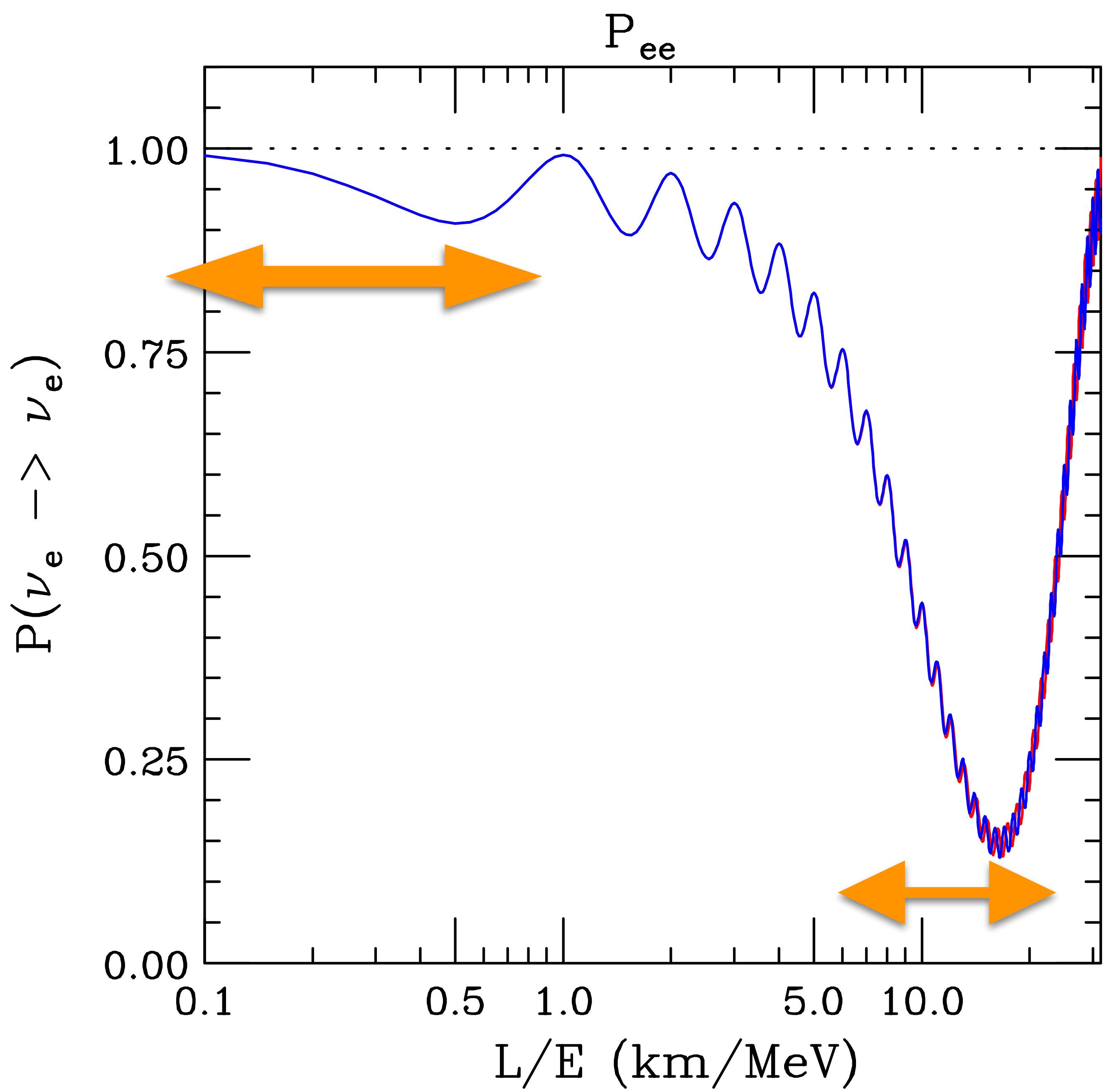
What is Δm^2_{ee} ?

Stephen Parke
Fermilab

SP arXiv:1601.07464

Daya Bay
RENO
D-Chooz



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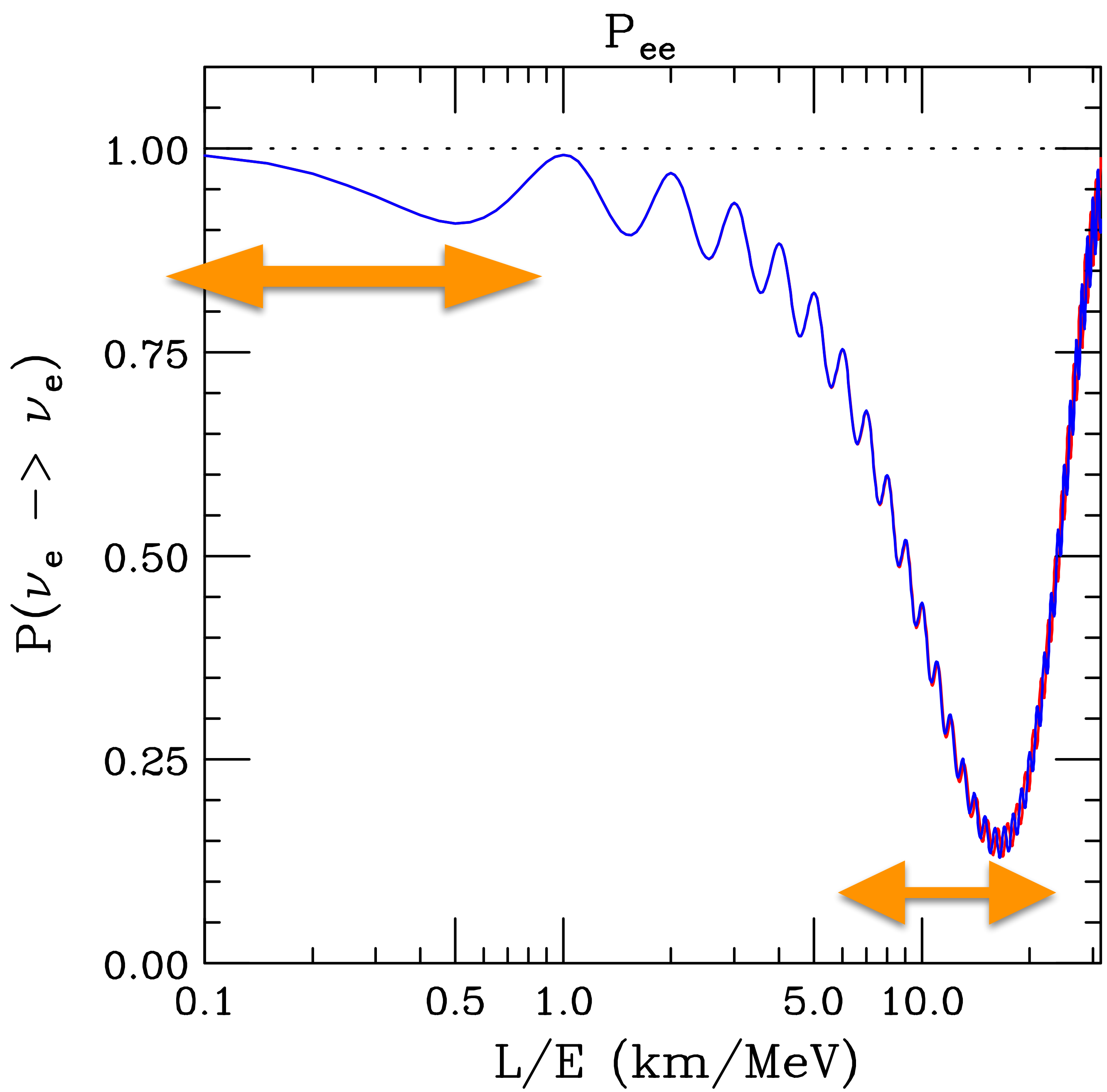


JUNO
RENO 50

Amplitude Modulation & Phase Advancement (NO) / Retardation (IO)

Reactor θ_{13} Experiments

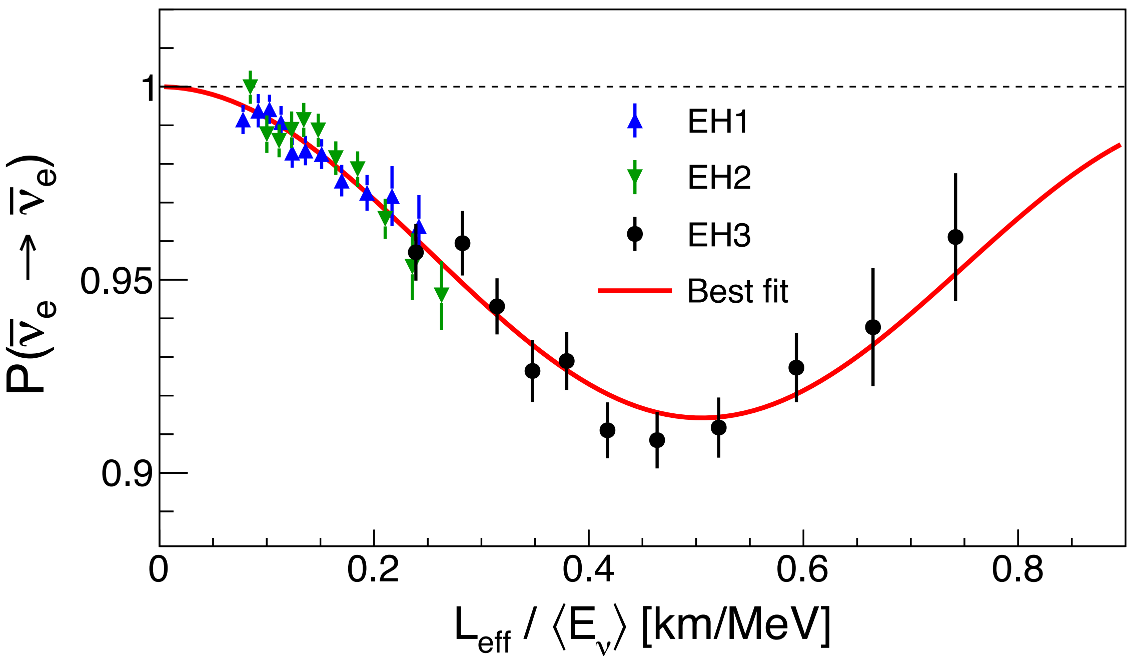
Daya Bay

RENO

Double Chooz

What is Δm^2_{ee} ?

from Daya Bay: arXiv:1505.03456



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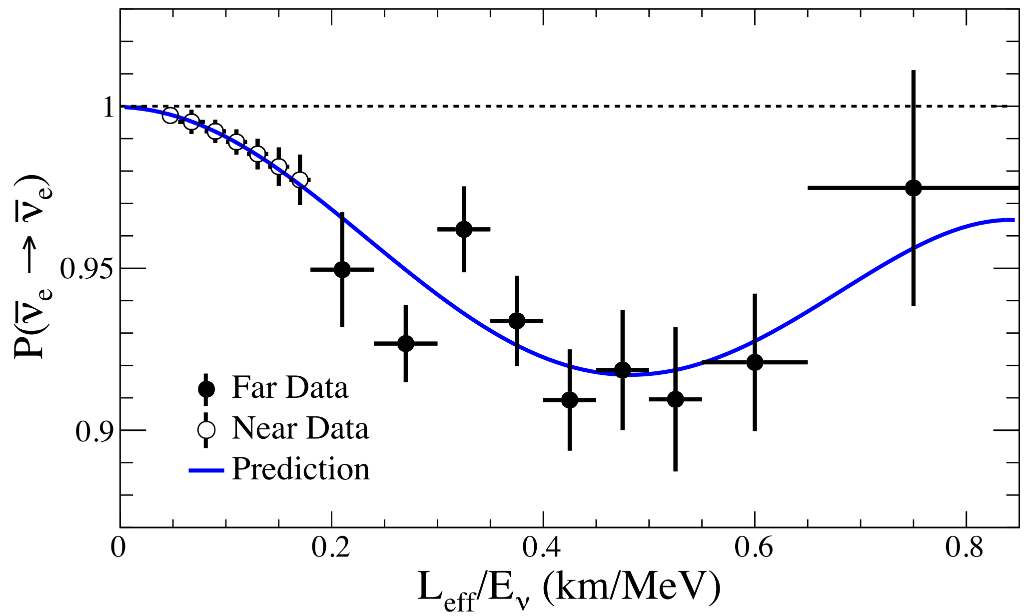

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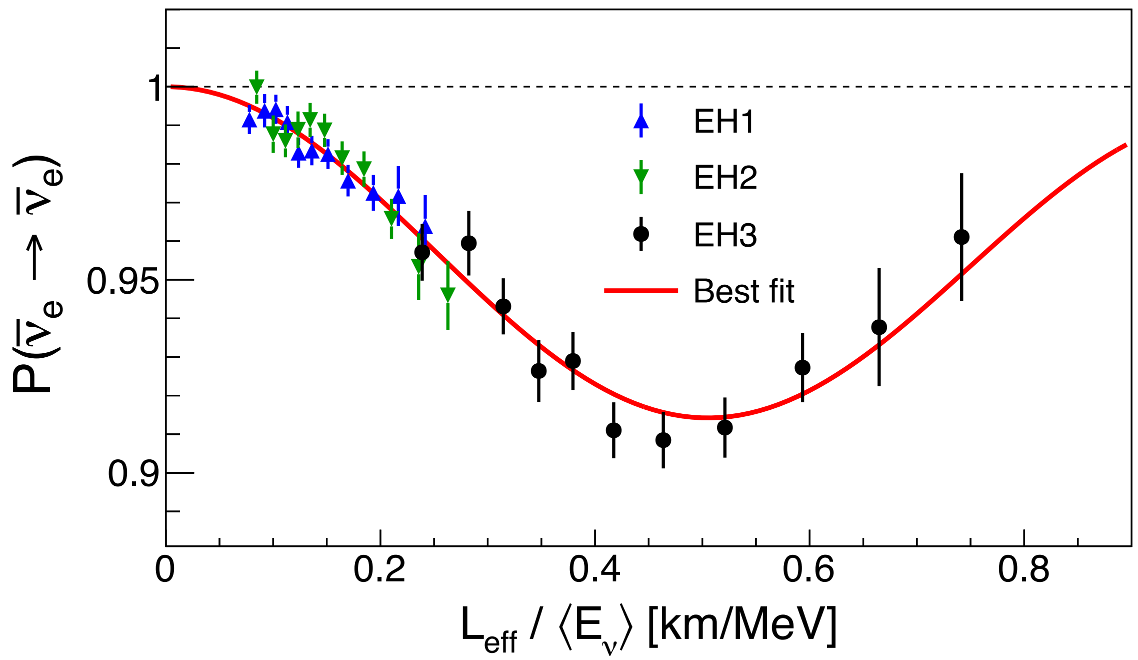

