Mixing and CP violation in the Bd and Bs systems in ATLAS

Evelina Bouhova-Thacker

On behalf of the ATLAS collaboration

HQL2016, Virginia Tech, Blacksburg, Virginia, 22-27 May, 2016

Topics

Measurement of the CP-violating phase ϕ_s and the B_s^0 meson decay width difference with $B_s^0 \to J/\psi\phi$ decays in ATLAS

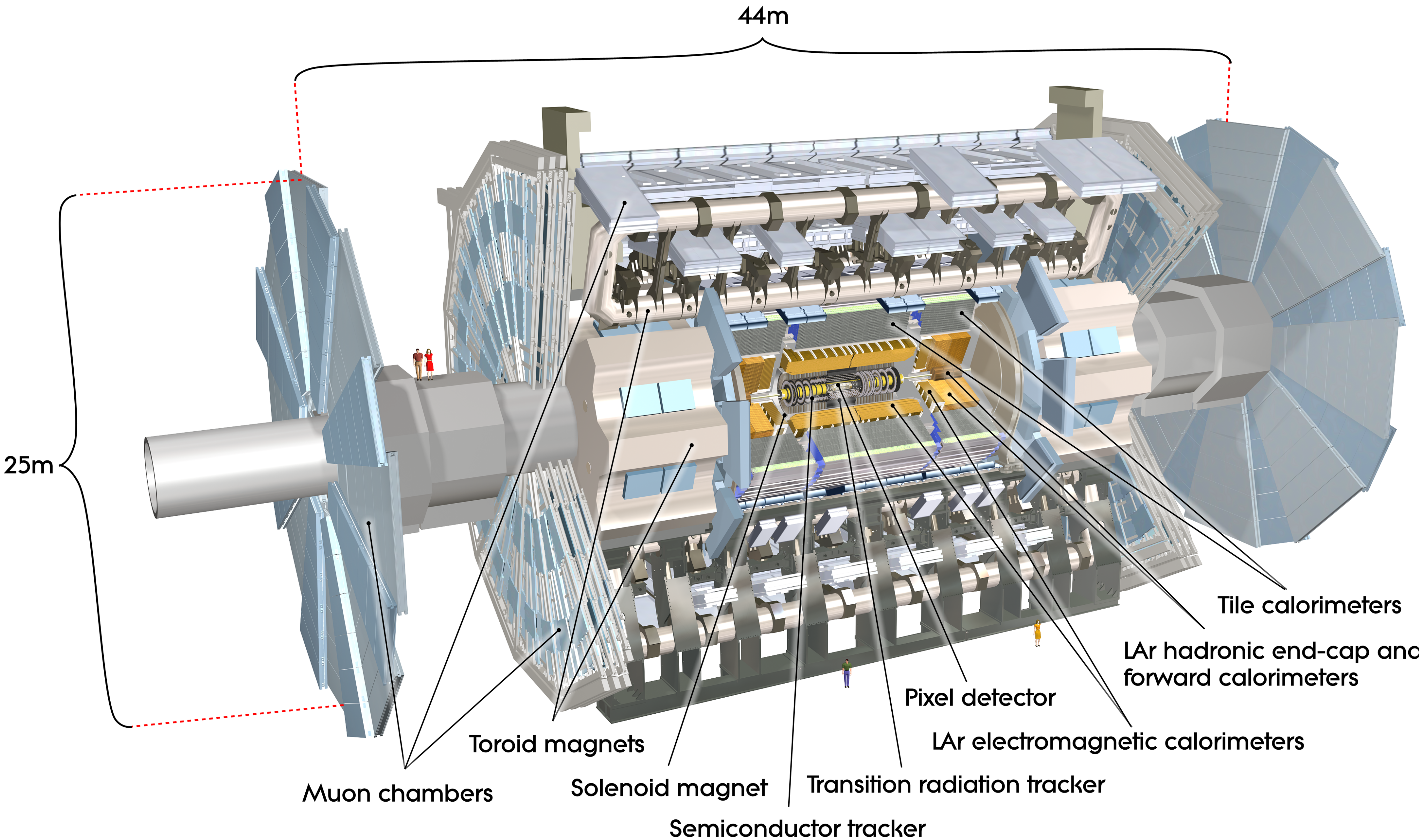
arXiv:1601.03297, Submitted to JHEP

Measurement of the relative width difference of the B^0-$\bar{B}^0$ system with the ATLAS detector

arXiv:1605.07485, Submitted to JHEP

New

The ATLAS Detector



Muon Spectrometer:
$|\eta| < 2.7$
air core toroids(average 0.5T)
gas-based muon chambers
Muon trigger and reconstruction
Momentum resolution < 10%
$up\ to\ E(\mu) \sim 1\ TeV$

3-level trigger, reducing the rate from 40 MHz to ~200 Hz

EM Calorimeter:
Pb-LAr accordion
e/γ trigger, identification and measurement
Resolution:
$\sigma/E \sim 10\%/\sqrt{E}$

Inner Detector:
Si Pixels, Si Strips, TRT straws
Precise tracking and vertexing
pT resolution:
$\sigma/p_T \sim 3.8 \times 10^{-4}\ p_T(GeV) \oplus 0.015$
$|\eta| < 2.5,\ \ B = 2T$

HAD Calorimetry:
$|\eta| < 5$
Fe/scintillator tiles (central), Cu/W-LAr (fwd)
Trigger and measurement of jets and MET
Resolution:
$\sigma/E \sim 50\%/\sqrt{E} \oplus 0.03$

