Lattice QCD Results for B-meson Semileptonic Decay Form Factors and Phenomenology

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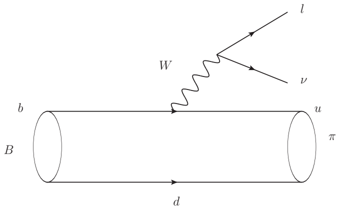
Fermilab

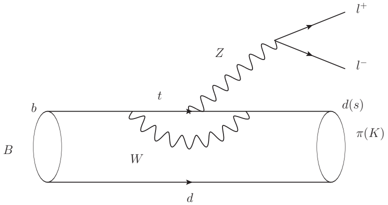
(Heavy Quarks and Leptons 2016)

05/27/2016

Theoretical Motivation



Tree-level diagram



Loop-level diagram

- B-meson semileptonic decays through tree-level diagram ($b \to ul\nu$). For example, $B \to \pi l\nu$, $B_s \to Kl\nu$
- B-meson semileptonic decays through loop-level diagram ($B \to K(\pi)l^+l^-$, $B \to K(\pi)\nu\bar{\nu}$)

Standard Model prediction

The effective Hamiltonian of the $b \to d(s) l^+ l^-$ transition under OPE with α_s and Λ / m_b corrections is:

$$\mathcal{H}_{\text{eff}} = -\frac{4 G_F}{\sqrt{2}} V^*_{td(s)} V_{tb} \sum_{i=0}^{10} C_i(\mu) O_i(\mu) + \ldots \tag{1}$$

the Standard Model prediction can be written in a generic form:

$$\text{Theo. pred.} = (\text{prefactors}) \times (\text{CKMfactor}) \times \langle f | \hat{O} | i \rangle \tag{2}$$

- Prefactors contain the Wilson coefficients (short distance physics).
- CKM factor depends on the processes.
- Lattice QCD calculates $\langle f | \hat{O} | i \rangle$ non-perturbatively from first principle. (long distance physics)

Hadronic matrix elements and form factors

- Matrix elements in $B \to K(\pi)ll$ and $B \to \pi l\nu$ processes:

$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle$, $\langle B(p)|\bar{s}\sigma^{\mu\nu} b|K(k)\rangle$

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = f_+(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu$$

$$= \sqrt{2m_B}\left[f_\parallel \frac{p^\mu}{m_B} + f_\perp k_\perp^\mu\right]$$

$$\begin{cases} f_\parallel(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}} \\ f_\perp(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}} \dfrac{1}{p_i} \end{cases}$$

$$\begin{cases} f_0(E_K) = \dfrac{2m_B}{m_B^2 - m_K^2}\left[(m_B - E_K) f_\parallel(E_K) + (E_K^2 - m_K^2) f_\perp(E_K)\right] \\ f_+(E_K) = \dfrac{1}{\sqrt{2m_B}}\left[f_\parallel(E_K) + (m_B - E_K) f_\perp(E_K)\right] \end{cases}$$

Hadronic matrix elements and form factors

Semileptonic $B \to K$ transition from tensor current:

$$q_\nu \langle K(k)|\bar{s}\sigma^{\mu\nu}b|B(p)\rangle = \frac{i f_T}{m_B + m_K}\left[q^2(p^\mu + k^\mu) - (m_B^2 - m_K^2)q^\mu\right]$$

Solve for f_T:

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle K(k)|ib\sigma^{0i}s|B(p)\rangle}{\sqrt{2m_B}k^i}$$