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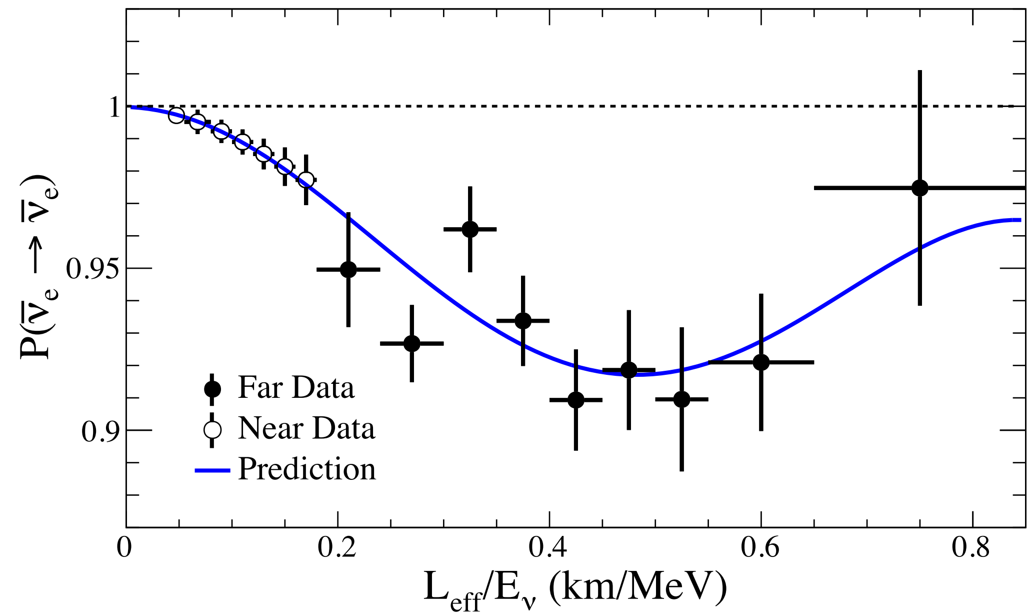

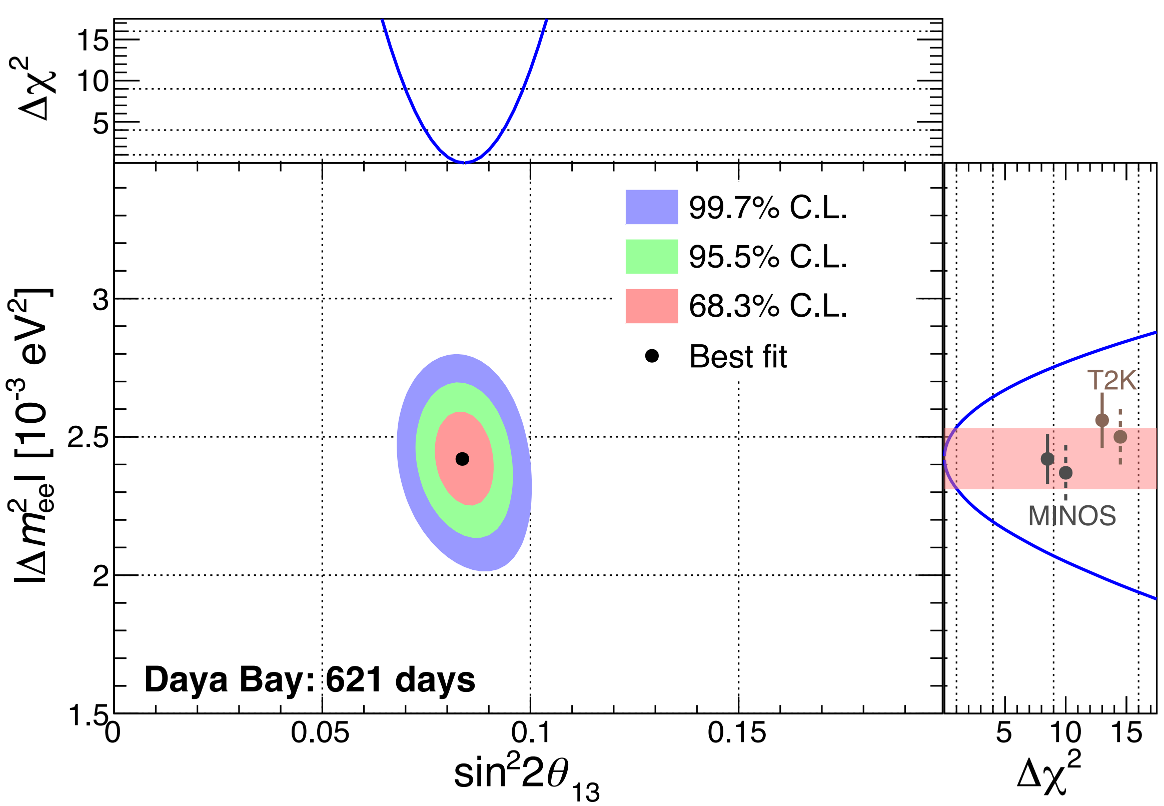

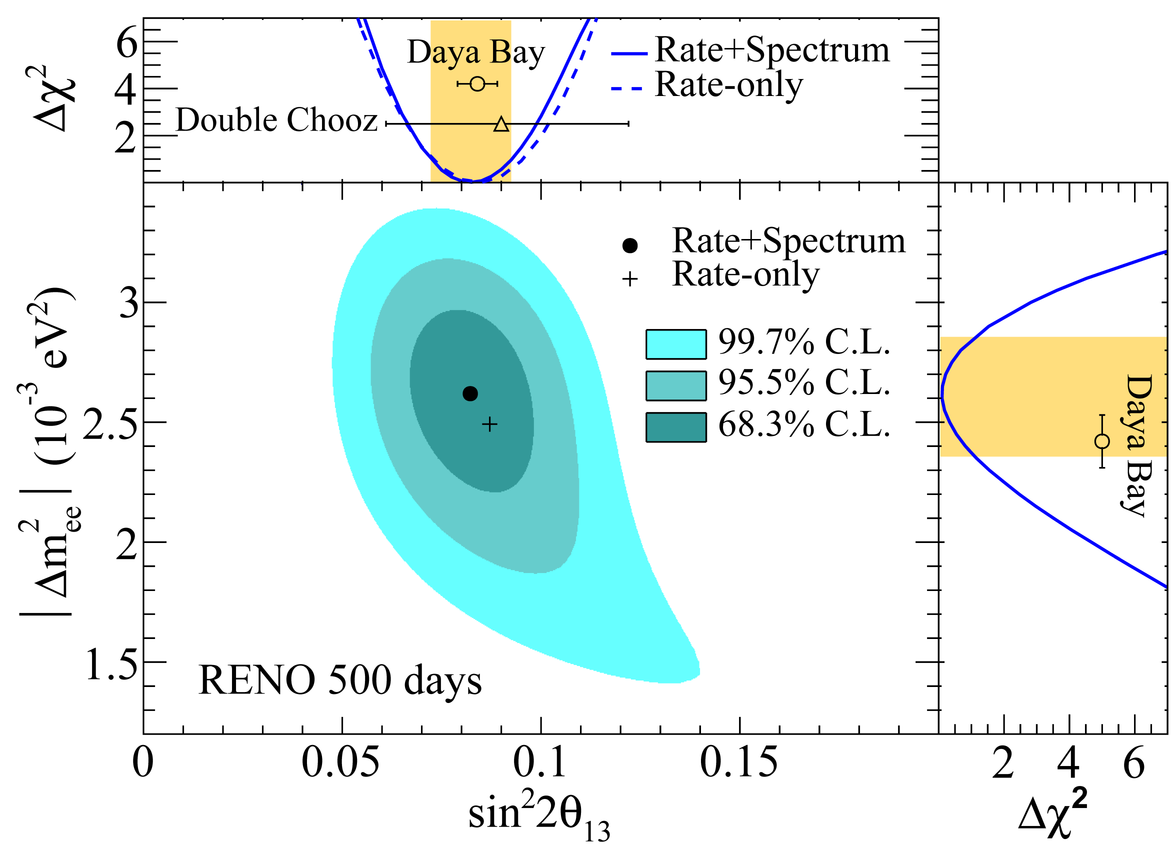

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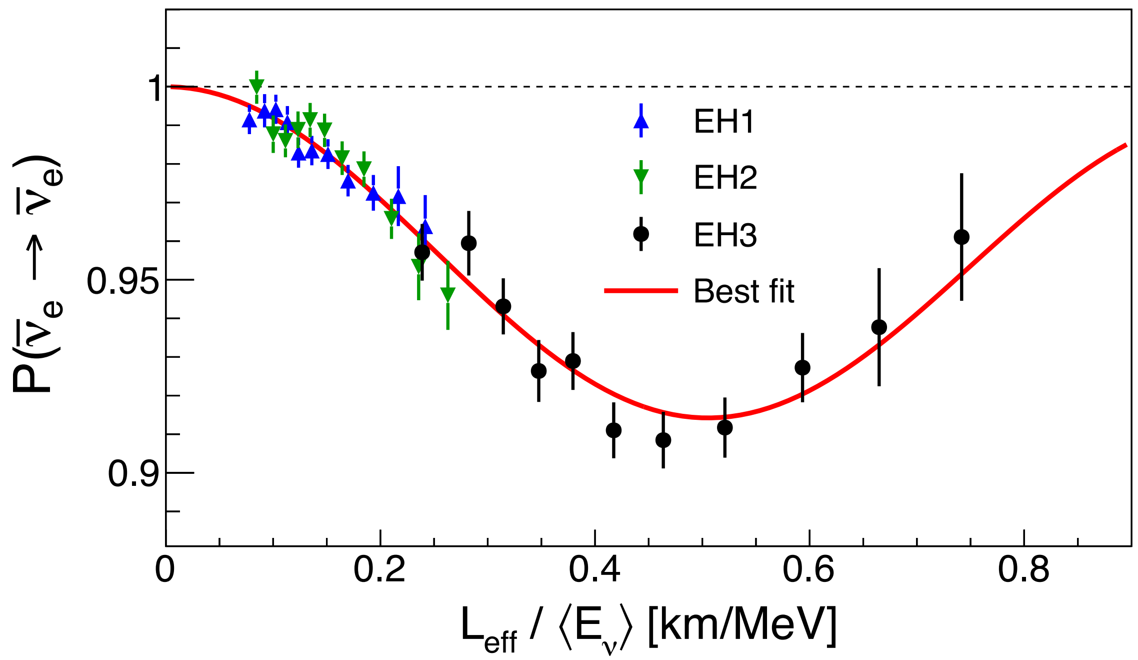

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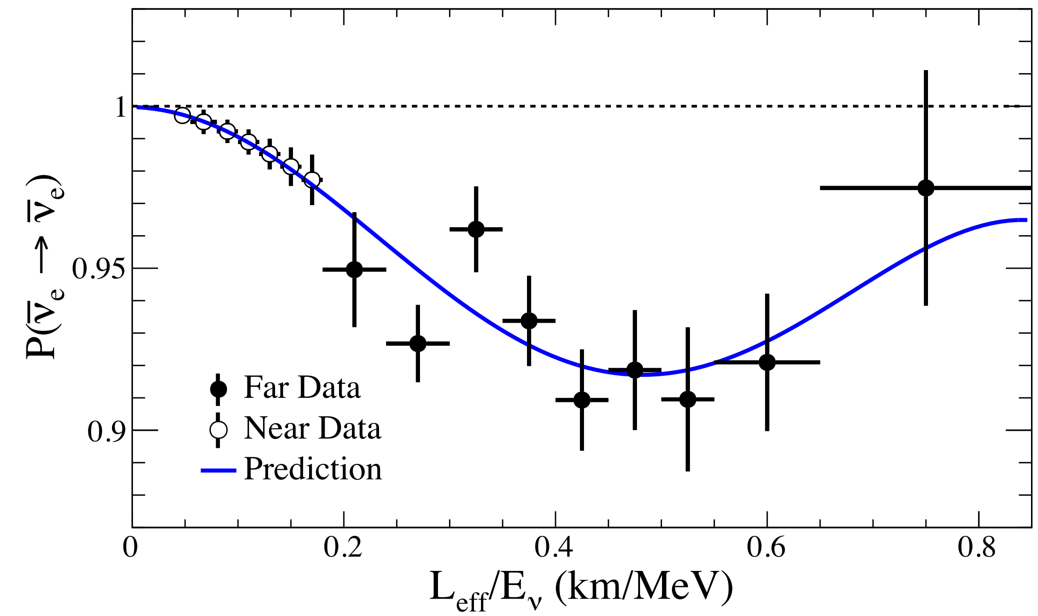

