Argon cross sections at low energies

Alessandro Lovato

In collaboration with:
Omar Benhar, Stefano Gandolfi, Joe Carlson, Steve Pieper, Noemi Rocco, and Rocco Schiavilla

Argonne NATIONAL LABORATORY

NUCLEI
Nuclear Computational Low-Energy Initiative

Towards the calculation of the Argon cross sections at low energies

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Introduction

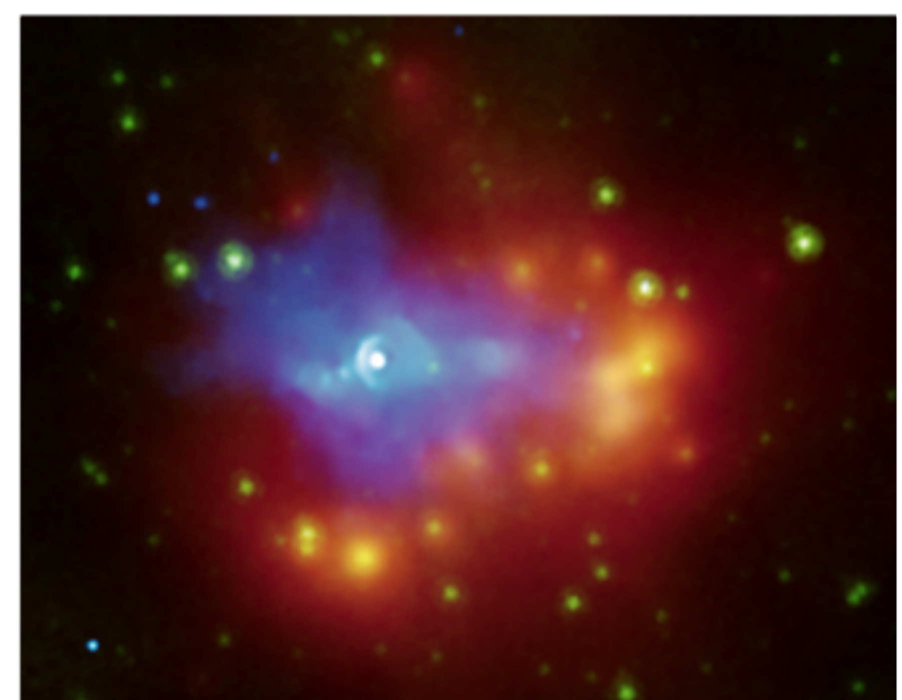
- Neutrino interactions with nuclei play a crucial role in supernovae, as they impact
    - The explosion
    - The nucleosynthesis

- Their detection is also affected by neutrino-nucleus interaction

- Neutrino emission is the main process driving the early stages of neutron stars' cooling.

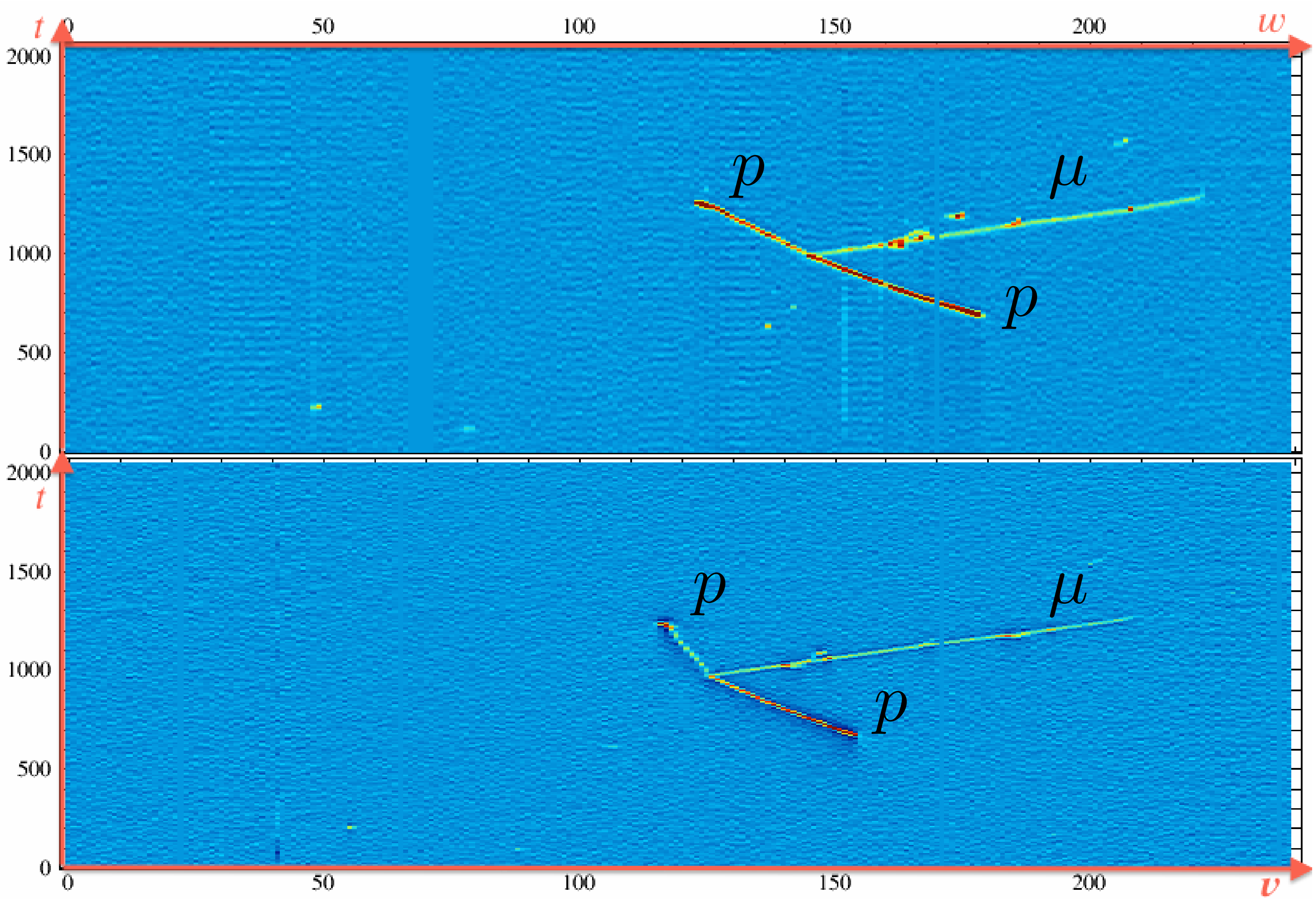
- The mechanisms for neutrino production depend on the nuclear equation of state, which is in turn dictated by nuclear interactions.

Neutrino interaction rate with nuclei and nuclear matter and equation of state (possibly at finite temperature) from the same nuclear dynamics, extensively tested on few-body nuclear systems



Argon: the beauty

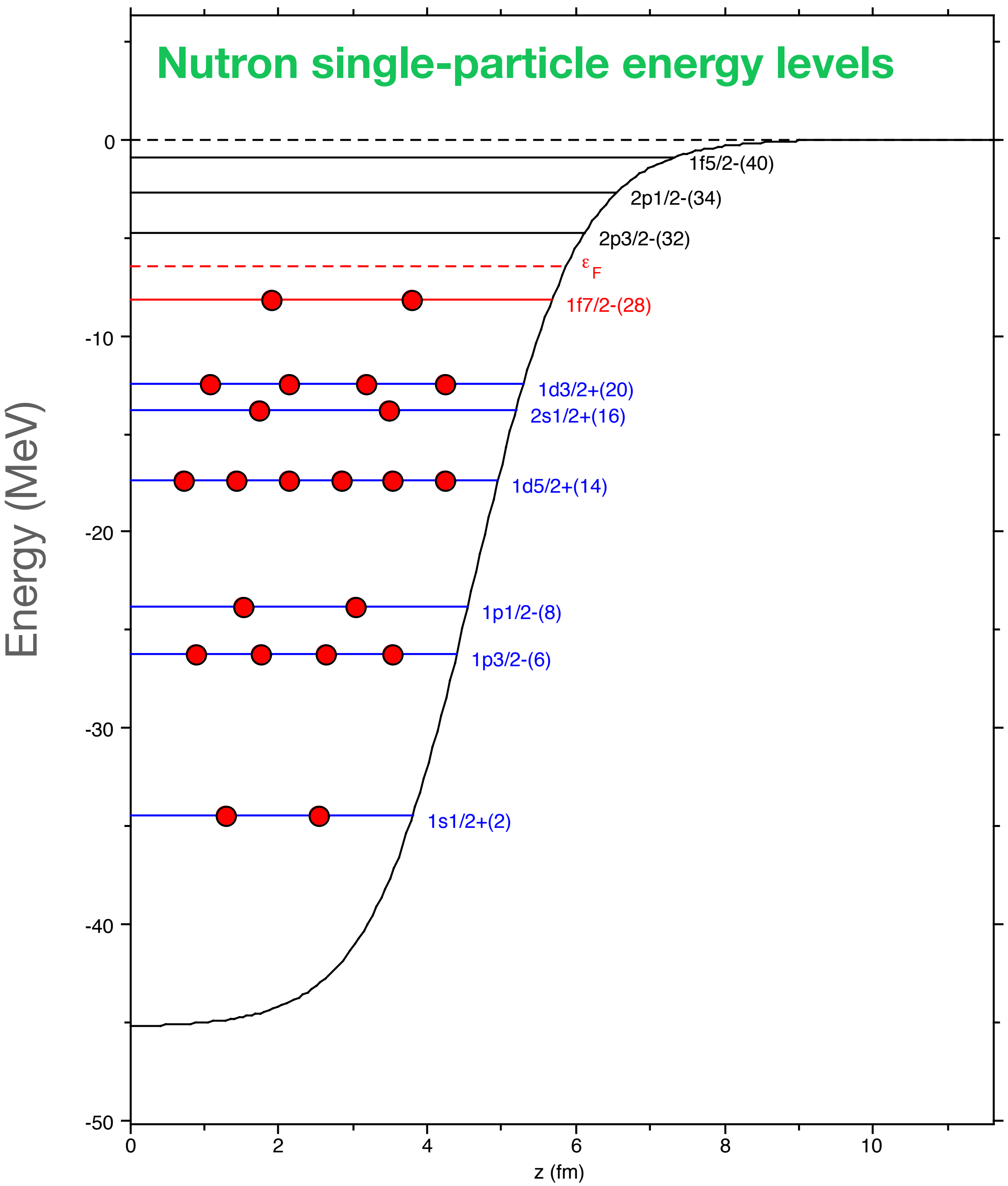
- Recently, the liquid Argon detector ArgoNeuT was able to elucidate the role of nuclear correlations in neutrino-nucleus scattering events.

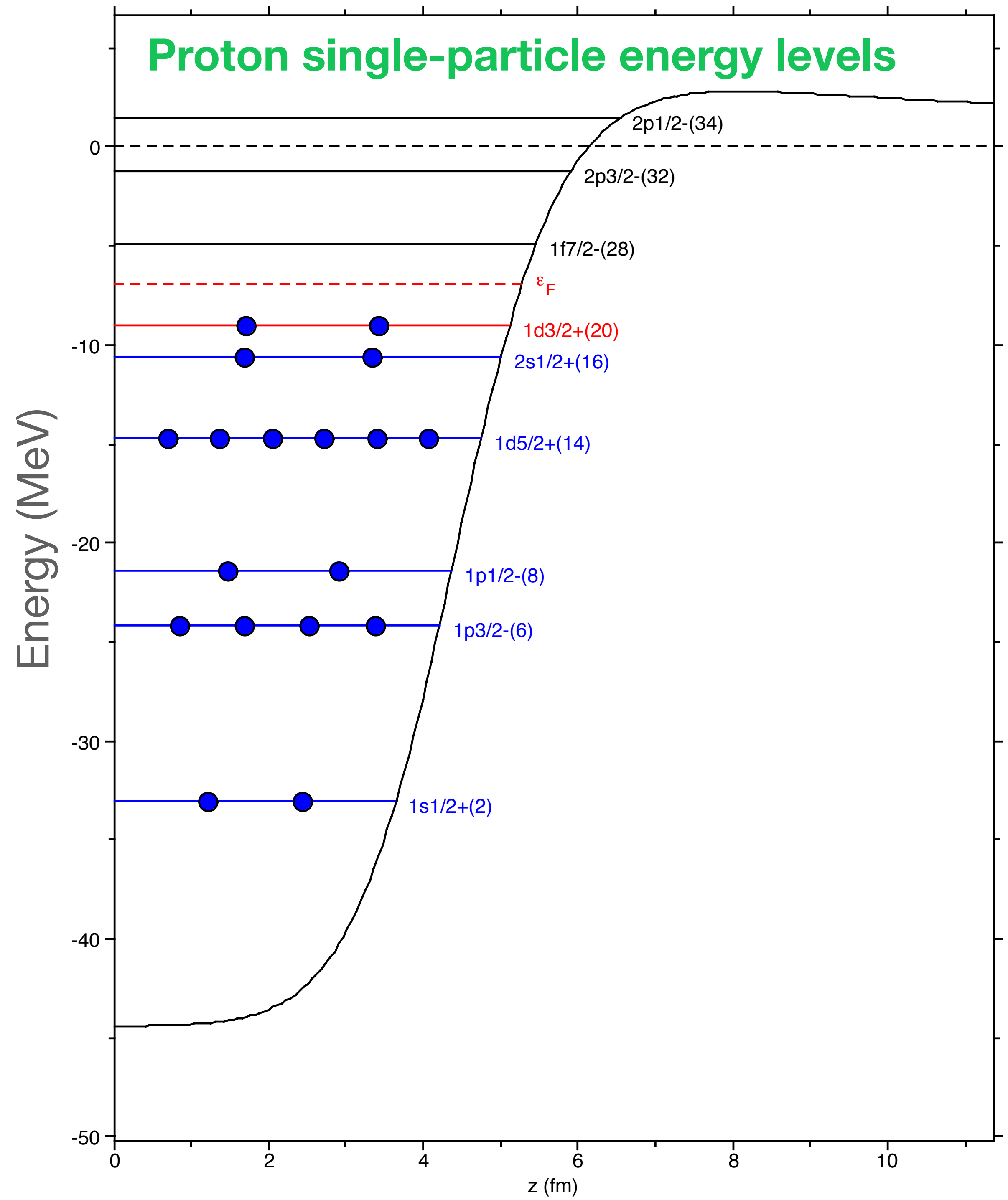


PRD 90, 012008 (2014)

Argon: the beast

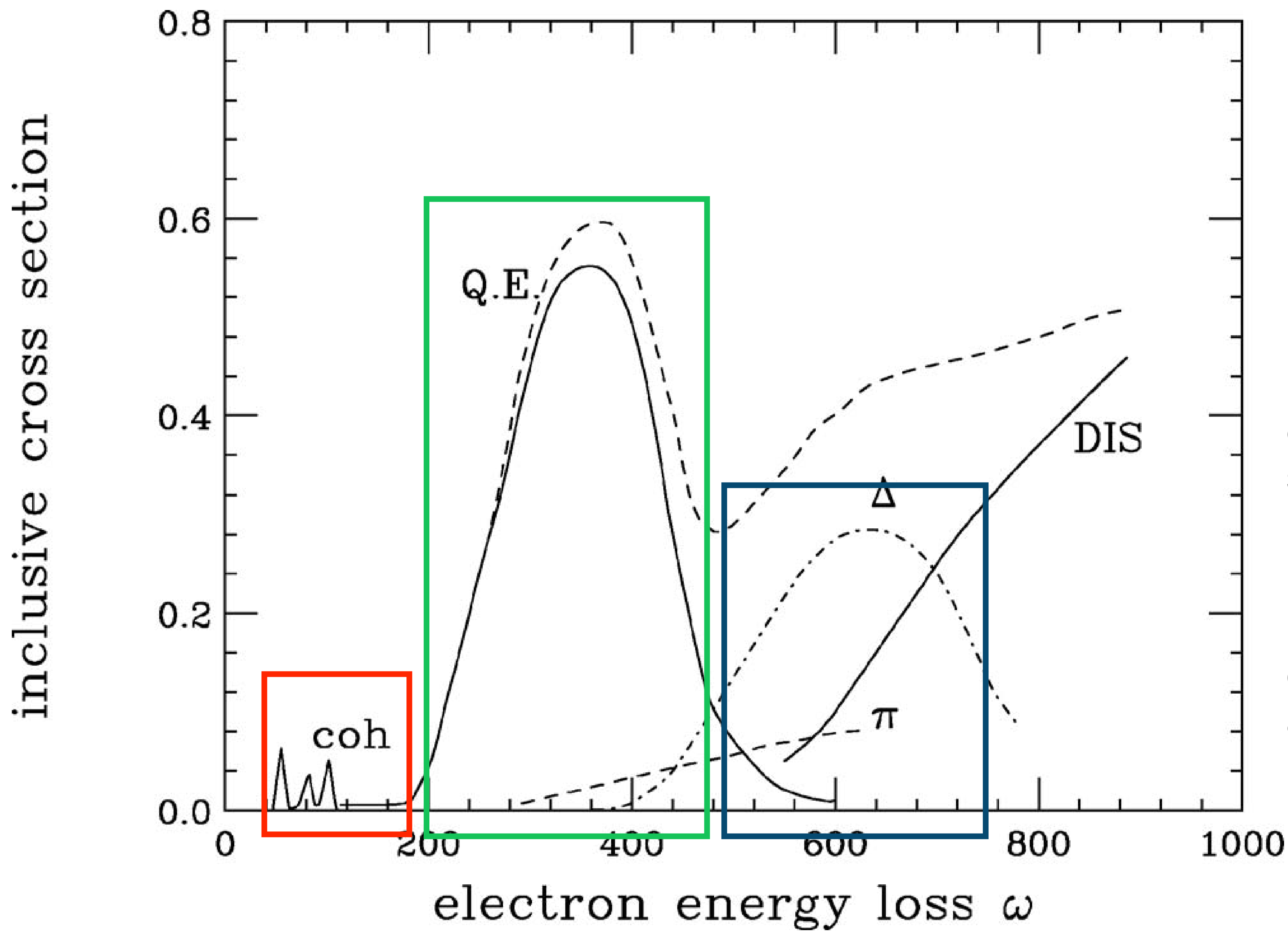
^{40}Ar is NOT a magic nucleus: open shells for neutrons and protons!





Electron-nucleus scattering

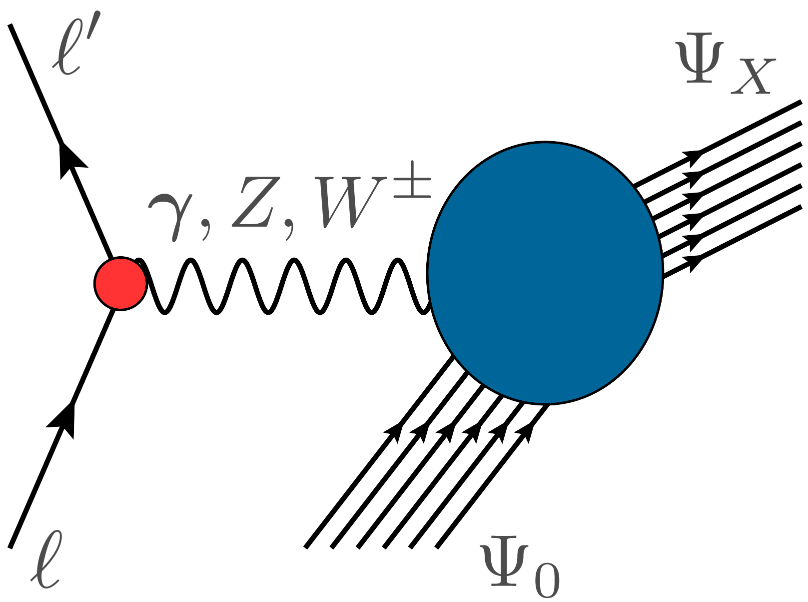
Schematic representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus and the hadronic final state is undetected can be written in terms of five response functions



$$\frac{d\sigma}{dE_{\ell'} d\Omega_\ell} \propto \left[v_{00} R_{00} + v_{zz} R_{zz} - v_{0z} R_{0z} + v_{xx} R_{xx} \mp v_{xy} R_{xy} \right]$$

- The response functions contains all the information on target structure and dynamics

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_\beta(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0)$$

- In (e, e') scattering interference $R_{xy} = 0$, $J_z(\mathbf{q}) \sim (\omega/q) J_0(\mathbf{q})$ and only the longitudinal, R_{00}, and the transverse R_{xx} response functions are left.

