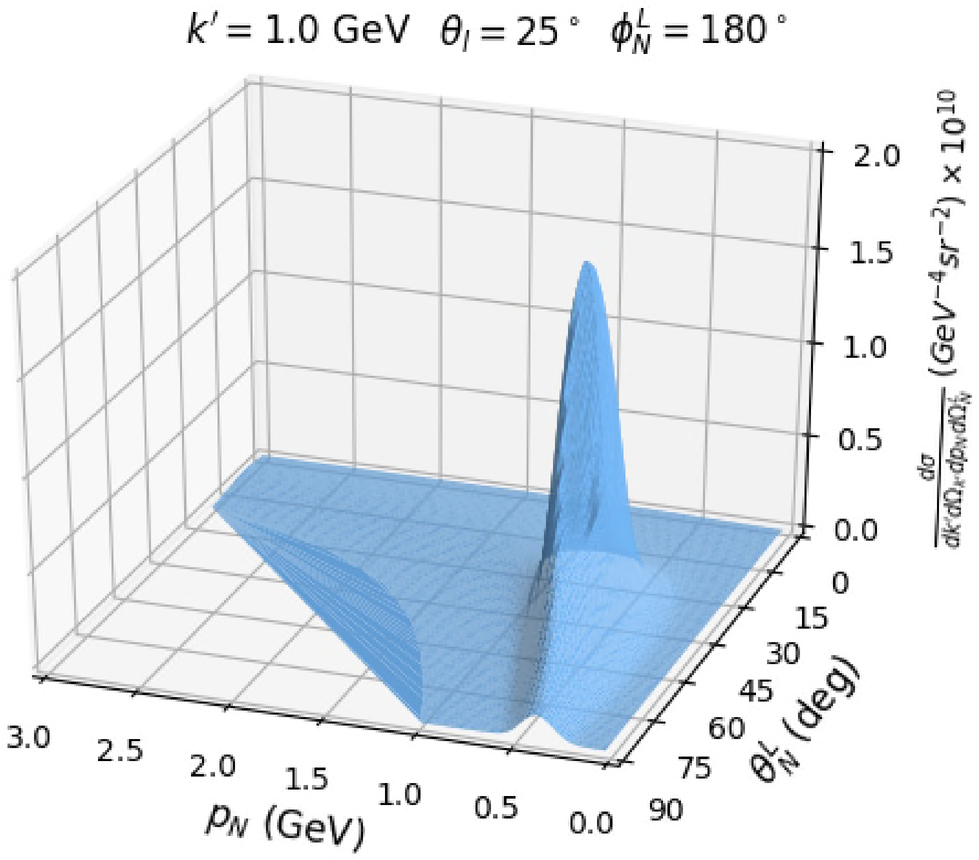
Longitudinal Response for Electron Scattering from ^{16}O