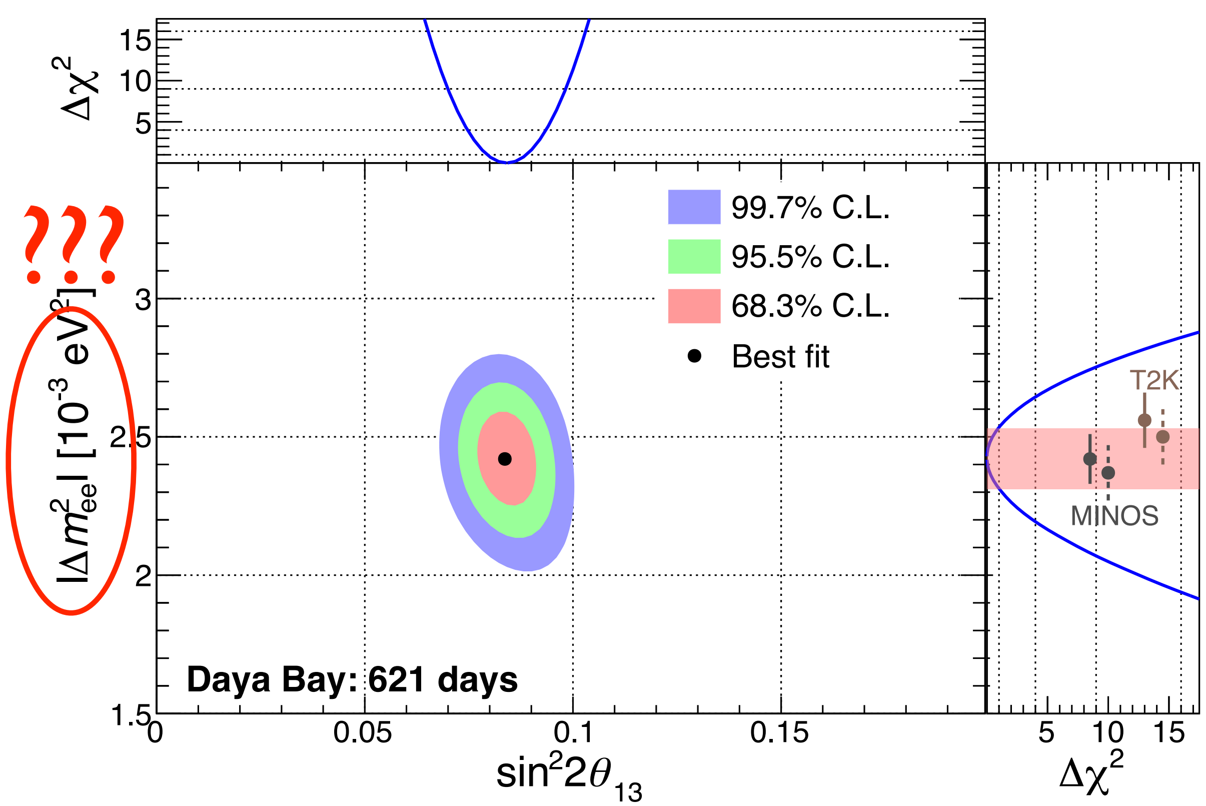

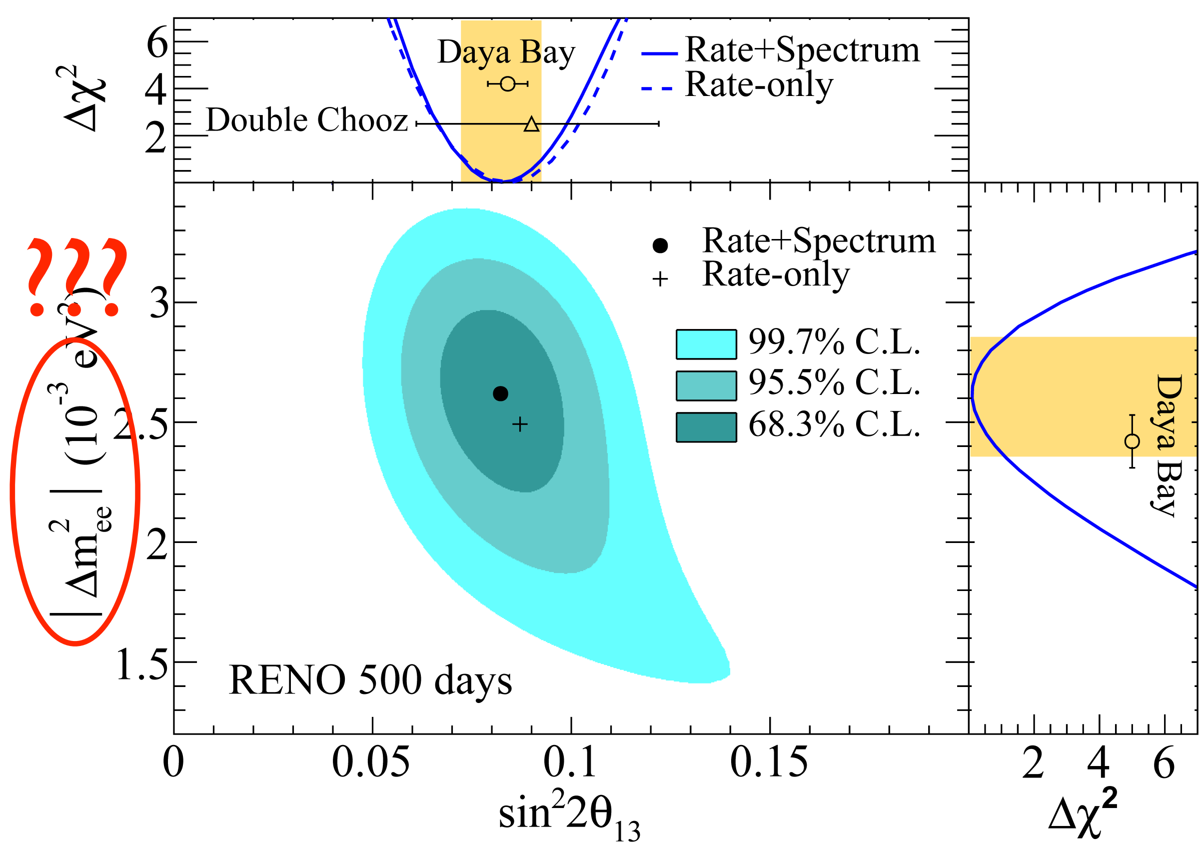

- **Daya Bay**

$$|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$$

4 % measurement !

Analysis of the relative antineutrino rates and energy spectra between detectors gave $\sin^2 2\theta_{13} = 0.084 \pm 0.005$ and $|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$ in the three-neutrino framework.

- **RENO**

$$\Delta m_{ee}^2 = [2.52 \pm 0.19(\mathrm{stat}) \pm 0.17(\mathrm{syst})]\ \mathrm{x}10^{-3}\ \mathrm{eV}^2$$

10 % measurement !

- **JUNO & RENO 50**

expect ~0.5 % measurement !!!

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NO: $|\Delta m^2_{31}| = (1 + 3\%)|\Delta m^2_{32}|$ IO: $|\Delta m^2_{31}| = (1 - 3\%)|\Delta m^2_{32}|$

Δm^2_{ee} is, obviously, some combination of Δm^2_{31} and Δm^2_{32} !

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But What Combination ?

Desirable Features:

- L/E independent
- Simple relationship to Δm^2_{31} and Δm^2_{32}
- Useful for experiments at any L

In vacuum the electron neutrino disappearance is

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$\begin{aligned} P &= 1 - 4|U_{e2}|^2|U_{e1}|^2 \sin^2\Delta_{21} \\ &\quad -4|U_{e3}|^2|U_{e1}|^2 \sin^2\Delta_{31} - 4|U_{e3}|^2|U_{e2}|^2 \sin^2\Delta_{32} \\ &= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \qquad (< 0.01) \\ &\quad - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}) \quad (< 0.1) \end{aligned}$$

(the mass ordering is built into the signs of Δm^2_{3i} and Δm^2_{rr})

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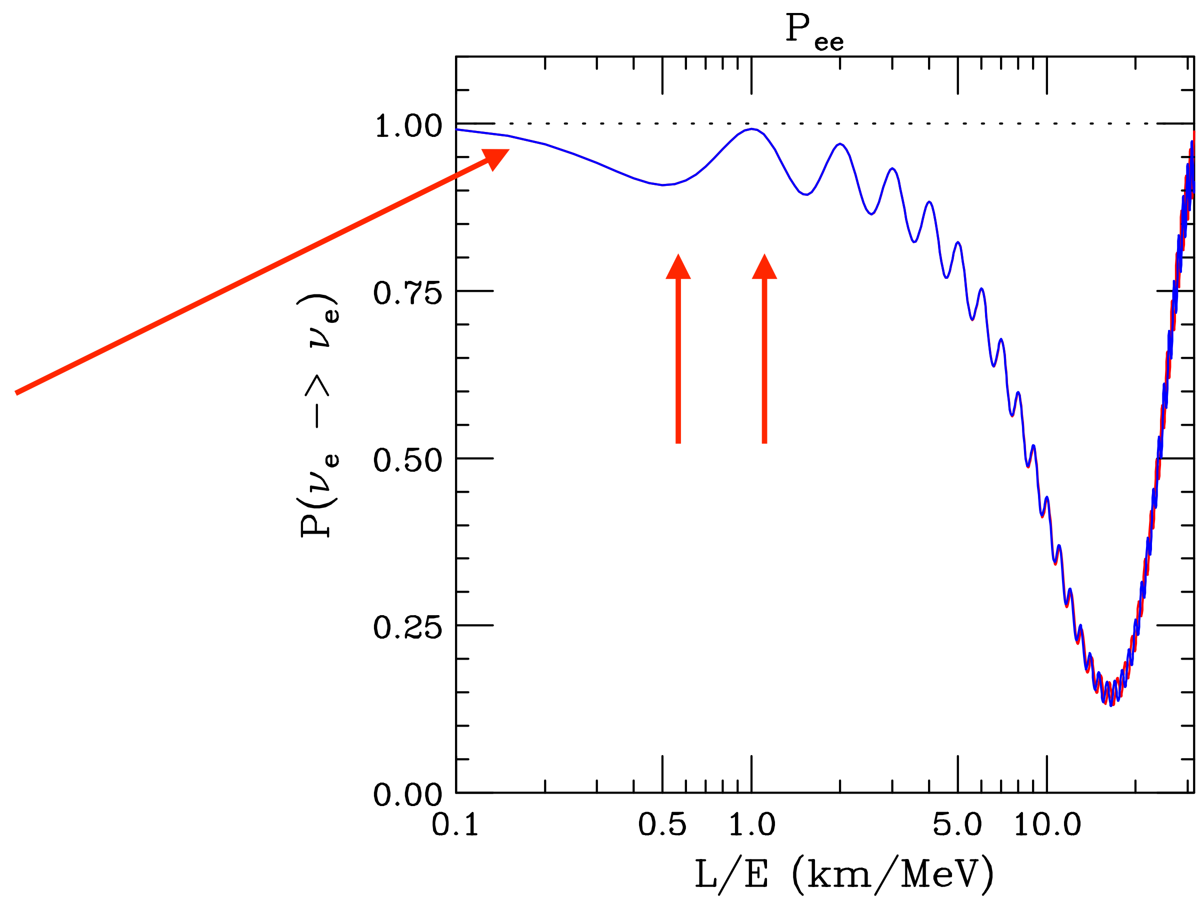
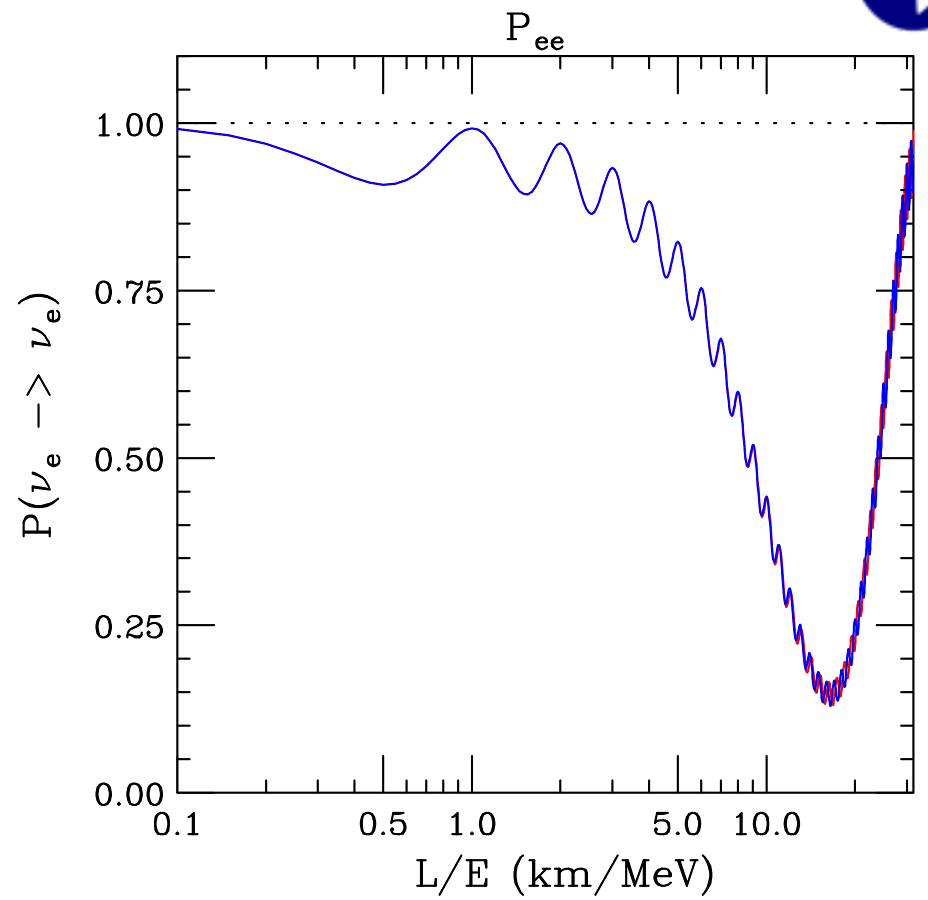
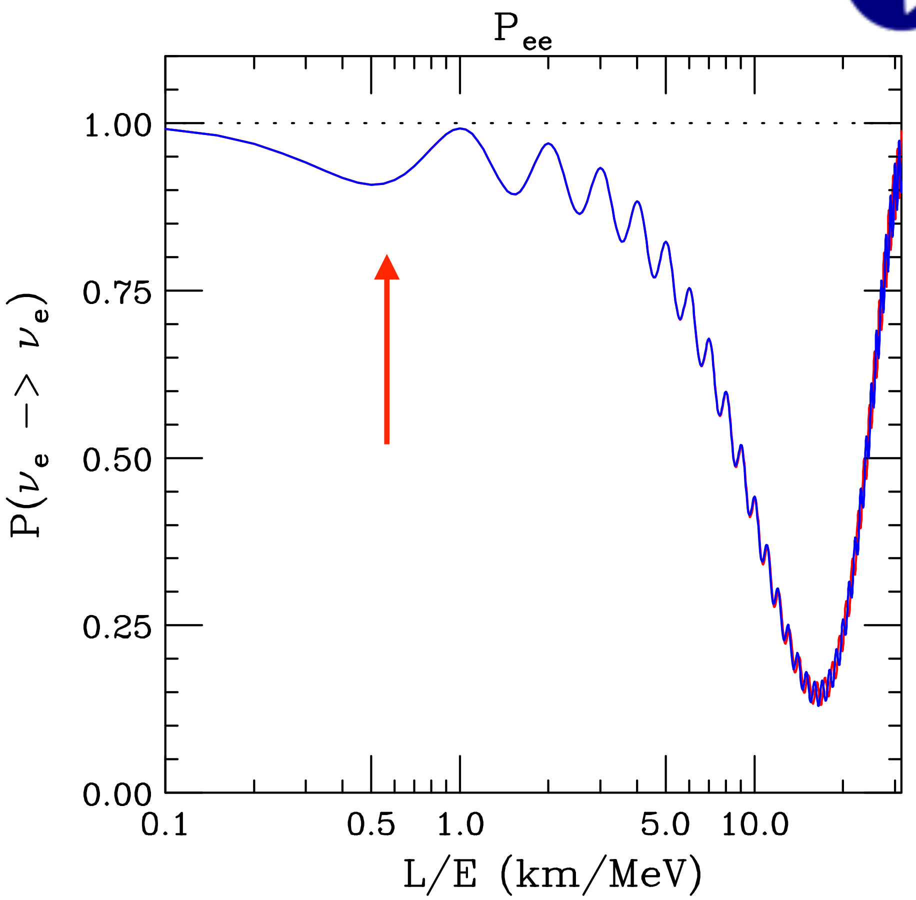




P_{ee}
$P(\nu_e \to \nu_e)$
1.00
0.75
0.50
0.25
0.00
0.1
0.5
1.0
5.0
10.0
L/E (km/MeV)

- Where is the first minima ?

$$\frac{L}{E} \approx \frac{2\pi}{(\ \cos^2\theta_{12}\ \ \Delta m^2_{31} + \sin^2\theta_{12}\ \ \Delta m^2_{32}\ )}$$