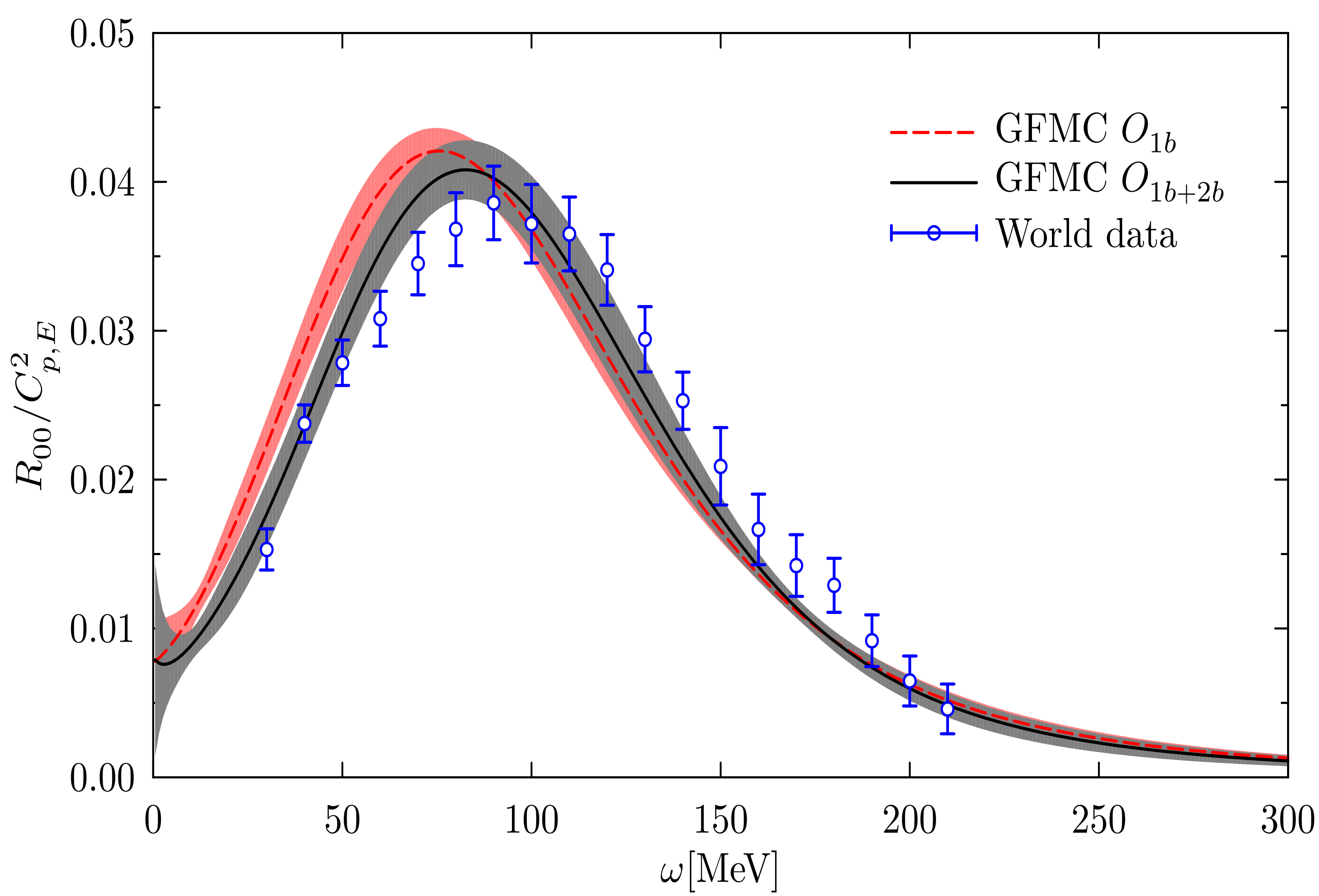
Our strategy: ab initio methods

We are aimed at computing the response functions of Argon in the broad kinematical region covered by neutrino experiments along with a realistic estimate of the theoretical uncertainty of the calculation.

Sources of theoretical uncertainty

- Modeling nuclear dynamics: nuclear potential, currents, form factors…
- Many-body technique: quantum Monte Carlo, spectral function, CBF effective interaction

In ab initio approaches these two sources of theoretical uncertainty are disentangled and can be properly estimated



Our strategy: ab initio methods

Green's function Monte Carlo (GFMC)

- Virtually exact up to the quasielastic region for $q \lesssim 500\mathrm{MeV}$
- Limited to nuclei large as ^{12}C

Auxiliary field diffusion Monte Carlo (AFDMC)

- Can be used to treat nuclei like ^{40}Ar (and bigger!) as well as nuclear matter
- Difficulties in extracting the response functions due to the large sign problem

Spectral function

- Fully relativistic kinematics and matrix elements for the current operators
- Reliable only for relatively large momentum transfer: $q \gtrsim 300\ \mathrm{MeV}$ (No collective modes!)

CBF effective interaction

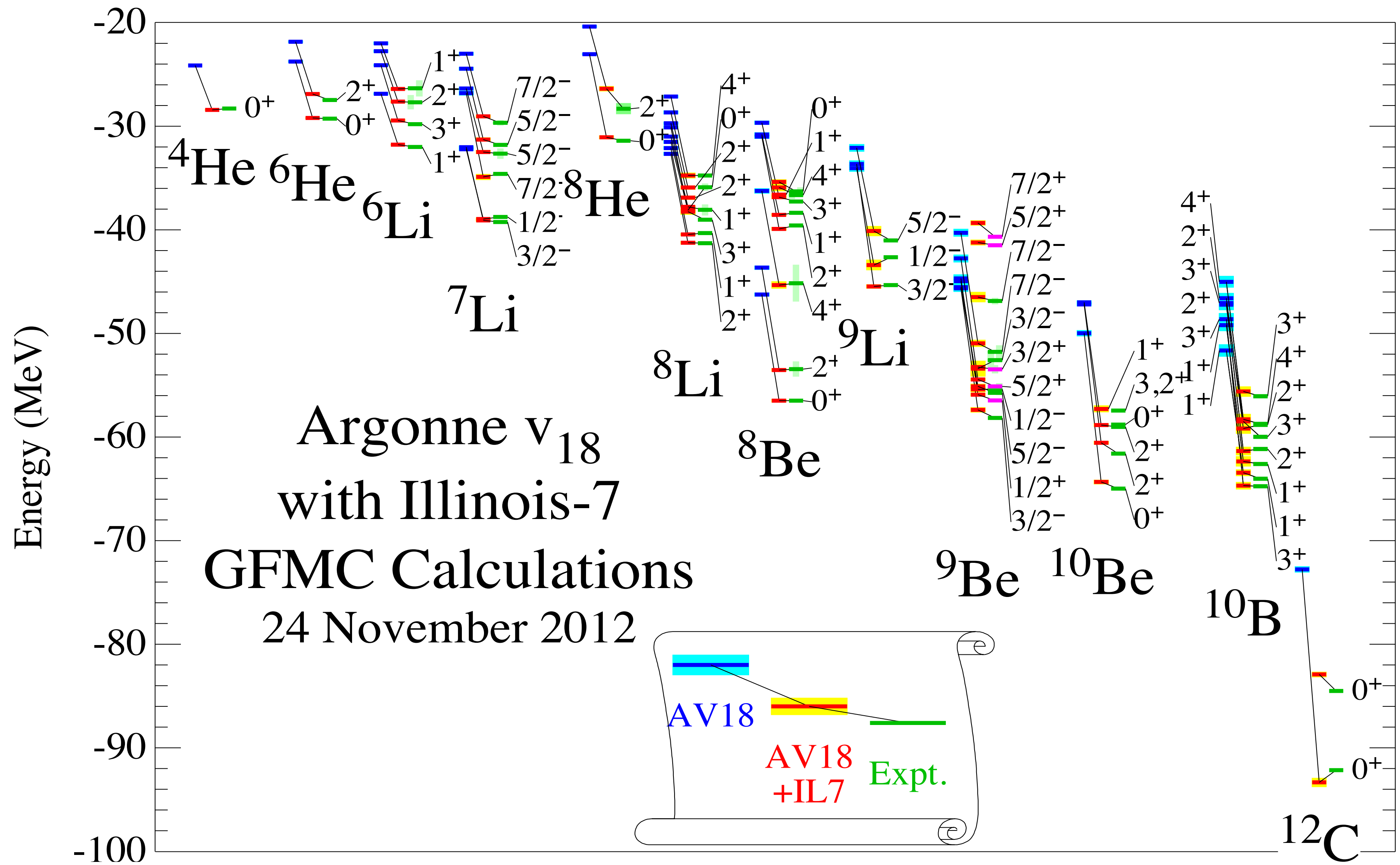
- Accurate for small values of momentum transfer (long and short-range correlations)
- Ideally suited for nuclear matter, but possible local density approximation implementation

Our strategy: ab initio methods

- Use GFMC whenever it is possible and seek a fruitful interplay between AFDMC, SF and CBF effective interaction approaches to estimate the systematic error of the many-body approach

All these many-body techniques are based on the same nuclear dynamics:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$



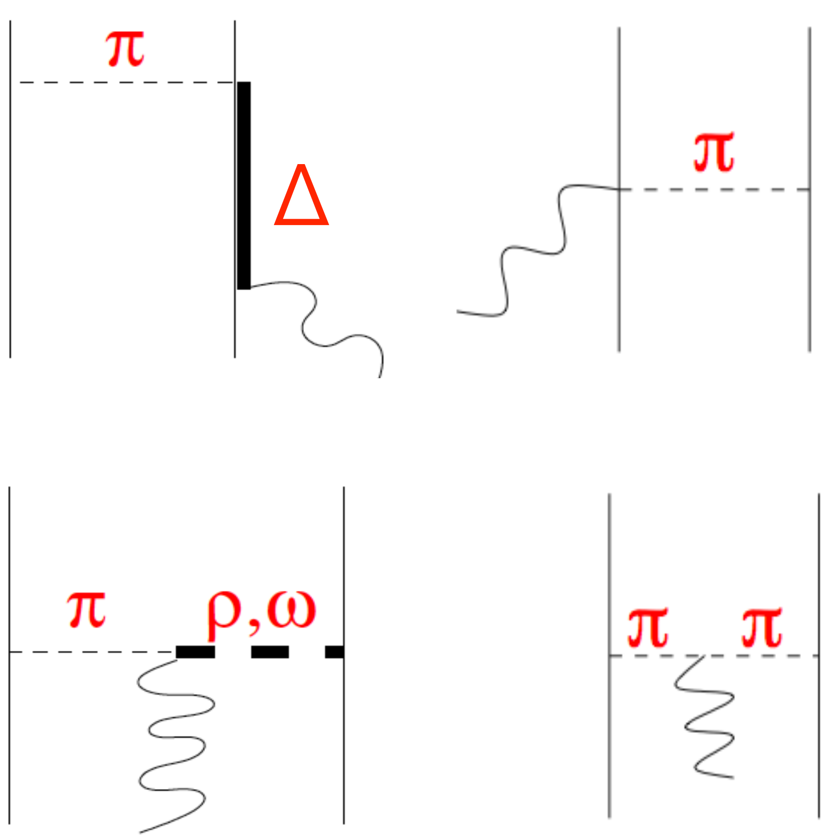
Our strategy: ab initio methods

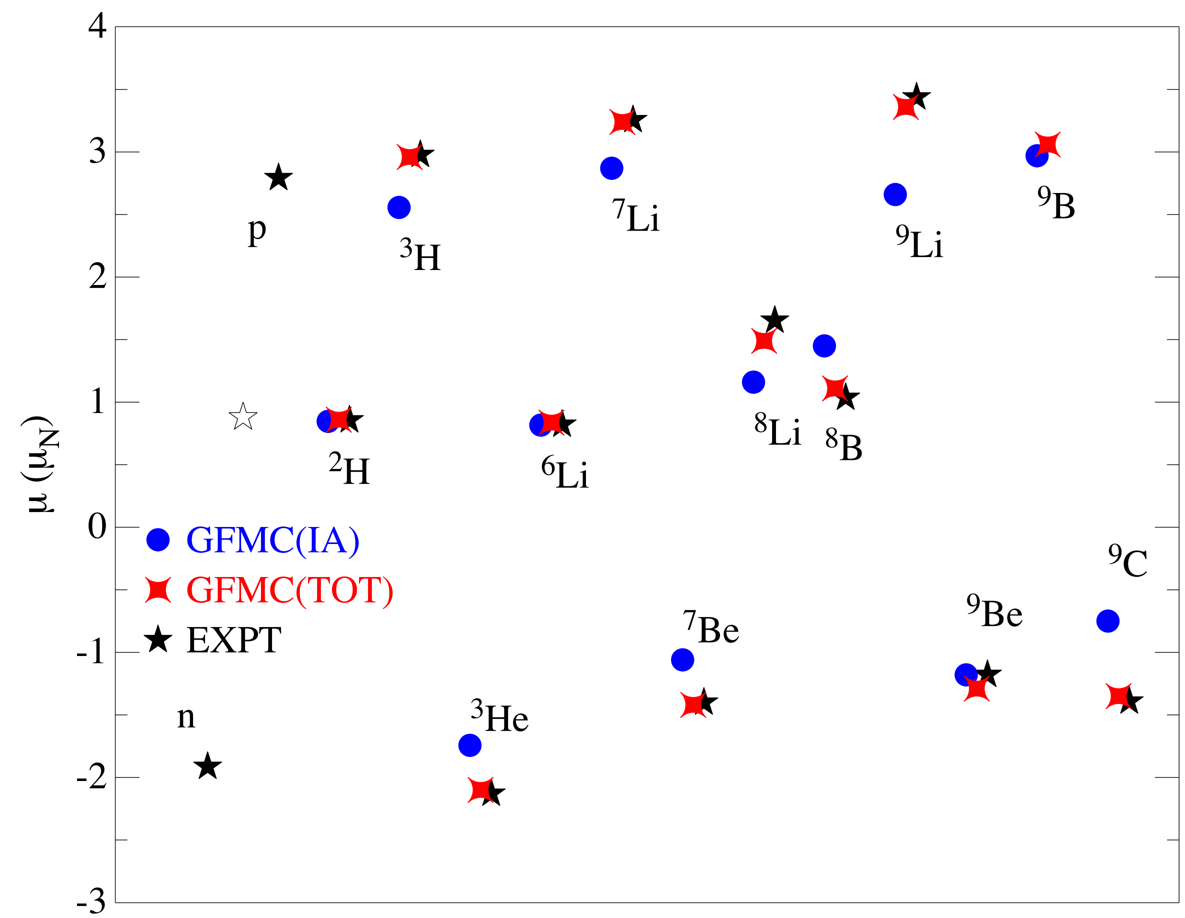
The nuclear electromagnetic current is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions. They account for processes in which the vector boson couples to the currents arising from meson exchange between two interacting nucleons.

- The inclusion of two-body currents is essential for low-momentum and low-energy transfer transitions.





Pastore at al., PRC 87, 035503 (2013)

Diffusion Monte Carlo

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting trial wave function.

- Any trial wave function can be expanded in the complete set of eigenstates of the the hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

which implies

$$\boxed{\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle}$$

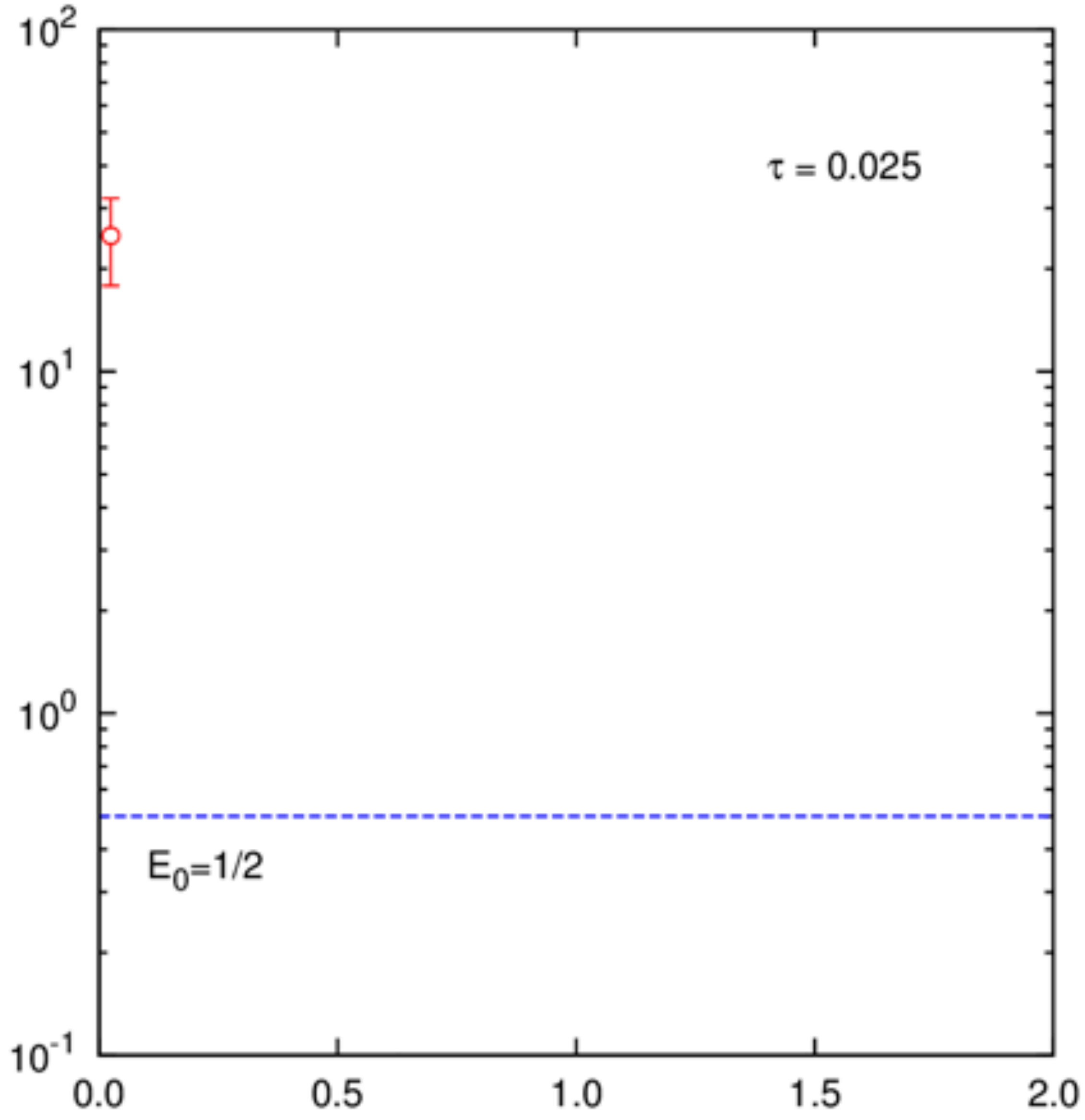
where τ is the imaginary time. Hence, GFMC and AFDMC project out the exact lowest-energy state, provided the trial wave function it is not orthogonal to the ground state.