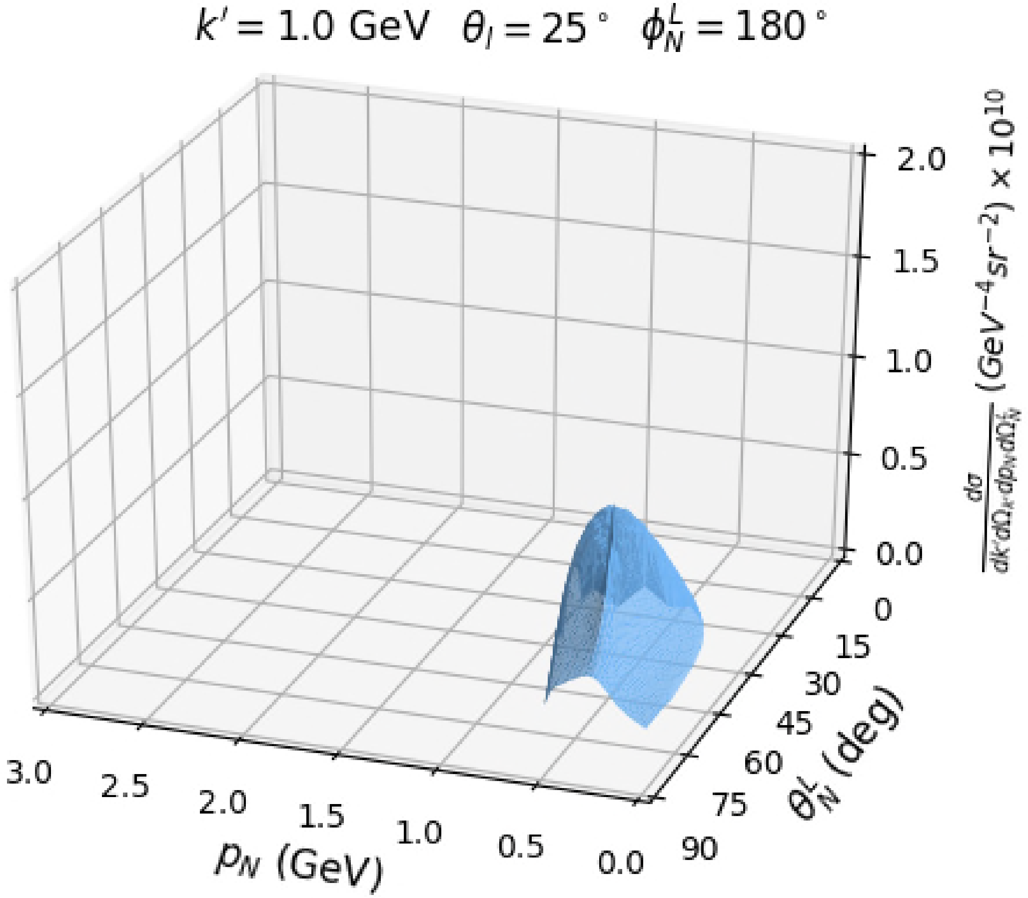


Semi-Inclusive CCν for ^{16}O

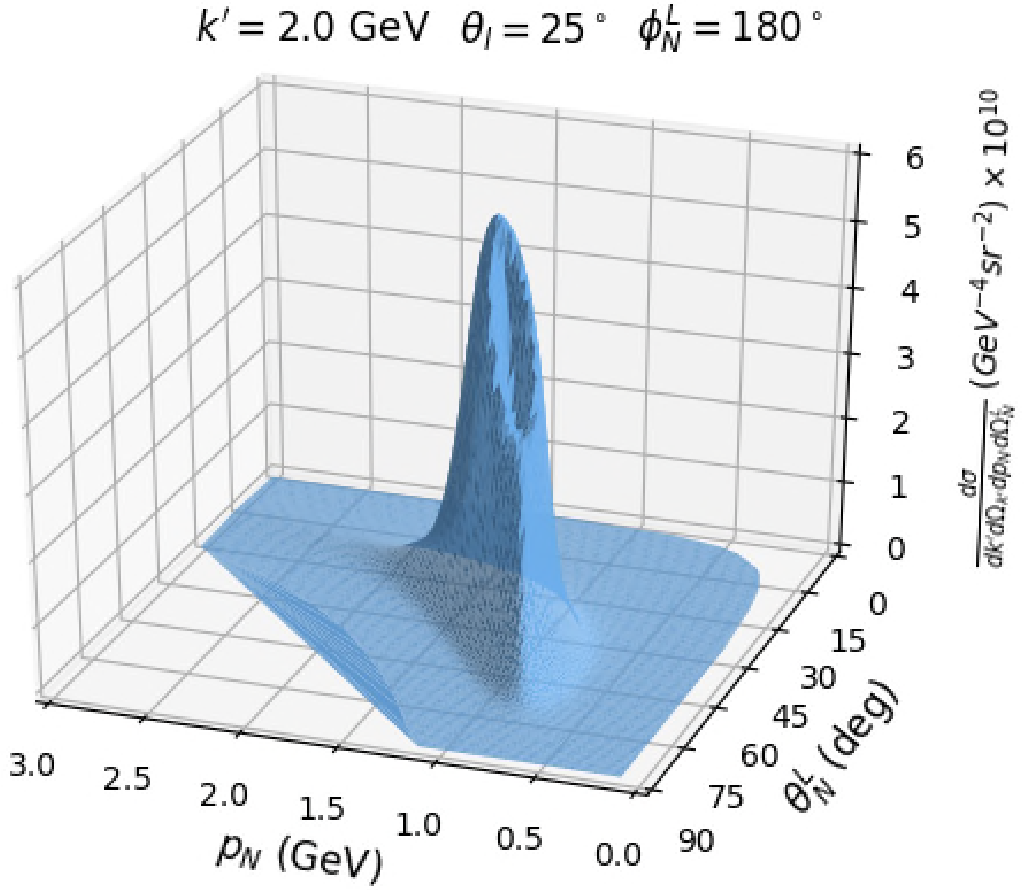
Spectral Function*



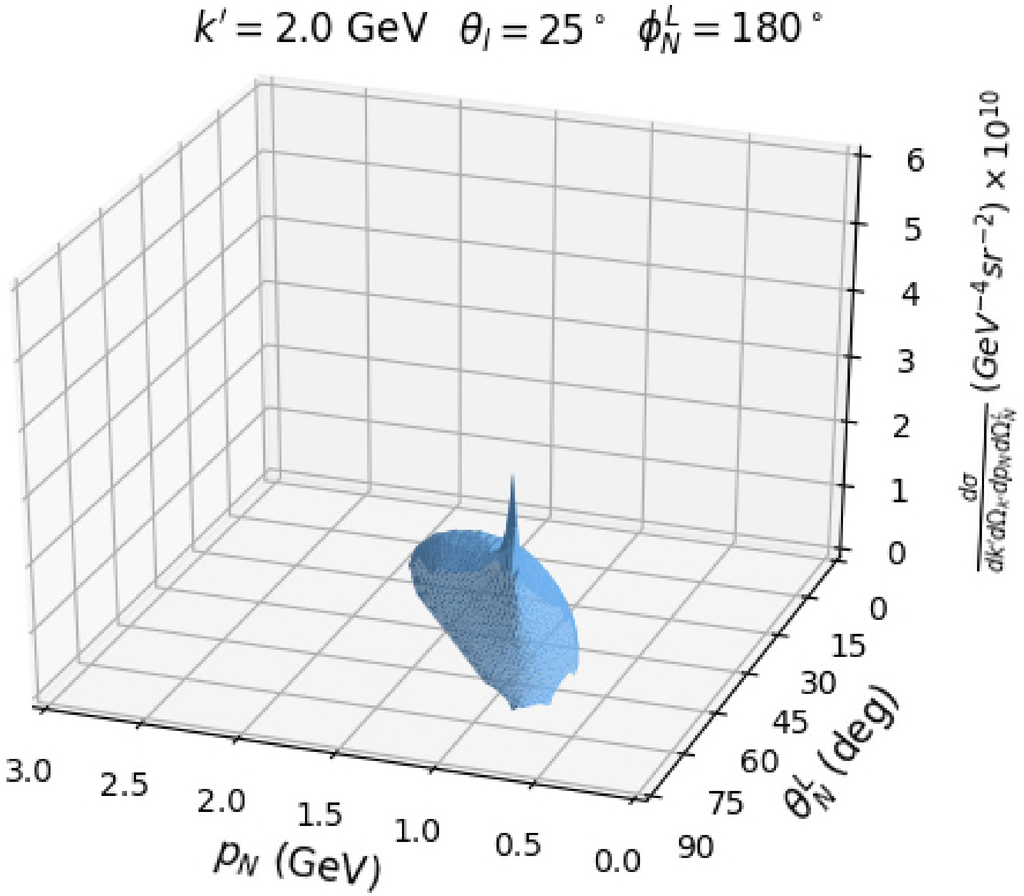
Local Density Spectral Function



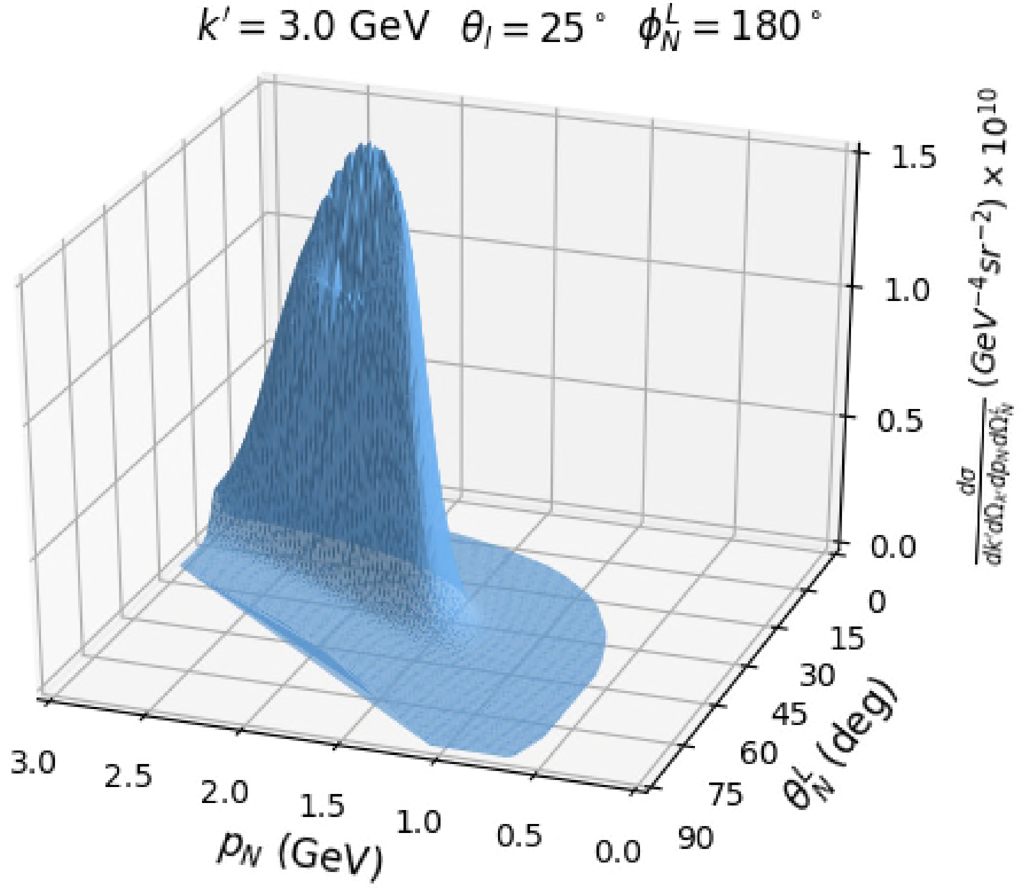
Spectral Function*



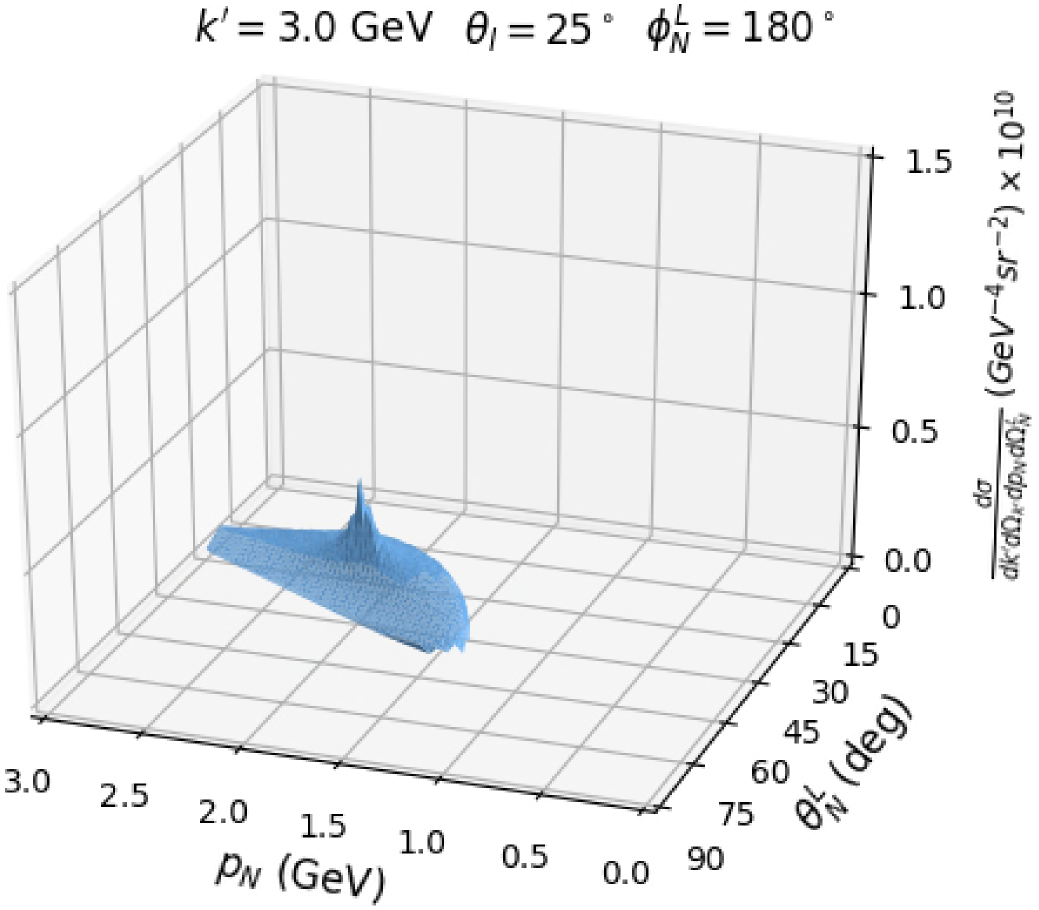
