Determination of V_{cb} and V_{ub}

From Semileptonic Decays

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UCSD

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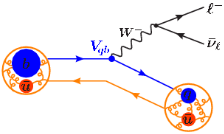
Fundamentals



- Decay rates of B and Λ_b are proportional to
  - $|V_{cb}|^2$ in decays $b \to c$
  - $|V_{ub}|^2$ in decays $b \to u$
- Single b-quark production rate also, but not as clean
- Semileptonic decays ($b \to (c,u)\ell\nu$) $\Rightarrow$ best theory control
- Empirically $|V_{cb}|^2 \sim 10^2 |V_{ub}|^2$
  $\Rightarrow b \to u\ell\nu$ "hides" under $b \to c\ell\nu$
- Concentrate on B decays; for Λ_b see talk by Svende Braun

Exclusive vs Inclusive

Exclusive:

- $\Gamma(B \to D\mu\nu)$, $\Gamma(B \to D^*\mu\nu)$
  - Need form factors: HQET, lattice, z-expansion
  - Large fractions: in small-velocity (SV) limit semileptonic saturated by $B \to D\mu\nu + B \to D^*\mu\nu$

    (Shifman & Voloshin, SJNP 47(1988)511; Boyd *et al* PRD54(1996)2081)
- $\Gamma(B \to \pi\mu\nu)$, $\Gamma(B \to \rho\mu\nu)$,...
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- $\Gamma(B \to X_c\ell\nu) \equiv \Gamma(B \to D\mu\nu) + \Gamma(B \to D^*\mu\nu) + \Gamma(B \to D\pi\ell\nu) + \cdots$
- Quark-hadron duality: $\Gamma(B \to X_c\ell\nu) \approx \Gamma(b \to c\ell\nu)$
  - (almost) from 1^{st} principles
  - integration over (some) phase space
  - holds for moments; characterize non-perturb effects
  - all analytic
- u hides under c

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Chay, BG, Georgi, PLB247(1990)399
Bigi *et al,* PRL71(1993)496
Bigi *et al,* PLB323(1994)408
Manohar, Wise, PRD49(1994)1310
Trott, PRD70(2004)073003
Bauer *et al,* PRD70(2004) 094017

- Combined OPE-HQET
- Differential rate $d\Gamma/dx$
- Need ≥ 1 integrated kinematic variable
- Expansion in Λ/m_b
- Corrections first at $(\Lambda/m_b)^2$
- Moments of the differential rate can kill perturbative part: zero-in on non-perturb terms
- Expansion also in $\alpha_s(m_b)$

$$\Gamma = \Gamma_0^{(0)} + \frac{\alpha_s}{\pi}\Gamma_0^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \Gamma_0^{(2)} + \left(\Gamma_\pi^{(0)} + \frac{\alpha_s}{\pi}\Gamma_\pi^{(1)}\right)\frac{\mu_\pi^2}{m_b^2} + \left(\Gamma_G^{(0)} + \frac{\alpha_s}{\pi}\Gamma_G^{(1)}\right)\frac{\mu_G^2}{m_b^2} + \Gamma_D^{(0)}\frac{\rho_D^3}{m_b^3} + \Gamma_{LS}^{(0)}\frac{\rho_{LS}^3}{m_b^3} + \cdots$$

Inclusive $b \to c\ell\nu$

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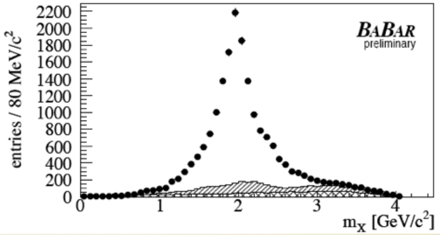
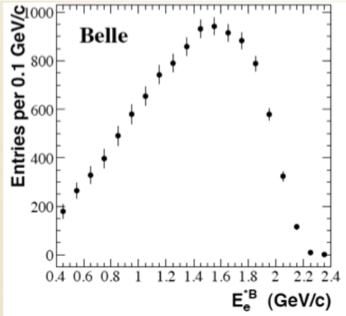
$$\mu_\pi^2 = -\langle B|\bar{b}(iD_\perp)^2 b|B\rangle$$
$$\mu_G^2 = -\langle B|\bar{b}(iD_\perp^\mu)(iD_\perp^\nu)\sigma_{\mu\nu} b|B\rangle$$