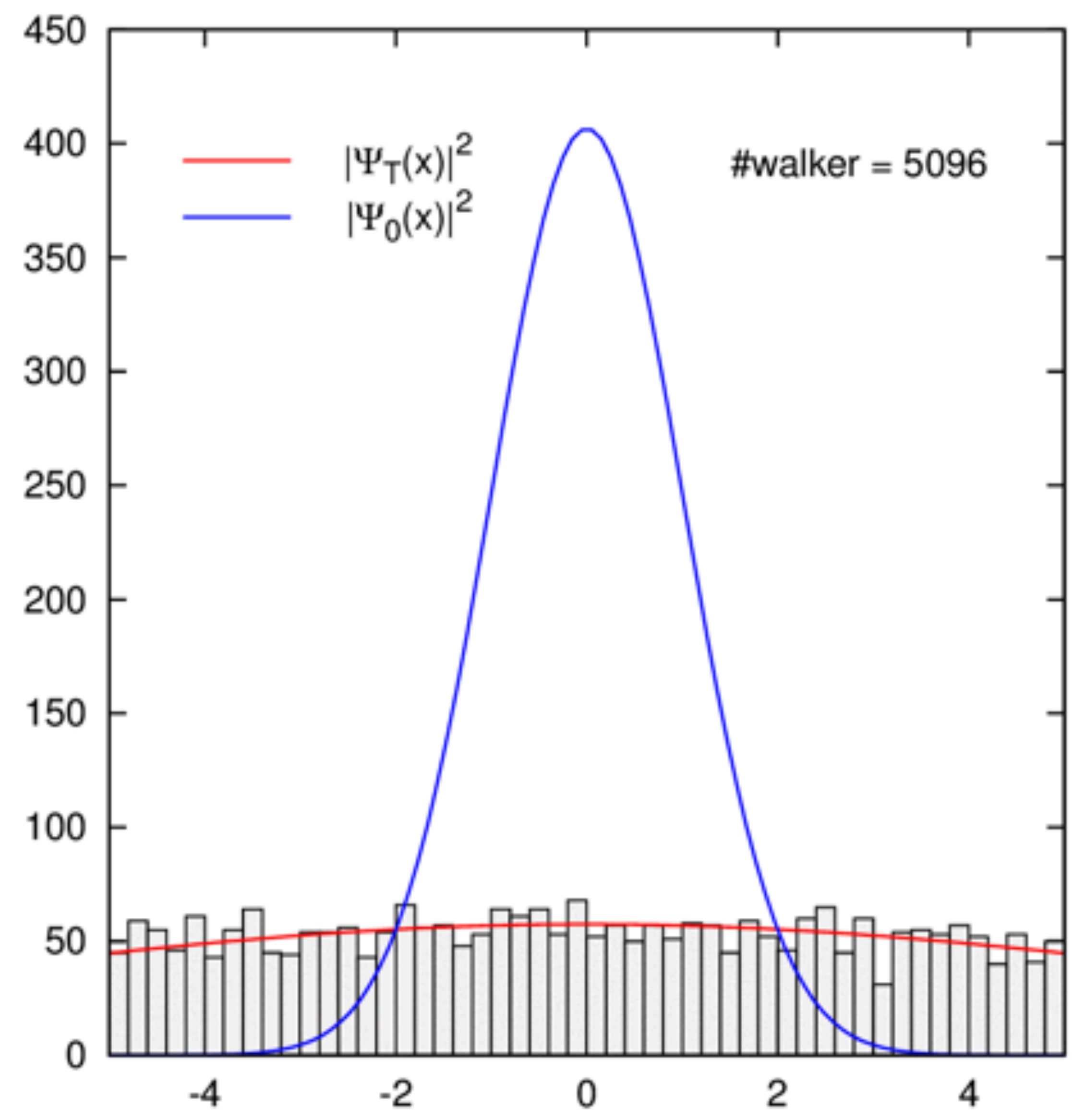
Diffusion Monte Carlo

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{x^2}{2}$$

$$\Psi_0(x) = e^{-x^2/2}$$

$$E_0 = 1/2$$





Euclidean response function

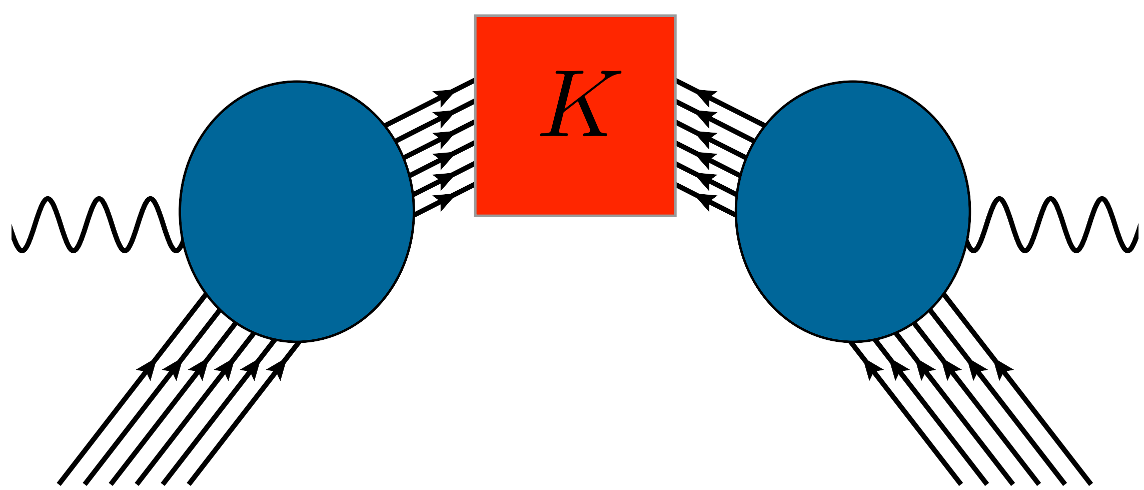
- The integral transform of the response function are generally defined as

$$E_{\alpha\beta}(\sigma, \mathbf{q}) \equiv \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q})$$

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) | \Psi_f \rangle \langle \Psi_f | J_\beta(\mathbf{q}) | \Psi_0 \rangle \delta(\omega - E_f + E_0)$$

- Using the completeness of the final states, they can be expressed in terms of ground-state expectation values

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \Psi_0 \rangle$$



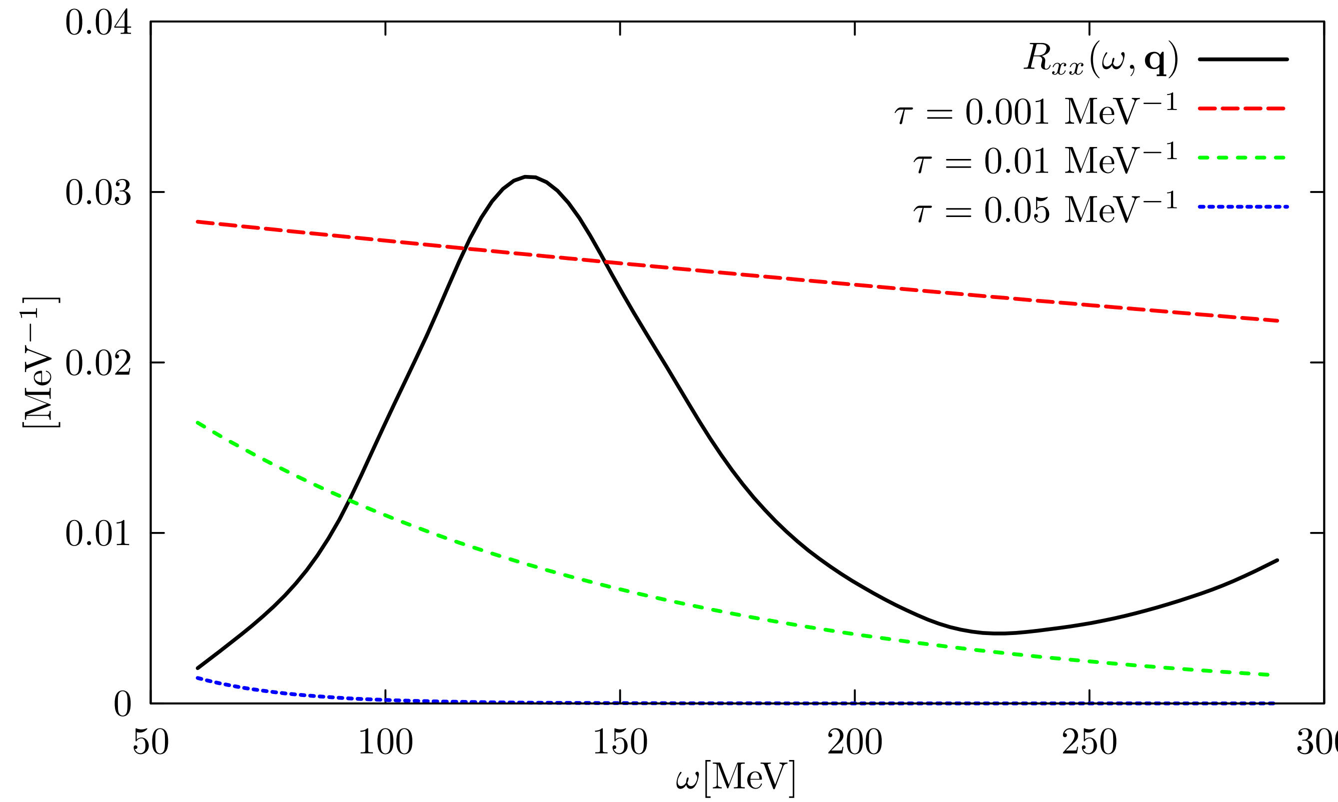
Euclidean response function

The the Kernel of the Euclidean response defines the Laplace transform

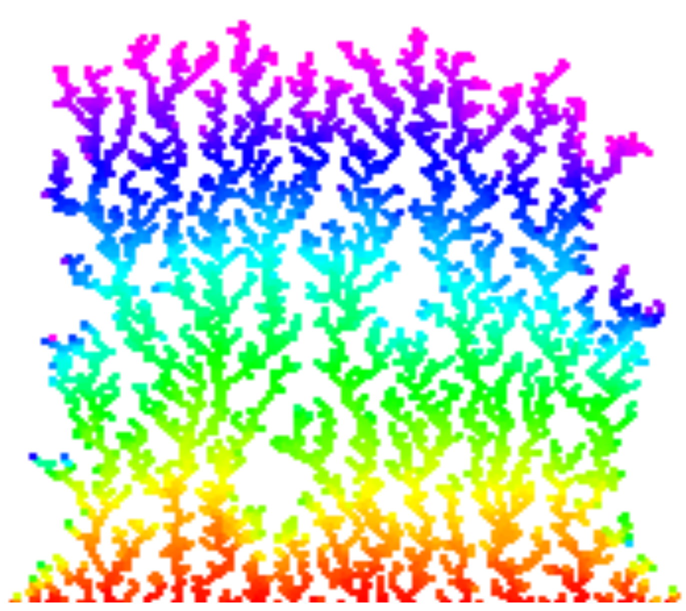
$$K(\tau, \omega) = e^{-\tau\omega}$$

$$E_{\alpha\beta}(\tau, \mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega, \mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed



The system is first heated up by the transition operator. How it cools down determines the Euclidean response of the system



$$\frac{\langle \Psi_0 | J^\dagger_\alpha(\mathbf{q}) e^{(H-E_0)\tau} J_\beta(\mathbf{q}) | \Psi_0 \rangle}{\langle \Psi_0 | e^{(H-E_0)\tau} | \Psi_0 \rangle}$$

Euclidean response function

Inverting the Euclidean response is an ill posed problem: any set of observations is limited and noisy and the situation is even worse since the kernel is a smoothing operator.

$$E_{\alpha\beta}(\tau, \mathbf{q}) \longrightarrow R_{\alpha\beta}(\omega, \mathbf{q})$$

Image reconstruction from incomplete and noisy data

S. F. Gull & G. J. Daniell*

Mullard Radio Astronomy Observatory, Cavendish Laboratory, Madingley Road, Cambridge, UK

Results are presented of a powerful technique for image reconstruction by a maximum entropy method, which is sufficiently fast to be useful for large and complicated images. Although our examples are taken from the fields of radio and X-ray astronomy, the technique is immediately applicable in spectroscopy, electron microscopy, X-ray crystallography, geophysics and virtually any type of optical image processing. Applied to radioastronomical data, the algorithm reveals details not seen by conventional analysis, but which are known to exist.

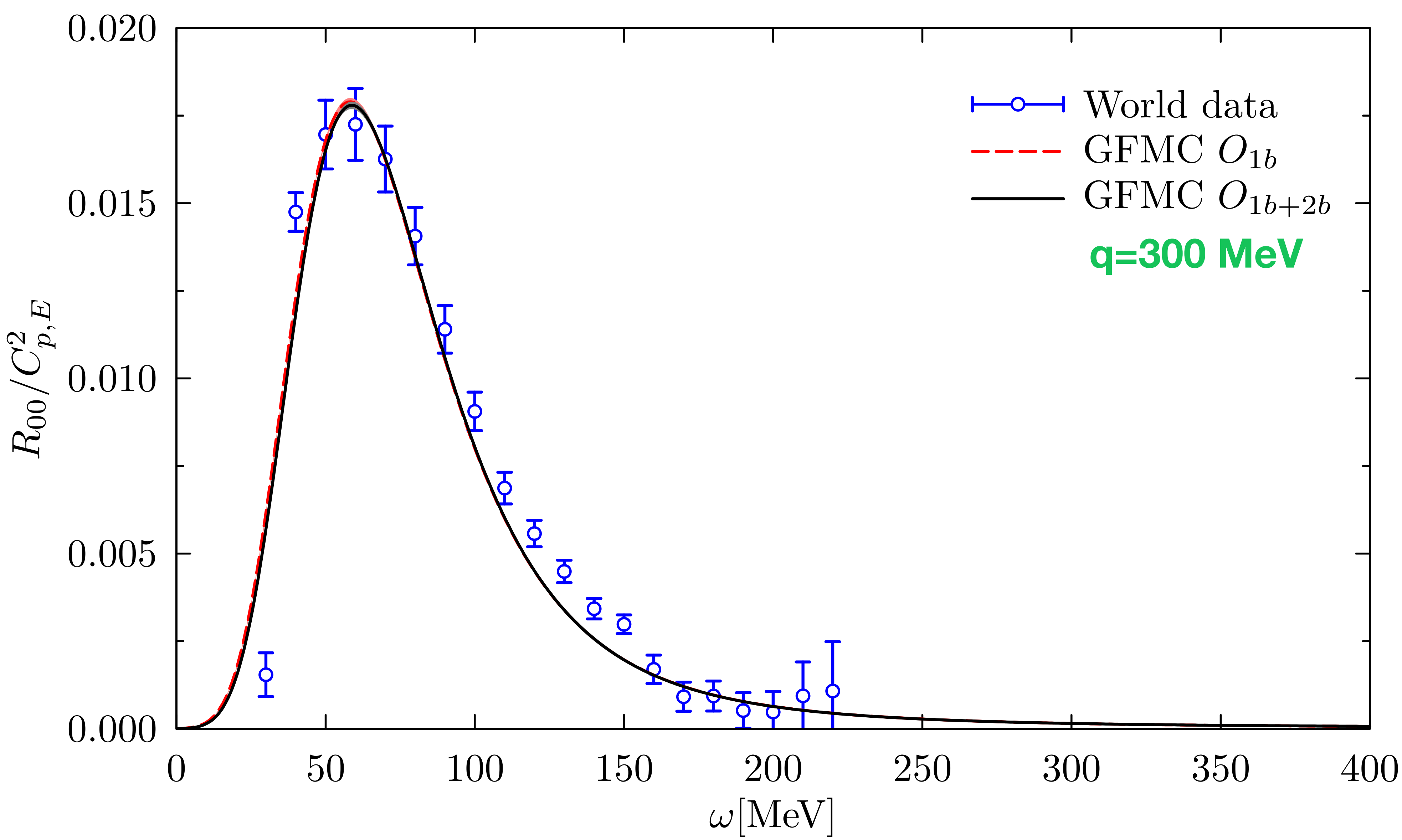
To avoid abstraction, we shall refer to our radioastronomical example. Starting with incomplete and noisy data, one can obtain by the Backus–Gilbert method a series of maps of the distribution of radio brightness across the sky, all of which are consistent with the data, but have different resolutions and noise levels. From the data alone, there is no reason to prefer any one of these maps, and the observer may select the most appropriate one to answer any specific question. Hence, the method cannot produce a unique 'best' map of the sky. There is no single map that is equally suitable for discussing both accurate flux measurements and source positions.

Nevertheless, it is useful to have a single general-purpose map of the sky, and the maximum-entropy map described here fulfils

Nature, 272, 688 (1978)

^{4}He electromagnetic response

Two-body currents do not provide significant changes in the longitudinal response.
The agreement with experimental data appears to be remarkably good.



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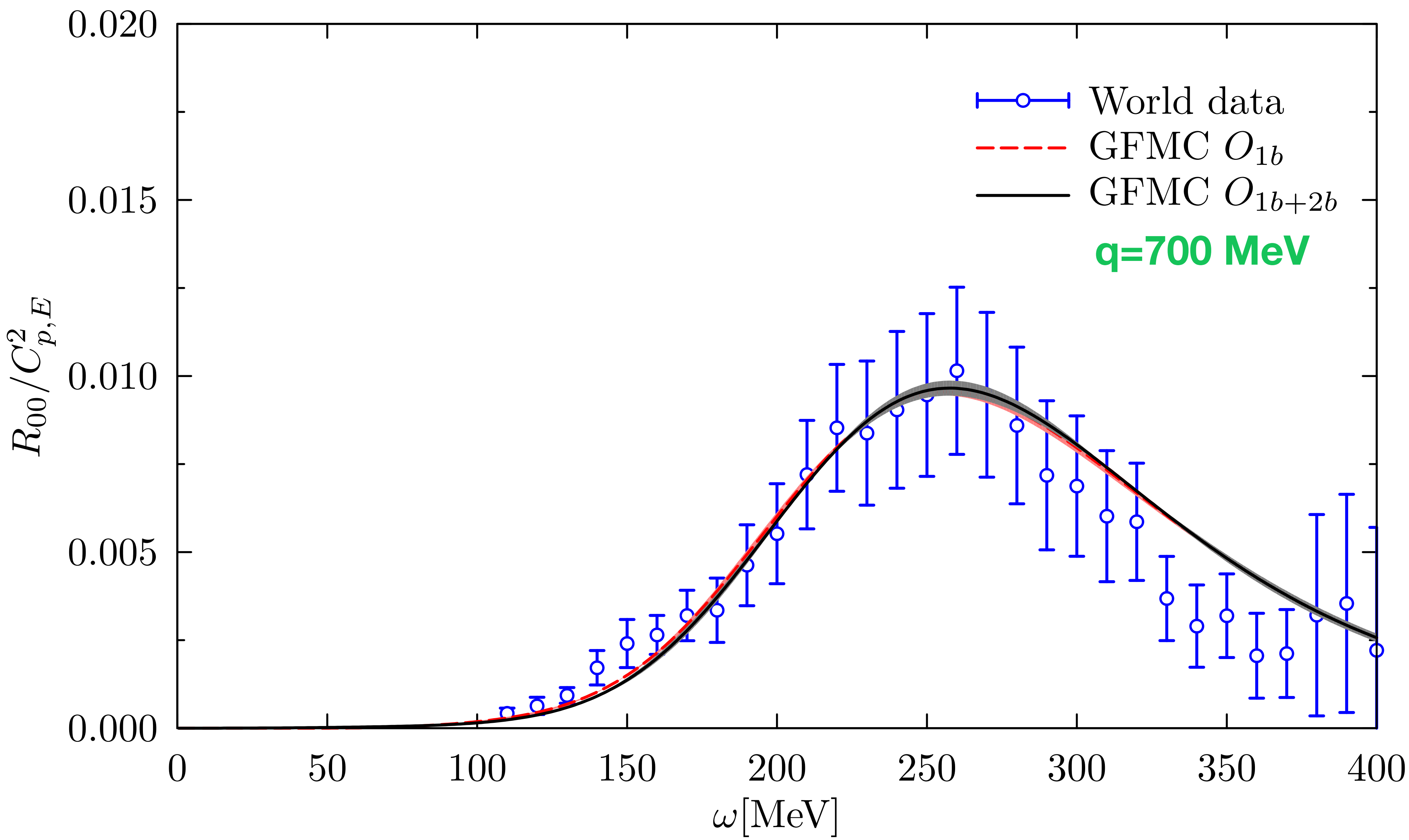
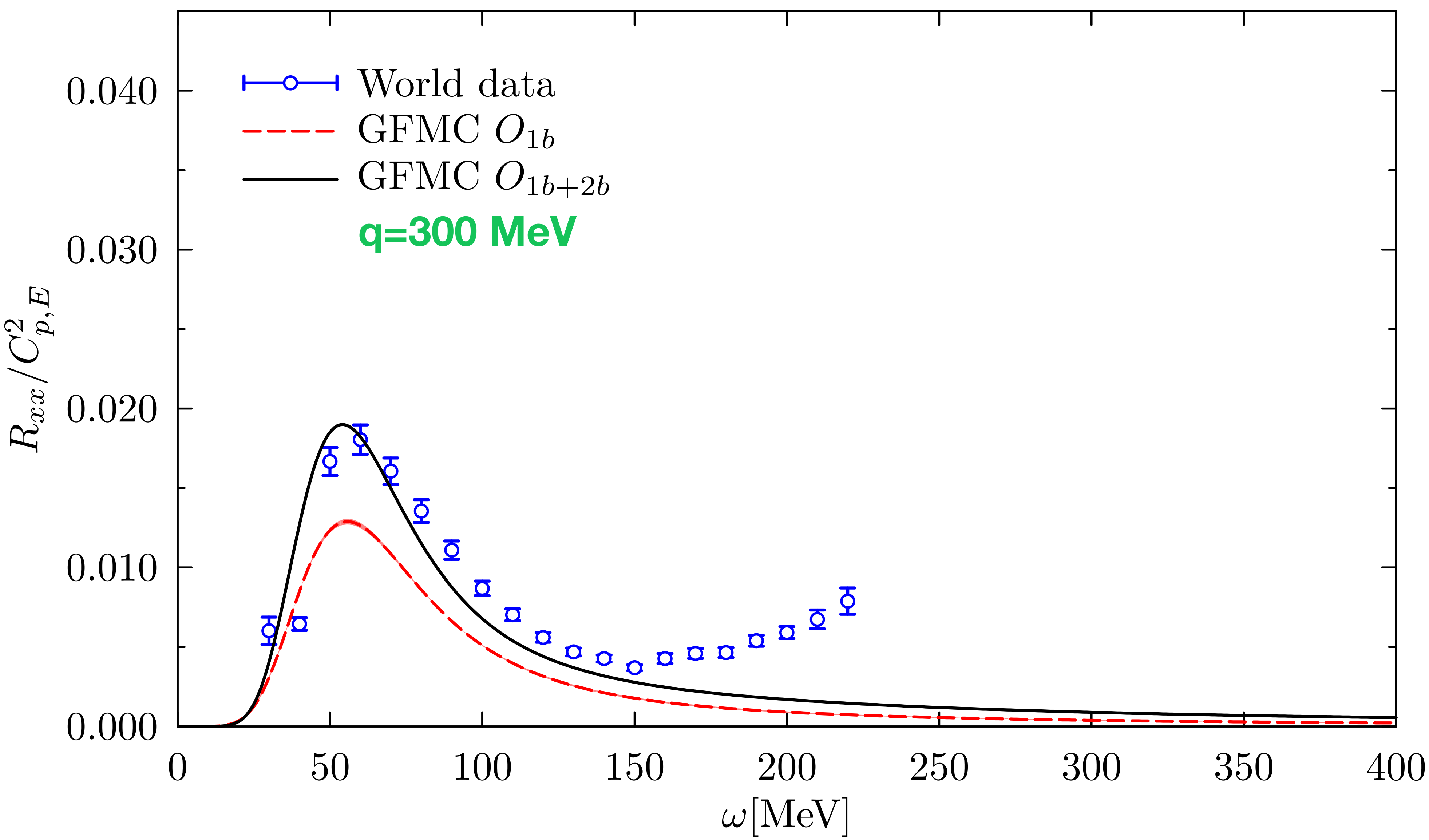


