Nuclear Matter Response to Low-Energy Neutrino Interactions

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Outline

- ⋆ Neutrino-nucleon interaction at low energy
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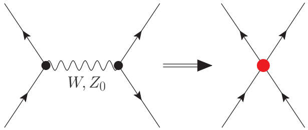
- ⋆ Models of nuclear structure and dynamics
  - ▷ Nuclear Hamiltonian
  - ▷ Correlated Basis Functions
- ⋆ Interaction effects
  - ▷ Mean Field
  - ▷ Correlations
  - ▷ Collective excitations
- ⋆ Summary & Outlook

Low-Energy Neutrino-Nucleon Interactions

- Neutrino interactions are mediated by the gauge bosons $W^\pm$ and Z_0, whose masses are in the range $\approx 80 - 90$ GeV
- In the regime of momentum transfer discussed in this talk, $q \sim 10$ MeV, Fermi theory of weak interactions works just fine



$$\mathcal{L}_F = \frac{G}{\sqrt{2}} J_{N\mu} J_\ell{}^\mu$$

$$J_\ell{}^\mu = \begin{cases} \bar{u}_{\ell^-}\gamma^\mu(1-\gamma_5)u_\nu & (CC) \\ \bar{u}_{\nu'}\gamma^\mu(1-\gamma_5)u_\nu & (NC) \end{cases}$$

- The nucleon current can be cast in the non relativistic limit

$$J_{N\mu} = \begin{cases} \bar{u}_p\gamma_\mu(1-g_A\gamma_5)u_n & \to & \chi^\dagger_{s_p}(g^0_\mu + g_A g^\mu_i \sigma_i)\chi_{s_n} & (CC) \\ \bar{u}_{n'}\gamma_\mu(1-c_A\gamma_5)u_n & \to & \chi^\dagger_{s'_n}(g^0_\mu + c_A g^\mu_i \sigma_i)\chi_{s_n} & (NC) \end{cases}$$

Neutrino-Nucleon x-Section

- Consider, as an example, the x-section of the neutral-current process $\nu(k) + n(p) \to \nu'(k') + n(p+q)$, $q = k - k'$

$$d\sigma \propto L_{\lambda\mu} W^{\lambda\mu}$$

  - $L_{\lambda\mu}$ specified by lepton kinematical variables

$$L_{\lambda\mu} = k_\lambda {k'}_\mu + k_\mu {k'}_\lambda - g_{\lambda\mu}(kk') + i\epsilon_{\lambda\rho\mu\sigma} k^\rho {k'}^\sigma$$

  - $W^{\lambda\mu}$ written in terms of five structure functions $W_i(q^2, (p \cdot q))$

$$W^{\lambda\mu} = -g^{\lambda\mu} W_1 + p^\lambda p^\mu \frac{W_2}{m_N^2} + i\,\varepsilon^{\lambda\mu\alpha\beta} q_\alpha p_\beta + \frac{W_3}{m_N^2} + q^\lambda q^\mu \frac{W_4}{m_N^2}$$
$$+ (p^\lambda q^\mu + p^\mu q^\lambda) \frac{W_5}{m_N^2}$$

- In the following, I will assume that the neutrino-nucleon cross section be known

NEUTRINO-NUCLEUS X-SECTION

- ⋆ Consider again a neutral current process

$$\nu + A \to \nu' + X$$

- ⋆ The nucleon tensor is replaced by the nuclear response tensor

$$W^{\lambda\mu} = \sum_n \langle 0|J^\lambda|n\rangle\langle n|J^\mu|0\rangle\delta^{(4)}(P_0 + q - P_n)$$

- ⋆ Interaction rate

$$W(\mathbf{q},\omega) \propto \frac{G_F}{4\pi^2} L_{\lambda\mu} W^{\lambda\mu} = \frac{G_F}{4\pi^2}\left[(1+\cos\theta)\mathcal{S}^\rho + \frac{{c_A}^2}{3}(3-\cos\theta)\mathcal{S}^\sigma\right]$$

where $\cos\theta = (\mathbf{k}\cdot\mathbf{k}')/(|\mathbf{k}||\mathbf{k}'|)$, while $\mathcal{S}^\rho$ and $\mathcal{S}^\rho$ are the nuclear responses in the density and spin-density channels, respectively.