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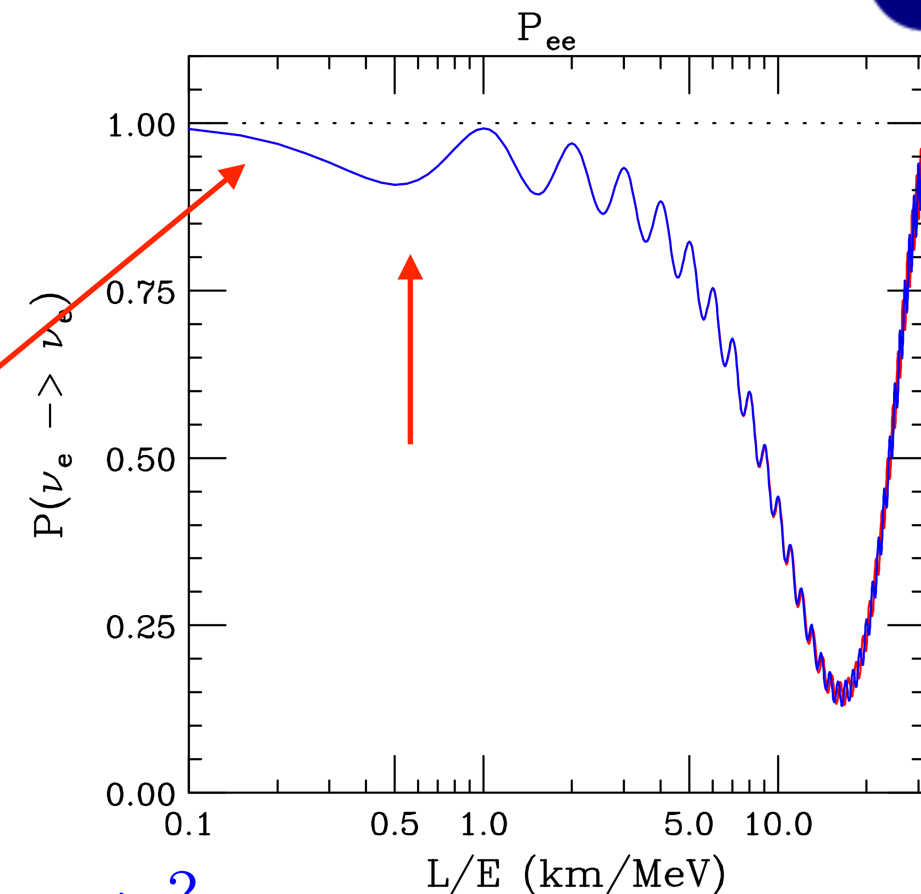

- As $L/E \to 0$

$$P \approx 1 - \sin^2 2\theta_{13} \left(\frac{(\cos^2\theta_{12}\ \Delta m^2_{31} + \sin^2\theta_{12}\ \Delta m^2_{32})\ L}{4E} \right)^2$$

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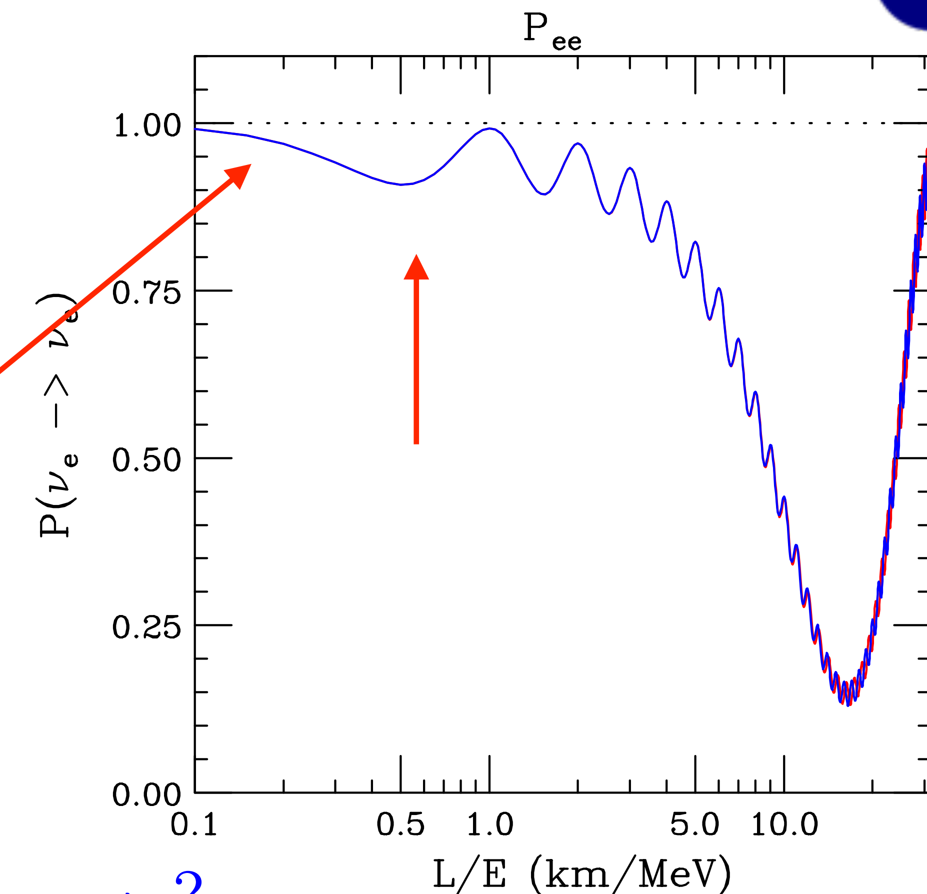

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So how about ?

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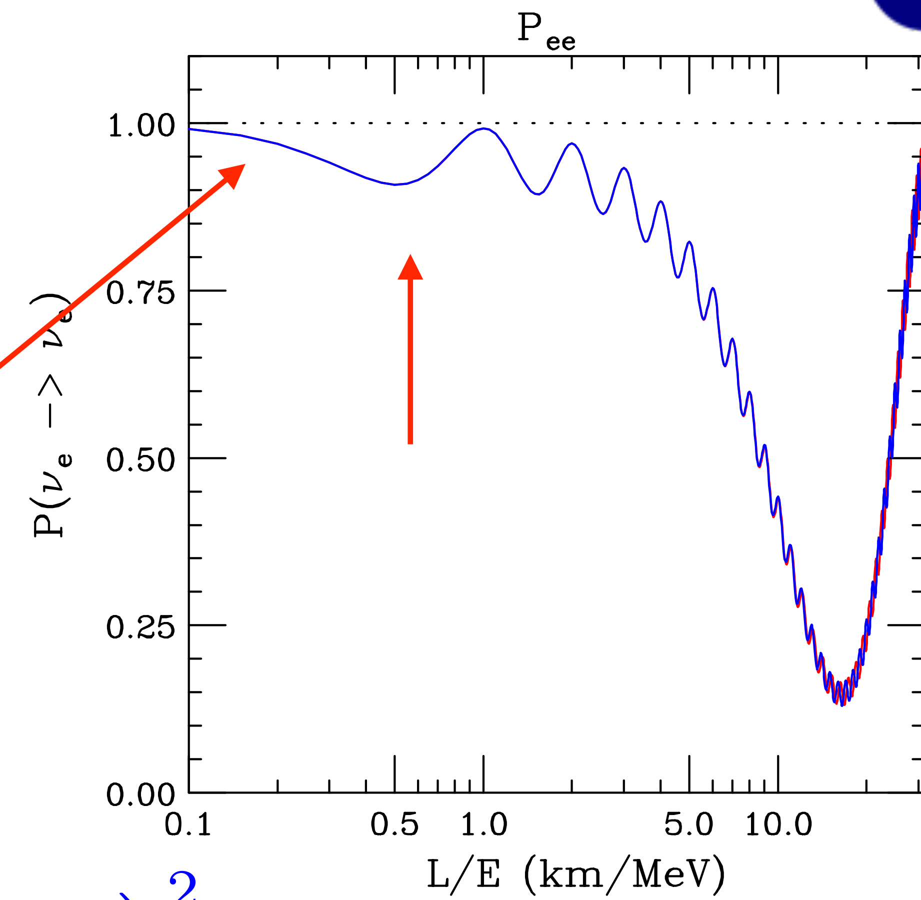

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So how about ?

$$\Delta m^2_{ee} = \cos^2\theta_{12}\ \Delta m^2_{31} + \sin^2\theta_{12}\ \Delta m^2_{32}$$

ν_e average !

$$P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{rr}$$

$$\boldsymbol{\Delta m^2_{rr} = (1-r)\,\Delta m^2_{31} + r\,\Delta m^2_{32}}$$

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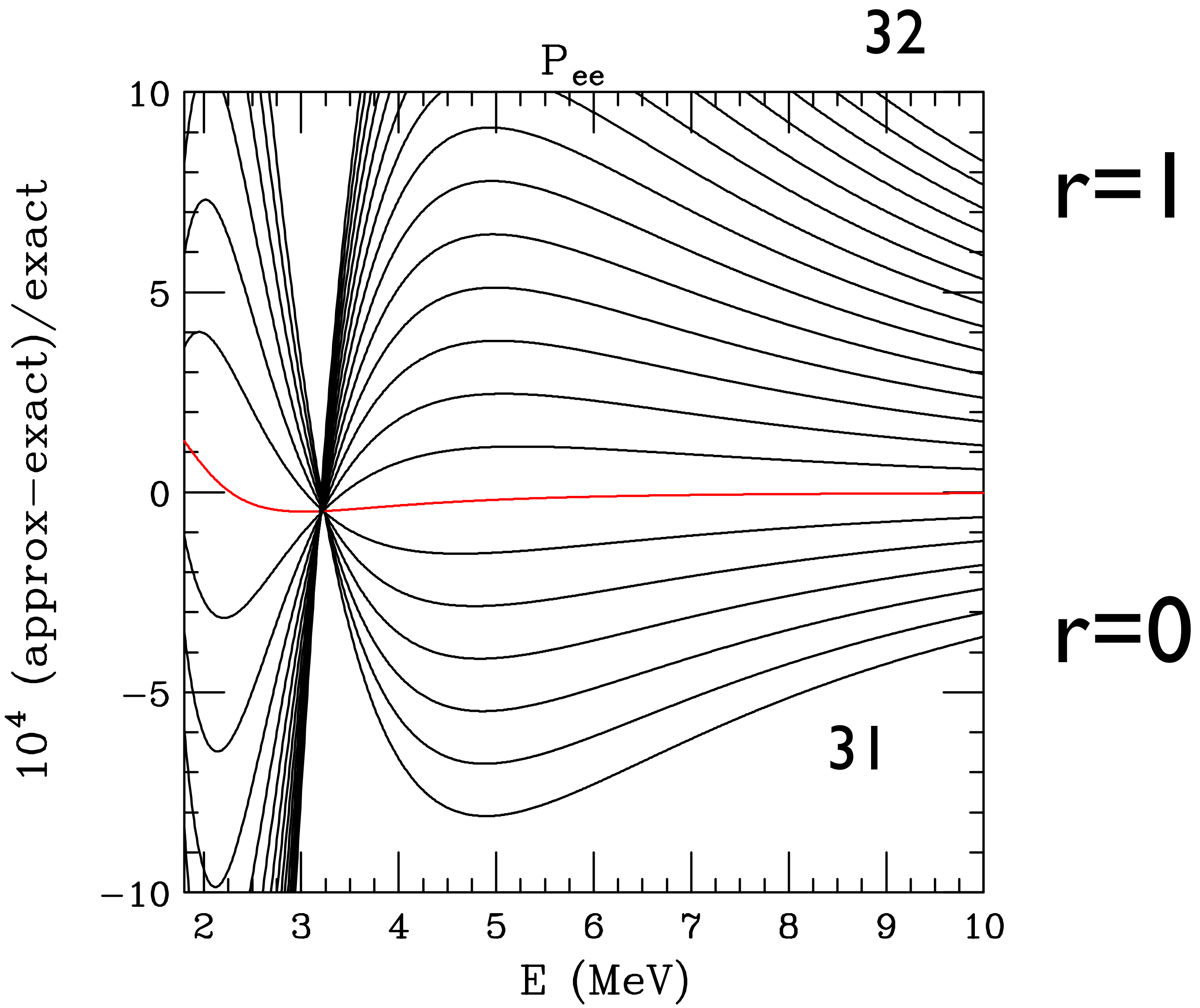

L=1.6 km

r=0, 0.05, 0.1 … 1.0

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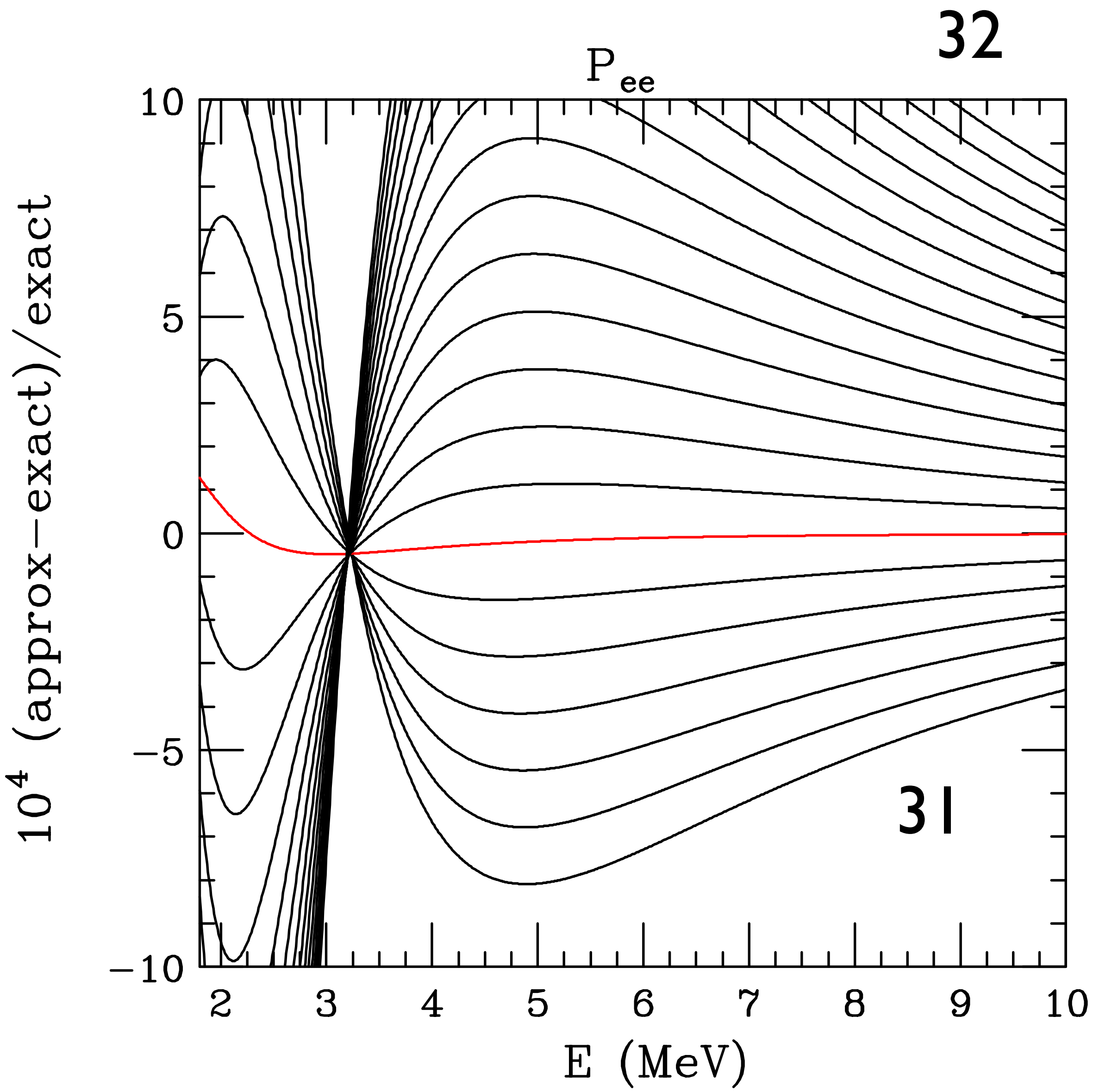


r=1

L=1.6 km

r=0, 0.05, 0.1 …1.0

r=0

$$r = \sin^2\theta_{12}$$

$$\boldsymbol{\Delta m^2_{ee} = \cos^2\theta_{12}\,\Delta m^2_{31} + \sin^2\theta_{12}\,\Delta m^2_{32}}$$

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

Note, the Δ_{21} terms vanishes !