^{4}He electromagnetic response

Two-body currents significantly enhance the transverse response function, not only in the dip region, but also in the quasielastic peak and threshold regions. They are needed for a better agreement with the experimental data.



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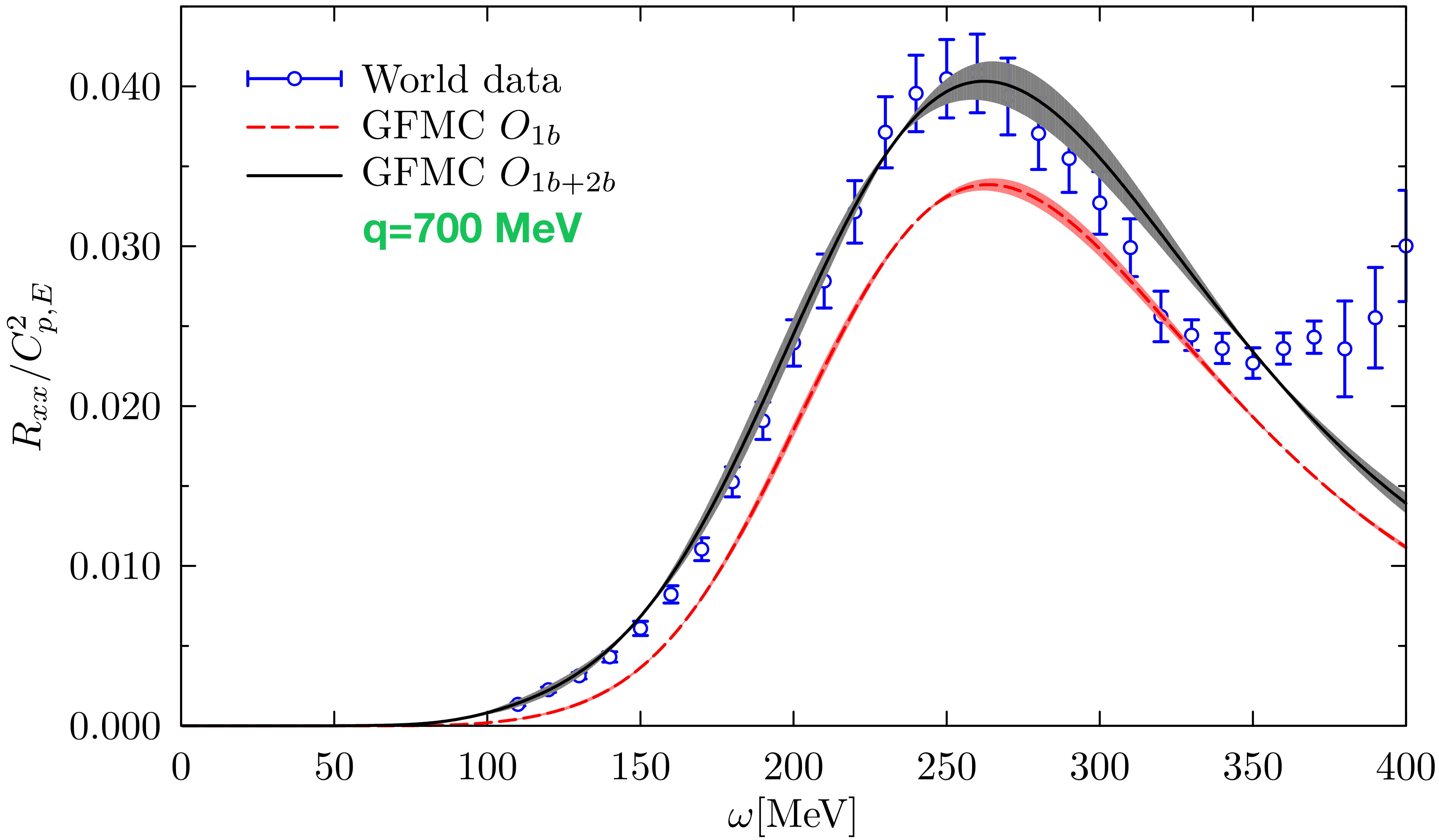
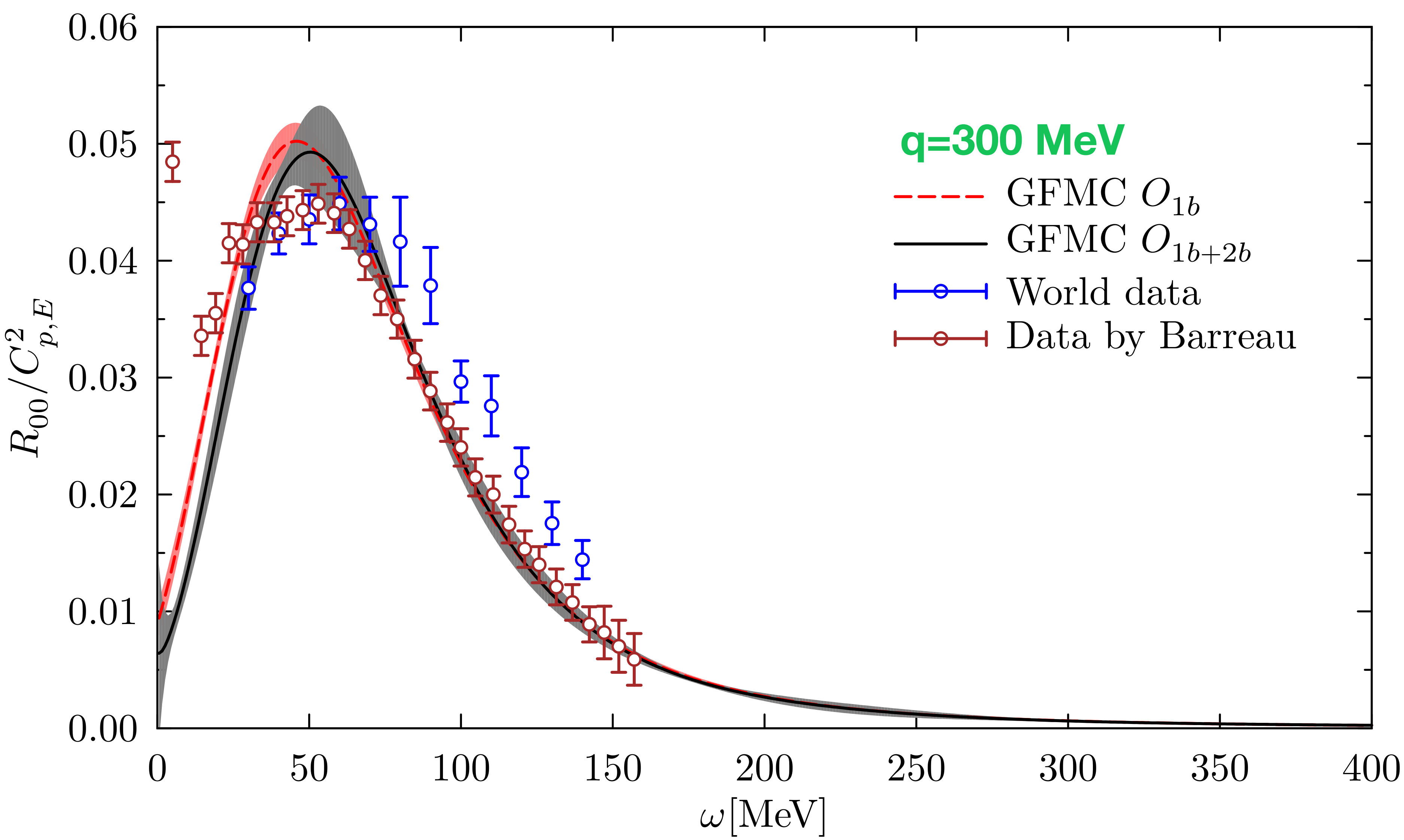


^{12}C electromagnetic response

- We were recently able to invert the electromagnetic Euclidean response of ^{12}C: first ab-initio calculation of the electromagnetic response of ^{12}C!

- Very good agreement with the experimental data. Small contribution from two-body currents.



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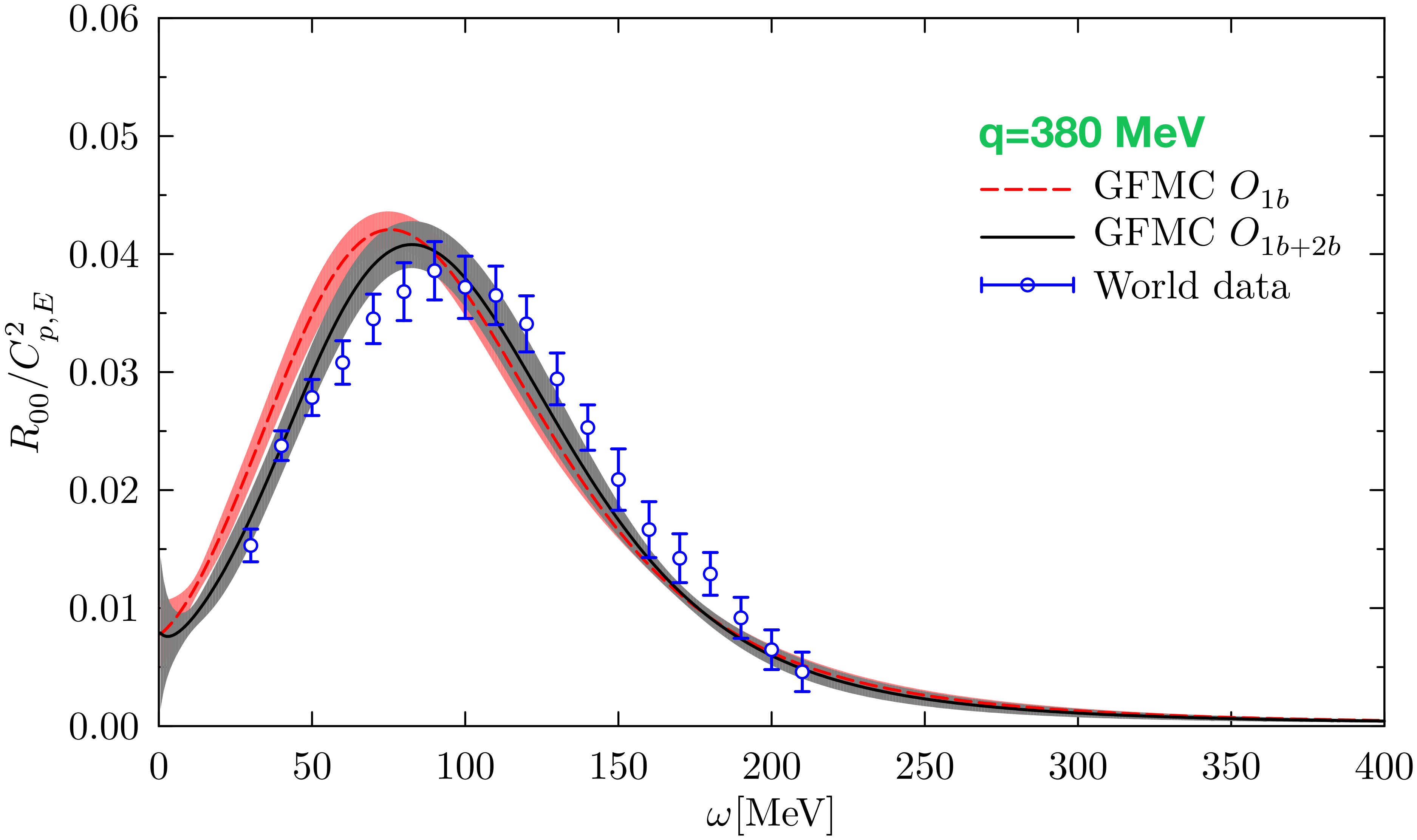

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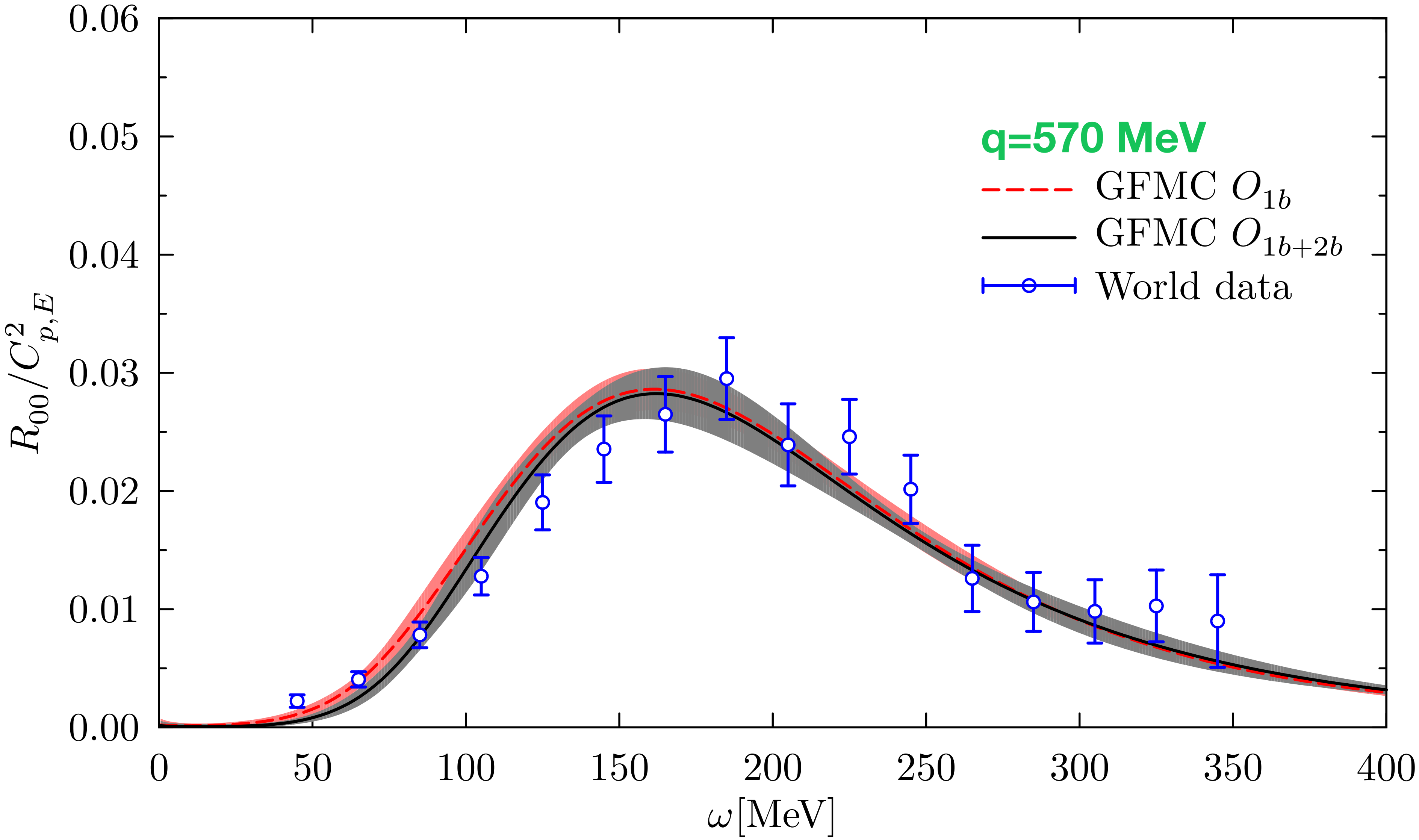