Nuclear Weak Responses at Low Energy

- density response

$$\mathcal{S}^{\rho} = \frac{1}{N} \sum_{n} |\langle 0|J_0|n\rangle\langle n|J_0|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- spin-density response ($\alpha,\ \beta = 1, \ldots 3$)

$$\mathcal{S}^{\rho} = \sum_{\alpha} \mathcal{S}^{\rho}_{\alpha\alpha}$$

$$\mathcal{S}^{\rho}_{\alpha\beta} = \frac{1}{N} \sum_{n} |\langle 0|J_\alpha|n\rangle\langle n|J_\beta|0\rangle \delta^{(4)}(P_0 + q - P_n)$$

- Neutral weak current

$$J_0 = \sum_i j_i^0 = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i} \quad , \quad J_\alpha = \sum_i j_i^\mu = \sum_i \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}_i}\sigma_\alpha$$

- Outstanding issues
  - Model nuclear dynamics (determine H)
  - Solve the many-body Schrödinger equation $H|n\rangle = E_n|n\rangle$

Modeling Nuclear Dynamics

- *ab initio* (bottom-up) approach

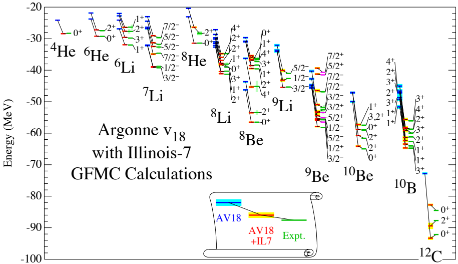
$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk}$$

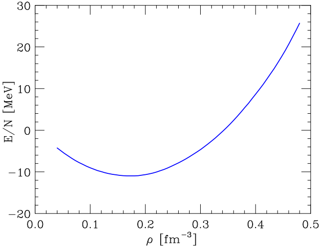
  - v_{ij} provides a very accurate descritpion of the two-nucleon system, and reduces to Yukawa's one-pion-exchange potential at large distances
  - inclusion of v_{ijk} needed to explain the ground-state energies of the three-nucleon systems
  - v_{ij} is spin and isospin dependent, and strongly repulsive at short distance
  - nuclear interactions can not be treated in perturbation theory in the basis of eigenstates of the non interacting system

- Mean field (independent particle) approximation

$$\left\{ \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk} \right\} \to \sum_i U_i$$

⋆ Quantum Monte Carlo and variational calculations performed using phenomenological nuclear Hamiltonians explain the energies of the ground- and low-lying excited states of nuclei with mass $A \leq 12$, as well as saturation of the equation of state of cold isospin-symmetric nuclear matter





Correlated Basis Functions

- ⋆ Replace the basis states of the non-interacting system with a set of correlated states

$$|n_0\rangle \to |n\rangle = \frac{F|n_0\rangle}{\langle n_0|F^\dagger F|n_0\rangle^{1/2}} = \frac{1}{\sqrt{\mathcal{N}_n}}\, F\,|n_0\rangle$$

$$F = \mathcal{S}\prod_{j>i} f_{ij}$$

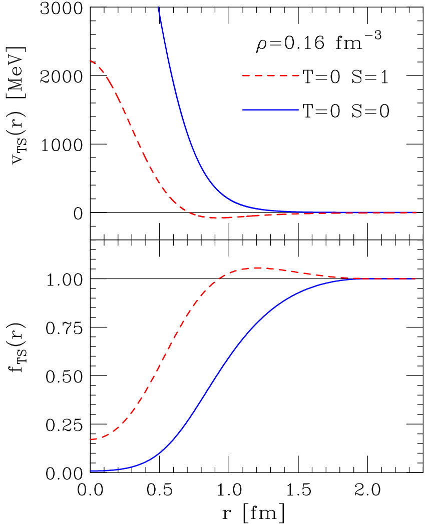
- ⋆ the structure of the two-nucleon correlation operator reflects the complexity of nuclear dynamics

$$f_{ij} = \sum_{S,T=0,1} [f_{TS}(r_{ij}) + \delta_{S1} f_{tT}(r_{ij}) S_{ij}] P_{ST}$$