Lattice ensembles used in $B \to K(\pi)$ works

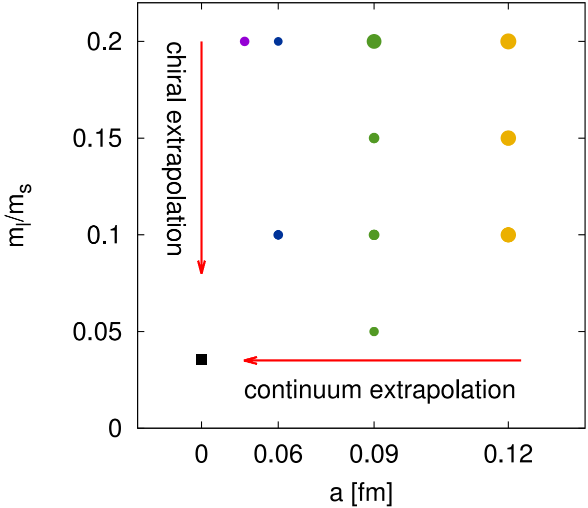
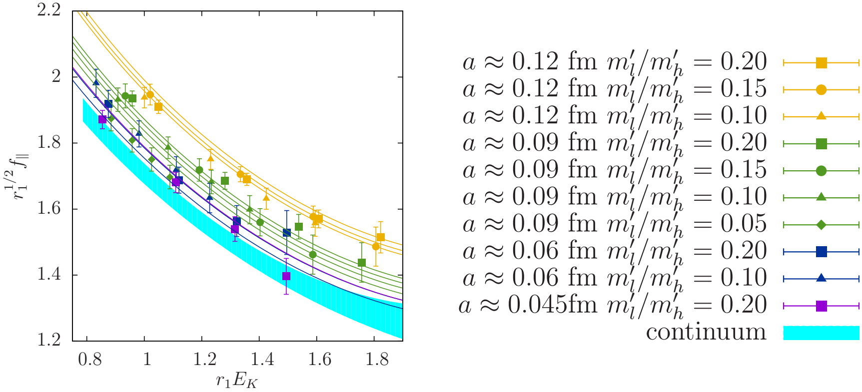


Figure : Ensembles of QCD gauge field configurations used in the simulations.

$f_\parallel$, $f_\perp$ chiral-continuum extrapolations



- Form factors defined in the continuum are extrapolated from lattice data.
- Lattice-QCD gives form factors in the low recoil ($E_{K(\pi)}$) region.
- z-expansion is used to extrapolate lattice-QCD results to the whole q^2 region; talk by Benjamin Grinstein (previous).

Form Factor Results From Lattice QCD

Form factor results from lattice QCD

- $B \to \pi l \nu$ and $B \to \pi ll$ form factors
- $B \to Kll$ form factors
- $B_s \to Kl\nu$ form factors

Lattice-QCD $B \to \pi l\nu$ and $B \to \pi ll$ form factors

$B \to \pi l\nu$ and $B \to \pi ll$ occur through $b \to ul\nu$ and $b \to dll$ transitions.
Results from RBC/UKQCD.

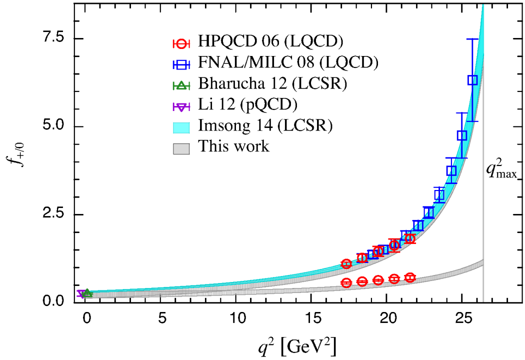


Figure : $B \to \pi l\nu$ form factors from arXiv:1501.05373.

- Fit together with experimental data and z-expansion fit, lattice QCD can get accurate form factors (f_+, f_0) in the whole q^2 region.

Lattice-QCD $B \to \pi l\nu$ and $B \to \pi ll$ form factors

Latest results from FNAL/MILC.