& coeff. of Δ_{21}^2 minimum !

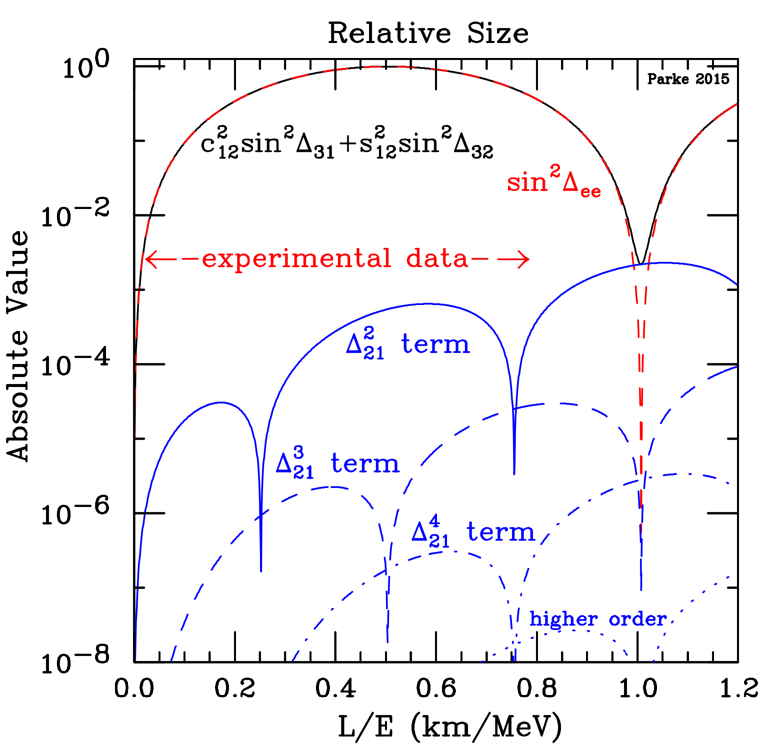
Taylor Series Expansion:

improvement

$\Delta_{31} = \Delta_{ee} + s_{12}^2\ \Delta_{21}$ and $\Delta_{32} = \Delta_{ee} - c_{12}^2\ \Delta_{21}$

$$
\begin{aligned}
c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{31} &= \sin^2 \Delta_{ee} + 0 + s_{12}^2 c_{12}^2 \Delta_{21}^2 \cos(2\Delta_{ee}) \\
&\quad - \frac{1}{6} \cos 2\theta_{12} \sin^2 2\theta_{12}\ \Delta_{21}^3\ \sin(2\Delta_{ee}) + \mathcal{O}(\Delta_{21}^4) \\
&= 1 + \mathcal{O}(10^{-3}) \pm \mathcal{O}(10^{-5}) \quad \text{at OM}
\end{aligned}
$$

Mass Ordering effects !



H. Nunokawa, S. J. Parke and R. Zukanovich Funchal,
"Another possible way to determine the neutrino mass hierarchy,"
Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

What Do the Experiments Do ?

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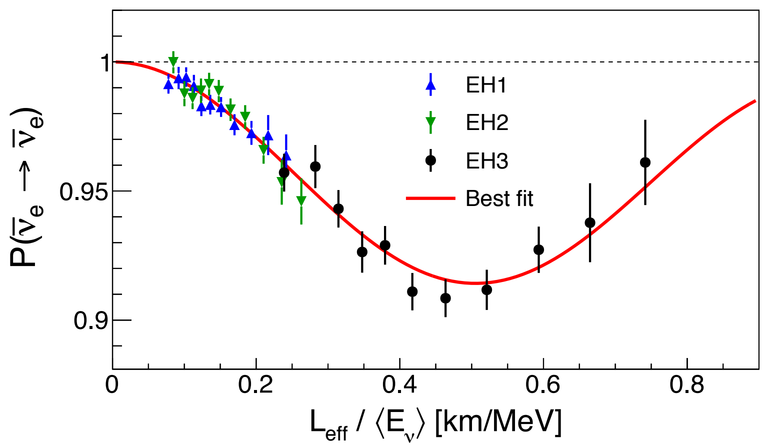

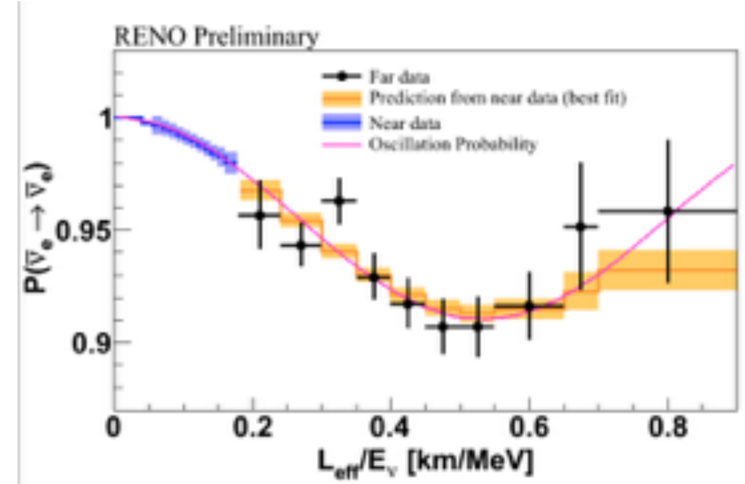

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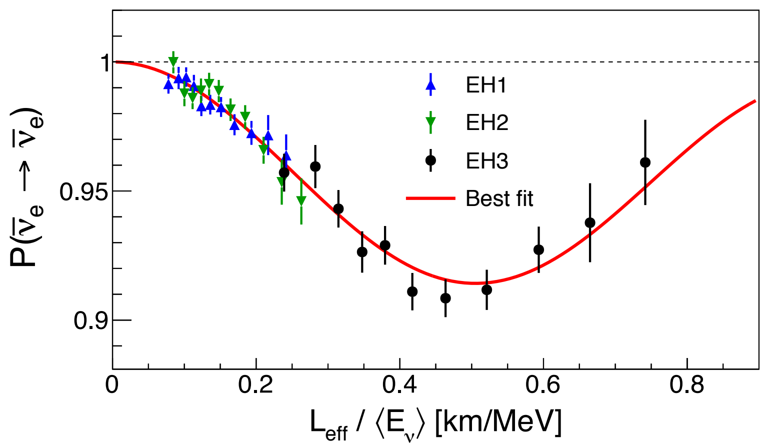

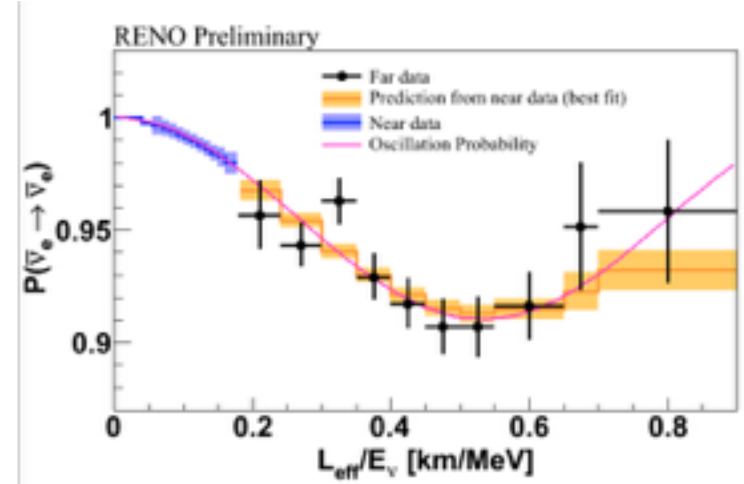

Daya Bay and RENO fit their L/E data to:

- $P_{ee} \approx 1 - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$

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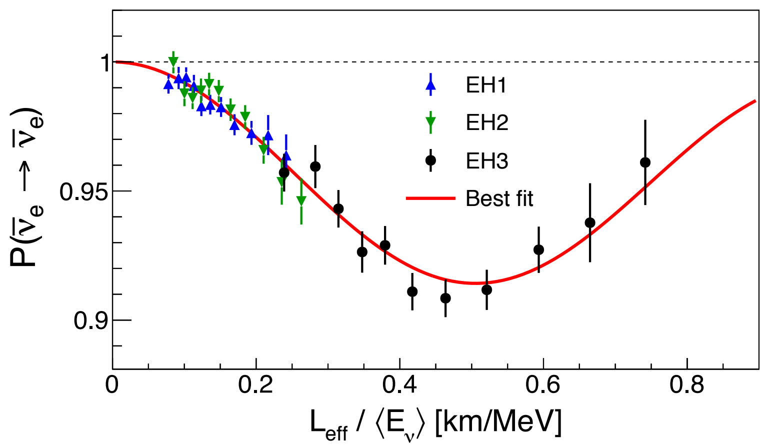

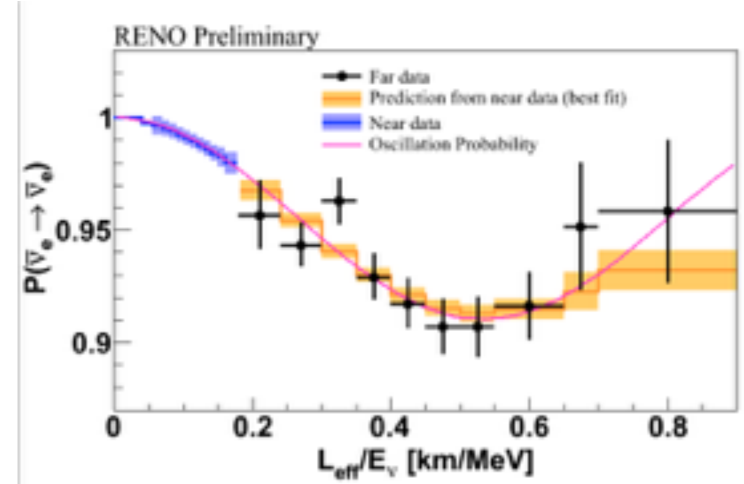

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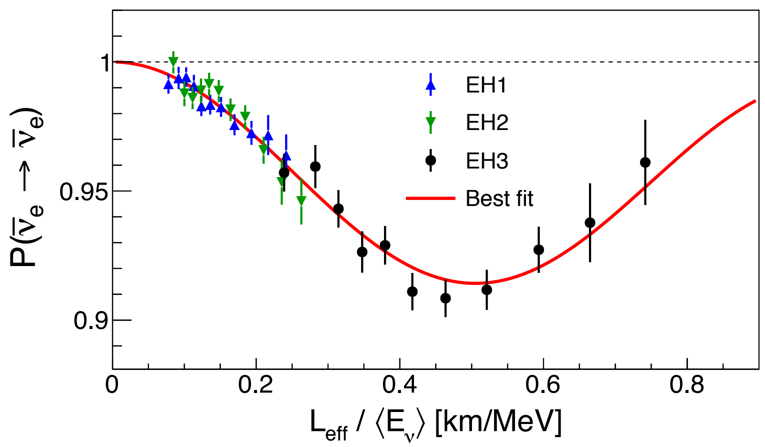

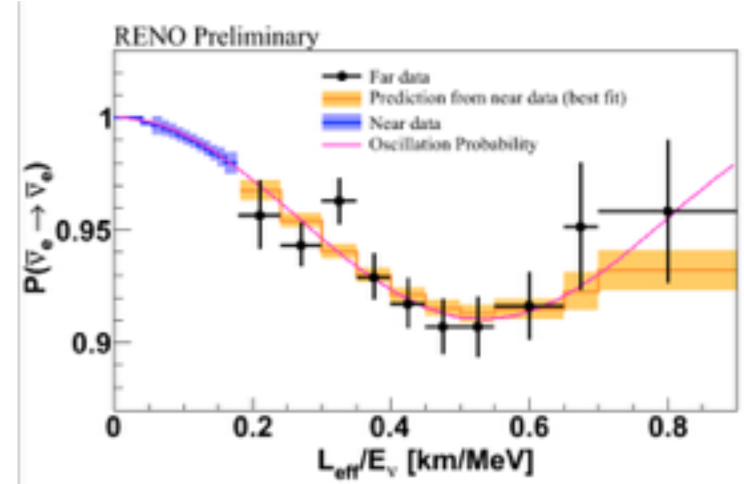

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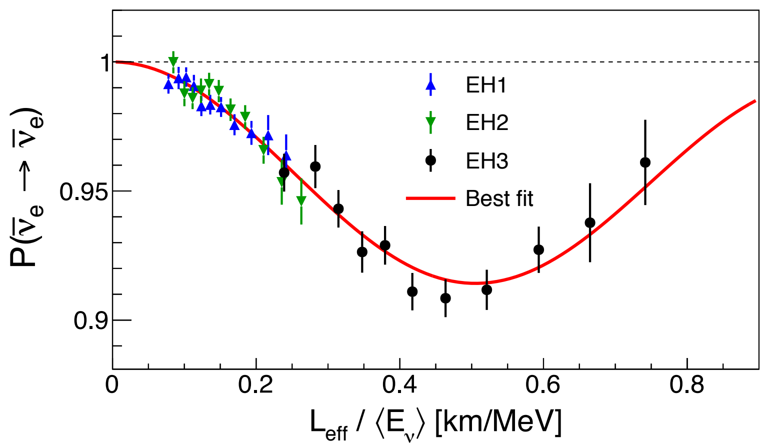

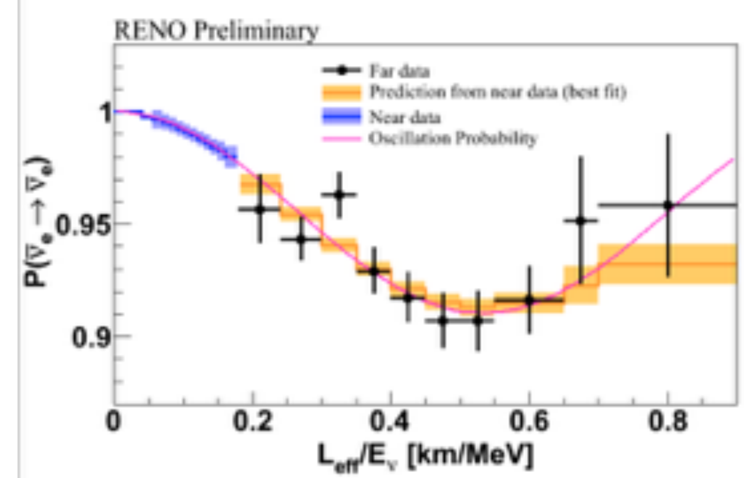

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- θ_{13} and $|\Delta m^2_{ee}|$ are only weakly dependent on solar parameters and are INDEPENDENT of mass ordering.
- Trivial to calculate $|\Delta m^2_{31}| = |\Delta m^2_{ee}| \pm \sin^2\theta_{12}\Delta m^2_{21}$ and $|\Delta m^2_{32}| = |\Delta m^2_{ee}| \mp \cos^2\theta_{12}\Delta m^2_{21}$ using mass ordering and solar parameters. But $|\Delta m^2_{31}|$ and $|\Delta m^2_{32}|$ are more dependent on solar parameters than $|\Delta m^2_{ee}|$!