P_{ST} spin − isospin projector operator , $S_{ij} = \sigma_i^\alpha \sigma_j^\beta \left(\frac{r_{ij}^\alpha r_{ij}^\beta}{r_{ij}^2} - \delta^{\alpha\beta}\right)$

- ⋆ shapes of $f_{TS}(r_{ij})$ and $f_{tT}(r_{ij})$ determined form minimization of the ground-state energy

NN Potential and Correlation Functions



Effective Interaction

- The effective interaction in nuclear matter at density ρ is defined through the relation [$k_F = (3\pi^2\rho/2)^{1/3}$]

$$\langle 0|H|0\rangle = \frac{3}{5}\frac{k_F^2}{2m} + \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle$$

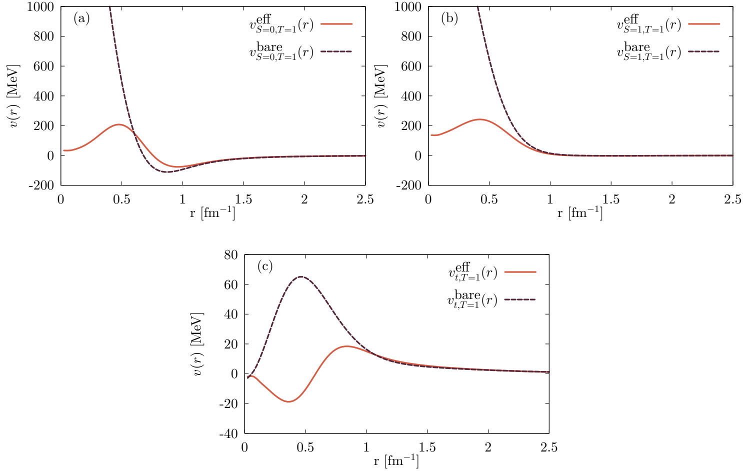
- unlike the bare NN potential, $V_{\rm eff}$ is well behaved, and can be used to perform perturbative calculations in the basis of eigenstates of the non interacting system
- the response can be also computed using the Fermi gas states and the corresponding effective operators, defined through

$$\langle n|J^\mu|0\rangle = \langle n_{FG}|J^\mu_{\rm eff}|0_{FG}\rangle$$

$$J^\mu_{\rm eff} = FJ^\mu F = J^\mu + \sum_{j>i}\{j^\mu_i + j^\mu_j, g_{ij}\} + \ldots$$

$$g_{ij} = f_{ij} - 1$$

EFFECTIVE INTERACTION



Effects of NN Interactions

- Mean field effects
  - Change of nucleon energy spectrum

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}' | V_{\text{eff}} | \mathbf{k}\mathbf{k}' \rangle_a$$

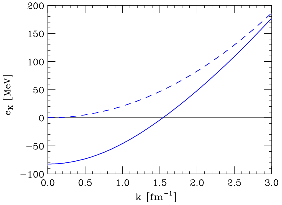
  - Effective mass

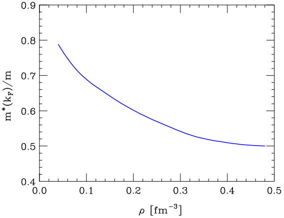
$$\frac{1}{m_k^\star} = \frac{1}{|\mathbf{k}|} \frac{de_k}{d|\mathbf{k}|}$$