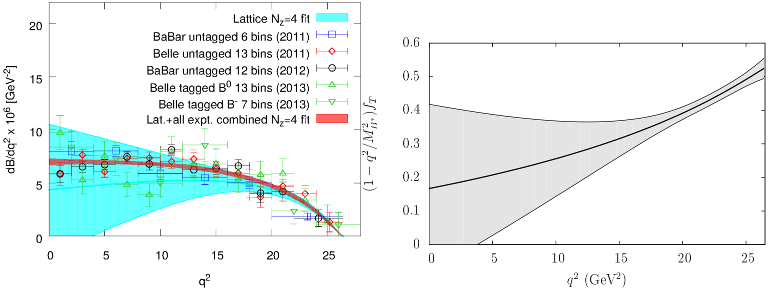


Figure : $B \to \pi l\nu$ and $B \to \pi ll$ form factors from arXiv:1509.06235 and arXiv:1507.01618.

- FNAL/MILC updated form factors with more statistics and ensembles.
- The first lattice calculation of f_T in $B \to \pi ll$ process is available.

Lattice-QCD $B \to \pi l \nu$ and $B \to \pi l l$ form factors

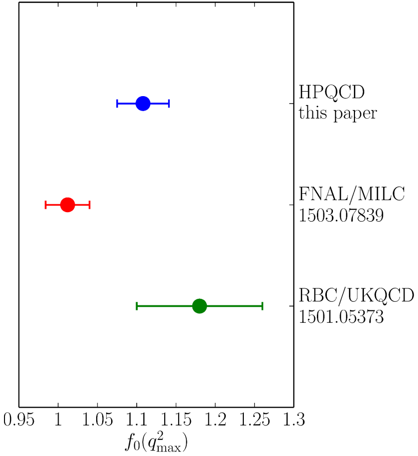


Figure : First lattice-QCD result on $B \to \pi l \nu$ $f_0(q^2_{\mathrm{max}})$ at zero recoil from physical u/d quark mass. (HPQCD, arXiv:1510.07446).

Lattice-QCD $B \to Kll$ form factors

$B \to Kll$ occurs through $b \to sll$ transitions.

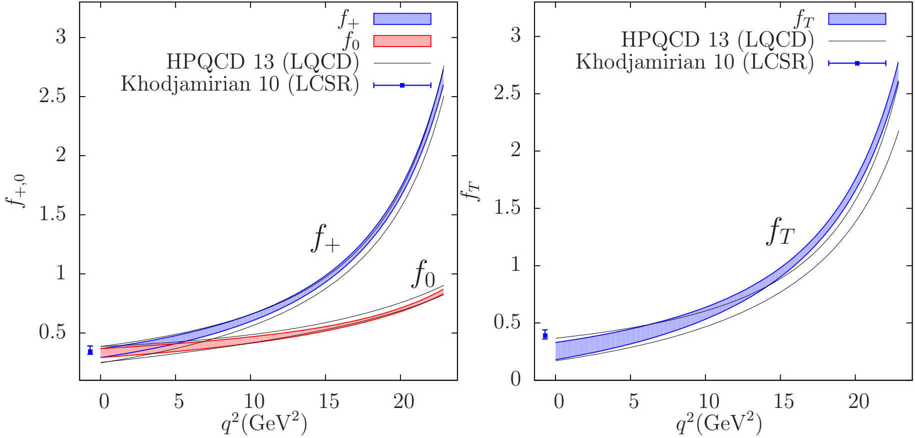
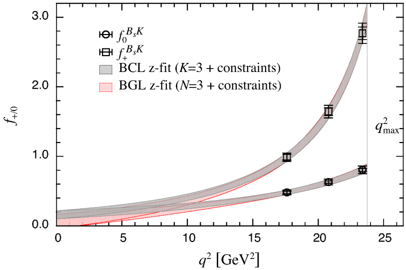


Figure : Comparison of the $B \to Kll$ form factors. Fermilab/MILC (Lattice), HPQCD (Lattice) and LCSR results are from arXiv:1509.06235, arXiv:1306.2384, and arXiv:1006.4945.