How Does Daya Bay Define Δm^2_{EE} ?

arXiv:1310.6732

ν_j and ν_i. Since $\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$ [1], the short-distance ($\sim$km) reactor $\overline{\nu}_e$ oscillation is due primarily to the Δ_{3i} terms and naturally leads to the definition of the effective mass-squared difference $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$ [11].

1505.03456v1

[8] $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$, where $\Delta_{ji} \equiv 1.267 \Delta m^2_{ji}(\mathrm{eV}^2)[L(\mathrm{m})/E(\mathrm{MeV})]$, and Δm^2_{ji} is the difference between the mass-squares of the mass eigenstates ν_j and ν_i.

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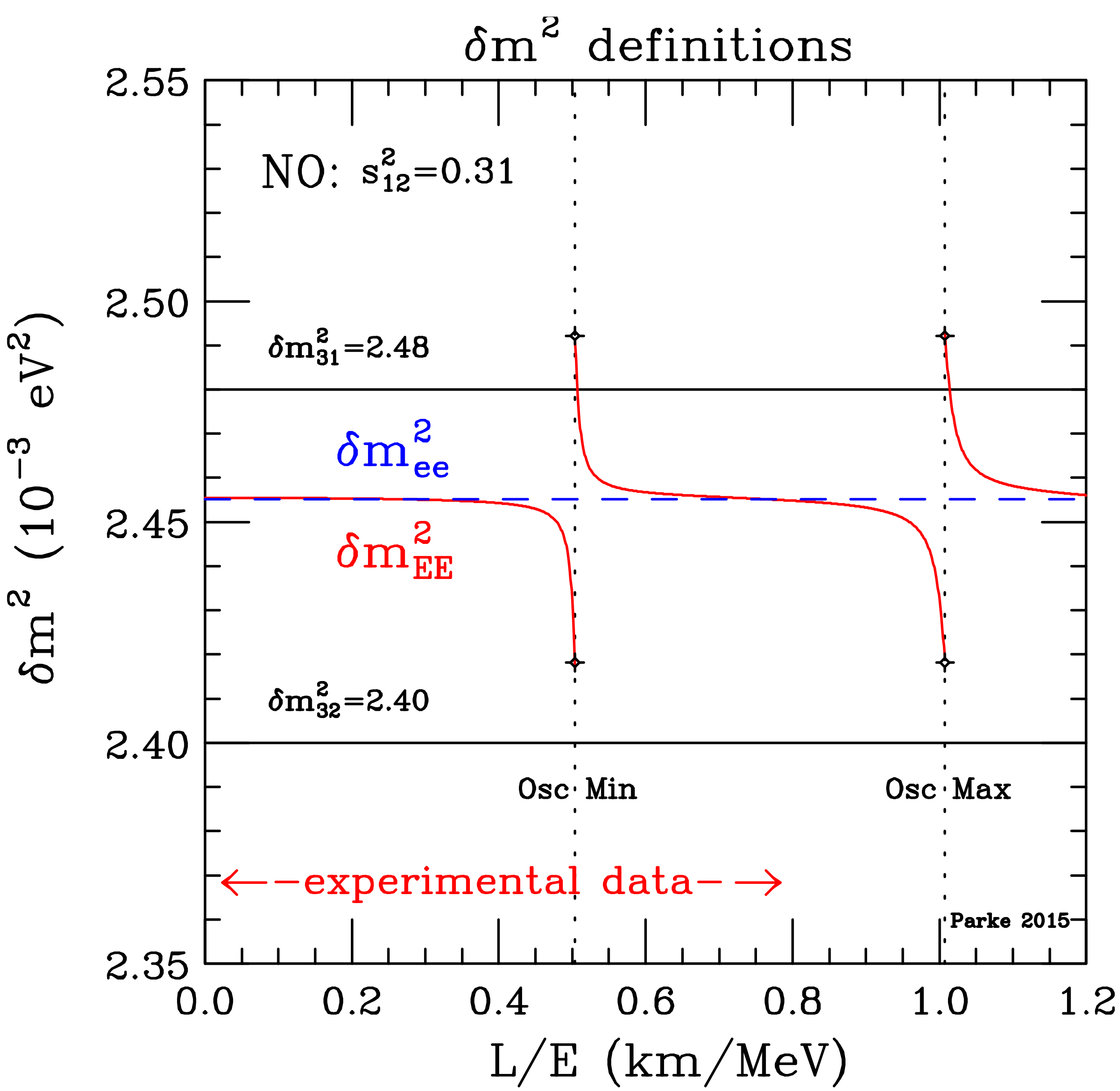

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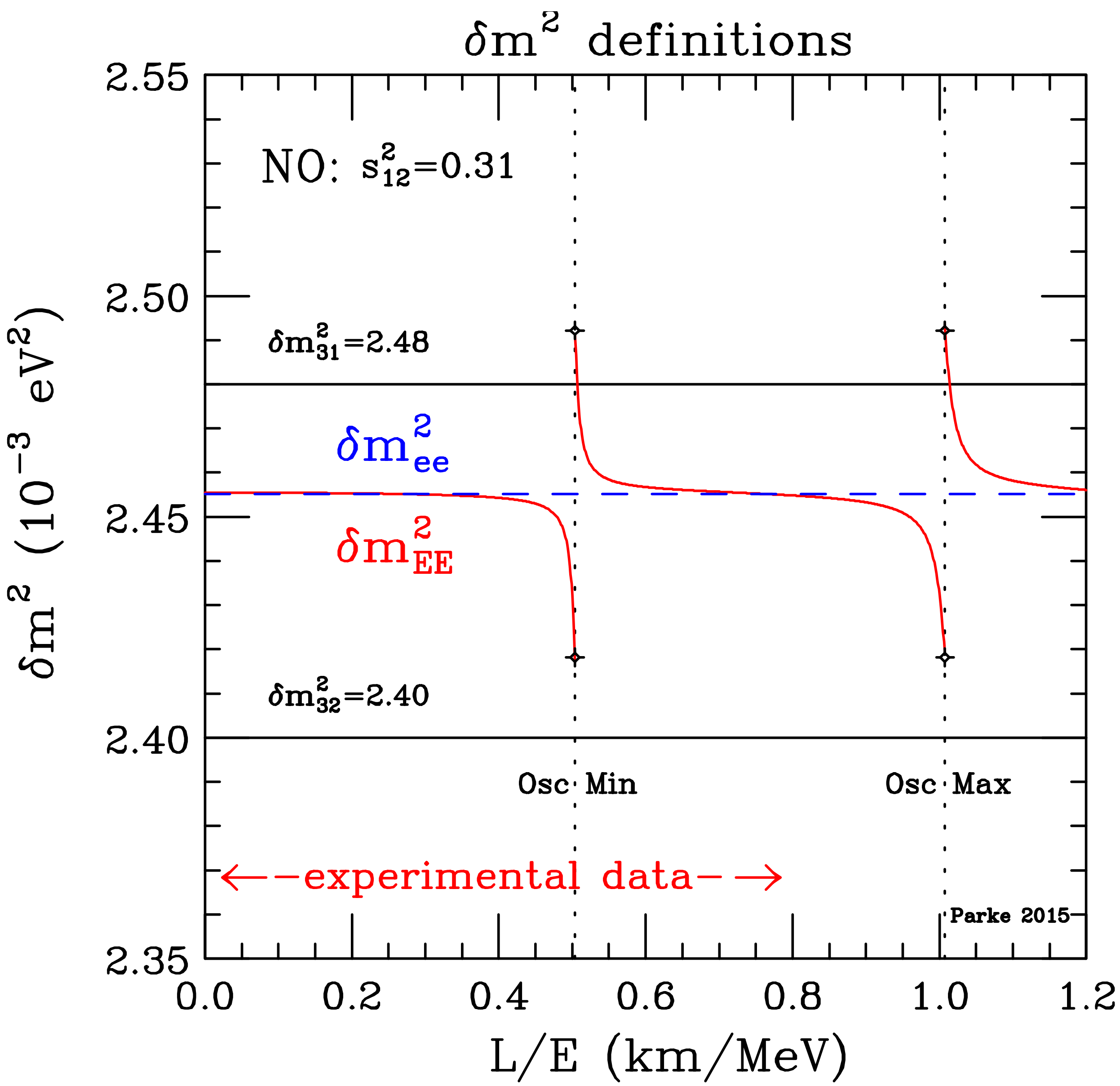

Maladies:

- L/E dependent
- no simple physical meaning !
- Discontinuous at Osc. Max./Min. (L/E ≈ 0.5, 1.0, .. km/MeV) 3% jump

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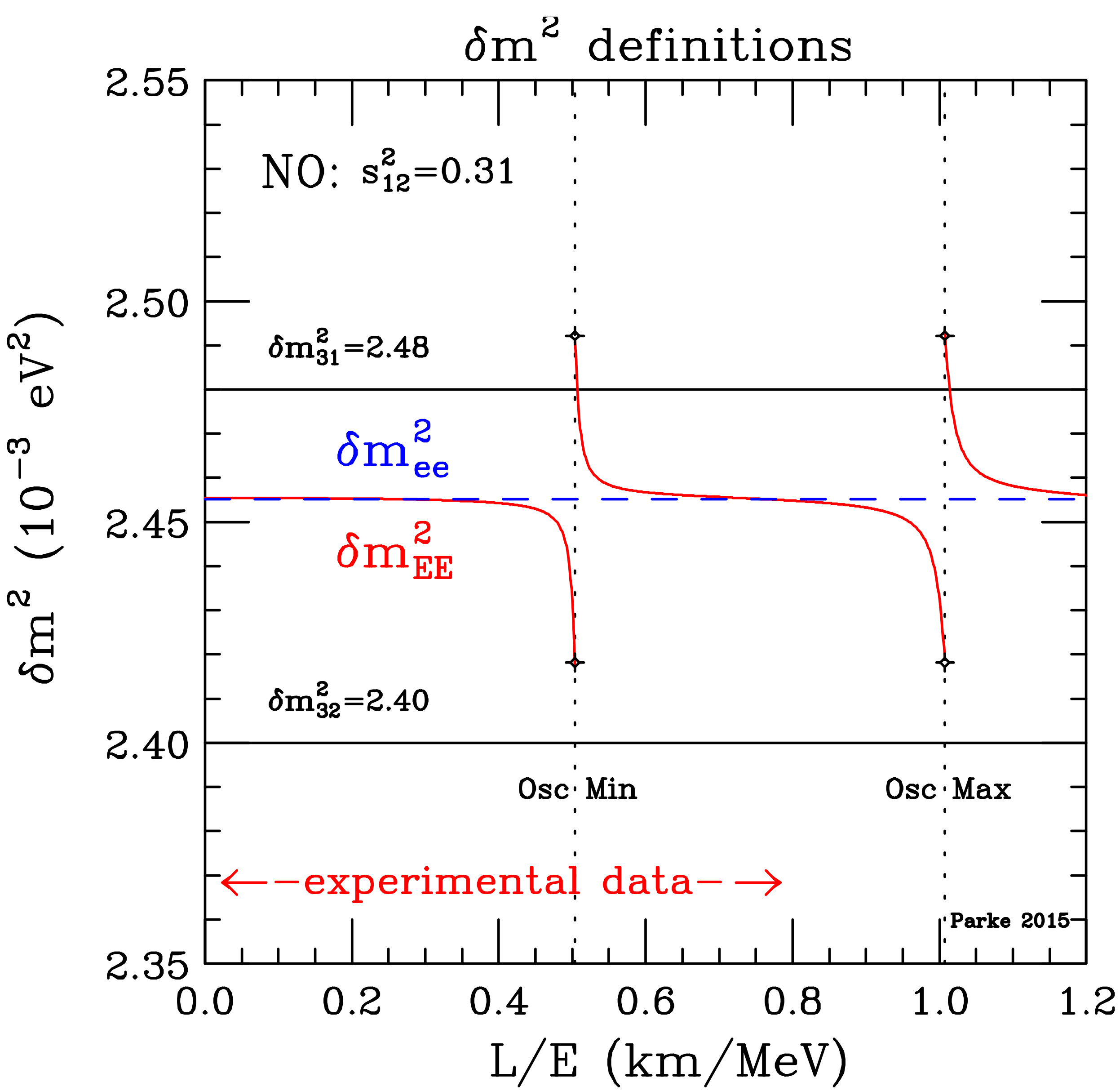

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Why???

RHS (1) never gets exactly to 1,
or back to 0
whereas LHS does !

eg $\sin^2(\frac{\pi}{2} \mp \epsilon) = 1 - \epsilon^2 + \mathcal{O}(\epsilon^4)$
with $\epsilon = s_{12}c_{12}\Delta_{21}$

After I pointing this out Daya Bay changed it's definition of Delta m^2_ee

[8] Δm^2_{ee} is an effective mass splitting that can be obtained by replacing $\cos^2\theta_{12}\sin^2\Delta_{31}+\sin^2\theta_{12}\sin^2\Delta_{32}$ with $\sin^2\Delta_{ee}$, where $\Delta_{ji}\equiv 1.267\Delta m^2_{ji}(\mathrm{eV}^2)[L(\ \mathrm{m})/E(\mathrm{MeV})]$, and Δm^2_{ji} is the difference between the mass-squares of the mass eigenstates ν_j and ν_i. To estimate the values of Δm^2_{31} and Δm^2_{32} from the measured value of Δm^2_{ee}, See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.115.111802.

Supplemental material: Why Δm^2_{ee} is used by Daya Bay

(The Daya Bay Collaboration)

(Dated: July 27, 2015)

This note describes the advantages of reporting the Daya Bay measurement of electron antineutrino disappearance in terms of an effective mass-squared difference Δm^2_{ee}, which is independent of the unknown ordering of neutrino masses and future improvements in our knowledge of the solar oscillation parameters.

INTRODUCTION

In the three-flavor framework, the survival probability of electron antineutrino is given by

$$\begin{aligned} &P(\overline{\nu}_e \to \overline{\nu}_e) \\ &\quad = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ &\qquad - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}), \quad (1) \end{aligned}$$

where $\Delta_x = \Delta m^2_x \frac{L}{4E}$. The three mass-squared differences are subject to the constraint $|\Delta m^2_{31}| = |\Delta m^2_{32}| \pm |\Delta m^2_{21}|$ where "+"("−") is for the normal(inverted) mass ordering (or hierarchy). Therefore, determination of Δm^2_{32} (or Δm^2_{31}) depends on knowledge of the mass ordering and solar oscillation parameters.

The Daya Bay experiment reports a precise measurement of the effective mass splitting Δm^2_{ee}, which is independent of our knowledge of the ordering and solar parameters. In this approach, we approximate the survival probability using

$$\begin{aligned} P(\overline{\nu}_e \to \overline{\nu}_e) \simeq & 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ & - \sin^2 2\theta_{13}\sin^2\Delta_{ee}. \qquad (2) \end{aligned}$$

Despite the advantage of using Δm^2_{ee} for the measurement, it has the disadvantage of not being a fundamental parameter. Therefore, we must determine a relation between Δm^2_{ee} and Δm^2_{32} given knowledge of the mass ordering and solar oscillation parameters.

