Lattice QCD Results for $B$-meson Semileptonic Decay Form Factors and Phenomenology

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Theoretical Motivation

- $B$-meson semileptonic decays through tree-level diagram ($b \rightarrow ul\nu$). For example, $B \rightarrow \pi l\nu$, $B_s \rightarrow Kl\nu$

- $B$-meson semileptonic decays through loop-level diagram ($B \rightarrow K(\pi)l^+l^-, B \rightarrow K(\pi)\nu\bar{\nu}$)
Standard Model prediction

The effective Hamiltonian of the $b \rightarrow d(s)l^+l^-$ transition under OPE with $\alpha_s$ and $\Lambda/m_b$ corrections is:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{td}^* V_{tb} \sum_{i=0}^{10} C_i(\mu) O_i(\mu) + \ldots$$  \hspace{1cm} (1)

the Standard Model prediction can be written in a generic form:

$$\text{Theo. pred.} = (\text{prefactors}) \times (\text{CKM factor}) \times \langle f|\hat{O}|i \rangle$$  \hspace{1cm} (2)

- Prefactors contain the Wilson coefficients (short distance physics).
- CKM factor depends on the processes.
- Lattice QCD calculates $\langle f|\hat{O}|i \rangle$ non-perturbatively from first principle. (long distance physics)
Hadronic matrix elements and form factors

- Matrix elements in $B \rightarrow K(\pi)ll$ and $B \rightarrow \pi l \nu$ processes:

$$\langle B(p)|\overline{b}\gamma^\mu s|K(k)\rangle, \langle B(p)|\overline{s}\sigma^{\mu\nu} b|K(k)\rangle$$

$$\langle B(p)|\overline{b}\gamma^\mu s|K(k)\rangle = f_+(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu$$

$$= \sqrt{2m_B} \left[ f_\parallel \frac{p^\mu}{m_B} + f_\perp k_\perp^\mu \right]$$

$$f_\parallel(E_K) = \frac{\langle B(p)|\overline{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}}$$

$$f_\perp(E_K) = \frac{\langle B(p)|\overline{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}} \frac{1}{p_i}$$

$$f_0(E_K) = \frac{2m_B}{m_B^2 - m_K^2} \left[ (m_B - E_K)f_\parallel(E_K) + (E_K^2 - m_K^2)f_\perp(E_K) \right]$$

$$f_+(E_K) = \frac{1}{\sqrt{2m_B}} \left[ f_\parallel(E_K) + (m_B - E_K)f_\perp(E_K) \right]$$
Semileptonic $B \rightarrow K$ transition from tensor current:

$$q_\nu \langle K(k) | \bar{s} \sigma^{\mu\nu} b | B(p) \rangle = \frac{if_T}{m_B + m_K} \left[ q^2 (p^\mu + k^\mu) - (m_B^2 - m_K^2) q^\mu \right]$$

Solve for $f_T$:

$$f_T = \frac{m_B + m_K}{\sqrt{2} m_B} \frac{\langle K(k) | i b \sigma^{0i} s | B(p) \rangle}{\sqrt{2} m_B k^i}$$
Lattice ensembles used in $B \rightarrow K(\pi)$ works

Figure: Ensembles of QCD gauge field configurations used in the simulations.
Form factors defined in the continuum are extrapolated from lattice data.

Lattice-QCD gives form factors in the low recoil ($E_K(\pi)$) region.

$z$-expansion is used to extrapolate lattice-QCD results to the whole $q^2$ region; talk by Benjamin Grinstein (previous).
Form factor results from lattice QCD

- $B \rightarrow \pi l \nu$ and $B \rightarrow \pi ll$ form factors
- $B \rightarrow K ll$ form factors
- $B_s \rightarrow K l \nu$ form factors
Lattice-QCD $B \to \pi l\nu$ and $B \to \pi ll$ form factors

$B \to \pi l\nu$ and $B \to \pi ll$ occur through $b \to ul\nu$ and $b \to dll$ transitions. Results from RBC/UKQCD.

![Plot](image-url)

**Figure**: $B \to \pi l\nu$ form factors from arXiv:1501.05373.

- Fit together with experimental data and $z$-expansion fit, lattice QCD can get accurate form factors $(f_+, f_0)$ in the whole $q^2$ region.
Lattice-QCD $B \to \pi \ell \nu$ and $B \to \pi \ell \ell$ form factors

Latest results from FNAL/MILC.

Figure: $B \to \pi \ell \nu$ and $B \to \pi \ell \ell$ form factors from arXiv:1509.06235 and arXiv:1507.01618.