Lattice-QCD $B_s \to K l \nu$ form factors

$B_s \to K l \nu$ occurs through $b \to u l \nu$ transitions.



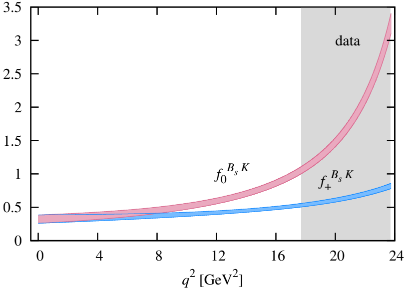


Figure : $B_s \to K l \nu$ form factors from RBC/UKQCD (left, arXiv:1501.05373) and HPQCD (right, arXiv:1406.2279).

Lattice-QCD $B_s \to K l \nu$ form factors

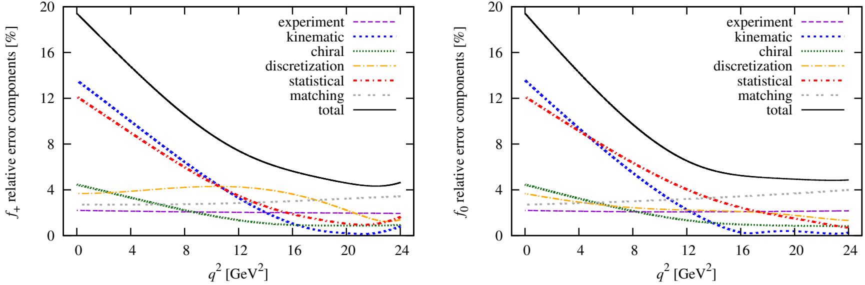


Figure : $B_s \to K l \nu$ f_+ and f_0 error budget. (HPQCD arXiv:1406.2279)

Lattice-QCD $B_s \to K l \nu$ form factors

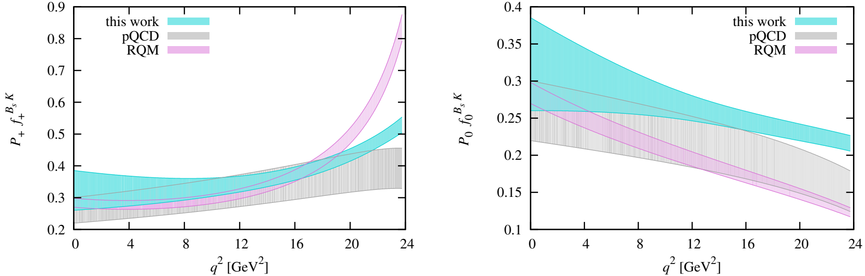


Figure : Comparison of lattice-QCD $B_s \to K$ form factors (arXiv:1406.2279) with those from a perturbative QCD model (arXiv:1207.0265) and the relativistic quark model (arXiv:1304.3255).

Semileptonic B-meson decay phenomenology

Impact of lattice-QCD form factors to Standard Model Phenomenology.

Tree-level process:

- $B \to \pi l \nu$ and $|V_{ub}|$ determination. (arXiv:1503.07839)

Loop-level process:

- $B \to K l^+ l^-$ ($l = e, \mu, \tau$) (arXiv:1509.06235, arXiv:1510.02349)
- $B \to \pi l^+ l^-$ ($l = e, \mu, \tau$) (arXiv:1503.07839, arXiv:1507.01618)
- $B \to \pi \nu \bar{\nu}$, $B \to K \nu \bar{\nu}$ (arXiv:1510.02349)

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

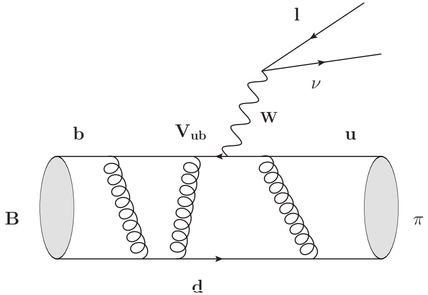


Figure : $B \to \pi l \nu$ exclusive decay process.