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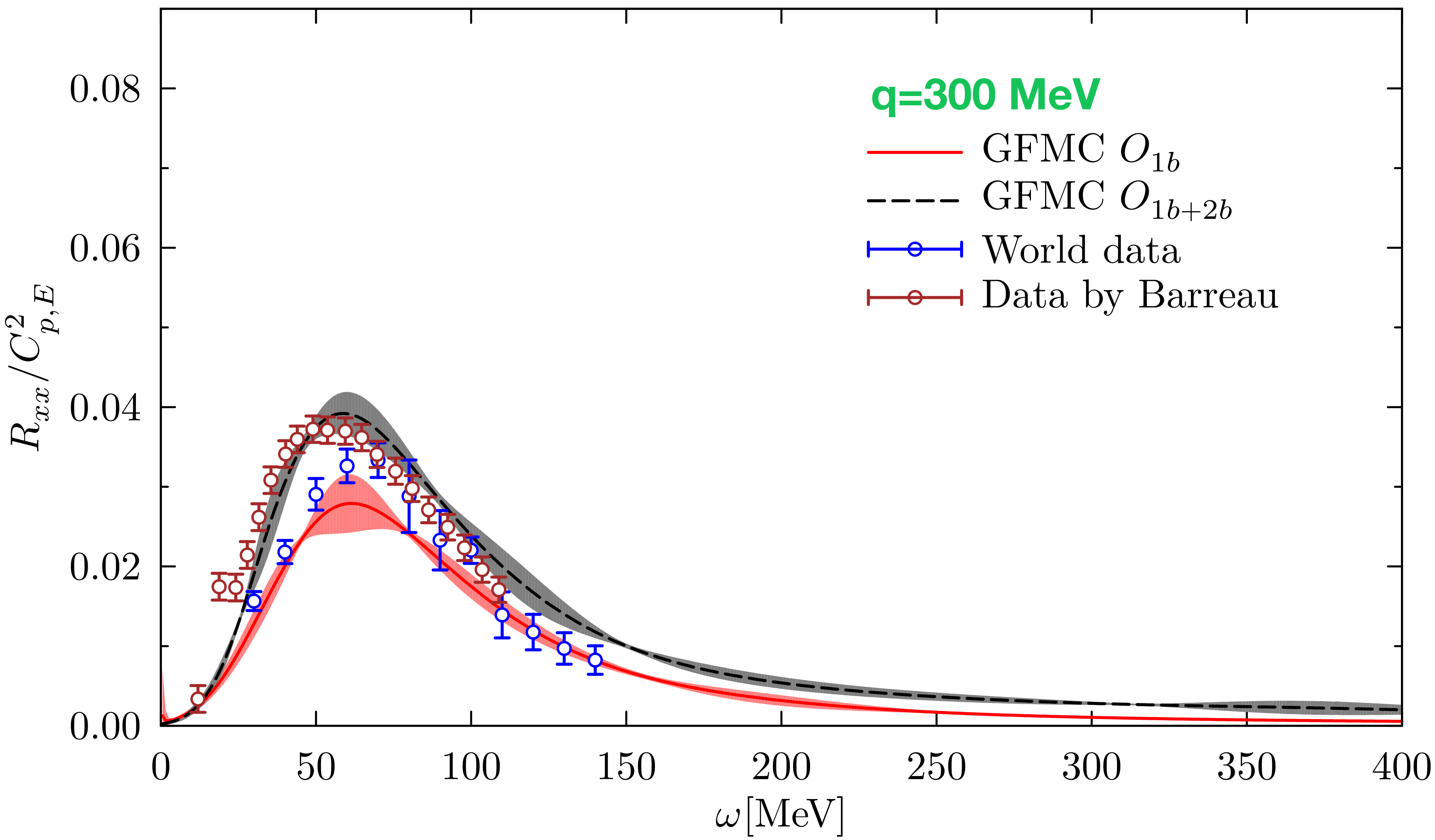

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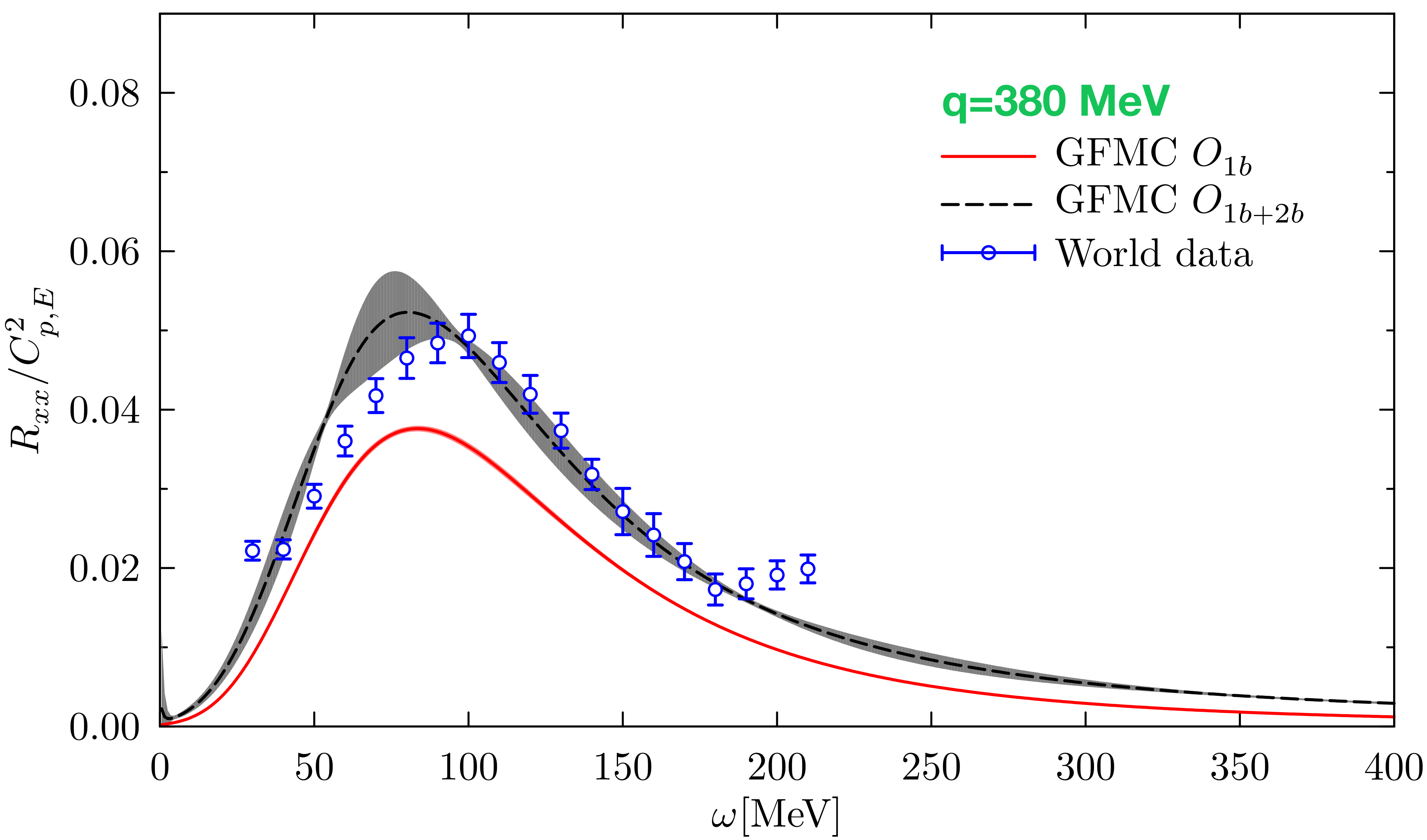
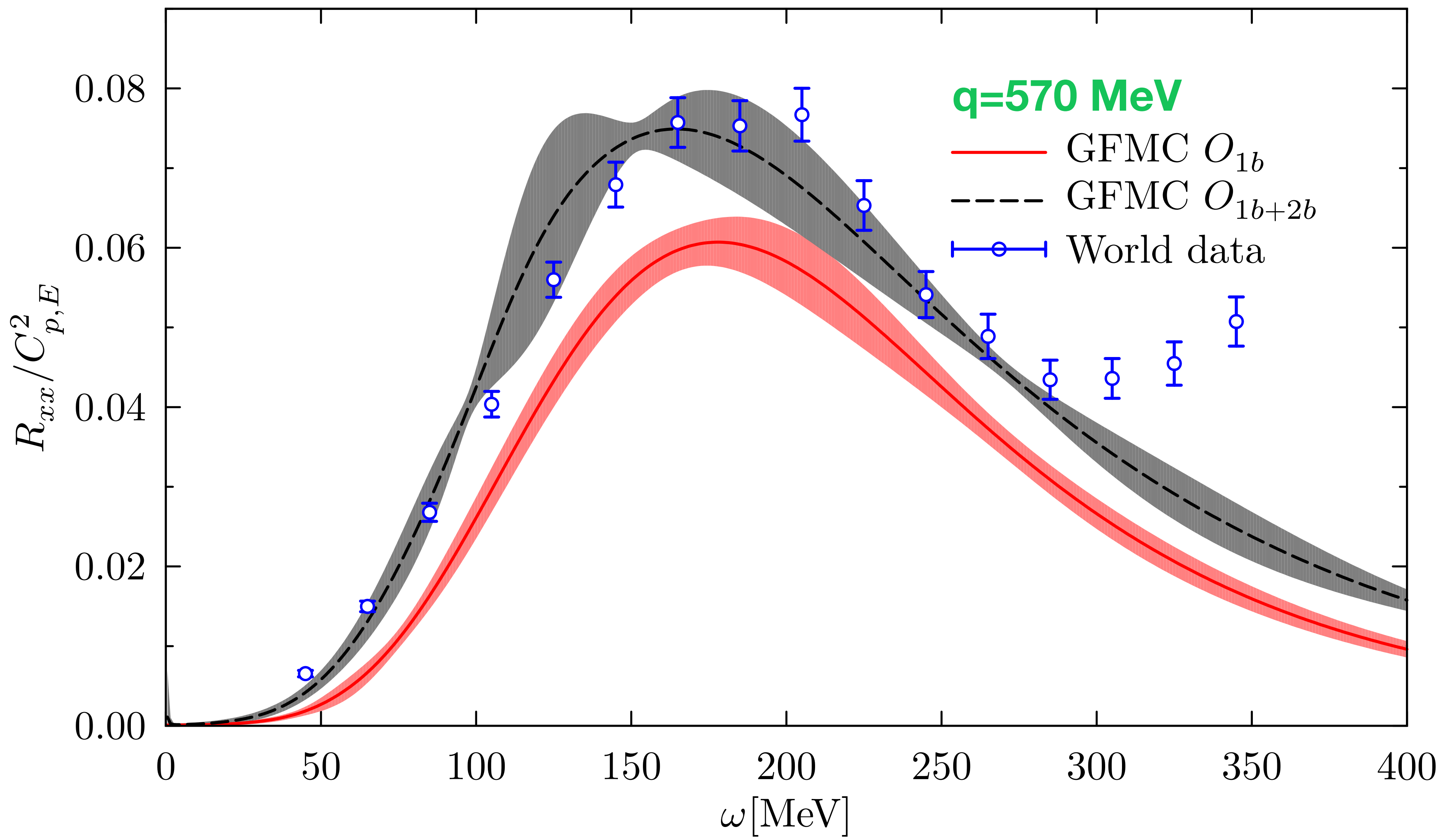


^{12}C electromagnetic response

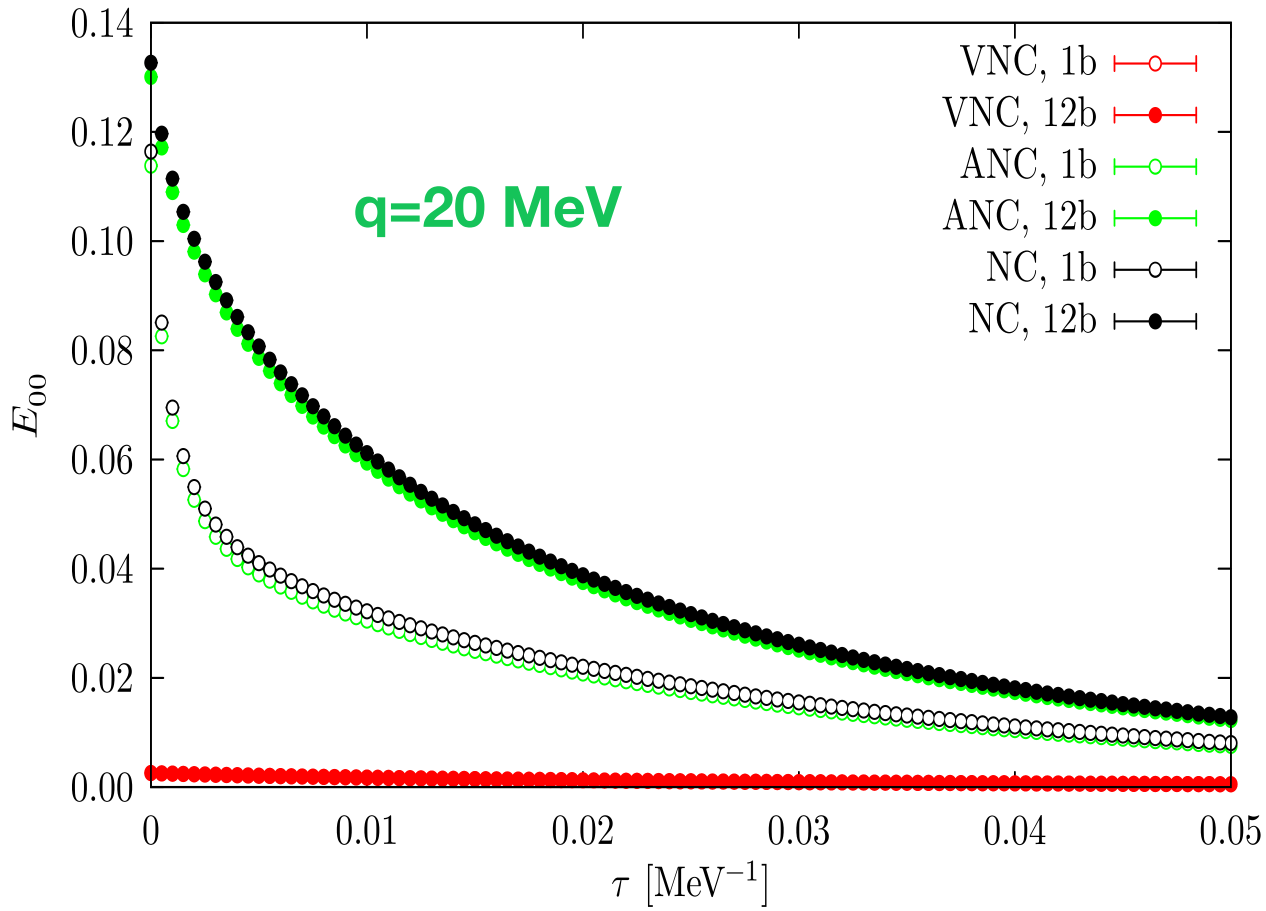
- We were recently able to invert the electromagnetic Euclidean response of ^{12}C: first ab-initio calculation of the electromagnetic response of ^{12}C!

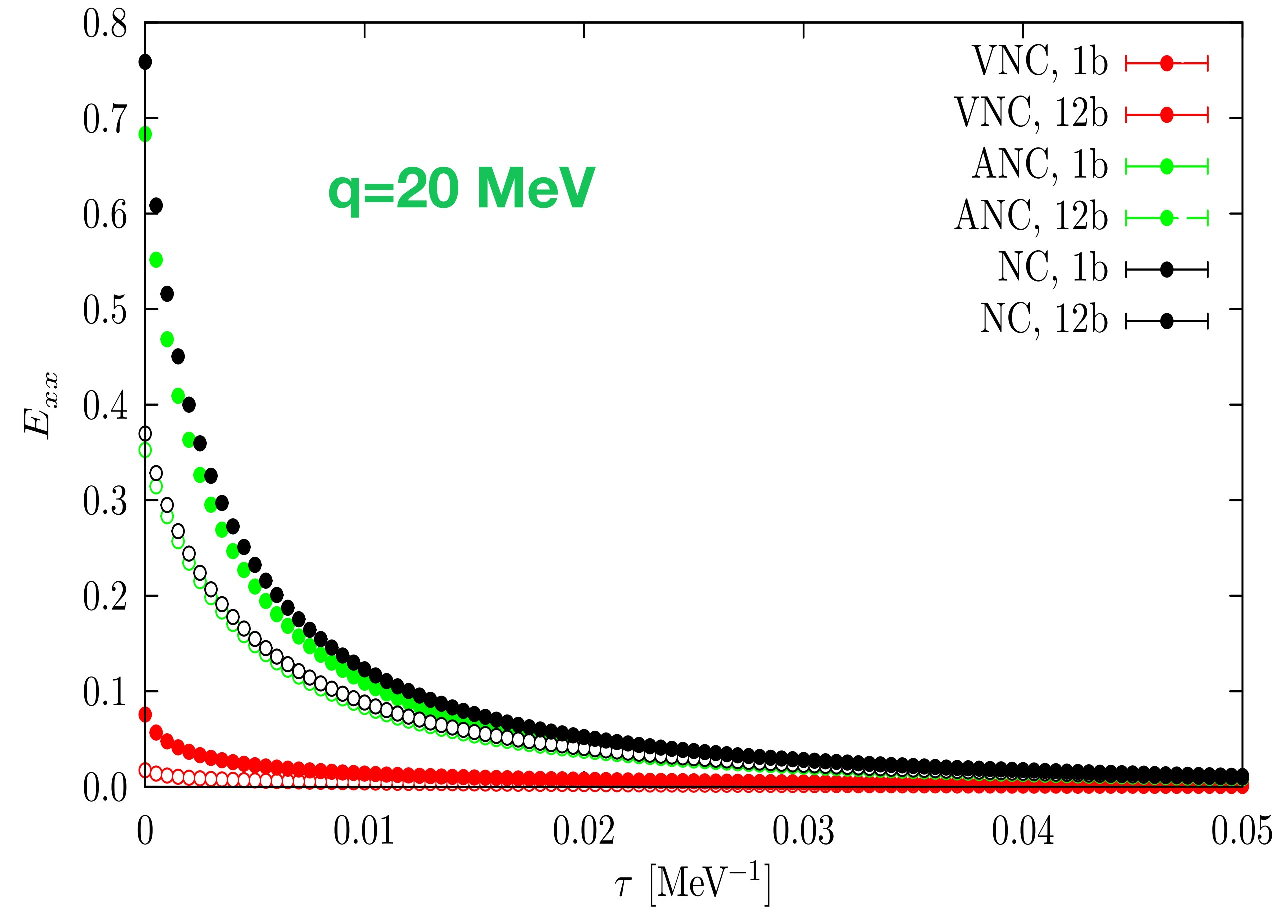
- Very good agreement with the experimental data once two-body currents are accounted for!



Neutral current response at low q

- Inelastic neutrino– ^{4}He reactions are supposed to play an important role in supernovae explosion
- Gazit and Barnea found very little effect from meson-exchange currents PRL 98, 192501 (2007)
- Preliminary results of GFMC calculations seem to indicate enhancement at high-energy transfer



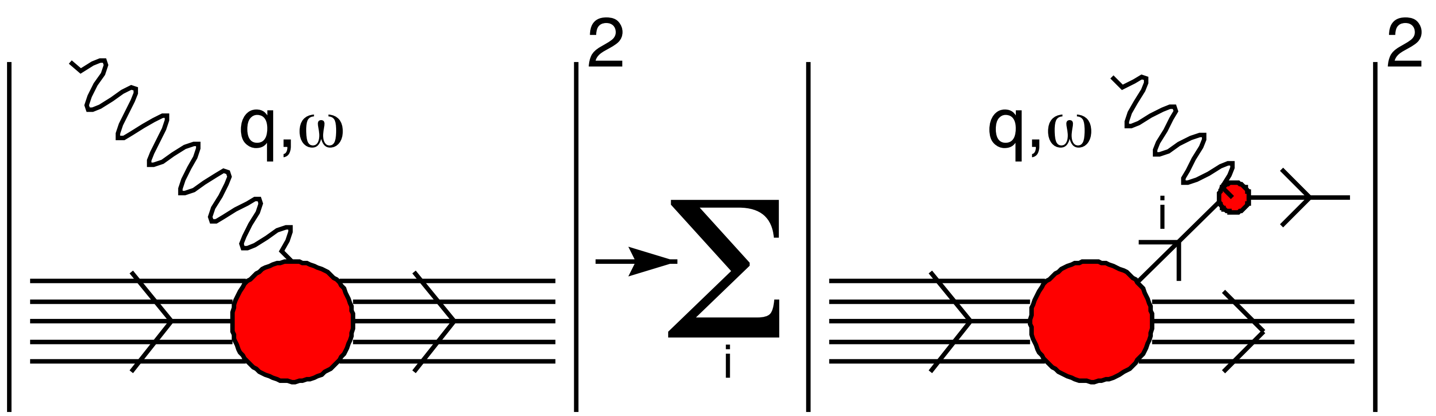


Spectral function approach

At large momentum transfer, scattering off a nuclear target reduces to the incoherent sum of scattering processes involving individual bound nucleons

$$J^\mu \to \sum_i j_i^\mu$$

$$|\Psi_f\rangle \to |\mathbf{p}\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-1}$$



$$\frac{d\sigma_{IA}}{d\Omega_{e'} dE_{e'}} = \int d^3p \, dE \, P(\mathbf{p}, E) \left[Z \frac{d\sigma_{ep}}{d\Omega_{e'} dE_{e'}} + (A - Z) \frac{d\sigma_{en}}{d\Omega_{e'} dE_{e'}} \right]$$