- Correlation effects
  - Effective operators couple the ground state to two-particle–two-hole (2p2h) final states, thus removing strength from the 1p1h sector

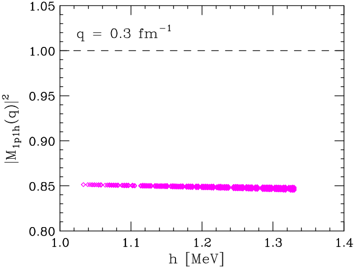
$$M_{2p2h} = \langle 2p2h | J^\mu_{\text{eff}} | 0 \rangle \neq 0 \to M_{1p1h} = \langle 1p1h | J^\mu_{\text{eff}} | 0 \rangle < \langle 1p1h | J^\mu | 0 \rangle$$

⋆ Nucleon energy spectrum and Effective mass in isospin-symmetric matter at equilibrium density



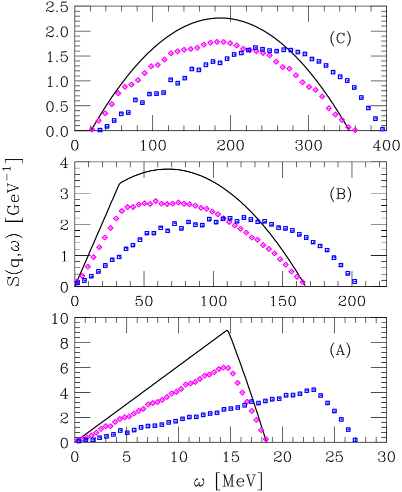


⋆ Quenching of Fermi transition strength in isospin-symmetric matter at equilibrium density



q-Evolution of Interaction Effects

- ⋆ Density response of isospin-symmetric matter at equilibrium density



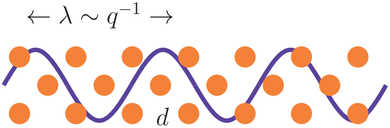
$|\mathbf{q}| = 3.0\ \mathrm{fm}^{-1}$

$|\mathbf{q}| = 1.8\ \mathrm{fm}^{-1}$

$|\mathbf{q}| = 0.3\ \mathrm{fm}^{-1}$

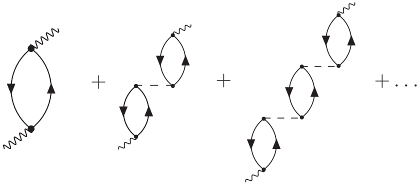
Long-Range Correlations

- At low momentum transfer the space resolusion of the neutrino becomes much larger than the average NN separation distance (~ 1.5 fm), and the interaction involves many nucleons



- Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

$$|n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i\rangle$$



Tamm-Dancoff (ring) Approximation

- ⋆ Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

$$|ph\rangle \to |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i|V_{\text{eff}}|h_j p_j)$$

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}'|V_{\text{eff}}|\mathbf{k}\mathbf{k}'\rangle_a$$

- ⋆ The appearance of an eigenvalue, ω_n, lying outside the particle-hole continuum signals the excitation of a collective mode