$$\frac{d\Gamma}{dq^2} \propto |V_{ub}|^2 |f_+(q^2)|^2 \quad \text{Exp.}$$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu$$

$$q^2 = (p_B - p_\pi)^2 = M_B^2 + M_\pi^2 - 2 M_B M_\pi E_\pi$$

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

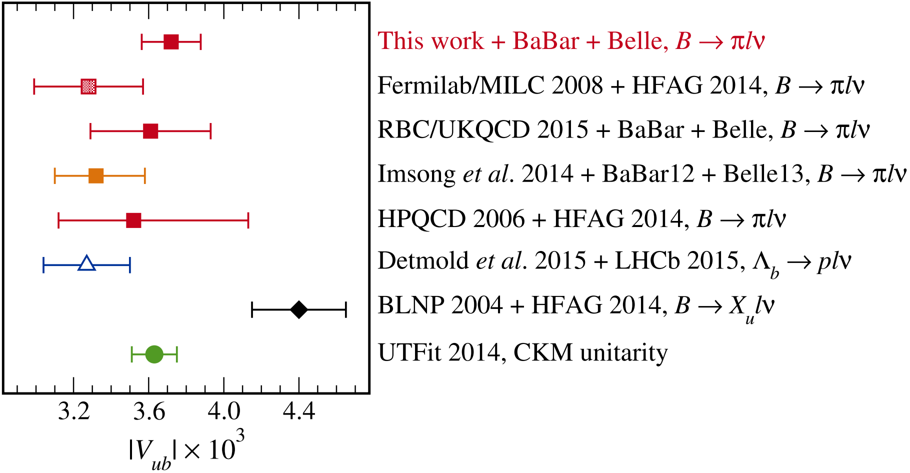


Figure : Comparison of the $|V_{ub}|$ results from different determinations. (arXiv:1503.07839)

$B_s \to Kl\nu$ semileptonic decay and $|V_{ub}|$

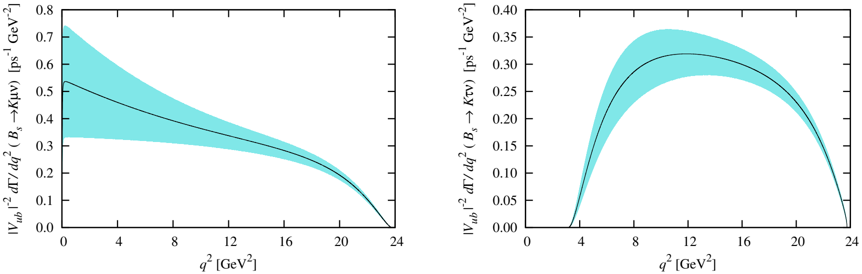
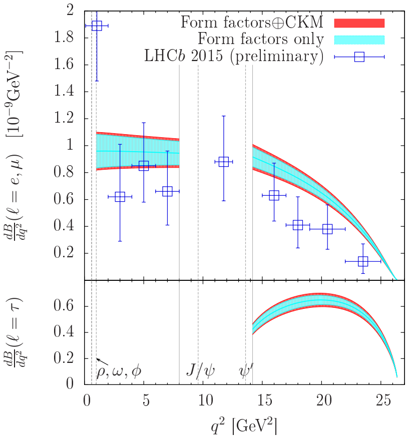
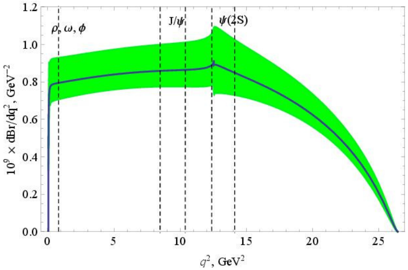


Figure : Theoretical predictions on differential decay rates, divided by $|V_{ub}|^2$, for $B_s \to K\mu\nu$ and $B_s \to K\tau\nu$. (arXiv:1406.2279)