Local Density Spectral Function

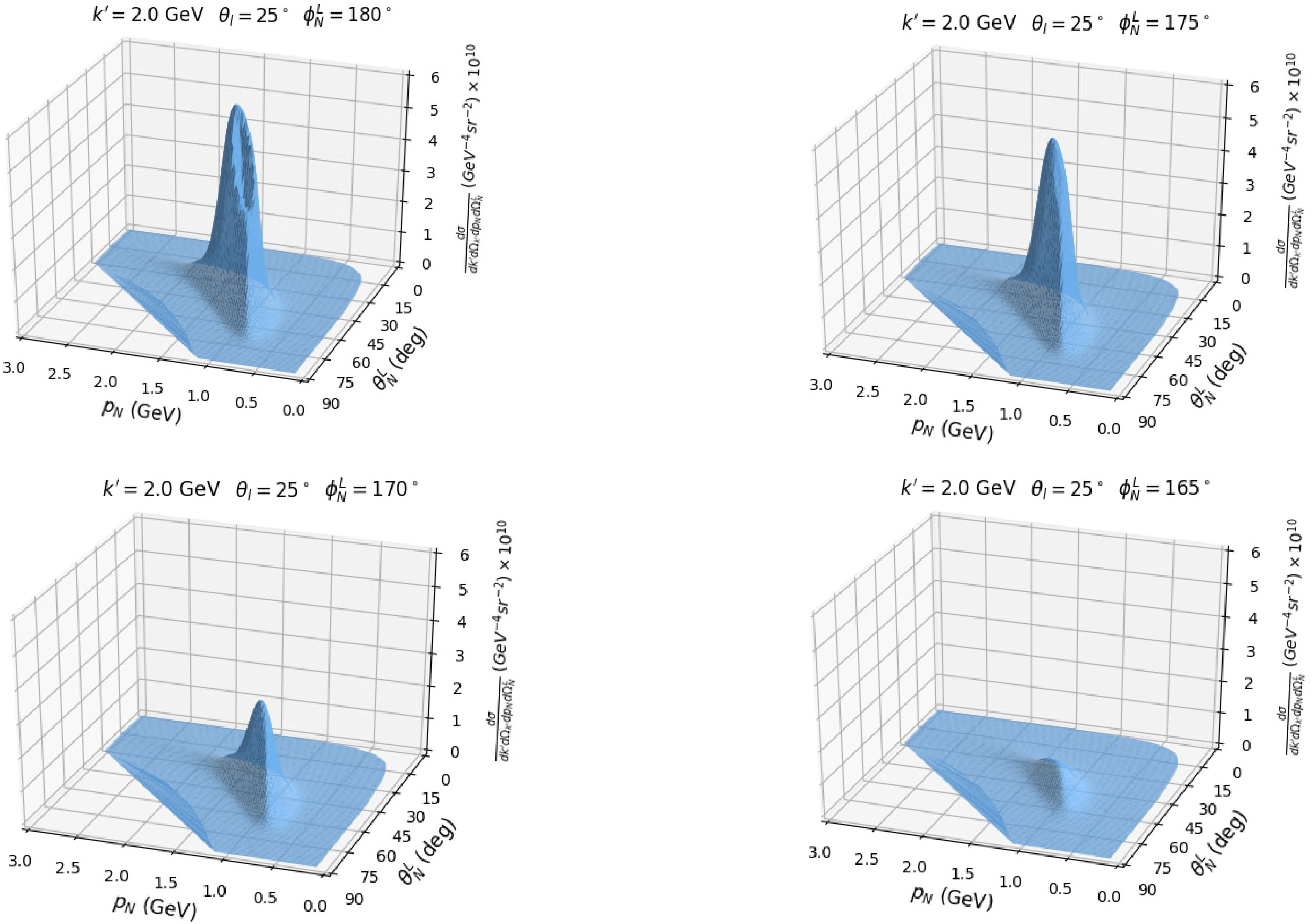


Spectral Function*



Local Density Spectral Function





$k' = 2.0$ GeV $\theta_l = 25^\circ$ $\phi_N^L = 180^\circ$
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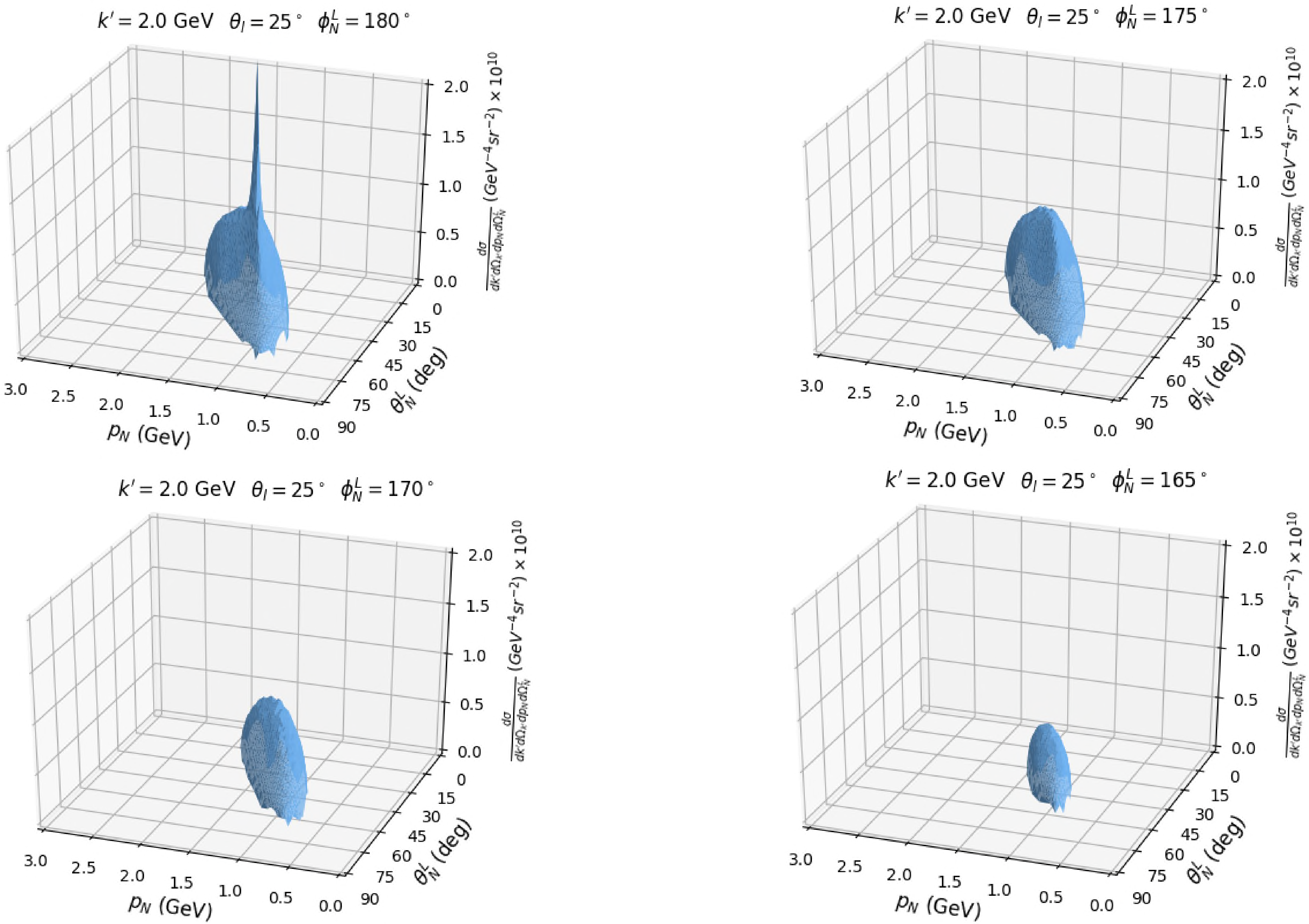