CP Violation in $B_s^0 \to J/\psi\phi$

CPV due to interference between:

direct decay $B_s^0 \to J/\psi\phi$

flavour oscillation $B_s^0 \to \bar{B}_s^0 \to J/\psi\phi$

CP violating phase ϕ_s weak phase difference between the $b \to c\bar{c}s$ decay amplitude and the B_s^0 - $\bar{B}_s^0$ mixing amplitude

SM prediction: $\phi_s \simeq -2\beta_s = 2\arg(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}) = -0.0363^{+0.0016}_{-0.0015}$ Phys. Rev. D 84 (2011) 033005

ϕ_s can be modified significantly by new physics: $\phi_s = \phi_s^{SM} + \phi_S^{NP}$

Precise measurements of ϕ_s by the LHC experiments have considerably reduced the possible amount of new physics contribution

B_s^0 meson decay width difference $\Delta\Gamma_s = \Gamma_L - \Gamma_H$

prediction: $\Delta\Gamma_s = 0.087 \pm 0.021\ \mathrm{ps}^{-1}$ arXiv:1102.4274 [hep-ph]

Analysis of ATLAS Run-1 Data

The analysis of the 2011 at $\sqrt{s} = 7\ \mathrm{TeV}$ and 2012 at $\sqrt{s} = 8\ \mathrm{TeV}$ data samples completed

The results obtained with the 2011 data published:

Phys. Rev. D90 (2014) 052007

$$\phi_s = 0.12 \pm 0.25(\mathrm{stat}) \pm 0.05(\mathrm{syst})$$

$$\Delta\Gamma_s = 0.053 \pm 0.021(\mathrm{stat}) \pm 0.010(\mathrm{syst})$$

The results obtained with the 2012 data released recently

arXiv:1601.03297

and are discussed in more detail

Combination of the 2011 and 2012 results

$B_s^0 \to J/\psi\phi$ Reconstruction

Events selected using dimuon triggers $p_T^{\mu} > 4 \text{ or } 6 \text{ GeV}$

J/ψ from fit to pairs of muons

$\phi \to K^+K^-$ reconstructed from pairs of oppositely charged tracks

$p_T > 1 \text{ GeV}$ $|\eta| < 2.5$

$B_s \to J/\psi\phi$ four track vertex fit

with J/ψ mass constraint

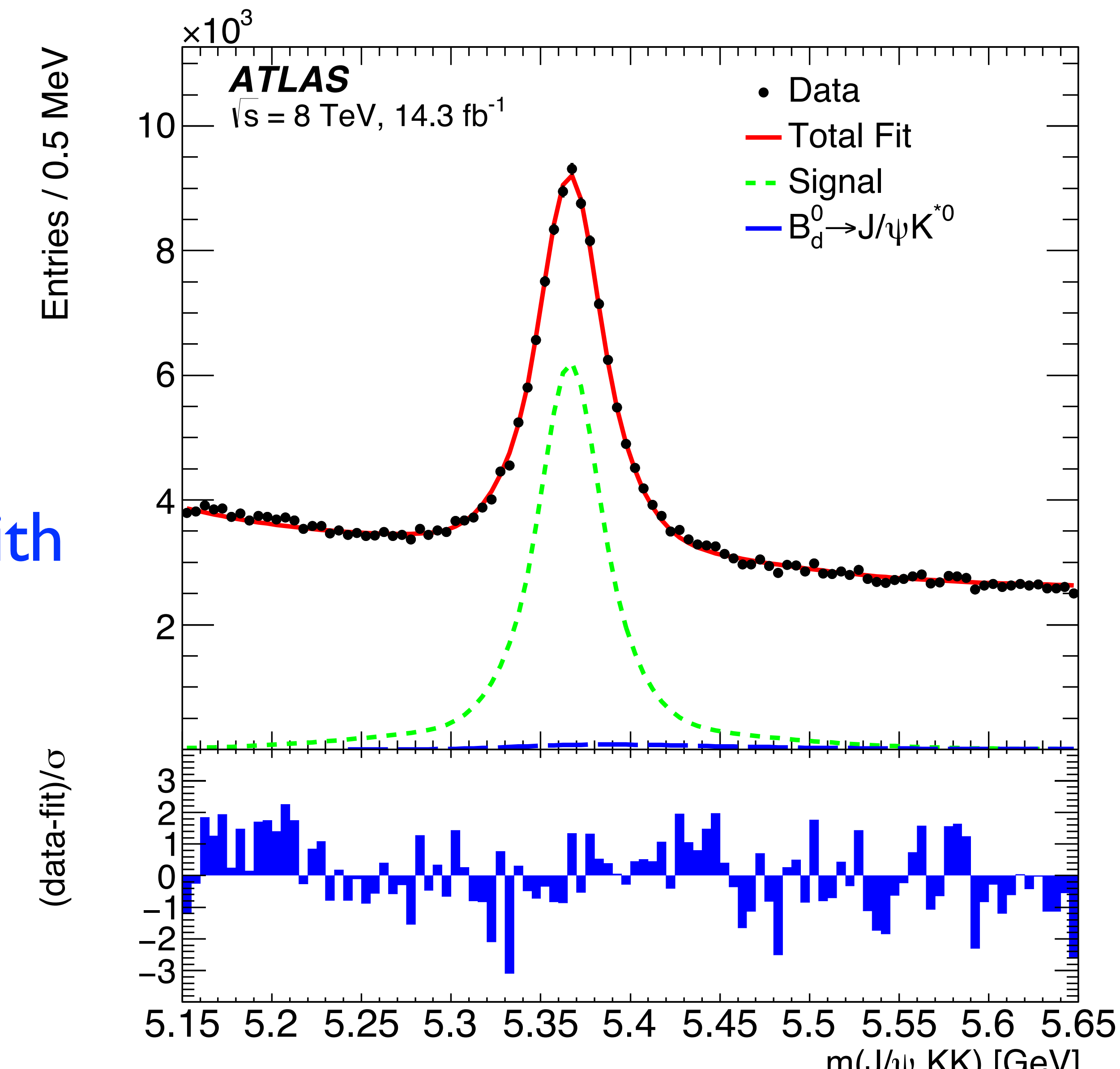
Lifetime-unbiased reconstruction

In total 376k event candidates selected with

$5.15 < m(J/\psi\phi) < 5.65 \text{ GeV}$

about 75k signal candidates

~3.5 times that of the 2011 sample



mass fit projection

$B_s^0 \to J/\psi\phi$ Flavour Tagging

The initial B_s^0 flavour is determined by the flavour of the opposite B meson

p_T weighted sum of charges from the decay

Muon or Electron - cone charge $\Delta R < 0.5$

Jet-charge - using jet with the highest value of b-tag weight

Calibrated with $B^{\pm} \to J/\psi K^{\pm}$

Tagger	Efficiency [%]	Dilution [%]	Tagging Power [%]
Combined μ	4.12 ± 0.02	47.4 ± 0.2	0.92 ± 0.02
Electron	1.19 ± 0.01	49.2 ± 0.3	0.29 ± 0.01
Segment-tagged μ	1.20 ± 0.01	28.6 ± 0.2	0.10 ± 0.01
Jet-charge	13.15 ± 0.03	11.85 ± 0.03	0.19 ± 0.01
Total	19.66 ± 0.04	27.56 ± 0.06	1.49 ± 0.02

$B_s^0 \to J/\psi\phi$ Fit Methodology

Time-dependent angular analysis

Observables:

- $J/\psi\phi$ mass and its uncertainty
- p_T of the B_s^0 candidate
- Proper decay length and its uncertainty
- Angles describing the kinematics of the B_s^0 decay
- Flavour tagging (applied probabilistically)

The properties of the $B_s^0 \to J/\psi\phi$ decay are obtained from an unbinned maximum likelihood fit

Physical parameters:

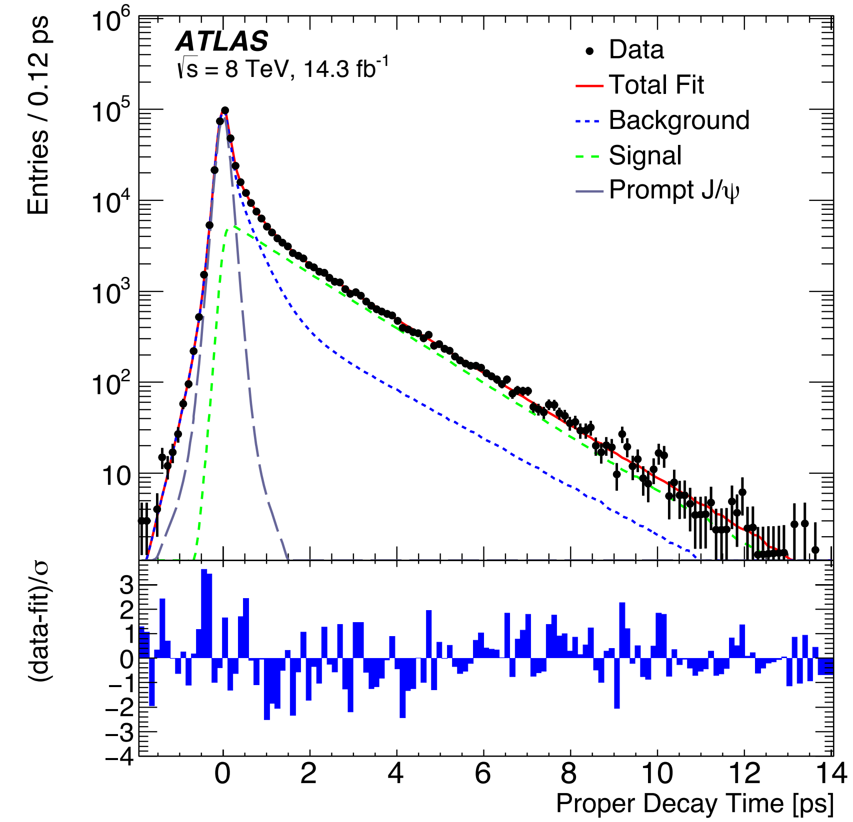
$$\phi_s,\ \Delta\Gamma_s,\ \Gamma_s$$

$$|A_0(0)|^2,\ |A_\parallel(0)|^2,\ |A_S(0)|^2,\ \delta_\perp,\ \delta_\parallel,\ \delta_\perp - \delta_S$$