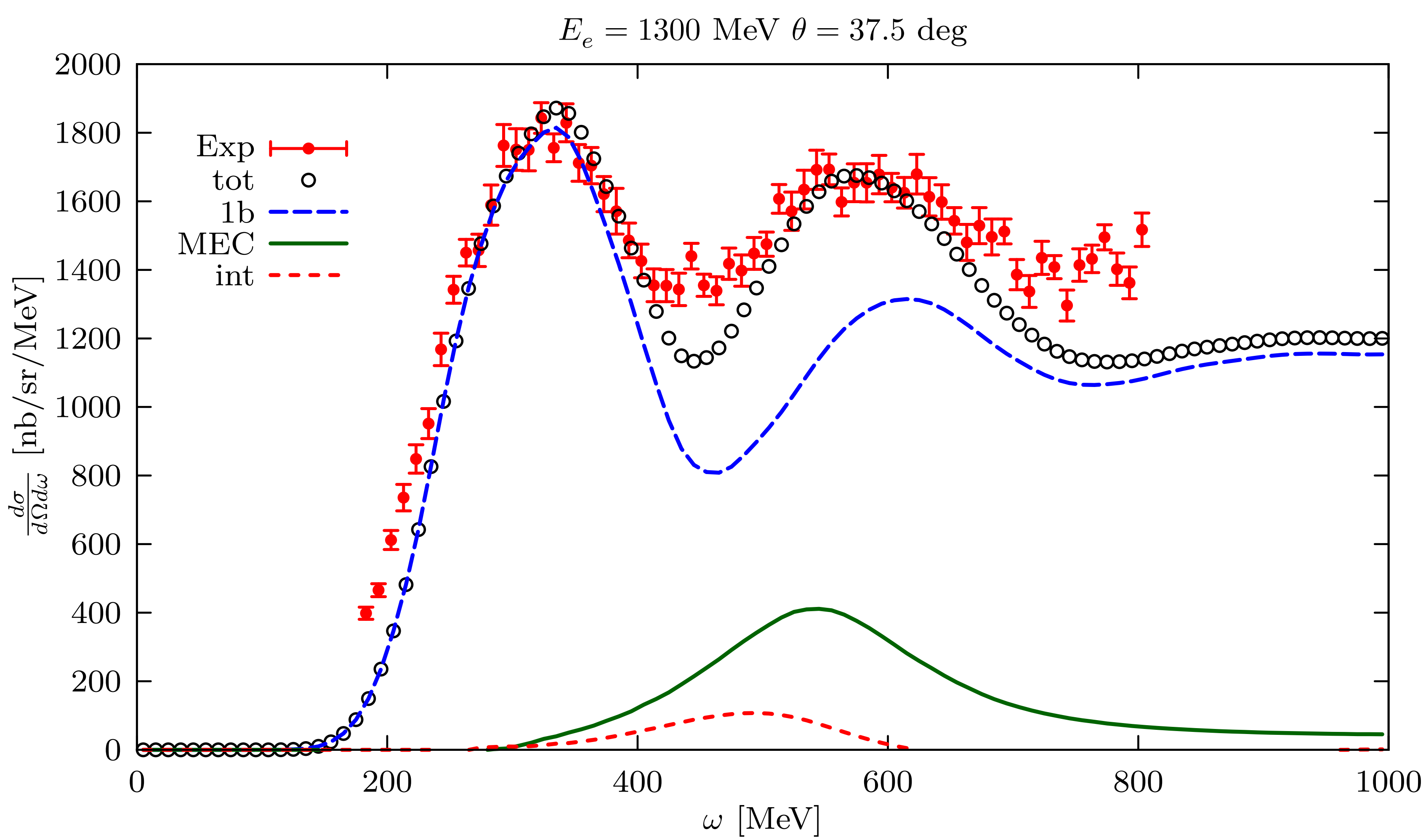
The spectral function yields the probability of removing a nucleon with momentum $\mathbf{p}$ from the target ground state leaving the residual system with excitation energy E .

Spectral function approach

Using relativistic MEC and realistic description of the nuclear ground state requires the extension of the factorization scheme to two-nucleon emission amplitude

$$|\Psi_f\rangle \to |\mathbf{p}\mathbf{p}'\rangle \otimes |\Psi_{\tilde{f}}\rangle_{A-2}$$

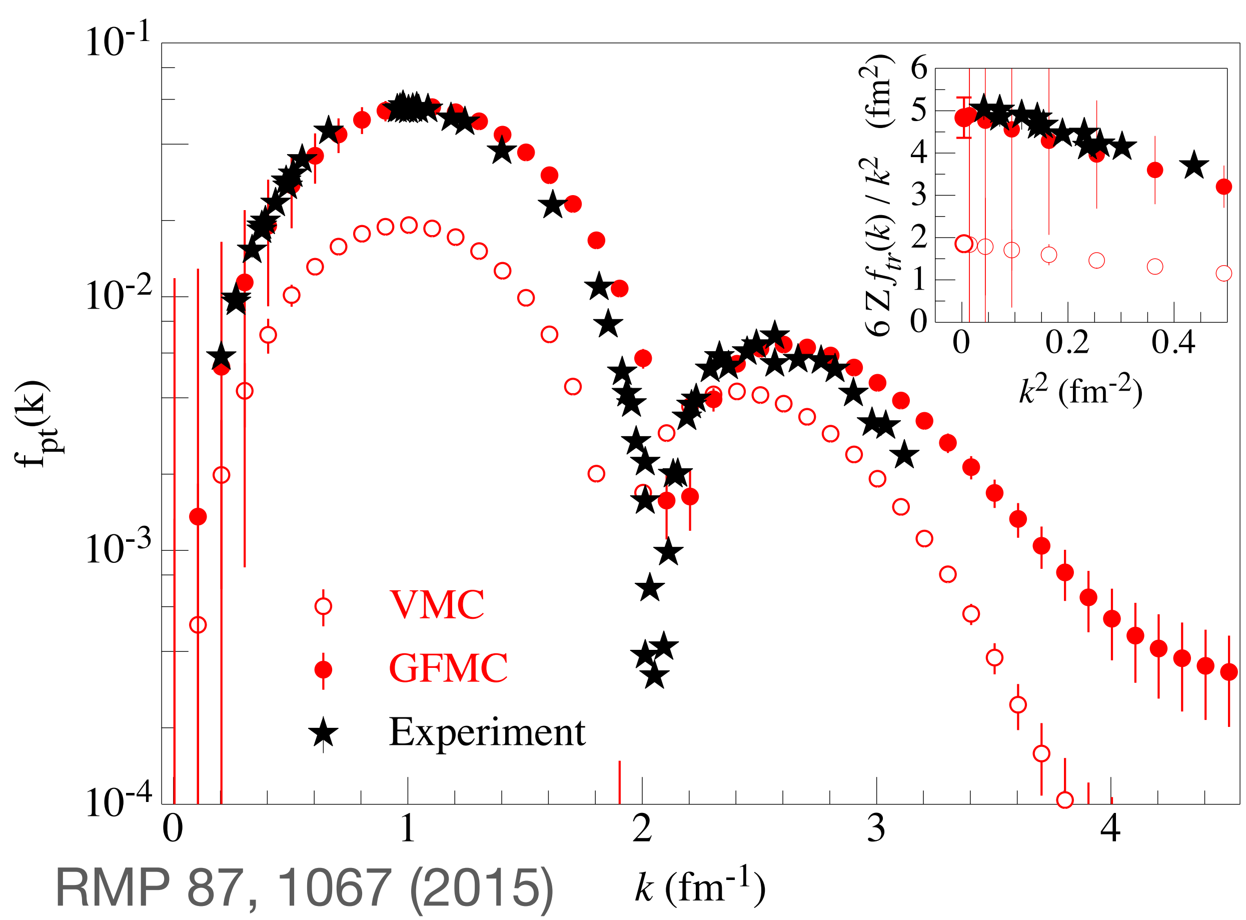
^{12}C calculations indicate a sizable enhancement of the electromagnetic transverse response



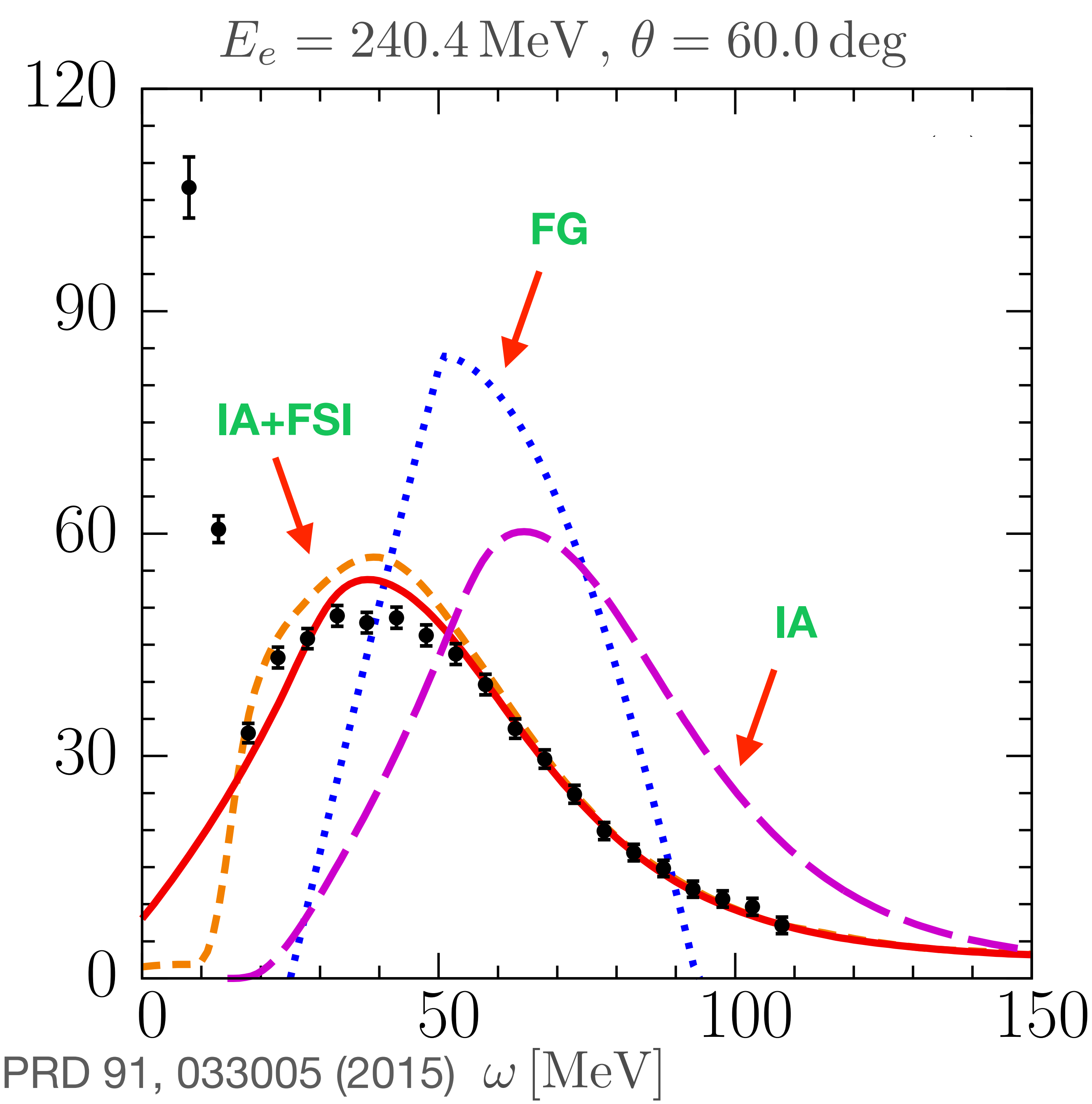
Spectral function approach

At lower momentum transfer the factorization ansatz is no longer reliable. However, if final state interactions are properly accounted for

- Satisfactory description of the quasi elastic region
- Missing elastic scattering and inelastic excitation of discrete nuclear states



RMP 87, 1067 (2015)

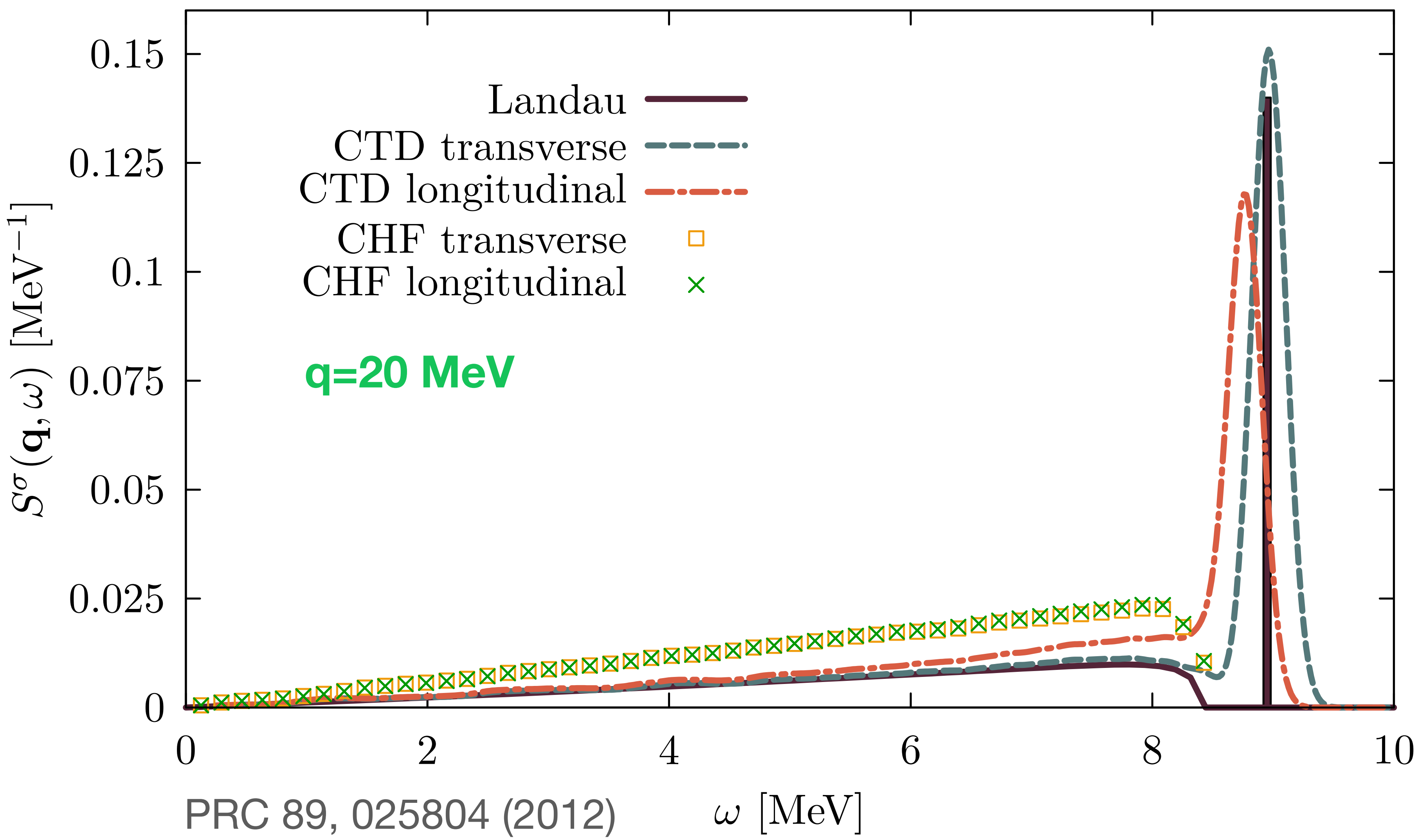


PRD 91, 033005 (2015)

CBF - Effective interaction approach

The effective interaction approach has shown to satisfactorily account for short- and long-range correlations (Omar's talk) in both symmetric nuclear matter and pure neutron matter

$$\langle \Psi_f | J_\alpha(\mathbf{q}) | \Psi_0 \rangle \to \langle \Phi_f | \mathcal{F}^\dagger \, J_\alpha(\mathbf{q}) \, \mathcal{F} | \Phi_0 \rangle$$



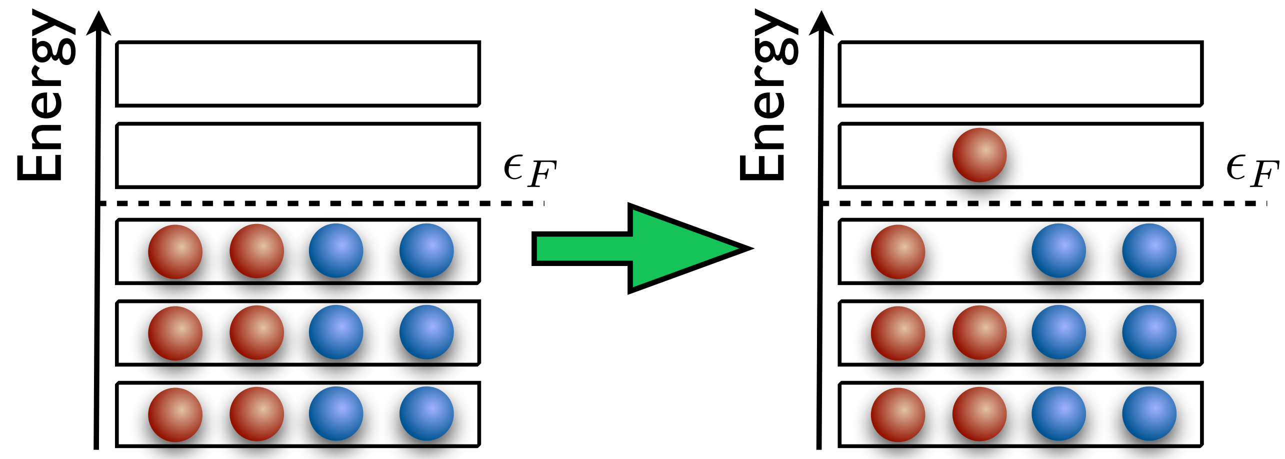
PRC 89, 025804 (2012)

CBF - Effective interaction approach

However, only correlated one-particle one-hole final state have been considered

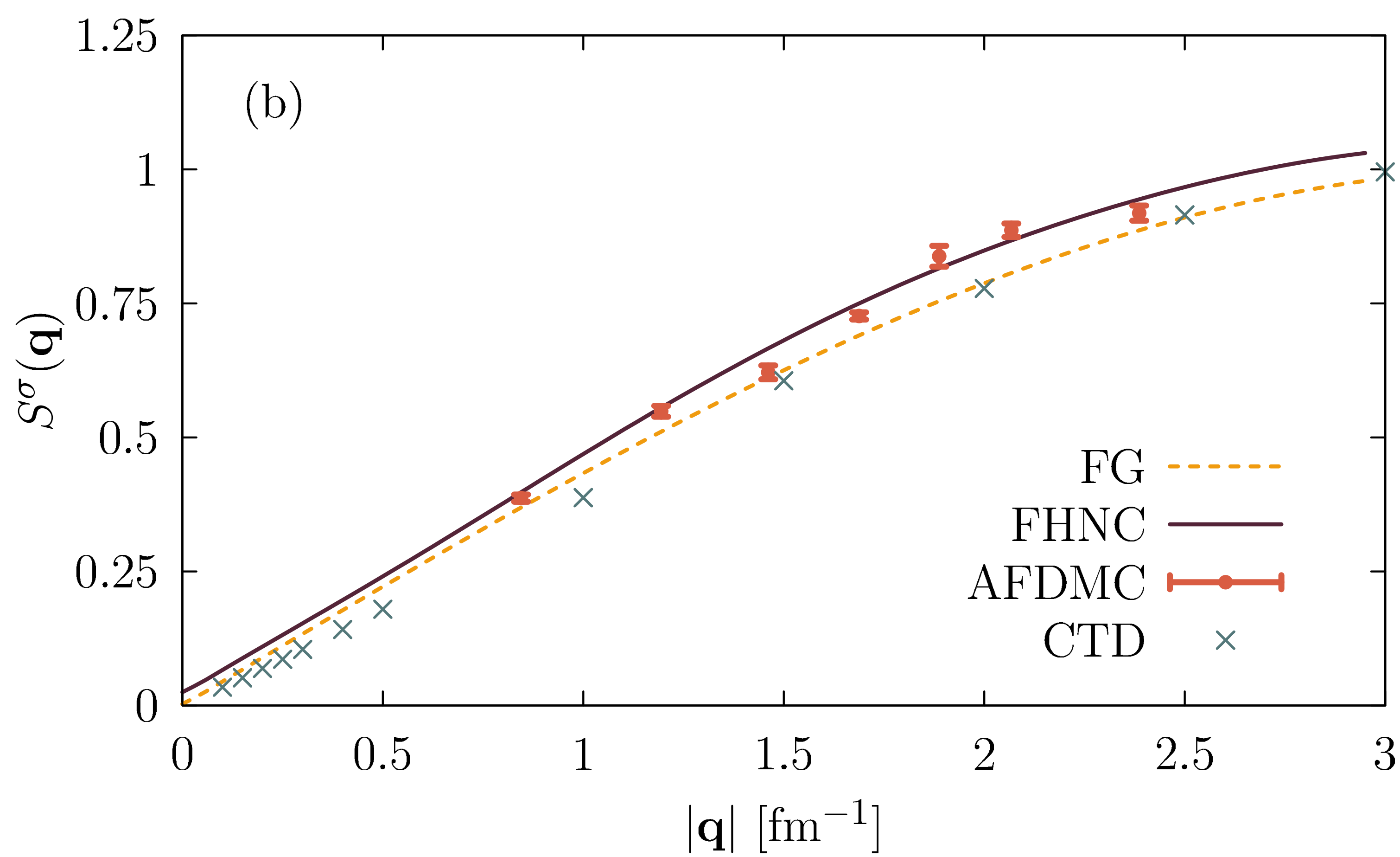
$$|\Phi_f\rangle = \sum_{p,h} C^f_{p,h} |\Phi_{p,h}\rangle$$

$$H^{\text{eff}} |\Phi_f\rangle = (E_0 + \omega_f) |\Phi_f\rangle$$



The missing strength due to many-particle many-hole states can be estimated using AFDMC