$$\Gamma = \Gamma_0^{(0)} + \frac{\alpha_s}{\pi}\Gamma_0^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \Gamma_0^{(2)} + \left(\Gamma_\pi^{(0)} + \frac{\alpha_s}{\pi}\Gamma_\pi^{(1)}\right)\frac{\mu_\pi^2}{m_b^2} + \left(\Gamma_G^{(0)} + \frac{\alpha_s}{\pi}\Gamma_G^{(1)}\right)\frac{\mu_G^2}{m_b^2} + \Gamma_D^{(0)}\frac{\rho_D^3}{m_b^3} + \Gamma_{LS}^{(0)}\frac{\rho_{LS}^3}{m_b^3} + \cdots$$

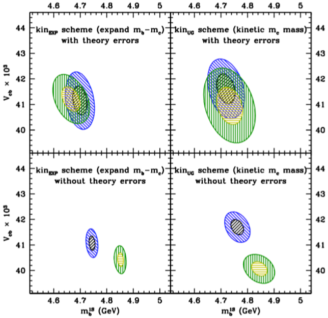
Recent fits:

- Six parameters: $m_{b,c}, \mu^2_{\pi,G}, \rho^3_{D,LS}$
- Includes corrections:
  α_s^2 at $(m_b)^0$
  α_s^1 at $(m_b)^{-2}$
- Rate $\Rightarrow |V_{cb}|^2$
- Moments $\Rightarrow$ non-pert parameters
- Same parameters as in
  - $b \to u\ell\nu$
  - Exclusive decays
  - Rare decays





Bauer *et al*, PRD70(2004) 094017



Gambino, Schwanda PRD89 (2014) 014022

m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\lvert V_{cb}\rvert$
4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86

Alberti *et al*, PRL114(2015)061802

m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

Exclusive: $B \to D^* \ell \nu$

$$\langle D^*(p',\epsilon)|\, V^\mu \,|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma$$

$$\langle D^*(p',\epsilon)|\, A^\mu \,|\bar{B}(p)\rangle = f_0\epsilon^{*\mu} + (\epsilon^*\cdot p)[a_+(p+p')^\mu + a_-(p-p')^\mu]$$

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2\sqrt{w^2-1}P(w)(\eta_{ew}\mathcal{F}(w))^2$$

- $w = p_B\cdot p_{D^*}/m_B m_{D^*}$
- $P(w)$ from phase space
- $\eta_{ew} = 1.007$ ew-correction
- HQS: $\mathcal{F}(1) = 1$ for $m_b = \infty$; corrections from μ_X^2, α_s
  - No $1/m_b$ corrections to FFs
  - Extrapolate to $w = 1$: see next slide
- Lattice $\Rightarrow \mathcal{F}(w)$; talk by Ran Zhou (next)

$$ff(w) = \mathcal{P}(z)\sum_{n=0}^{\infty} a_n z^n, \quad \sum_{n=0}^{\infty} a_n^2 \leq 1, \qquad z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$

- Uses: extrapolation, lattice fit
- Ingredients: Analyticity, crossing symmetry, unitarity
- $\mathcal{P}(z)$: computable (Blaschke, QCD)
- Physical region: $0 \leq z \leq 0.056$
- Not just a "parametrization:" $n \geq 2$ terms give no more than 1%

BGL:
Boyd, BG, Lebed, PLB353 (1995) 306;
Nuovo Cim. A109 (1996) 863;
NPB 461 (1996) 493;
PRD56 (1997) 6895-6911

CLN:
Caprini, Neubert, PLB380 (1996) 376;
C, Lellouch, N, NPB530 (1998) 153

FNAL/MILC:
PRD89 (2014) 114504

HFAG: $|V_{cb}|\mathcal{F}(1) = 35.90(45)\times 10^{-3}$, FNAL/MILC: $\mathcal{F}(1) = 0.906(13)$

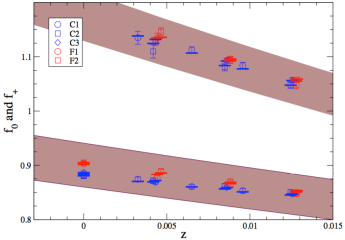
$$|V_{cb}| = 39.04(49)_{\rm exp}(53)_{\rm lat}(19)_{\rm QED}\times 10^{-3}$$