In the following sections, we are going to address the following two questions:

- Is Eq. 2 good enough at the current experimental precision?
- How can we estimate the value of Δm^2_{32} once the value of Δm^2_{ee} is obtained?

MATHEMATICAL DERIVATION

Using the relation $|\Delta m^2_{31}| = |\Delta m^2_{32}| \pm |\Delta m^2_{21}|$, Eq. 1 can be written as,

$$\begin{aligned} P(\overline{\nu}_e \to \overline{\nu}_e) = & 1 - 2s^2_{13}c^2_{13} \\ & + 2s^2_{13}c^2_{13}\sqrt{1 - 4s^2_{12}c^2_{12}\sin^2\Delta_{21}}\cos(2\Delta_{32} \pm \phi) \\ & - 4c^4_{13}s^2_{12}c^2_{12}\sin^2\Delta_{21}, \qquad (3) \end{aligned}$$

where $s_x = \sin\theta_x$, $c_x = \cos\theta_x$, and $\phi = \arctan\left(\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \tan^2\theta_{12}}\right)$. The last term of the above formula is the so-called "solar term" that governs the reactor antineutrino oscillation at O(100) km. For the L/E range covered by Daya Bay, $4s^2_{12}c^2_{12}\sin^2\Delta_{21} \ll 1$. Thus, Eq. 3 can be approximated as,

$$\begin{aligned} & P(\overline{\nu}_e \to \overline{\nu}_e) \\ & \simeq 1 - 4s^2_{13}c^2_{13}\left[\frac{1 - \cos(2\Delta_{32} \pm \phi)}{2}\right] - (solar\ term) \\ & = 1 - \sin^2 2\theta_{13}\sin^2(\Delta_{32} \pm \phi/2) - (solar\ term). \qquad (4) \end{aligned}$$

By comparing Eq. 4 with Eq. 2, we obtain the expression relating Δm^2_{ee} to Δm^2_{32} (or Δm^2_{31})

$$\begin{aligned} |\Delta m^2_{ee}| &= |\Delta m^2_{32}| \pm \Delta m^2_\phi/2 \qquad (5) \\ &= |\Delta m^2_{31}| \mp (|\Delta m^2_{21}| - \Delta m^2_\phi/2), \qquad (6) \end{aligned}$$

where $\Delta m^2_\phi = \phi \times \frac{4E}{L}$.

NUMERICAL EVALUATION

By definition, Δm^2_ϕ is a function of L/E. Using the current values of $\Delta m^2_{21} = 7.50 \times 10^{-5}$ eV^2 and $\sin^2 2\theta_{12} = 0.857$ [1], Fig. 1 shows the value of $\Delta m^2_\phi/2$ as a function of energy for L = 1.6 km. We find that $\Delta m^2_\phi/2 \simeq 5.17 \times 10^{-5}$ eV^2 is essentially a constant in our L/E region, and numerically identical to $\cos^2\theta_{12}\Delta m^2_{21}$. Thus, this definition of Δm^2_{ee} is similar to the definition introduced in Ref. [2]:

$$\begin{aligned} \Delta m^2_{\rm eff}|_e &= \cos^2\theta_{12}|\Delta m^2_{31}| + \sin^2\theta_{12}|\Delta m^2_{32}| \qquad (7) \\ &= |\Delta m^2_{32}| \pm \cos^2\theta_{12}\Delta m^2_{21}. \qquad (8) \end{aligned}$$

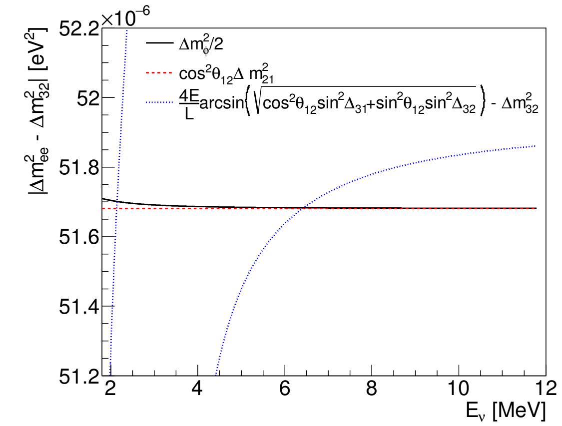


FIG. 1. Values of $\Delta m^2_\phi/2 = |\Delta m^2_{ee} - \Delta m^2_{32}|$ (black solid line) at L = 1.6 km as a function of the neutrino energy, with $\Delta m^2_{21} = 7.50 \times 10^{-5}$ eV^2 and $\sin^2 2\theta_{12} = 0.857$ [1]. For comparison, calculations based on other definitions of Δm^2_{ee}, $\Delta m^2_{ee} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$ (red dashed line) and $\Delta m^2_{ee} = \frac{4E}{L}\arcsin\left[\sqrt{\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}}\right]$ (blue dotted line) are also shown.

Figure 2 is a comparison of the approximated formula with $\Delta m^2_\phi/2 = 5.17 \times 10^{-5}$ eV^2,

$$\begin{aligned} P_{ee} \simeq & 1 - \sin^2 2\theta_{13}\sin^2\left[(\Delta m^2_{32} + 5.17 \times 10^{-5}\ \text{eV}^2)\frac{L}{4E}\right] \\ & - (solar\ term), \qquad (9) \end{aligned}$$

to the three-flavor formula, Eq. 1. In this comparison, L = 1.6 km, $\sin^2 2\theta_{13} = 0.09$, $\Delta m^2_{32} = 2.44 \times 10^{-3}$ eV^2, and normal mass hierarchy are the inputs. The agreement between the two, better than 10^{-4}, is excellent and exceeds the achievable experimental precision.

This study demonstrates that, once we obtain the value of $|\Delta m^2_{ee}|$ using Eq. 2, we can reliably deduce the values of $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$ using Eqs. 5 and 6 with

$$\Delta m^2_\phi/2 \simeq \cos^2\theta_{12}\Delta m^2_{21}. \qquad (10)$$

Using the current values of θ_{12} and Δm^2_{21}, $\Delta m^2_\phi/2 \simeq 5.17 \times 10^{-5}$eV^2, and $(|\Delta m^2_{21}| - \Delta m^2_\phi/2) \simeq 2.33 \times 10^{-5}$ eV^2.

It is important to point out that the exact solution of $\sin^2(\Delta m^2_{ee}\frac{L}{4E}) = \cos^2\theta_{12}\sin^2(\Delta m^2_{31}\frac{L}{4E}) + \sin^2\theta_{12}\sin^2(\Delta m^2_{32}\frac{L}{4E})$ was never used to extract the value of Δm^2_{32} or Δm^2_{31} from the measured Δm^2_{ee} in Daya Bay.

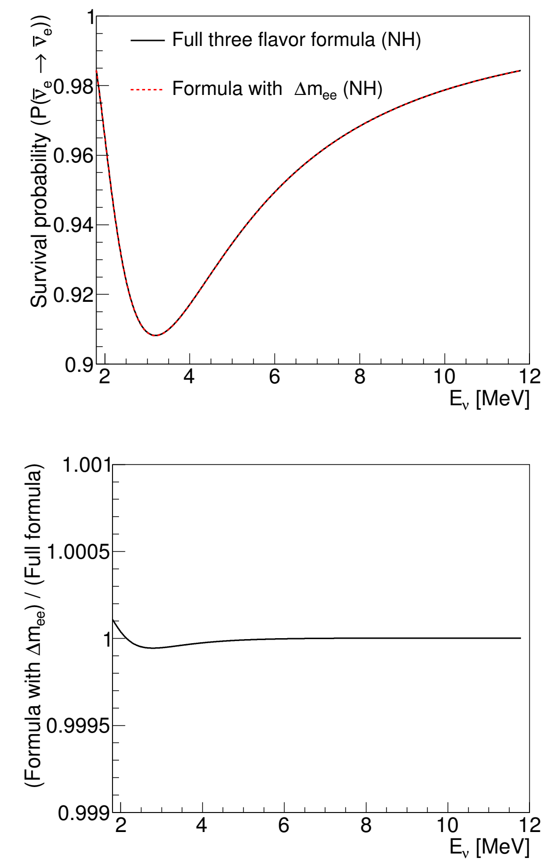


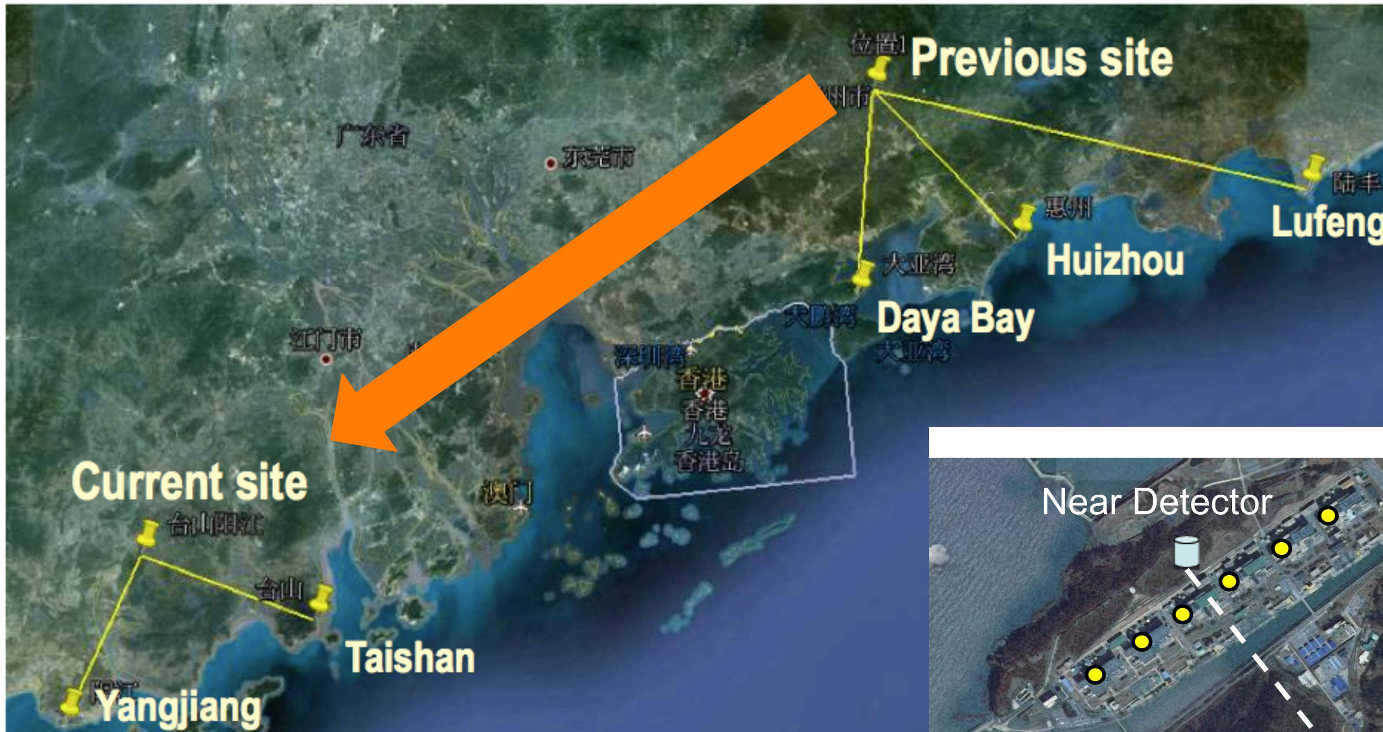
FIG. 2. Comparison of the survival probability at L = 1.6 km between the approximated formula with $\Delta m^2_{ee} = \Delta m^2_{32} + 5.17 \times 10^{-5}$ eV^2 and the exact three-flavor formula (Eq. 1). The oscillation parameters used in this comparison are $\sin^2 2\theta_{13} = 0.09$ and $\Delta m^2_{32} = 2.44 \times 10^{-3}$ eV^2 under the normal mass hierarchy assumption. The top panel shows the survival probabilities calculated with the two formulae, and the bottom panel shows the ratio of the two.

[1] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012), Section 13.

[2] H. Nunokawa, S. J. Parke, and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005).

Future Experiments:

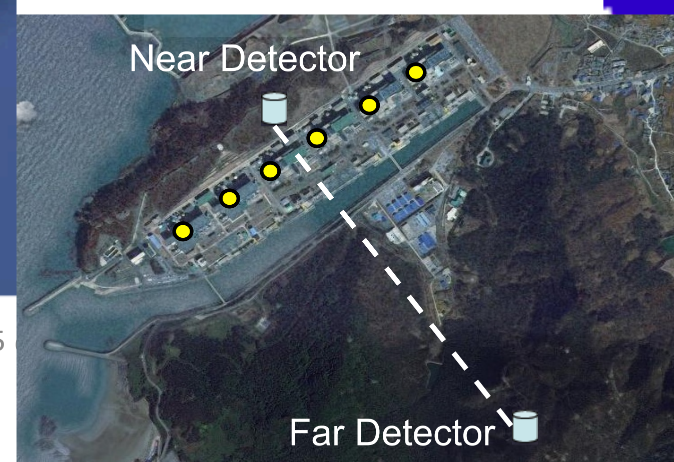
JUNO



Seon-Hee Seo, SNU

FPCP 2015

✧ Baseline difference should be < 500 m.