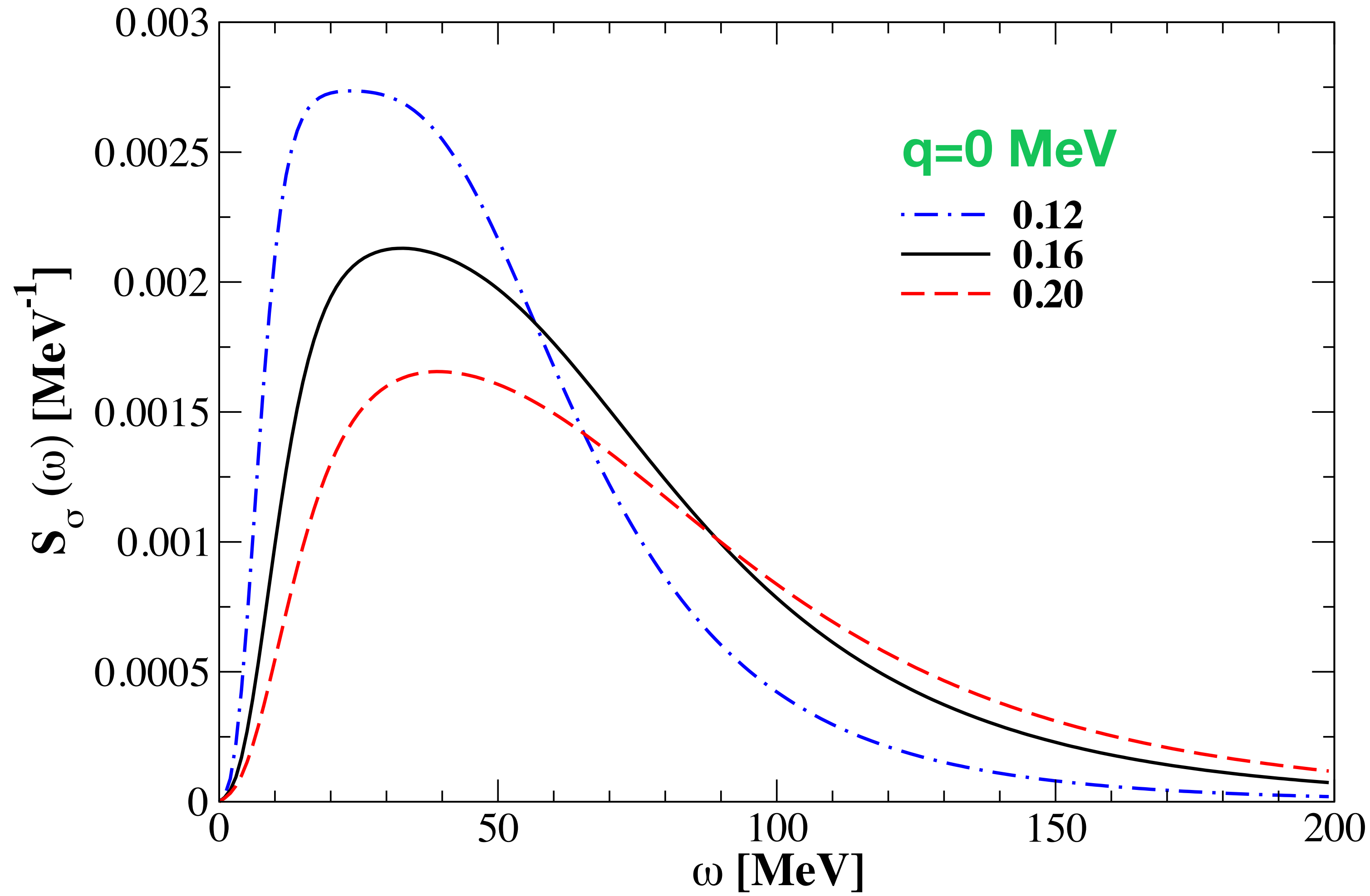
Contribution from (mainly) 2particle-2hole states important at low momentum transfer



AFDMC

AFDMC will be exploited to compute the Euclidean response functions of density and spin-transition operators in neutron matter, relevant for neutrino propagation

From the energy weighted sum rules computed with AFDMC it was possible to gather information on the spin-density response of neutron matter at zero momentum transfer



PRC 87, 025802 (2012)

Conclusions

- For relatively large momentum transfer, the two-body currents enhancement is effective in the entire energy transfer domain.

- For low momentum transfer, two-body currents enhancement is more pronounced in the high energy transfer region.

- ^{4}He and ^{12}C results for the electromagnetic response obtained using Maximum Entropy technique are in very good agreement with experimental data.

- Fruitful interplay between GFMC, SF, CBF effective interaction and AFDMC approaches. This is possible as they are all based on the same model of nuclear dynamics.

- We are tackling the computation of the neutrino-Argon cross section using different approaches and benchmarking them were possible. However,

It is a very difficult problem, especially for the low-energy transitions to nuclear excited states

Future developments

The results we obtained are very nice, but are not yet completely satisfactory

- The continuity equation only constraints the longitudinal components of the current
- The transverse component and the axial terms are phenomenological (the coupling constant is fitted on the tritium beta-decay)
- Two- and three- body forces not fully consistent

$$\boxed{\text{Within this framework, the theoretical error arising from modeling the nuclear dynamics cannot be properly assessed!}}$$

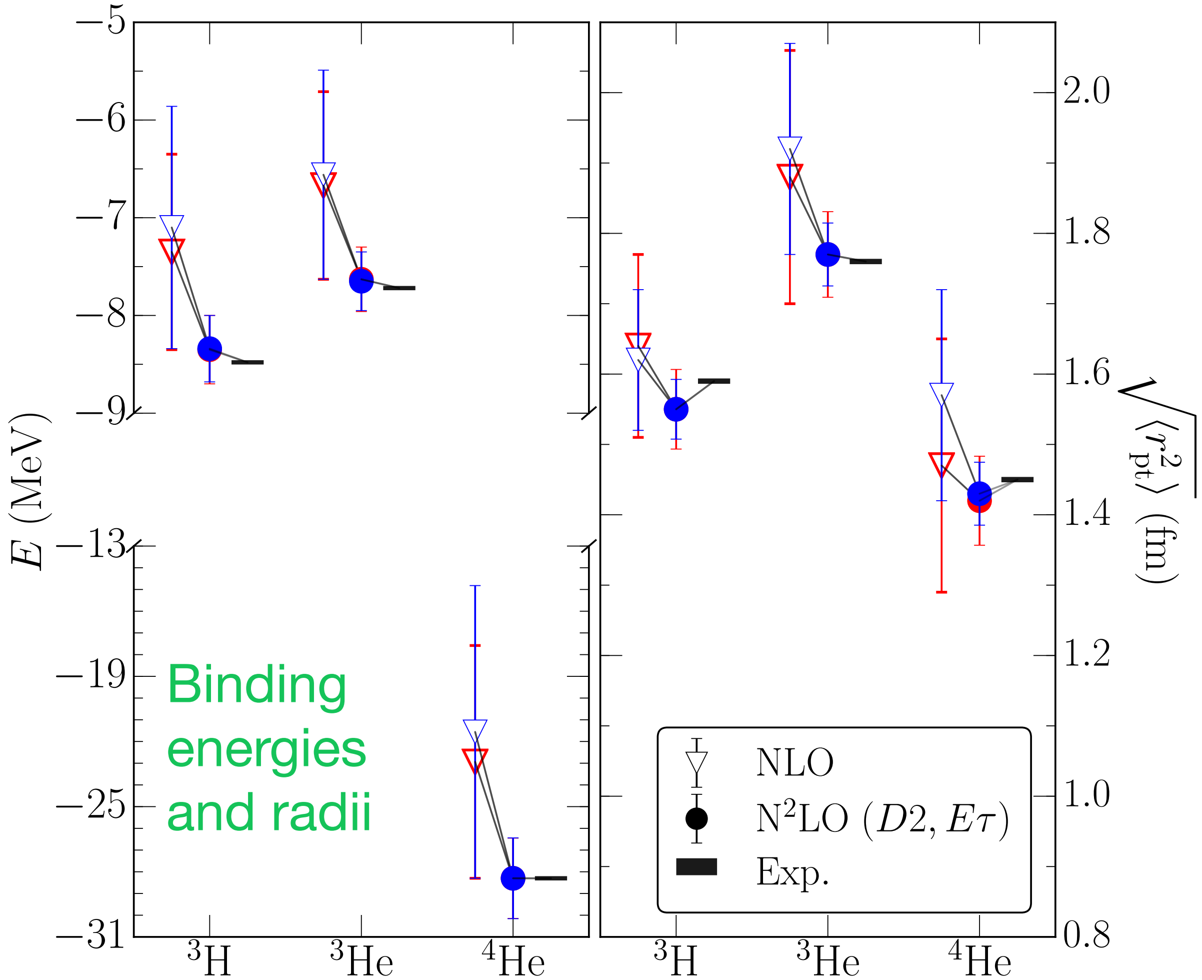
Chiral effective field theory (χEFT) has witnessed much progress during the two decades since the pioneering papers by Weinberg (1990, 1991, 1992)

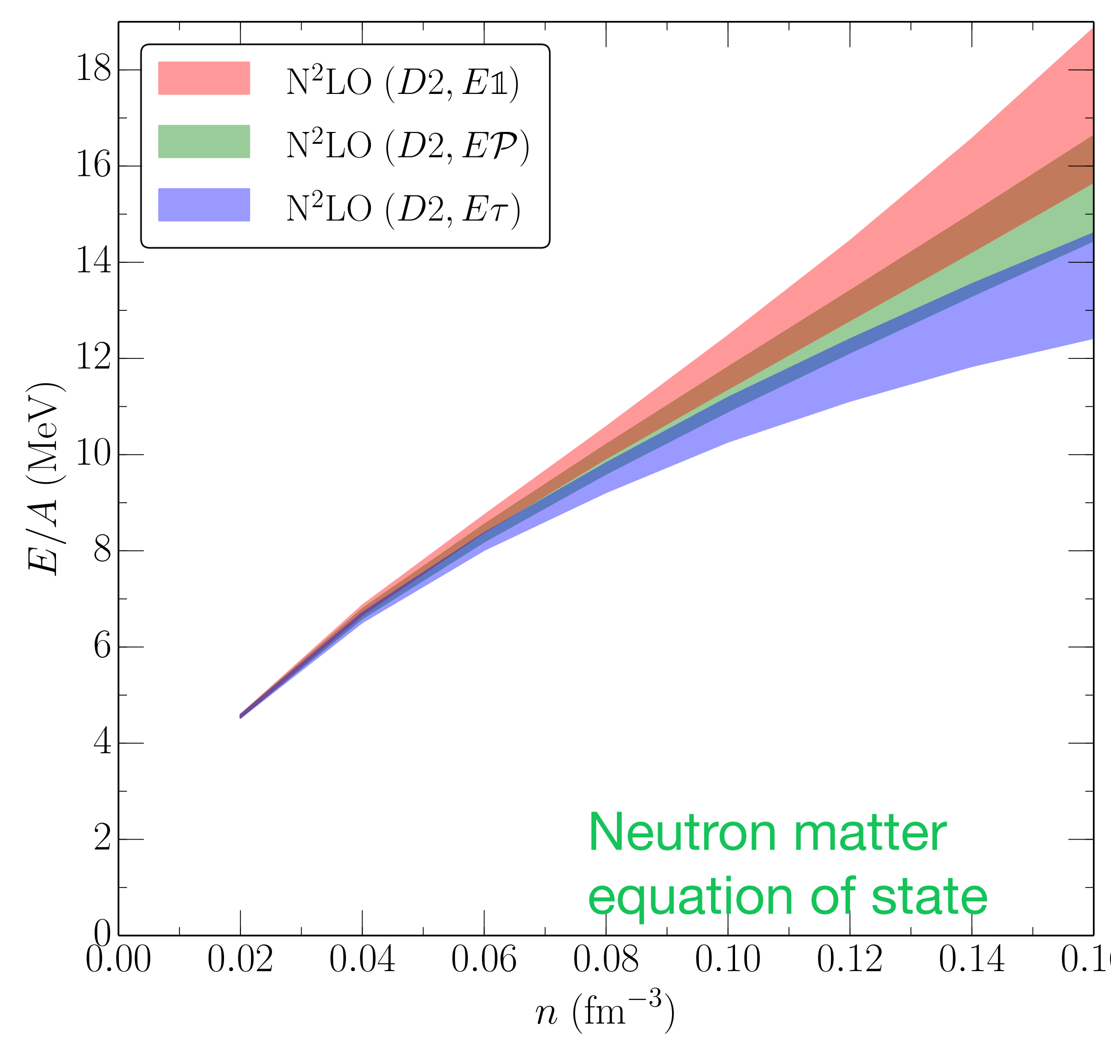
In χEFT, the symmetries of quantum chromodynamics (QCD), in particular its approximate chiral symmetry, are employed to systematically constrain classes of Lagrangians describing the interactions of baryons with pions as well as the interactions of these hadrons with electroweak fields

Chiral EFT

Recently chiral nuclear interactions have been developed that are local up to next-to-next-to-leading order (N2LO). These interactions employ a different regularization scheme from previous chiral interactions, with a cutoff in the relative NN momentum.

They are therefore fairly simple to treat with standard QMC techniques to calculate properties of nuclei and neutron matter,

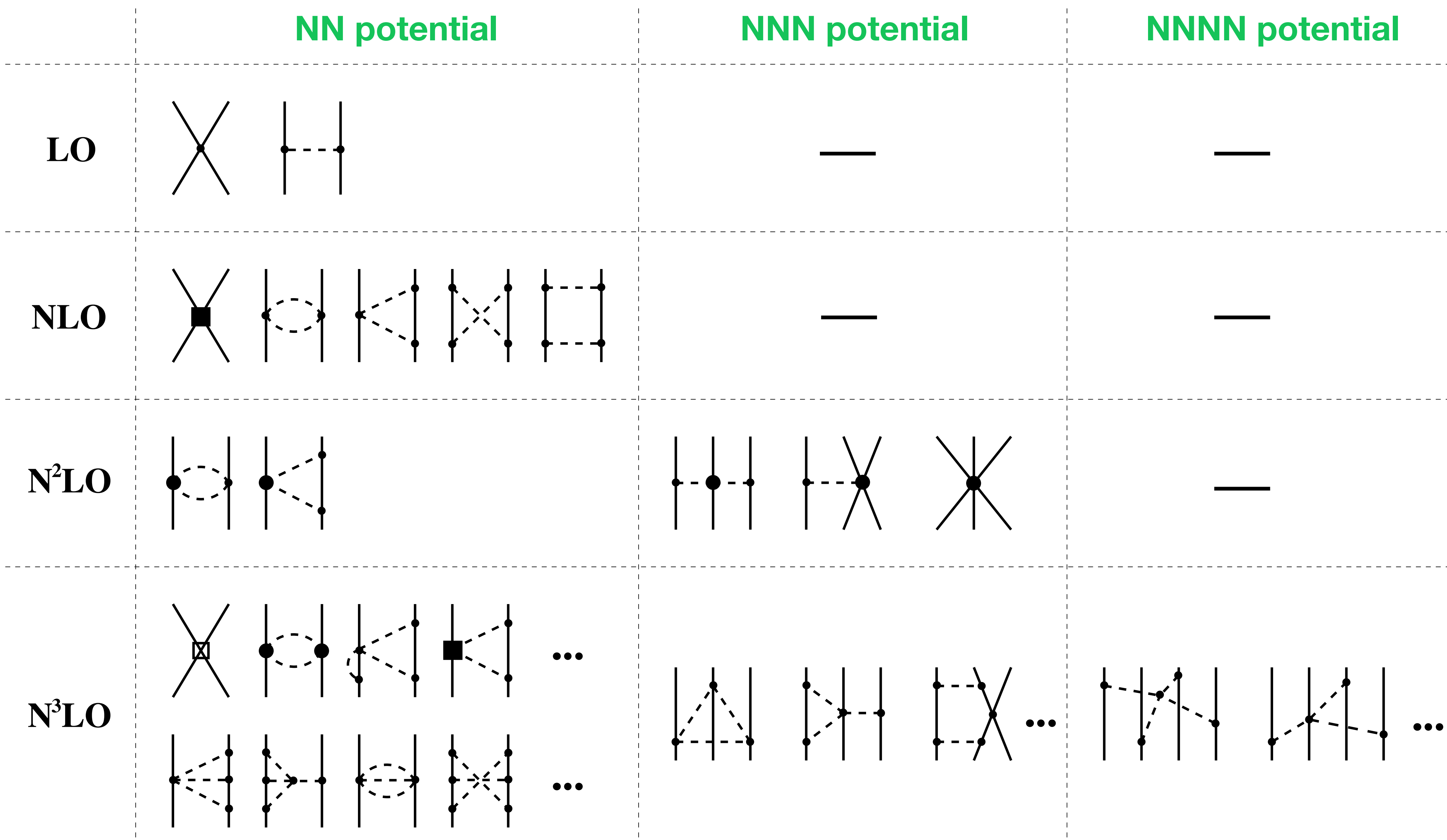




Lynn, Tews et al. arXiv:1509.03470

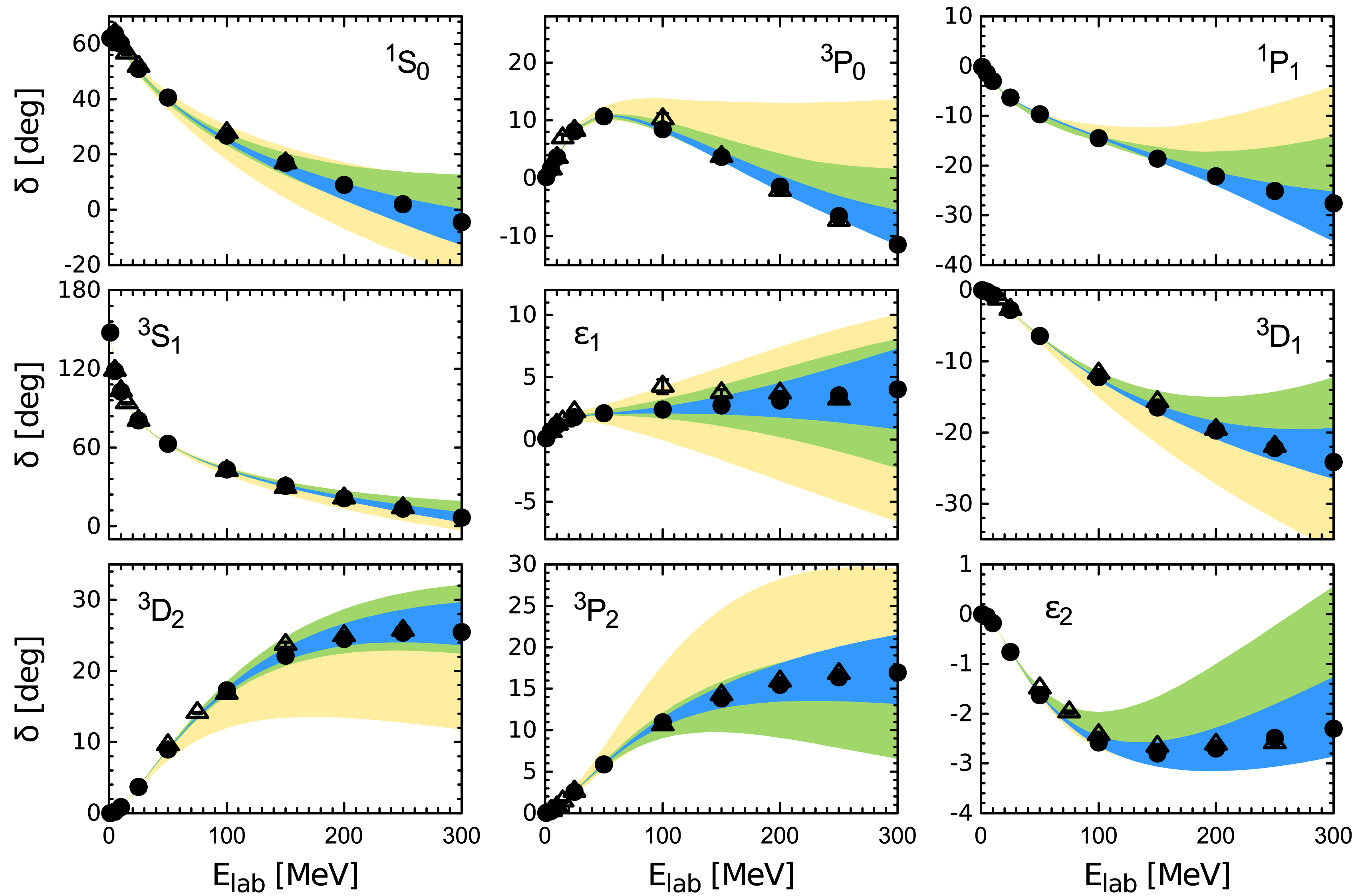
Chiral EFT

Within χ EFT two- and three- body potentials and currents can be consistently derived and obey a power counting scheme



Chiral EFT

χEFT provides a framework to derive consistent many-body forces and currents and the tools to rigorously estimate their uncertainties, along with a systematic prescription for reducing them.