Exclusive: $B \to D\ell\nu$

$$\langle D(p')|V^\mu|B(p)\rangle = f_+(p+p')^\mu + (f_0 - f_+)\frac{m_B^2 - m_D^2}{q^2}q^\mu,\ q = p - p',\ f = f(q^2)$$

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3(\sqrt{w^2-1})^3(\eta_{\rm ew}\mathcal{G}(w))^2$$

Story in pictures: (HPQCD – PRD92 (2015) 054510)



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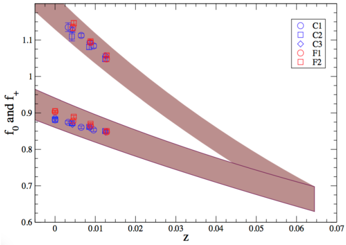

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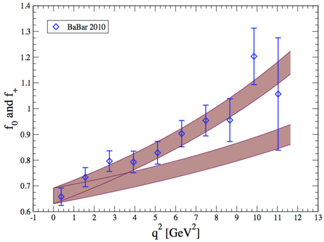

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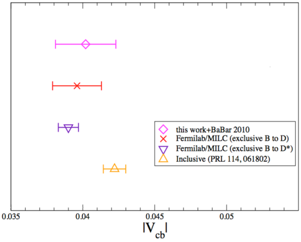
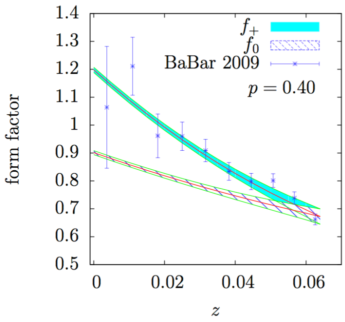
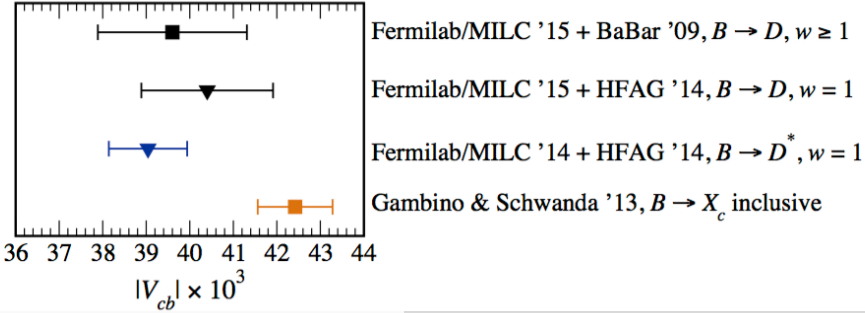


MILC PRD92 (2015) 034506

MILC uses BGL for fit, else no control of theoretical errors.

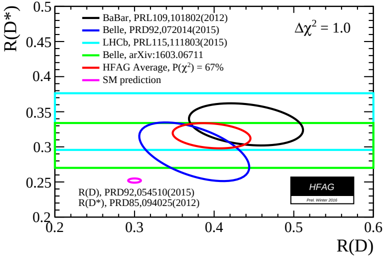




A global fit to $B \to D l \nu$

- $|V_{cb}|$=40.62(0.98) 10^{-3} preliminary (BGL,N=2)
- **$|V_{cb}|$=40.49(0.99) 10^{-3}** preliminary (BGL,N=3,4)
- based on z-expansion with unitarity constraints using Boyd,Grinstein,Lebed & Caprini,Lellouch,Neubert 1997
- assumes no correlation between FNAL and HPQCD, 3% syst error on Babar data, correct treatment of last bin, no finite size bin effect, updated Belle results 1510.03657
- CLN parameterization gives $|V_{cb}|$ = 40.85(95)10^{-3} but terrible fit (p-value < 10^{-5}) *when lattice results for f_0 are included.*
  We are getting too precise for CLN!!
- Non-zero recoil lattice results are crucial: only zero recoil leads to $|V_{cb}|$=39.6(2.1) 10^{-3} (BGL) 40.0(1.1) 10^{-3} (CLN)
- Very precise **R(D)=0.302(3),** 1.9σ from HFAG average