RENO θ_{13}

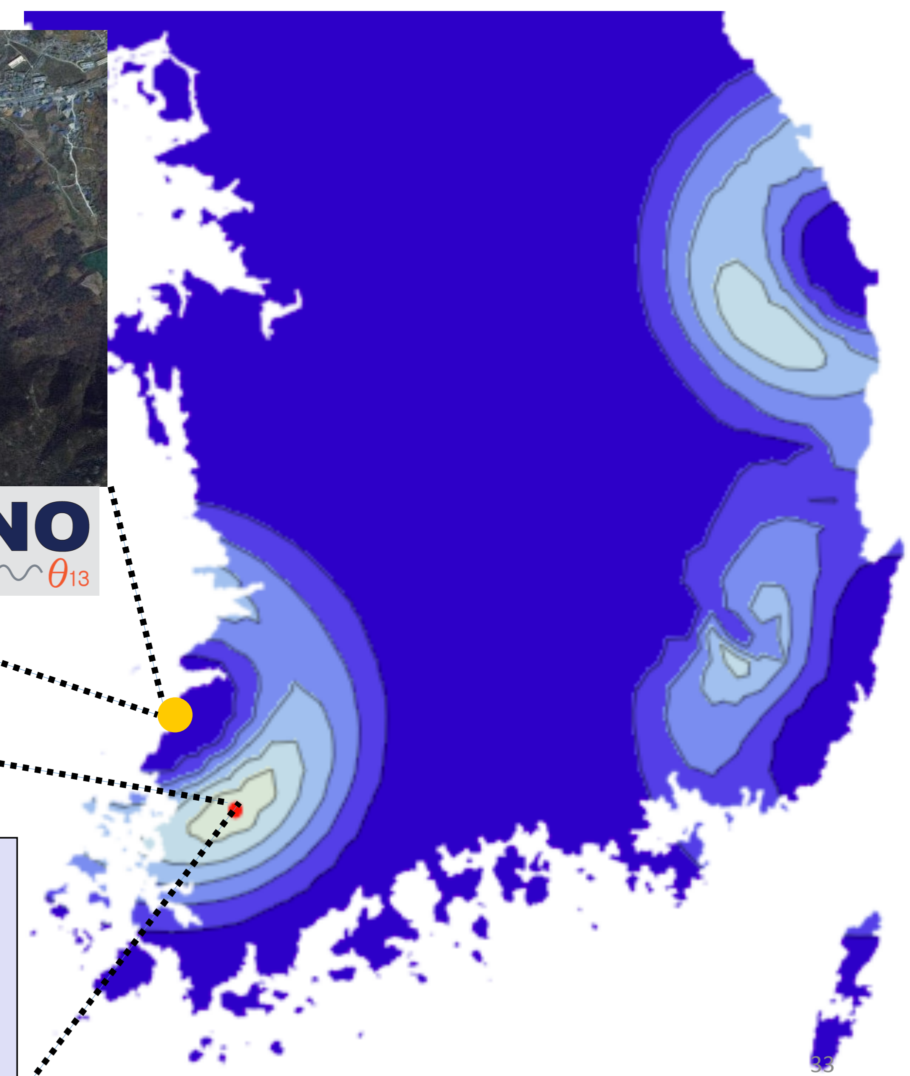
(NEAR Detector)

(FAR Detector)

RENO-50

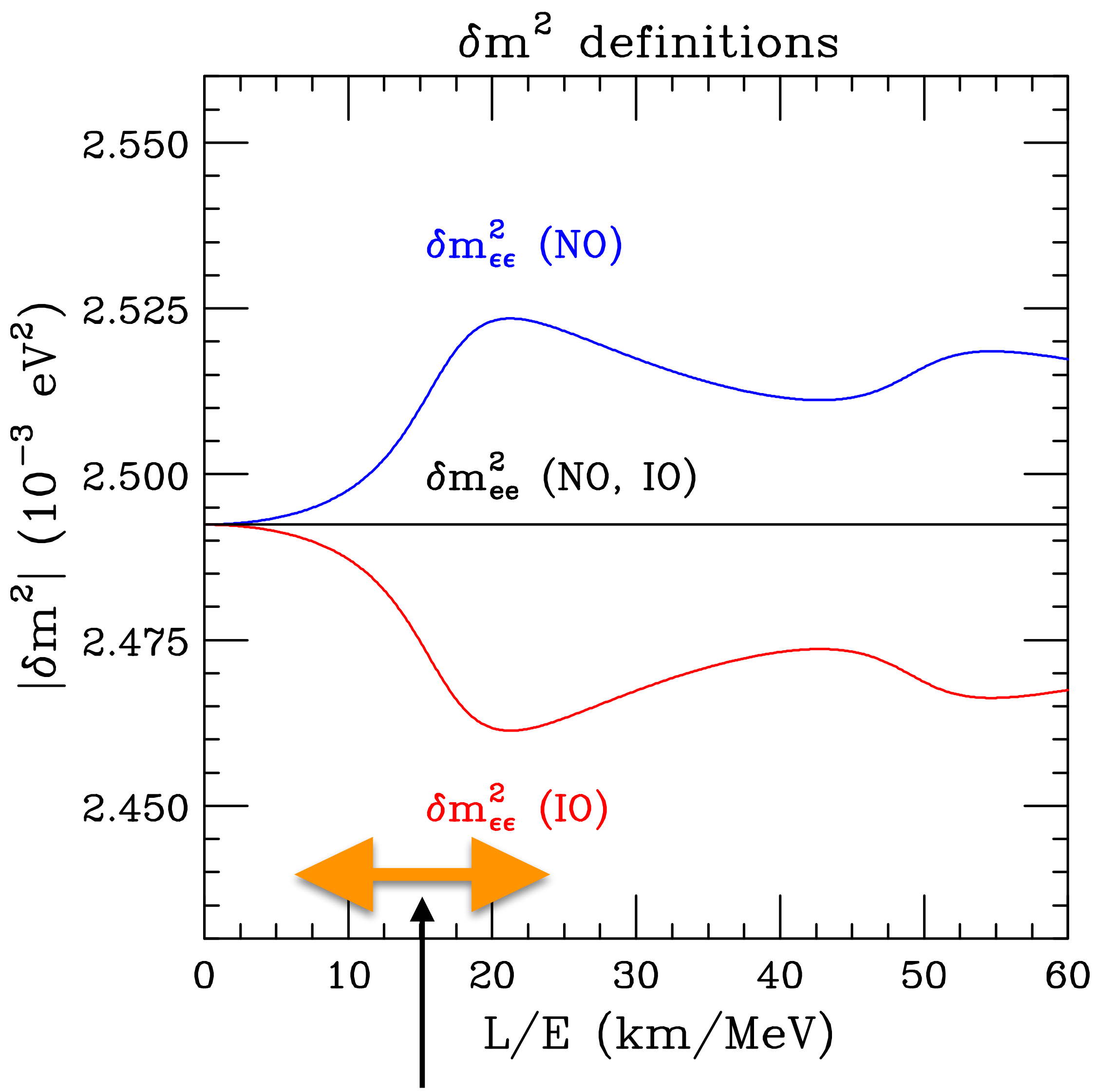
10 kton LS Detector
~47 km from YG reactors

Mt. Guemseong (450 m)
~900 m.w.e. overburden



New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E \to 0}$)



JUNO & RENO50

Identical for L/E < 5 -10 km/MeV !

Significant ($\sim 1\%$) L/E
and
Mass Ordering Dependence!

expect ~0.5 % measurement !!!

What does RENO use?

(arXiv:1511.05849)

Using the $\overline{\nu}_e$ survival probability P [8], reactor experiments with a baseline distance of $\sim$1 km can determine the mixing angle θ_{13} and an effective squared-mass-difference $\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$ [9].

$$
\begin{aligned}
1 - P &= \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}) \\
&\quad + \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\
&\approx \sin^2 2\theta_{13}\sin^2\Delta_{ee} + \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}, \quad (1)
\end{aligned}
$$

Double Chooz ?

Neutrino Propogation in Matter

if one choses to write the Hamiltonian using Δm^2_{ee} and Δm^2_{21} then the Hamiltonian is simpler than with any other choice !

See Hisakazu Minakata and SP arXiv:1505.01826

old

$$\tilde{H}(x) = \frac{\Delta m^2_{31}}{2E}\left\{\begin{bmatrix} \frac{a(x)}{\Delta m^2_{31}} + s^2_{13} & 0 & c_{13}s_{13} \\ 0 & 0 & 0 \\ c_{13}s_{13} & 0 & c^2_{13} \end{bmatrix} + \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\begin{bmatrix} s^2_{12}c^2_{13} & c_{12}s_{12}c_{13} & -s^2_{12}c_{13}s_{13} \\ c_{12}s_{12}c_{13} & c^2_{12} & -c_{12}s_{12}s_{13} \\ -s^2_{12}c_{13}s_{13} & -c_{12}s_{12}s_{13} & s^2_{12}s^2_{13} \end{bmatrix}\right\}$$

new

$$\tilde{H} = \frac{1}{2E}\begin{pmatrix} \lambda_a & & s_{13}c_{13}\Delta m^2_{ee} \\ & \lambda_b & \\ s_{13}c_{13}\Delta m^2_{ee} & & \lambda_c \end{pmatrix} + \epsilon s_{12}c_{12}\frac{\Delta m^2_{ee}}{2E}\begin{pmatrix} & c_{13} & \\ c_{13} & & -s_{13} \\ & -s_{13} & \end{pmatrix}$$

$$\lambda_a = a + (s^2_{13} + \epsilon s^2_{12})\Delta m^2_{ee}\,,$$
$$\lambda_b = \epsilon c^2_{12}\Delta m^2_{ee}\,,$$
$$\lambda_c = (c^2_{13} + \epsilon s^2_{12})\Delta m^2_{ee}\,,$$

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

$$\Delta m^2_{ee} \equiv c^2_{12}\, \Delta m^2_{31} + s^2_{12}\, \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

- Has a direct, simple, physical interpretation:
  Δm^2_{ee} is "the ν_e weighted average of Δm^2_{31} and Δm^2_{32},"
  since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c^2_{12} : s^2_{12}$.

$\Delta m^2_{ee} \equiv c^2_{12}\,\Delta m^2_{31} + s^2_{12}\,\Delta m^2_{32}$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

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  Δm^2_{ee} is "the ν_e weighted average of Δm^2_{31} and Δm^2_{32},"
  since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c^2_{12} : s^2_{12}$.

- Can be used in short baseline reactor experiments, $L/E < 1$ km/MeV, using the approximate oscillation probability,

$$P(\bar{\nu}_e \to \bar{\nu}_e) \;\approx\; 1 - 4c^4_{13}s^2_{12}c^2_{12}\sin^2\Delta_{21} - 4s^2_{13}c^2_{13}\sin^2\Delta_{ee},$$

  which is accurate to better than one part in 10^4.

$$\Delta m_{ee}^2 \equiv c_{12}^2 \, \Delta m_{31}^2 + s_{12}^2 \, \Delta m_{32}^2$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

- Has a direct, simple, physical interpretation:
  Δm_{ee}^2 is "the ν_e weighted average of Δm_{31}^2 and Δm_{32}^2,"
  since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c_{12}^2 : s_{12}^2$.

- Can be used in short baseline reactor experiments, $L/E < 1$ km/MeV, using the approximate oscillation probability,

  $$P(\bar{\nu}_e \to \bar{\nu}_e) \;\approx\; 1 - 4c_{13}^4 s_{12}^2 c_{12}^2 \sin^2 \Delta_{21} - 4s_{13}^2 c_{13}^2 \sin^2 \Delta_{ee},$$

  which is accurate to better than one part in 10^4.

- Can be used in medium baseline reactor experiments, $L \approx 50$ km, and determines the Δm^2 of the fundamental oscillation. The advancement or retardation of the phase of this fundamental oscillation determines the neutrino mass ordering.

What is Δm^2_{ee} ?

SP arXiv:1601.07464

- the ν_e weighted average Δm^2_{31} and Δm^2_{32}
- $\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$
- Useful at any L/E, and especially at

  < 0.8 km/MeV (Daya Bay, RENO, DC)

  $6 - 25$ km/MeV (JUNO, RENO-50)

backup

$$
\begin{aligned}
P_{ee} &= 1-\cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\
&\quad -\sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31}+\sin^2\theta_{12}\sin^2\Delta_{32})
\end{aligned}
$$

$$
\begin{aligned}
&= 1-\cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\
&\quad -\frac{1}{2}\sin^2 2\theta_{13}\left(1-\sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos\Omega\right)
\end{aligned}
$$

(+/- mass ordering)

$$
\begin{aligned}
\text{where}\quad \Omega &= 2|\Delta_{ee}|\pm\phi \\
\text{with}\quad \phi &= \{\arctan(\cos 2\theta_{12}\tan\Delta_{21})-\Delta_{21}\cos 2\theta_{12}\}
\end{aligned}
$$

H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **74**, 053008 (2006) [hep-ph/0607284].

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

$$= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \sqrt{(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos\Omega\right)$$

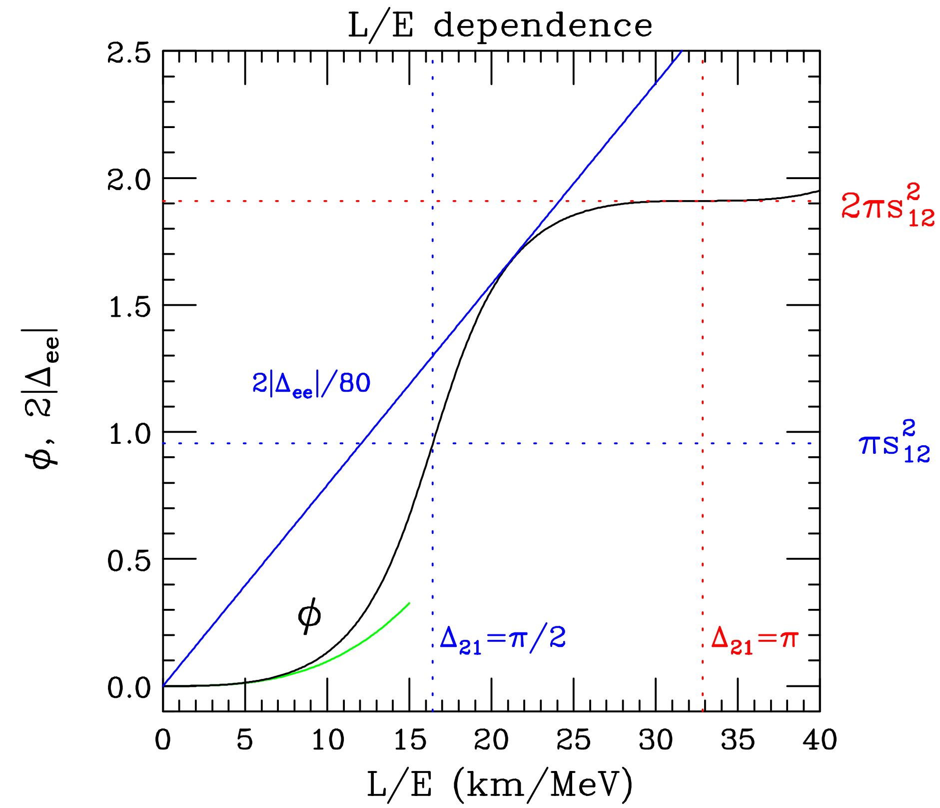
(+/- mass ordering)

$$\text{where} \quad \Omega = 2|\Delta_{ee}| \pm \phi$$

$$\text{with} \quad \phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$$

Note:

- Separation $2\Delta_{ee}$ and ϕ is unique !
- $2\Delta_{ee}$ is linear in L/E and depends on an atm Δm^2.
- $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$ starts at $\Delta_{21}^3 \sim (L/E)^3$ and depends on only solar parameters, staircase function.



H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **74**, 053008 (2006) [hep-ph/0607284].

An overview of the Daya Bay Reactor Neutrino Experiment

Co-spokepersons of Daya Bay

Jun Cao[a], Kam-Biu Luk[b]

[a]*Institute of High Energy Physics, Beijing, China*
[b]*Department of Physics, University of California and Lawrence Berkeley National Laboratory, Berkeley, California, USA*

distance ($\mathcal{O}(km)$) from the reactors. In the three-neutrino framework, the survival probability is given by

$$P = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}\,, \qquad (2)$$

where $\sin^2\Delta_{ee} \equiv \cos^2\theta_{12}\sin^2\Delta_{31}+\sin^2\theta_{12}\sin^2\Delta_{32}$ and $\Delta_{ji} \equiv 1.267\Delta m^2_{ji}L/E$. Δm^2_{ji} is the mass-squared difference in eV^2, E is the energy of the $\overline{\nu}_e$ in MeV, and L is the distance in meters from the production point.

mass splitting are reviewed. Current precision on $\sin^2 2\theta_{13}$ and $|\Delta m^2_{\rm ee}|$ are 6% and 4.5%, respectively. The projected precisions are shown in Fig. 5. The Daya Bay experiment is expected to operate until 2020; by then, the precision is $\sim 3\%$ for both $\sin^2 2\theta_{13}$ and $|\Delta m^2_{\rm ee}|$. Daya Bay has also obtained