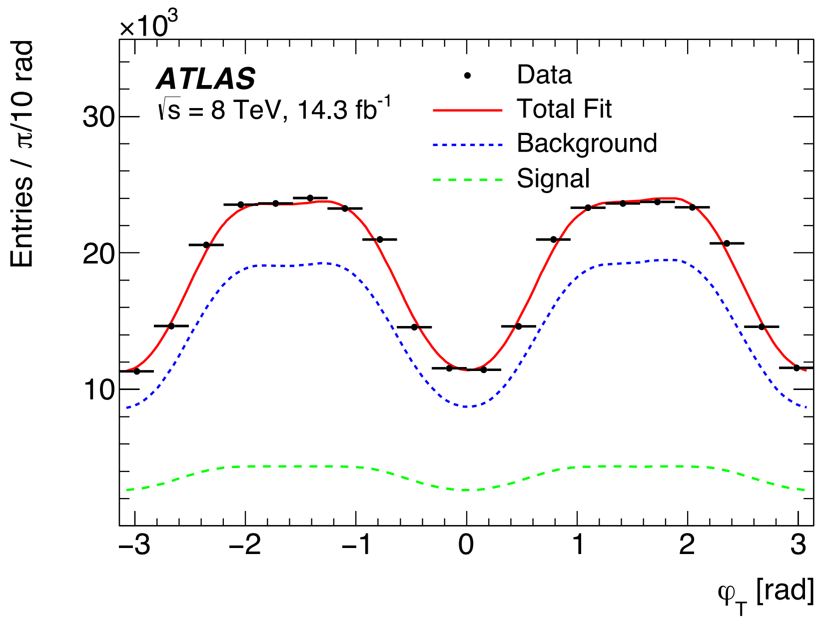
$B_s^0 \to J/\psi\phi$ 2012 Fit Results

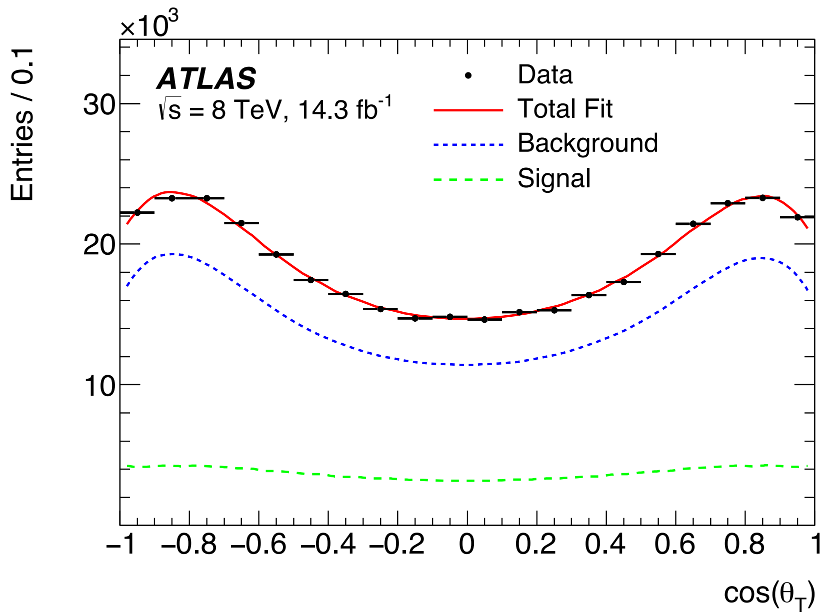
$$\phi_s = -0.123 \pm 0.089(\text{stat}) \pm 0.041(\text{syst})$$

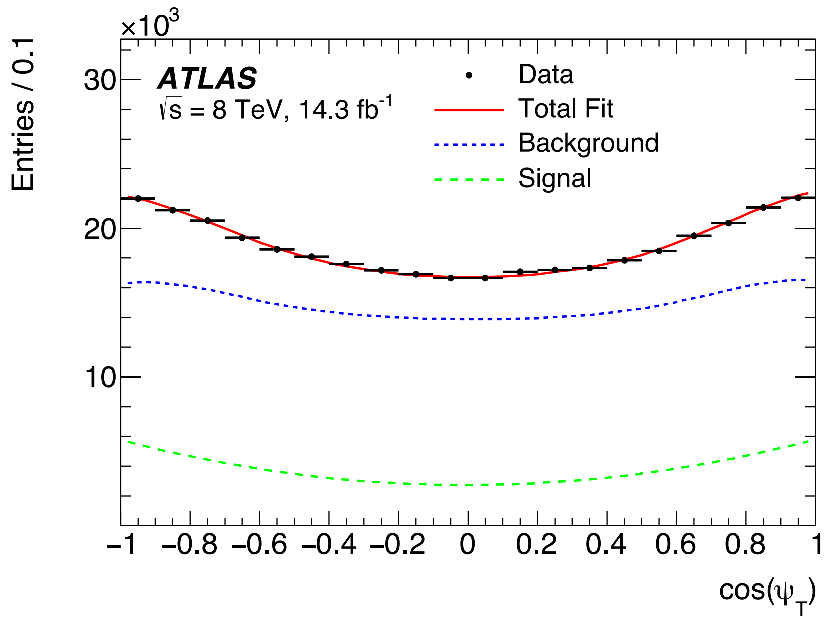
$$\Delta\Gamma_s = 0.096 \pm 0.013(\text{stat}) \pm 0.007(\text{syst})$$



The fit projections of the mass, lifetime and angles demonstrates the good quality of the fit