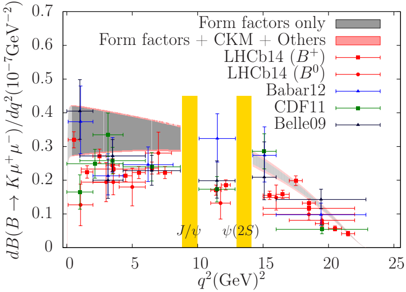
Standard Model predictions





- Left: FNAL/MILC $B \to \pi$ lattice data + exp (arXiv:1503.07839)
- Right: (arXiv:1312.2523): old FNAL/MILC $B \to \pi$ lattice data (arXiv:0811.3640) + HPQCD's $B \to K$ lattice data(arXiv:1306.2384) + exp + LCSR + model

Standard Model predictions of $B \to Kl^+l^-$ process



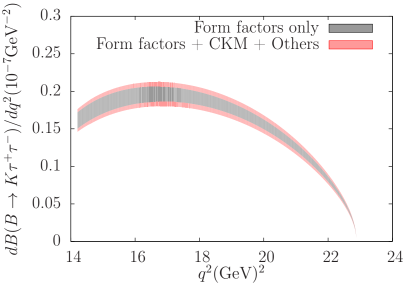
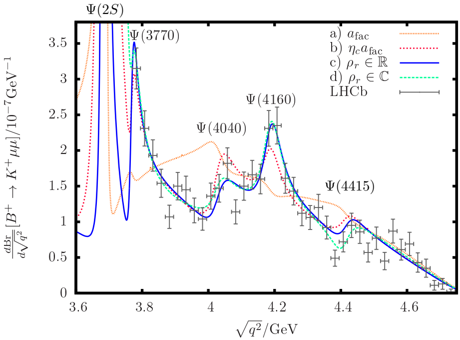


Figure : Standard-Model differential branching fraction (gray band) for $B \to K\mu^+\mu^-$ decay (left) and $B \to K\tau^+\tau^-$ (right) using the form factors obtained from lattice QCD. Experimental results for $B \to K\mu^+\mu^-$ are from Belle (arXiv:0904.0770), CDF (arXiv:1107.3753), BaBar (arXiv:1204.3933), and LHCb (arXiv:1403.8044). The BaBar, Belle, and CDF experiments report isospin-averaged measurements, while LHCb separately reports results for B^+ and B^0 decays.

Resonance states in $B \to Kl^+l^-$ process



- The resonance states could have contribution to the final result of dB/dq^2. The plot is quoted from arXiv:1406.0566.
- We focus on the results in larger bins.

Lepton flavor violation in $B \to K(\pi)ll$ process

Lepton-flavor-violating effect in the $B \to K(\pi)ll$ process is defined as:

$$R^{\mu,e}(q_1^2, q_2^2) = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2}{\int_{q_1^2}^{q_2^2} dq^2 \, d\mathrm{BR}(B \to Ke^+e^-)/dq^2}, \tag{3}$$

- $R^{\mu,e}$ is close 1 in Standard Model for $B \to Kll$ and $B \to \pi ll$ processes.
- BaBar found $R_K^{\mu,e} = 1.00(^{+31}_{-25})(7)$ in the union of $[0.1, 8.12]\mathrm{GeV}^2$ and $[10.11, q^2_{\mathrm{max}}]\mathrm{GeV}^2$. (arXiv:1204.3933)
- Bell found $R_K^{\mu,e} = 1.03(^{+19}_{-6})$ in the full q^2 range. (arXiv:0904.0770)
- LHCb found $R_K^{\mu,e} = 0.745(^{+90}_{-74})(36)$ which is 2.6σ away from 1. (arXiv:1406.6482)
- New physics models and lepton flavor violation. (arXiv:1411.0565, arXiv:1508.07009, *etc.*)