- FNAL/MILC updated form factors with more statistics and ensembles.
- The first lattice calculation of $f_T$ in $B \to \pi \ell \ell$ process is available.
Lattice-QCD $B \to \pi l \nu$ and $B \to \pi l l$ form factors

**Figure:** First lattice-QCD result on $B \to \pi l \nu f_0(q^2_{\text{max}})$ at zero recoil from physical u/d quark mass. (HPQCD, arXiv:1510.07446).
Lattice-QCD $B \to K\ell\ell$ form factors

$B \to K\ell\ell$ occurs through $b \to s\ell\ell$ transitions.

**Figure:** Comparison of the $B \to K\ell\ell$ form factors. Fermilab/MILC (Lattice), HPQCD (Lattice) and LCSR results are from arXiv:1509.06235, arXiv:1306.2384, and arXiv:1006.4945.
Lattice-QCD $B_s \rightarrow Kl\nu$ form factors

$B_s \rightarrow Kl\nu$ occurs through $b \rightarrow ul\nu$ transitions.

![Graph of form factors](image)

**Figure:** $B_s \rightarrow Kl\nu$ form factors from RBC/UKQCD (left, arXiv:1501.05373) and HPQCD (right, arXiv:1406.2279).
Lattice-QCD $B_s \rightarrow Kl\nu$ form factors

**Figure**: $B_s \rightarrow Kl\nu$ $f_+$ and $f_0$ error budget. (HPQCD arXiv:1406.2279)
Lattice-QCD $B_s \to K\ell\nu$ form factors

**Figure:** Comparison of lattice-QCD $B_s \to K$ form factors (arXiv:1406.2279) with those from a perturbative QCD model (arXiv:1207.0265) and the relativistic quark model (arXiv:1304.3255).
Semileptonic $B$-meson decay phenomenology

Impact of lattice-QCD form factors to Standard Model Phenomenology.

Tree-level process:
- $B \rightarrow \pi l \nu$ and $|V_{ub}|$ determination. (arXiv:1503.07839)

Loop-level process:
- $B \rightarrow Kl^+l^-$ ($l = e, \mu, \tau$) (arXiv:1509.06235, arXiv:1510.02349)
- $B \rightarrow \pi l^+l^-$ ($l = e, \mu, \tau$) (arXiv:1503.07839, arXiv:1507.01618)
- $B \rightarrow \pi\nu\bar{\nu}$, $B \rightarrow K\nu\bar{\nu}$ (arXiv:1510.02349)
$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

**Figure**: $B \to \pi l \nu$ exclusive decay process.

\[
\frac{d\Gamma}{dq^2} \propto |V_{ub}|^2 |f_+(q^2)|^2 \quad \text{Exp.}
\]

\[
\langle \pi | V^\mu | B \rangle = f_+(q^2) \left[ p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu
\]

\[
q^2 = (p_B - p_\pi)^2 = M_B^2 + M_\pi^2 - 2M_B M_\pi E_\pi
\]
$B \rightarrow \pi l \nu$ semileptonic decay and $|V_{ub}|$

This work + BaBar + Belle, $B \rightarrow \pi l \nu$

Fermilab/MILC 2008 + HFAG 2014, $B \rightarrow \pi l \nu$

RBC/UKQCD 2015 + BaBar + Belle, $B \rightarrow \pi l \nu$

Imsong et al. 2014 + BaBar12 + Belle13, $B \rightarrow \pi l \nu$

HPQCD 2006 + HFAG 2014, $B \rightarrow \pi l \nu$

Detmold et al. 2015 + LHCb 2015, $\Lambda_b \rightarrow pl \nu$

BLNP 2004 + HFAG 2014, $B \rightarrow X_u l \nu$

UTFit 2014, CKM unitarity

**Figure**: Comparison of the $|V_{ub}|$ results from different determinations. (arXiv:1503.07839)
$B_s \rightarrow Kl\nu$ semileptonic decay and $|V_{ub}|$

Figure: Theoretical predictions on differential decay rates, divided by $|V_{ub}|^2$, for $B_s \rightarrow K\mu\nu$ and $B_s \rightarrow K\tau\nu$. (arXiv:1406.2279)
Standard Model predictions

- Left: FNAL/MILC $B \to \pi$ lattice data + exp (arXiv:1503.07839)
- Right: (arXiv:1312.2523): old FNAL/MILC $B \to \pi$ lattice data (arXiv:0811.3640) + HPQCD’s $B \to K$ lattice data (arXiv:1306.2384) + exp + LCSR + model

Form factors $\oplus$ CKM
Form factors only
LHCb 2015 (preliminary)
Standard Model predictions of $B \rightarrow Kl^+l^-$ process

**Figure:** Standard-Model differential branching fraction (gray band) for $B \rightarrow K\mu^+\mu^-$ decay (left) and $B \rightarrow K\tau^+\tau^-$ (right) using the form factors obtained from lattice QCD. Experimental results for $B \rightarrow K\mu^+\mu^-$ are from Belle (arXiv:0904.0770), CDF (arXiv:1107.3753), BaBar (arXiv:1204.3933), and LHCb (arXiv:1403.8044). The BaBar, Belle, and CDF experiments report isospin-averaged measurements, while LHCb separately reports results for $B^+$ and $B^0$ decays.
The resonance states could have contribution to the final result of $dB/dq^2$. The plot is quoted from arXiv:1406.0566.