Epelbaum et al., arXiv:1412.0142

QMC allows to disentangle the theoretical uncertainty arising from the nuclear interaction from the one associated with the many-body computational scheme.

Thank you

Maximum entropy algorithm

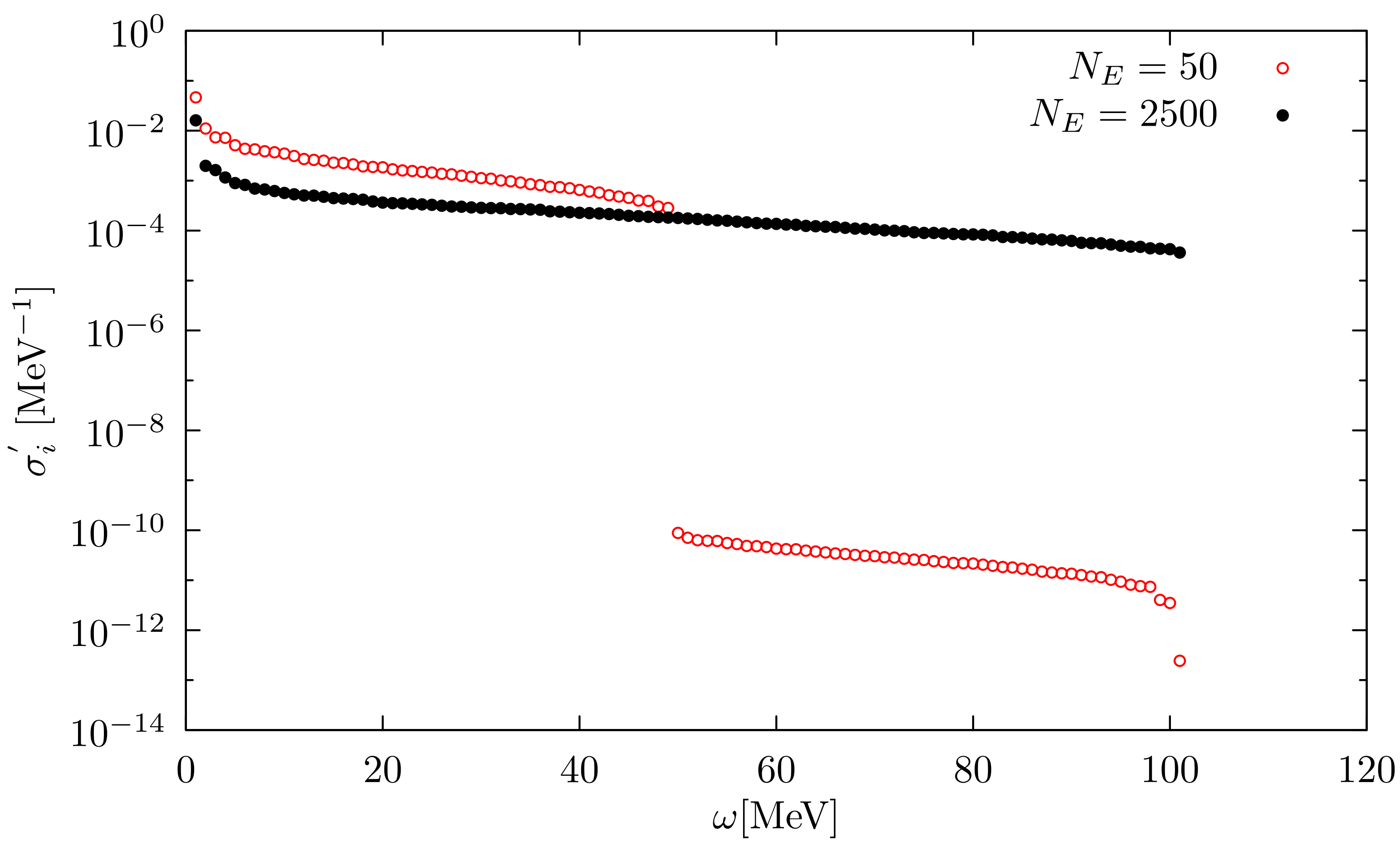
We estimate the mean and the covariance matrix from N_E Euclidean responses

$$\bar{E}(\tau_i) = \frac{1}{N}\sum_n E^n(\tau_i) \qquad C(\tau_i,\tau_j) = \frac{1}{N(N-1)}\sum_n (\bar{E}^n(\tau_i) - E^n(\tau_i))(\bar{E}^n(\tau_j) - E^n(\tau_j))$$

- The covariance matrix in general is NOT diagonal, and it is convenient to diagonalize it

$$(\mathbf{U}^{-1}\mathbf{C}\mathbf{U})_{ij} = \sigma_i'^2 \delta_{ij}$$

- If N is not sufficiently large, the spectrum of the covariance eigenvalues becomes pathological.



Maximum entropy algorithm

- The likelihood is defined in terms of the covariance matrix

$$\chi^2 = \sum_{i,j=1}^{N_\tau} \left(\overline{E}_i - E_i\right) \left(C^{-1}\right)_{ij} \left(\overline{E}_j - E_j\right) \qquad E_i = \sum_{j=1}^{N_\omega} K_{ij}\, R_j$$

- We rotate both the data and the kernel in the diagonal representation of the covariance matrix

$$\mathbf{K}' = \mathbf{U}^{-1}\mathbf{K} \qquad \bar{\mathbf{E}}' = \mathbf{U}^{-1}\bar{\mathbf{E}} \qquad \longleftrightarrow \qquad (\mathbf{U}^{-1}\mathbf{C}\mathbf{U})_{ij} = \sigma_i'^{\,2}\delta_{ij}$$

- The likelihood can be written in terms of the statistically independent measurements and the rotated kernel

$$\chi^2 = \frac{1}{N_\tau} \sum_i \frac{(\sum_j K'_{ij} R_j - \bar{E}'_i)^2}{\sigma_i'^{\,2}}$$

Maximum entropy algorithm

Maximum entropy approach can be justified on the basis of Bayesian inference. The best solution will be the one that maximizes the conditional probability

$$Pr[R|\bar{E}] = \frac{Pr[\bar{E}|R]\, Pr[R]}{Pr[\bar{E}]}$$

- The evidence is merely a normalization constant

$$Pr[\bar{E}] = \int \mathcal{D}R\, Pr[\bar{E}|R]\, Pr[R]$$

- When the number of measurements becomes large, the asymptotic limit of the likelihood function is

$$Pr[\bar{E}|R] = \frac{1}{Z_1} e^{-L[R]} = \frac{1}{Z_1} e^{-\frac{1}{2}\chi^2[R]} \qquad \chi^2 = \frac{1}{N_\tau} \sum_i \frac{(\sum_j K'_{ij} R_j - \bar{E}'_i)^2}{\sigma'^2_i}$$

Limiting ourselves to the minimization of the χ^2, we implicitly make the assumption that the prior probability is important or unknown.

Maximum entropy algorithm

Since the response function is nonnegative and normalizable, it can be interpreted as a probability distribution function.

The principle of maximum entropy states that the values of a probability function are to be assigned by maximizing the entropy expression

$$S[R] \equiv -\int d\omega (R(\omega) - D(\omega) - R(\omega) \ln[R(\omega)/D(\omega)]) \quad \longleftrightarrow \quad D(\omega)\text{: Default model}$$

The prior probability then reads

$$Pr[R] = \frac{1}{Z_2} e^{\alpha S[R]}$$

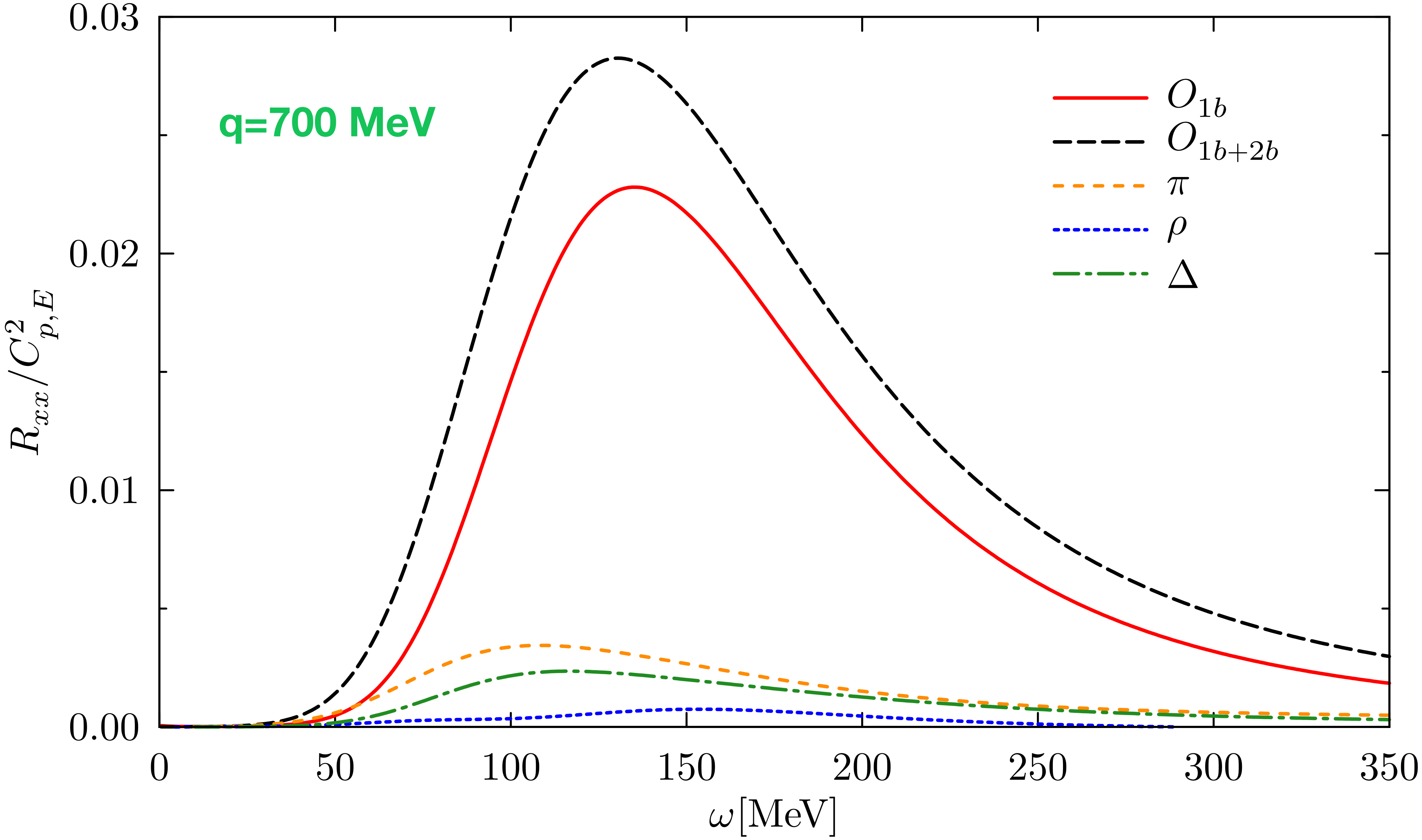
and the posterior probability can be rewritten as

$$Pr[R|\bar{E}] = \frac{e^{-Q[R]}}{Z_1 Z_2 Pr[\bar{E}]} \quad \longleftrightarrow \quad Q[R] \equiv \frac{1}{2}\chi^2[R] - \alpha S[R]$$

Regularization parameter

^{4}He electromagnetic response

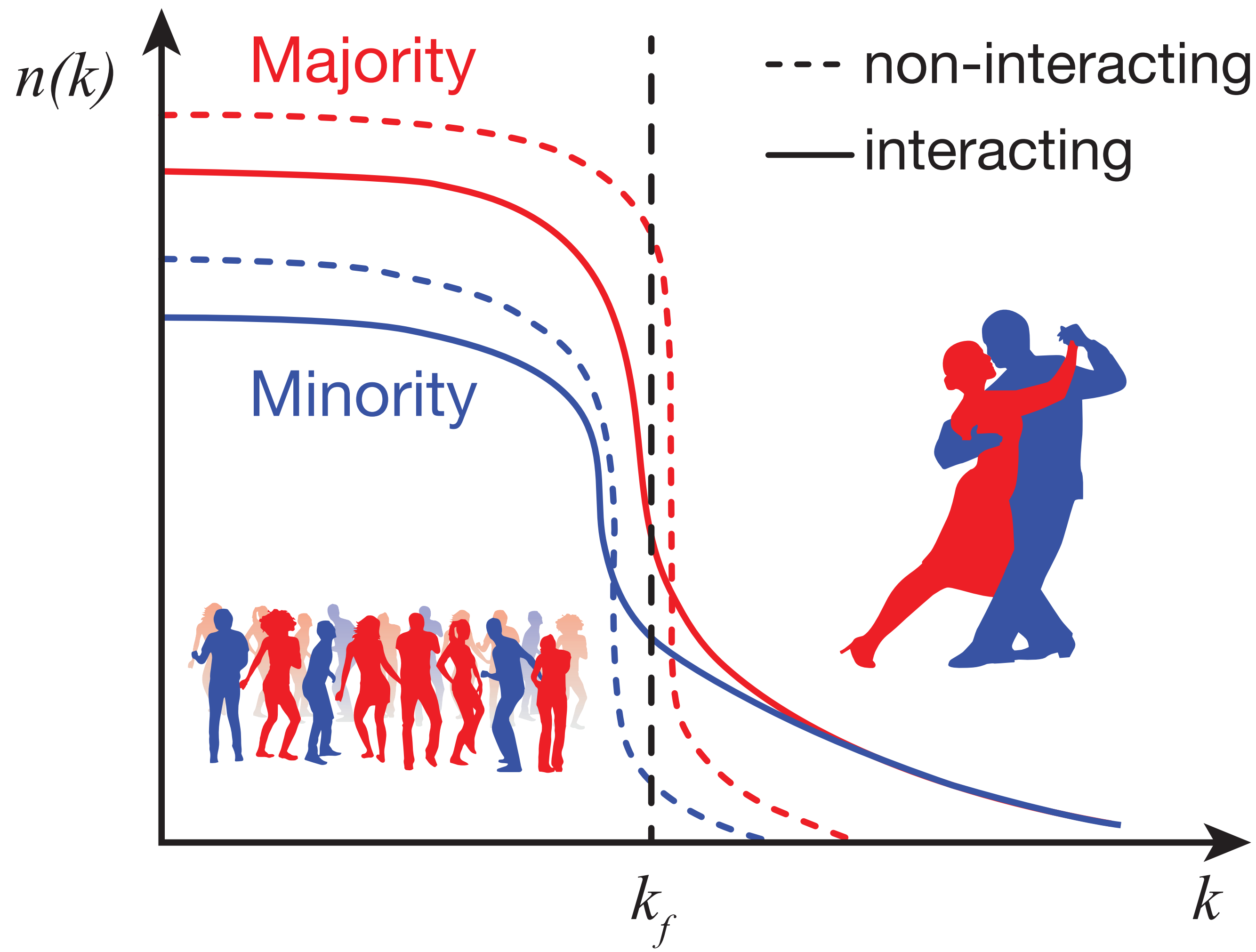
The enhancement is driven by process involving one-pion exchange and the excitation of the Delta degrees of freedom

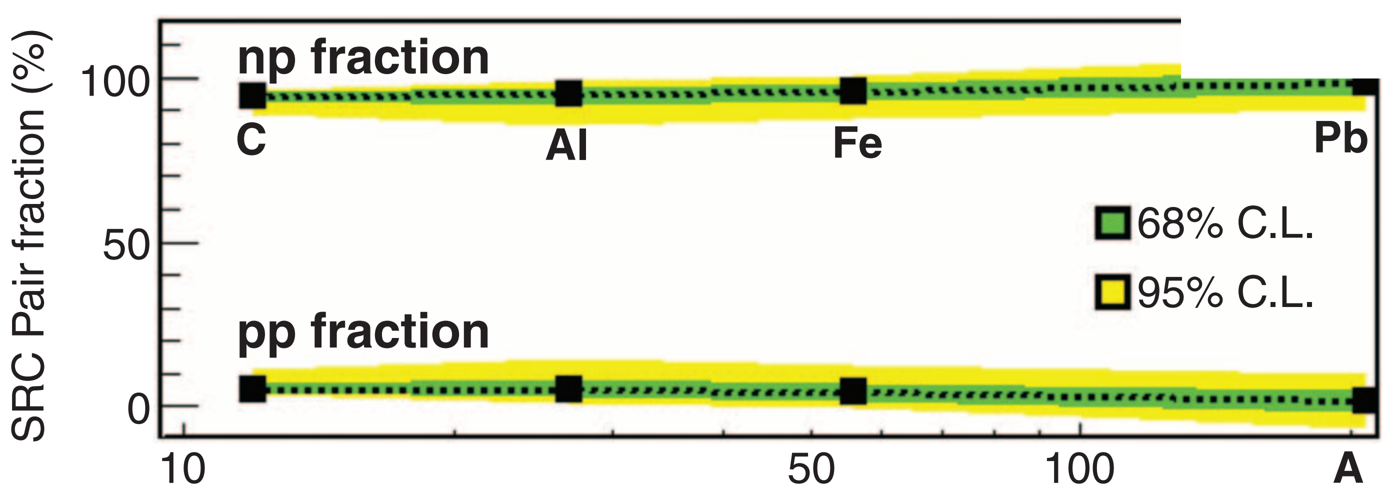


Nuclear correlations

- Nuclear interaction creates short-range correlated pairs of unlike fermions with large relative momentum and pushes fermions from low momenta to high momenta creating a “high-momentum tail.”

- Like in a dance party with a majority of girls, where boy-girl interactions will make the average boy dance more than the average girl





- Even in neutron-rich nuclei, protons have a greater probability than neutrons to have momentum larger than the Fermi momentum.

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