Lepton favor violation in $B \to K(\pi)ll$ process

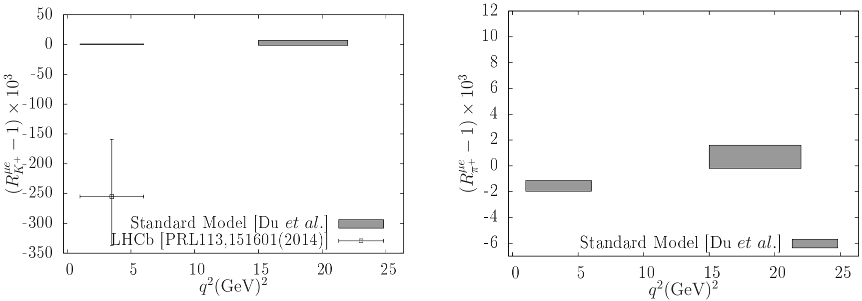


Figure : Standard-Model lepton-flavor-violating ratios $\left(R^{\mu e}_{K^+} - 1\right)$ (left) and $\left(R^{\mu e}_{\pi^+} - 1\right)$ (right) for $(q^2_{\min}, q^2_{\max}) = (1\mathrm{GeV}^2, 6\mathrm{GeV}^2)$ and $(15\mathrm{GeV}^2, 22\mathrm{GeV}^2)$ using the lattice form factors. Our result is consistent with HPQCD's (arXiv:1306.0434), but different from LHCb's experimental data.

Other Important Topics

Heavy to light meson semileptonic decay form factors:

- $D \to \pi l \nu$ ($c \to d$) form factors at zero recoil. (arXiv:1206.4936)
- $D \to K l \nu$ ($c \to s$) form factors at nonzero recoil. (arXiv:1305.1462)

Heavy to heavy meson semileptonic decay form factors:

- $B \to D l \nu$ ($b \to c$) form factors at zero recoil. (arXiv:1503.07237, arXiv:1505.03925)
- $B \to D^* l \nu$ ($b \to c$) form factors at nonzero recoil. (arXiv:1403.0635)

Light to light meson semileptonic decay form factors:

- $K \to \pi l \nu$ ($s \to u$) form factors. (arXiv:1312.1228)

Summary

- The form factors in the semileptonic B, D and K-meson decay processes can be computed by lattice-QCD accurately.
- The latest form factors calculated from lattice-QCD enable us to calculated Standard Model predictions more accurately.
- More results on the these semileptonic decay form factors from improved lattice actions will be available in the next few years.

Backup Slides

Standard Model predictions

Theoretical prediction of dB/dq^2 in high q^2 region:

$$\frac{dB}{dq^2} = \frac{G_F^2 \alpha^2 |V_{tb} V_{td}^*|^2}{2^7 \pi^5} |\mathbf{k}| \beta_+ \left\{ \frac{2}{3} |\mathbf{k}|^2 \beta_+^2 \left| C_{10}^{\text{eff}} f_+(q^2) \right|^2 \right.$$
$$+ \frac{m_l^2 (M_B^2 - M_K^2)^2}{q^2 M_B^2} \left| C_{10}^{\text{eff}} f_0(q^2) \right|^2$$
$$\left. + |\mathbf{k}|^2 \left[1 - \frac{1}{3} \beta_+^2 \right] \left| C_9^{\text{eff}} f_+(q^2) + 2 C_7^{\text{eff}} \frac{m_b}{M_B + M_K} f_T(q^2) \right|^2 \right\}, \qquad (4)$$

where G_F, α, and V_{tq} are the Fermi constant, the (QED) fine structure constant, and CKM matrix elements, respectively, $|\mathbf{k}| = \sqrt{E_K^2 - M_K^2}$ is the kaon momentum in the B-meson rest frame, and $\beta_+^2 = 1 - 4m_l^2/q^2$, with m_l the lepton mass.