We focus on the results in larger bins.
Lepton flavor violation in $B \to K(\pi)ll$ process

Lepton-flavor-violating effect in the $B \to K(\pi)ll$ process is defined as:

$$R_{\mu,e}^{\mu,e}(q_1^2, q_2^2) = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, dBR(B \to K\mu^+\mu^-)/dq^2}{\int_{q_1^2}^{q_2^2} dq^2 \, dBR(B \to K\ell^+\ell^-)/dq^2},$$

(3)

- $R_{\mu,e}^{\mu,e}$ is close 1 in Standard Model for $B \to K\ell\ell$ and $B \to \pi\ell\ell$ processes.
- BaBar found $R_{K}^{\mu,e} = 1.00(^{+31}_{-25})(7)$ in the union of $[0.1, 8.12]\text{GeV}^2$ and $[10.11, q_{\text{max}}^2]\text{GeV}^2$. (arXiv:1204.3933)
- Bell found $R_{K}^{\mu,e} = 1.03(^{+19}_{-6})$ in the full $q^2$ range. (arXiv:0904.0770)
- LHCb found $R_{K}^{\mu,e} = 0.745(^{+90}_{-74})(36)$ which is $2.6\sigma$ away from 1. (arXiv:1406.6482)
Lepton favor violation in $B \rightarrow K(\pi)ll$ process

Figure: Standard-Model lepton-flavor-violating ratios $(R_{K^+} - 1)$ (left) and $(R_{\pi^+} - 1)$ (right) for $(q_{\text{min}}^2, q_{\text{max}}^2) = (1\text{GeV}^2, 6\text{GeV}^2)$ and $(15\text{GeV}^2, 22\text{GeV}^2)$ using the lattice form factors. Our result is consistent with HPQCD’s (arXiv:1306.0434), but different from LHCb’s experimental data.
Other Important Topics

Heavy to light meson semileptonic decay form factors:
- $D \to \pi l\nu$ ($c \to d$) form factors at zero recoil. (arXiv:1206.4936)
- $D \to Kl\nu$ ($c \to s$) form factors at nonzero recoil. (arXiv:1305.1462)

Heavy to heavy meson semileptonic decay form factors:
- $B \to Dl\nu$ ($b \to c$) form factors at zero recoil. (arXiv:1503.07237, arXiv:1505.03925)
- $B \to D^*l\nu$ ($b \to c$) form factors at nonzero recoil. (arXiv:1403.0635)

Light to light meson semileptonic decay form factors:
- $K \to \pi l\nu$ ($s \to u$) form factors. (arXiv:1312.1228)
Summary

- The form factors in the semileptonic $B$, $D$ and $K$-meson decay processes can be computed by lattice-QCD accurately.
- The latest form factors calculated from lattice-QCD enable us to calculate Standard Model predictions more accurately.
- More results on these semileptonic decay form factors from improved lattice actions will be available in the next few years.
Backup Slides
Standard Model predictions

Theoretical prediction of $dB/dq^2$ in high $q^2$ region:

$$
\frac{dB}{dq^2} = \frac{G_F^2 \alpha^2 |V_{tb} V_{td}^*|^2}{2^7 \pi^5} |\mathbf{k}| \beta_+ \left\{ \frac{2}{3} |\mathbf{k}|^2 \beta_+^2 \left| C_{10}^{\text{eff}} f_+(q^2) \right|^2 \\
+ \frac{m_l^2 (M_B^2 - M_K^2)^2}{q^2 M_B^2} \left| C_{10}^{\text{eff}} f_0(q^2) \right|^2 \\
+ |\mathbf{k}|^2 \left[ 1 - \frac{1}{3} \beta_+^2 \right] \left| C_9^{\text{eff}} f_+(q^2) + 2 C_7^{\text{eff}} \frac{m_b}{M_B + M_K} f_T(q^2) \right|^2 \right\}, \quad (4)
$$

where $G_F$, $\alpha$, and $V_{tq}$ are the Fermi constant, the (QED) fine structure constant, and CKM matrix elements, respectively, $|\mathbf{k}| = \sqrt{E_K^2 - M_K^2}$ is the kaon momentum in the $B$-meson rest frame, and $\beta_+^2 = 1 - 4m_l^2/q^2$, with $m_l$ the lepton mass.