"$R_{D^{(*)}}$ anomaly" in $B \to D^{(*)}\ell\nu$: $\quad R_{D^{(*)}} = \frac{\mathcal{B}(\bar{B}\to D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(\bar{B}\to D^{(*)}\ell\bar{\nu})}$



- ***Excesses*** observed at more than 4σ

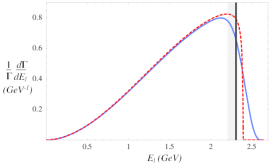
	$R(D)$	$R(D^*)$
BaBar	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018$
Belle	$0.375^{+0.064}_{-0.063} \pm 0.026$	$0.293^{+0.039}_{-0.037} \pm 0.015$
LHCb		$0.336 \pm 0.027 \pm 0.030$
Exp. average	0.388 ± 0.047	0.321 ± 0.021
SM expectation	0.300 ± 0.010	0.252 ± 0.005
Belle II, 50 ab^{-1}	± 0.010	± 0.005

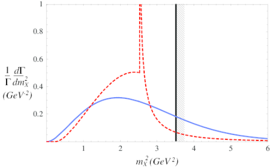
T. Freytsis *et al.* 1506.08896

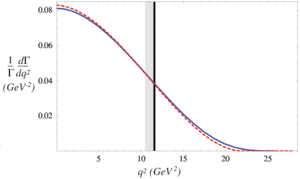
Inclusive: $B \to X_u \ell \nu$

The problem:

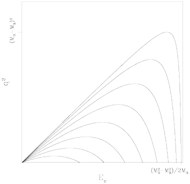
- $q^2 > (m_B - m_D)^2$ less sensitive to shape function (SF); sensitive to m_b, $(\Lambda/m_b)^3$ corrs, low rate, missing E
- $m_X < m_D$, sensitive to SF, higher rate
- $E_\ell > (m_b^2 - m_D^2)/2m_B$ sensitive to SF, low rate, simplest

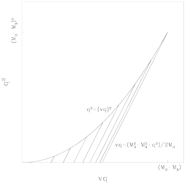




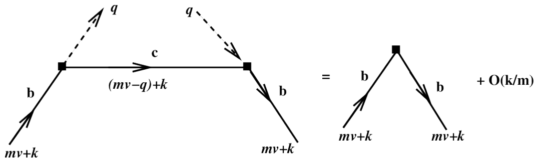


Lines of constant m_X
$(vq = E_\ell + E_\nu)$





Illustrate problem: $B \to X_s \gamma$



- Expand in $k/m_b \sim \Lambda_{\rm QCD}/m_b$ but $q/m_b \sim 1$.

$$\bar{\Gamma}\frac{i}{m_b \not{v} - \not{q} + \not{k} + i\epsilon}\Gamma = \frac{i}{m_b^2 - 2m_b q_0 + i\epsilon}\bar{\Gamma}(m_b\not{v} - \not{q})\bar{\Gamma} + \cdots$$

- Series in poles $q_0 - m_b/2$; endpoint region is $q_0 \approx m_b/2$
- $\Rightarrow$ retain all orders; $x = 2q_0/m_b$, $\mathrm{Im}\frac{1}{1-x-i\epsilon} = \pi\delta(1-x)$

$$\frac{d\Gamma}{dx} \propto \left(\delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2}\delta'(1-x) + \cdots\right) = f(1-x)$$

- Non-perturbative SF $2m_B f(\omega) = \langle B|\bar{h}_v \delta(\omega + in\cdot D)h_v|B\rangle$, $(n^2 = 0, n\cdot v = 1)$
(M. Neubert, PRD49(1994)4623, Bigi et al., IJMP A9(1994)2467)
- Endpoint spectra in $B \to X_u \ell \nu$ given in terms of $f(\omega)$ too
- Some analysis eliminate $f(\omega)$ from combined $B \to X_s \gamma$ and $B \to X_u \ell \nu$ analysis; other model SF subject to moment constraints