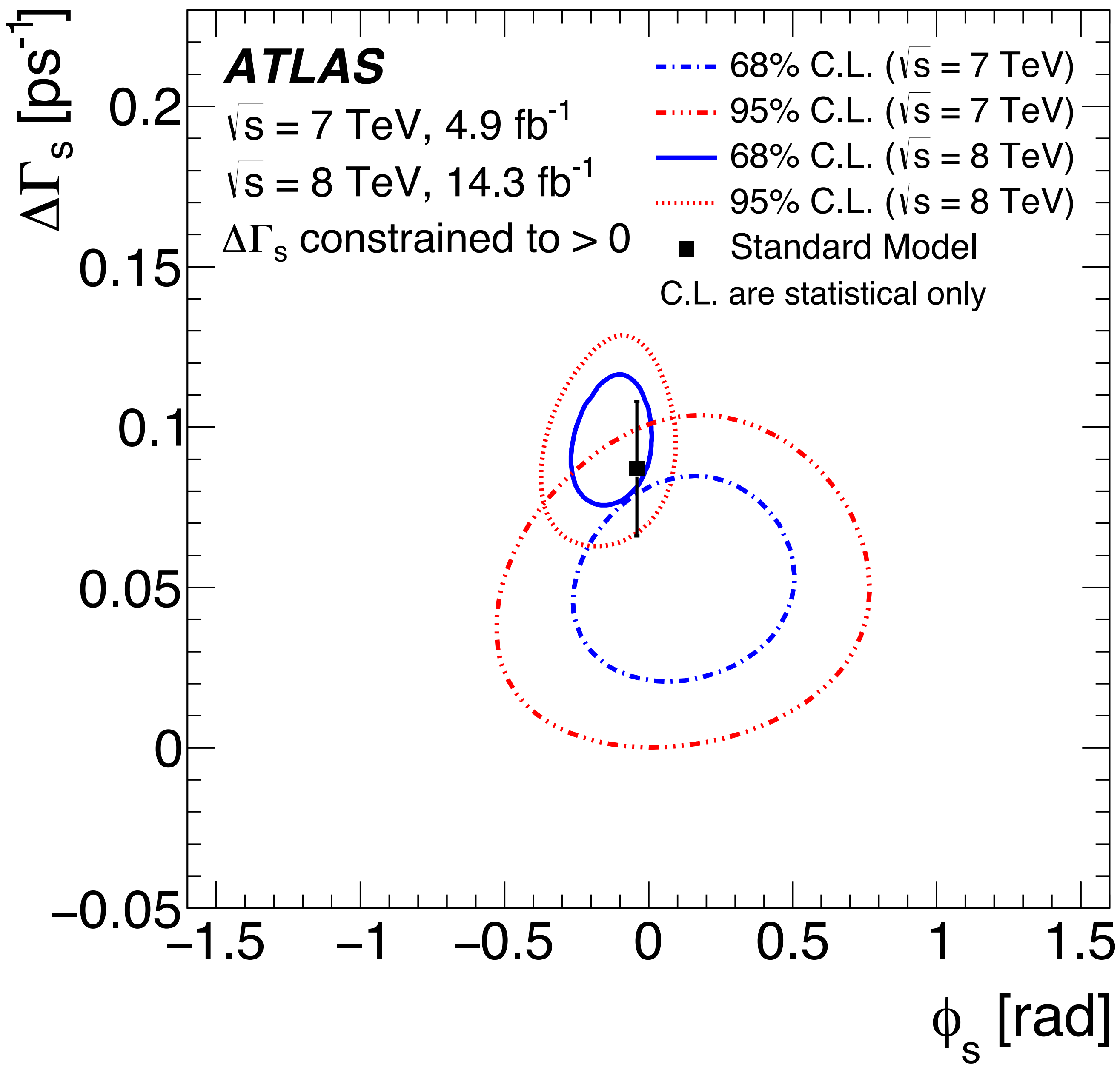


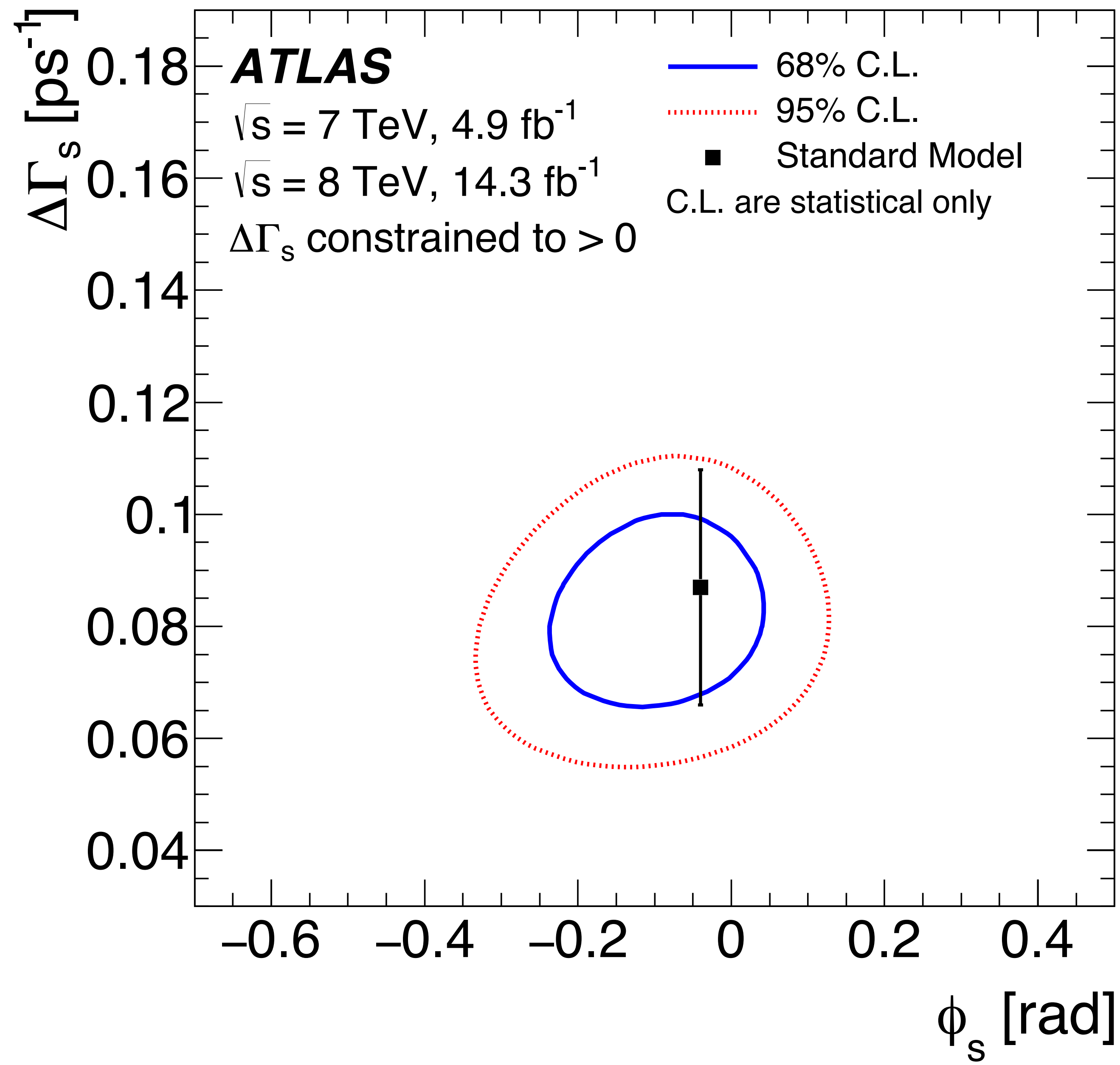


Combination of 7 TeV And 8 TeV Results

$$\phi_s = -0.098 \pm 0.084(\text{stat}) \pm 0.040(\text{syst})$$

$$\Delta\Gamma_s = -0.083 \pm 0.011(\text{stat}) \pm 0.007(\text{syst})$$

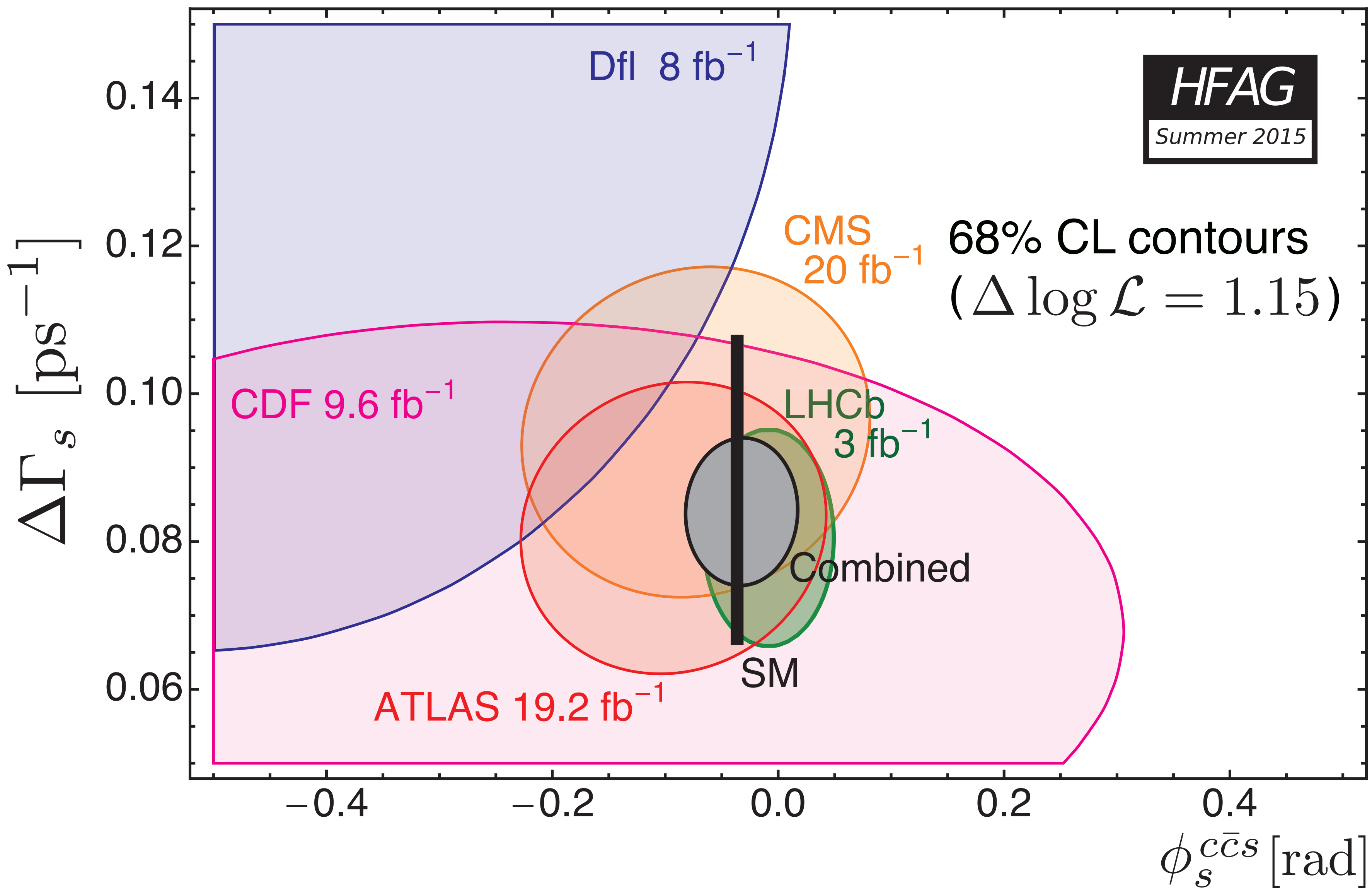




The 2011 and 2012 results are consistent

The result agrees well with the SM prediction

Comparison with Other Experiments



ATLAS contour using 2015 ATLAS Preliminary results
current contour (using arXiv:1601.03297) is slightly different

Existing measurements consistent between each other and with the SM

Room for New Physics in CPV ϕ_s

Need Run-2 and LHC upgrade

Width Difference of B_d^0

New

$\Delta\Gamma_d$ is one of the parameters describing the time evolution of the B_d system

SM prediction: $\Delta\Gamma_d/\Gamma_d = (0.42 \pm 0.08) \times 10^{-2}$ arXiv: 1102.4274 [hep-ph]