$k' = 2.0$ GeV $\theta_l = 25^\circ$ $\phi_N^L = 165^\circ$
p_N (GeV)
θ_N^L (deg)
$\frac{d\sigma}{dk' d\Omega_{k'} dp_N d\Omega_N^L}$ $(GeV^{-4} sr^{-2}) \times 10^{10}$



$k' = 2.0$ GeV $\theta_l = 25^\circ$ $\phi_N^L = 180^\circ$
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$k' = 2.0$ GeV $\theta_l = 25^\circ$ $\phi_N^L = 165^\circ$
p_N (GeV)
θ_N^L (deg)
$\frac{d\sigma}{dk' d\Omega_{k'} dp_N d\Omega_N^L}$ $(GeV^{-4} sr^{-2}) \times 10^{10}$

Summary and Conclusions

- Models that appear to fit inclusive cross sections may disagree significantly in predicting semi-inclusive cross sections.
- At this time the spectral function approach seems to be the best method for providing input for event generators.
- Calculations of spectral functions based on realistic nucleon-nucleon interactions may be possible in the near future using modern approaches to the many-body problem.
- Some contributions from collective modes at low three-momentum can not be reproduced within the current event generator format.
- This matter needs to be given careful consideration and should be the subject of discussions between the theory community and the writers of event generators.