QCD Calculation	Phase Space Region	$\Delta\Gamma_{\text{theory}}$ (ps^{-1})	$\lvert V_{ub}\rvert(10^{-3})$
	$M_X \leq 1.55$ GeV	$39.3^{+4.7}_{-4.3}$	$4.17 \pm 0.15 \pm 0.12^{+0.24}_{-0.24}$
	$M_X \leq 1.70$ GeV	$46.1^{+5.0}_{-4.4}$	$3.97 \pm 0.17 \pm 0.14^{+0.20}_{-0.20}$
	$P_+ \leq 0.66$ GeV	$38.3^{+4.7}_{-4.3}$	$4.02 \pm 0.18 \pm 0.16^{+0.24}_{-0.23}$
BLNP	$M_X \leq 1.70$ GeV, $q^2 \geq 8$ GeV^2	$23.8^{+3.0}_{-2.4}$	$4.25 \pm 0.19 \pm 0.13^{+0.23}_{-0.25}$
	$M_X - q^2$, $p^*_\ell > 1.0$ GeV	$62.0^{+6.2}_{-5.0}$	$4.28 \pm 0.15 \pm 0.18^{+0.18}_{-0.20}$
	$p^*_\ell > 1.0$ GeV	$62.0^{+6.2}_{-5.0}$	$4.30 \pm 0.18 \pm 0.21^{+0.18}_{-0.20}$
	$p^*_\ell > 1.3$ GeV	$52.8^{+5.3}_{-4.3}$	$4.29 \pm 0.18 \pm 0.20^{+0.19}_{-0.20}$
	$M_X \leq 1.55$ GeV	$35.3^{+3.3}_{-3.5}$	$4.40 \pm 0.16 \pm 0.12^{+0.24}_{-0.19}$
	$M_X \leq 1.70$ GeV	$42.0^{+4.8}_{-4.8}$	$4.16 \pm 0.18 \pm 0.14^{+0.26}_{-0.22}$
	$P_+ \leq 0.66$ GeV	$36.9^{+5.5}_{-5.8}$	$4.10 \pm 0.19 \pm 0.17^{+0.37}_{-0.28}$
DGE	$M_X \leq 1.70$ GeV, $q^2 \geq 8$ GeV^2	$24.4^{+2.4}_{-2.0}$	$4.19 \pm 0.19 \pm 0.12^{+0.18}_{-0.19}$
	$M_X - q^2$, $p^*_\ell > 1.0$ GeV	$58.7^{+3.5}_{-3.2}$	$4.40 \pm 0.16 \pm 0.18^{+0.12}_{-0.13}$
	$p^*_\ell > 1.0$ GeV	$58.7^{+3.5}_{-3.2}$	$4.42 \pm 0.19 \pm 0.23^{+0.13}_{-0.13}$
	$p^*_\ell > 1.3$ GeV	$50.4^{+3.3}_{-3.0}$	$4.39 \pm 0.19 \pm 0.20^{+0.15}_{-0.14}$
	$M_X \leq 1.55$ GeV	$41.0^{+4.6}_{-3.8}$	$4.08 \pm 0.15 \pm 0.11^{+0.20}_{-0.21}$
	$M_X \leq 1.70$ GeV	$46.8^{+4.2}_{-3.6}$	$3.94 \pm 0.17 \pm 0.14^{+0.16}_{-0.17}$
	$P_+ \leq 0.66$ GeV	$44.0^{+8.6}_{-6.3}$	$3.75 \pm 0.17 \pm 0.15^{+0.30}_{-0.32}$
GGOU	$M_X \leq 1.70$ GeV, $q^2 \geq 8$ GeV^2	$24.7^{+3.2}_{-2.4}$	$4.17 \pm 0.18 \pm 0.12^{+0.22}_{-0.25}$
	$M_X - q^2$, $p^*_\ell > 1.0$ GeV	$60.2^{+3.0}_{-2.5}$	$4.35 \pm 0.16 \pm 0.18^{+0.09}_{-0.10}$
	$p^*_\ell > 1.0$ GeV	$60.2^{+3.0}_{-2.5}$	$4.36 \pm 0.19 \pm 0.23^{+0.09}_{-0.10}$
	$p^*_\ell > 1.3$ GeV	$51.8^{+2.8}_{-2.3}$	$4.33 \pm 0.18 \pm 0.20^{+0.10}_{-0.11}$
	$M_X \leq 1.55$ GeV	$47.1^{+5.2}_{-4.3}$	$3.81 \pm 0.14 \pm 0.11^{+0.18}_{-0.20}$
	$M_X \leq 1.70$ GeV	$52.3^{+5.4}_{-4.5}$	$3.73 \pm 0.16 \pm 0.13^{+0.17}_{-0.18}$
	$P_+ \leq 0.66$ GeV	$48.9^{+5.6}_{-4.6}$	$3.56 \pm 0.16 \pm 0.15^{+0.18}_{-0.19}$
ADFR	$M_X \leq 1.70$ GeV, $q^2 \geq 8$ GeV^2	$30.9^{+3.0}_{-2.5}$	$3.74 \pm 0.16 \pm 0.11^{+0.16}_{-0.17}$
	$M_X - q^2$, $p^*_\ell > 1.0$ GeV	$62.0^{+5.7}_{-5.0}$	$4.29 \pm 0.15 \pm 0.18^{+0.18}_{-0.19}$
	$p^*_\ell > 1.0$ GeV	$62.0^{+5.7}_{-5.0}$	$4.30 \pm 0.19 \pm 0.23^{+0.18}_{-0.19}$
	$p^*_\ell > 1.3$ GeV	$53.3^{+5.1}_{-4.4}$	$4.27 \pm 0.18 \pm 0.19^{+0.18}_{-0.19}$