The current experimental uncertainty is too large to perform a stringent test of the SM prediction: World average $\Delta\Gamma_d/\Gamma_d = (0.1 \pm 1.0) \times 10^{-2}$

Relatively large variation of $\Delta\Gamma_d$ due to a possible New Physics contribution would not contradict other existing SM results arXiv:1404.2531 [hep-ph]

Additional measurements are needed to constrain this quantity and verify or disprove the SM prediction

$\Delta\Gamma_d$ Measurement Method

$\Delta\Gamma_d$ is obtained from the ratio of the proper decay time distributions of $B^0 \to J/\psi K_S$ and $B^0 \to J/\psi K^{*0}$ decays

$J/\psi \to \mu^+\mu^-$ $\quad$ $K_S \to \pi^+\pi^-$ $\quad$ $K^{*0} \to K^+\pi^-$

The decay time distribution of $B^0 \to J/\psi K_S$ depends on $\Delta\Gamma_d$:

$$\Gamma[t, J/\psi K_S] \propto \mathrm{e}^{-\Gamma_d t} \left[\cosh\frac{\Delta\Gamma_d t}{2} + \cos(2\beta)\sinh\frac{\Delta\Gamma_d t}{2} - A_{\mathrm{P}}\sin(2\beta)\sin(\Delta m_d t)\right]$$

t is the proper decay time of the B^0 meson

β is the angle of the CKM unitarity triangle

A_{P} is the B^0 meson production asymmetry

The decay time distribution of $B^0 \to J/\psi K^{*0}$ is almost not sensitive on $\Delta\Gamma_d$:

$$\Gamma[t, J/\psi K^{*0}] \propto \mathrm{e}^{-\Gamma_d t}\cosh\frac{\Delta\Gamma_d t}{2}$$

The $B^0 \to J/\psi K^{*0}$ and $\bar{B}^0 \to J/\psi \bar{K}^{*0}$ decays are added together

$\Delta\Gamma_d$ Measurement Method

$\Delta\Gamma_d$ is determined from the ratio of the two decay modes, which helps to reduce the systematic uncertainties of the measurement

The factor $\mathrm{e}^{-\Gamma_d t}$ cancels in the ratio, increasing the sensitivity to $\Delta\Gamma_d$

Same number of charged particles in the two decay modes

Some residual differences due to the displaced K_S vertex

Both decay modes are triggered by the same dimuon triggers and the other particles from the B^0 decay are not used in the trigger or the proper decay time determination

The measurement is performed using the B^0 proper decay length $L_{\rm prop}^B$

$$L_{\rm prop}^B = \frac{(x^{J/\psi} - x^{\rm PV})p_x^B + (y^{J/\psi} - y^{\rm PV})p_y^B}{(p_{\rm T}^B)^2} m_{B^0}$$

The origin of the B^0 $(x^{\rm PV}, y^{\rm PV})$ is measured using a PV fit in which the decay products of the B^0 are removed

The primary vertex which has the smallest $|\delta z|$ relative to the B^0 trajectory is selected as the PV of B^0 production

The position of the B^0 decay is defined by the J/ψ decay vertex $(x^{J/\psi}, y^{J/\psi})$

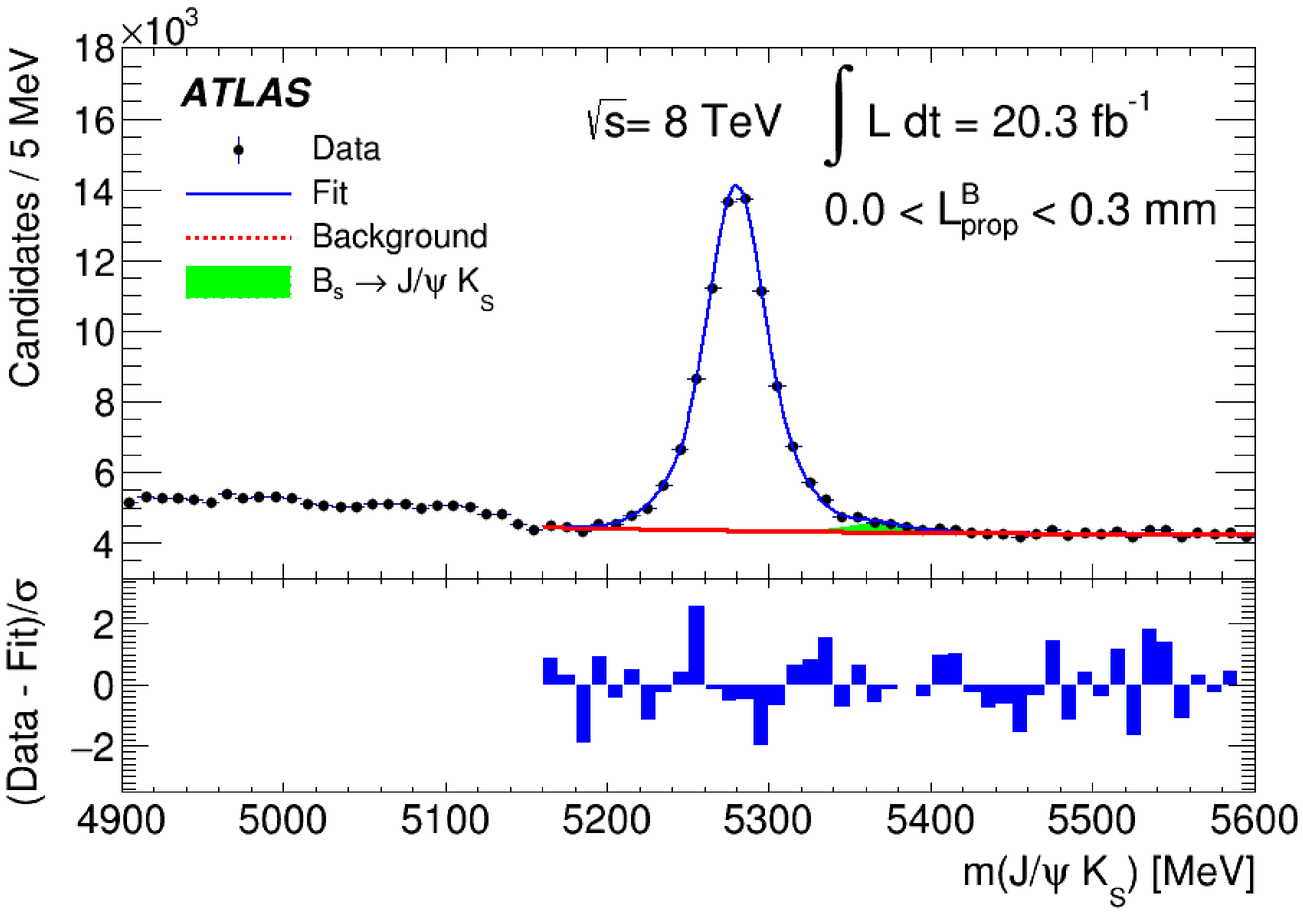
The B^0 momentum is determined by the vertex fit; world average used for the mass