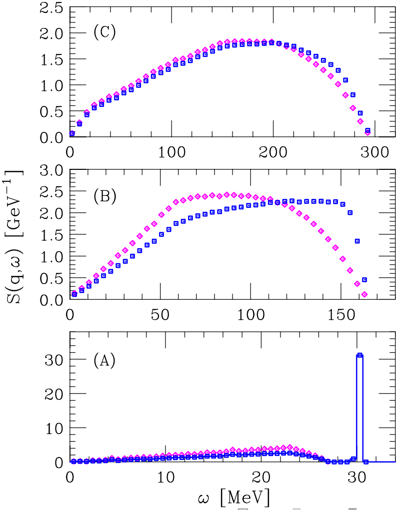
Effects of Long-Range Correlations

- ⋆ Density response of isospin-symmetric nuclear matter at equilibrium density

$|\mathbf{q}| \approx 480\,\mathrm{MeV}$

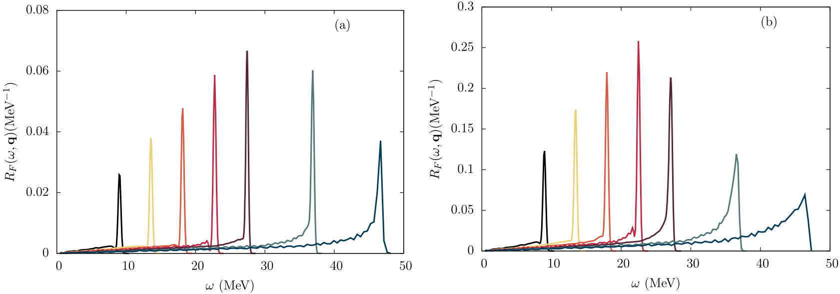
$|\mathbf{q}| \approx 300\,\mathrm{MeV}$

$|\mathbf{q}| \approx 60\,\mathrm{MeV}$



Excitation of collective modes

- ⋆ Density (a) and spin-density (b) responses of isospin-symmetric nuclear matter at equilibrium density



- ⋆ $|\mathbf{q}|$ = 0.1, 0.15, 0.20, 0.25, 0.30, 0.40 and 0.50 fm^{-1}

SUMMARY

- ⋆ A consistent theoretical framework for the description of the weak response of nuclear matter at low energy is available
- ⋆ Recent studies have signifcantly advanced the understanding of the relevant reaction mechanisms, pinning down the role of both short- and long-range correlations
- ⋆ The generalization to atomin nuclei, while being demanding from the computational point of view, does not involve conceptual difficulties
- ⋆ As an intermediate step, nuclear cross sections can be estimated exploiting nuclear matter results within the conceptual framework of the local density approximation

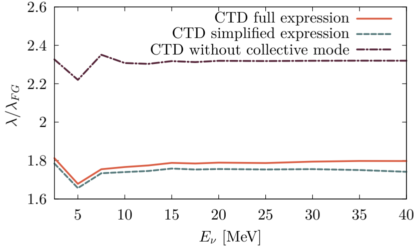
Neutrino mean free path in neutron matter

- The mean free path of non degenerate neutrinos at zero temperature is obtained from

$$\frac{1}{\lambda} = \frac{G_F^2}{4}\rho \int \frac{d^3q}{(2\pi)^3} \left[(1+\cos\theta)S(\mathbf{q},\omega) + \mathbf{C_A^2}(\mathbf{3}-\cos\theta)\mathcal{S}(\mathbf{q},\omega)\right]$$

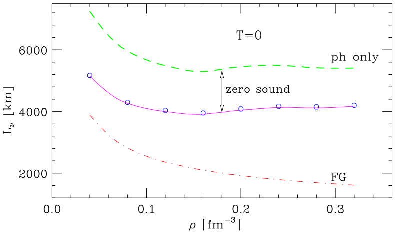
where S and $\mathcal{S}$ are the density (Fermi) and spin (Gamow Teller) response, respectively

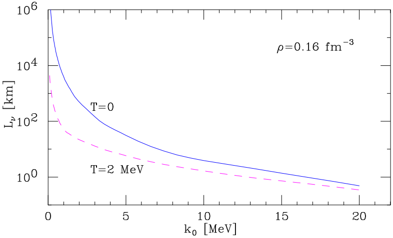
A Lovato et al, arXiv 1310.0510 [nucl-th]



- Both short and long range correlations important

⋆ Mean free path of a non degenerate neutrino in neutron matter. Left: density-dependence at $k_0 = 1$ MeV and $T = 0$; Right: energy dependence at $\rho = 0.16\ \mathrm{fm}^{-3}$ and $T = 0, 2$ MeV





⋆ Density and temperature dependence of the mean free path of a non degenerate neutrino at $k_0 = 1$ MeV and $\rho = 0.16$ fm^{-3}