BaBar PRD86 (2012) 032004

BLNP: Bosch, *et al* PRD 72 (2005)073006
DGE: "dressed gluon exponentiation" Andersen, Gardi, JHEP 0601 (2006)097
ADFR: Aglietti, *et al* Eur. Phys. J. C 59(2009)831
GGOU: Gambino, *et al* JHEP 0710 (2007)058

PDG: $$|V_{ub}| = (4.41 \pm 0.15_{\text{exp}}{}^{+0.15}_{-0.17}{}_{\text{th}}) \times 10^{-3}$$

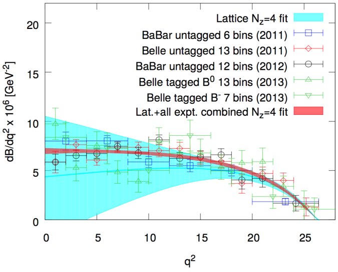
Exclusive $|V_{ub}|$: $B \to \pi\ell\nu$

Much as in $B \to D\ell\nu$

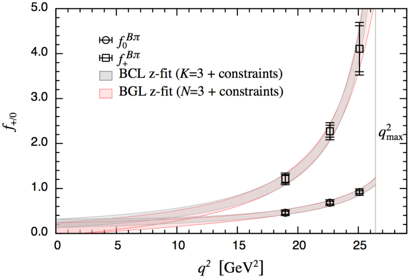
$$\langle\pi(p')|V^\mu|B(p)\rangle = f_+(p+p')^\mu + (f_0 - f_+)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{ub}|^2|p_\pi|^3|f_+(q^2)|^2$$

- $f_{+,0}(q^2)$ from Lattice: next talk Ran Zhou
  UKQCD PRD91(2015)074510; FNAL/MILC PRD92(2015)014024
- Interpolation: z-fit
  Boyd, BG, Lebed, PRL74(1995)4603; Bourrely, Caprini, Lellouch PRD79(2009)013008
- Fit lattice and data to z-fit Arnesen *et al*, PRL95(2005)071802



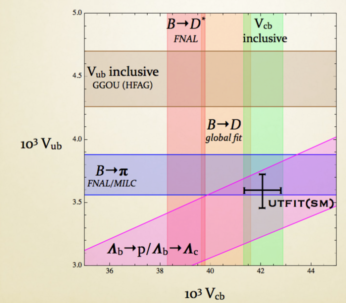
FNAL/MILC



UKQCD

Figure cum Conclusions

Gambino, Beauty 2016



- $|V_{cb}|$ incl. vs D^*(FNAL/MILC) is $\sim 8\%(\sim 3\sigma)$
- RH currents won't do

$$|V_{cb}|_{\rm incl} = |V_{cb}|(1+\tfrac{1}{2}\epsilon^2)$$
$$|V_{cb}|_{D^*} = |V_{cb}|(1+\epsilon)$$
$$|V_{cb}|_{D} = |V_{cb}|(1-\epsilon)$$

- More general NP dim-6 ops can't either
  Crivellin, Pokorski 1407.1320
- Tension decreased on $|V_{ub}|$ Bernlochner, Ligeti, Turczyk, PRD90(2014)094003