$\Delta\Gamma_d$ Signal Extraction

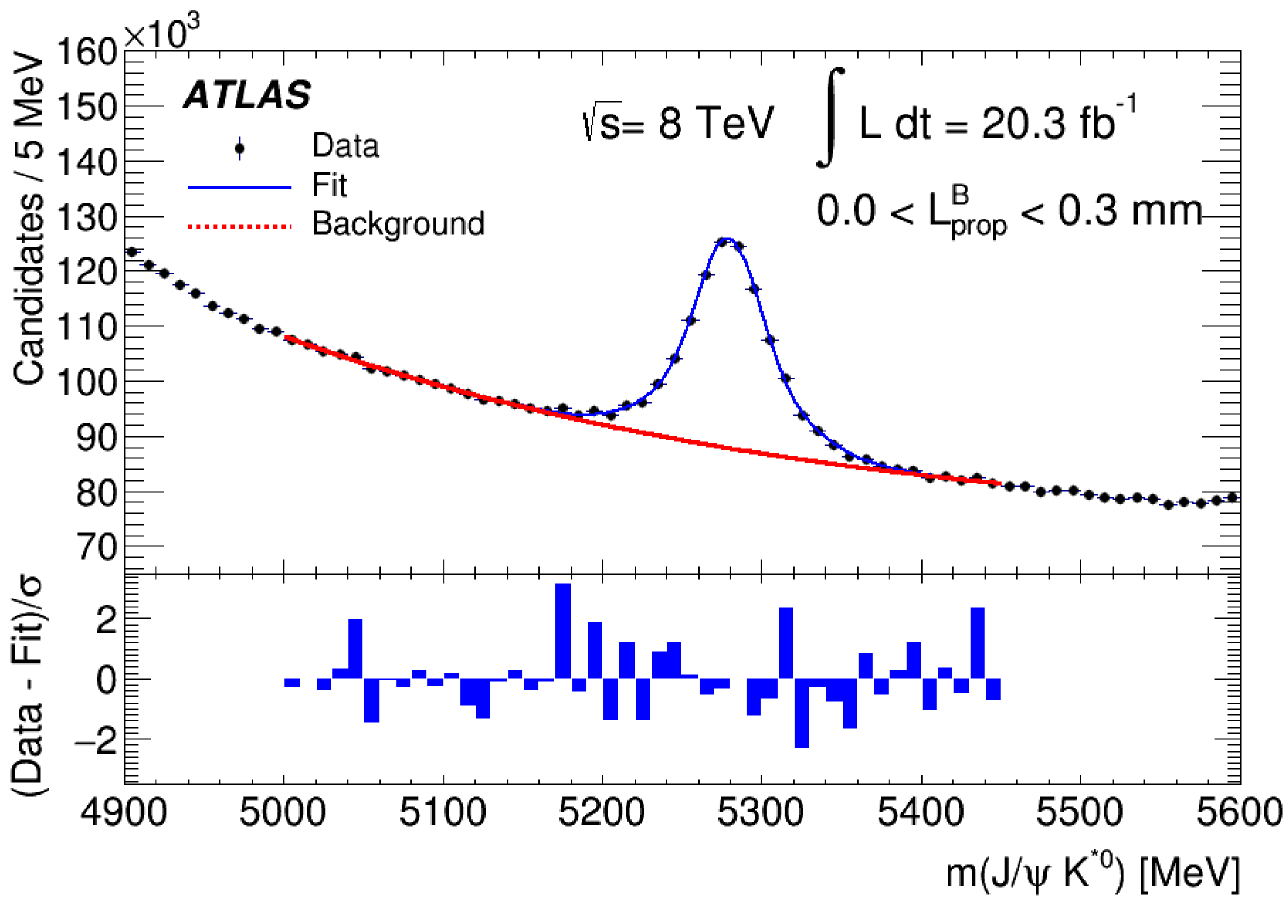
The range of L_{prop}^{B} is divided into 10 bins between -0.3 and 6.0 mm

The number of signal decays for each decay mode is determined from a fit to the invariant mass distribution in each bin

$B^0 \to J/\psi K_S$



$B^0 \to J/\psi K^{*0}$



Total number of $B^0 \to J/\psi K_S$ decays: ~28k in 2011, ~ 111k in 2012

Total number of $B^0 \to J/\psi K^{*0}$ decays: ~129k in 2011, ~ 556k in 2012

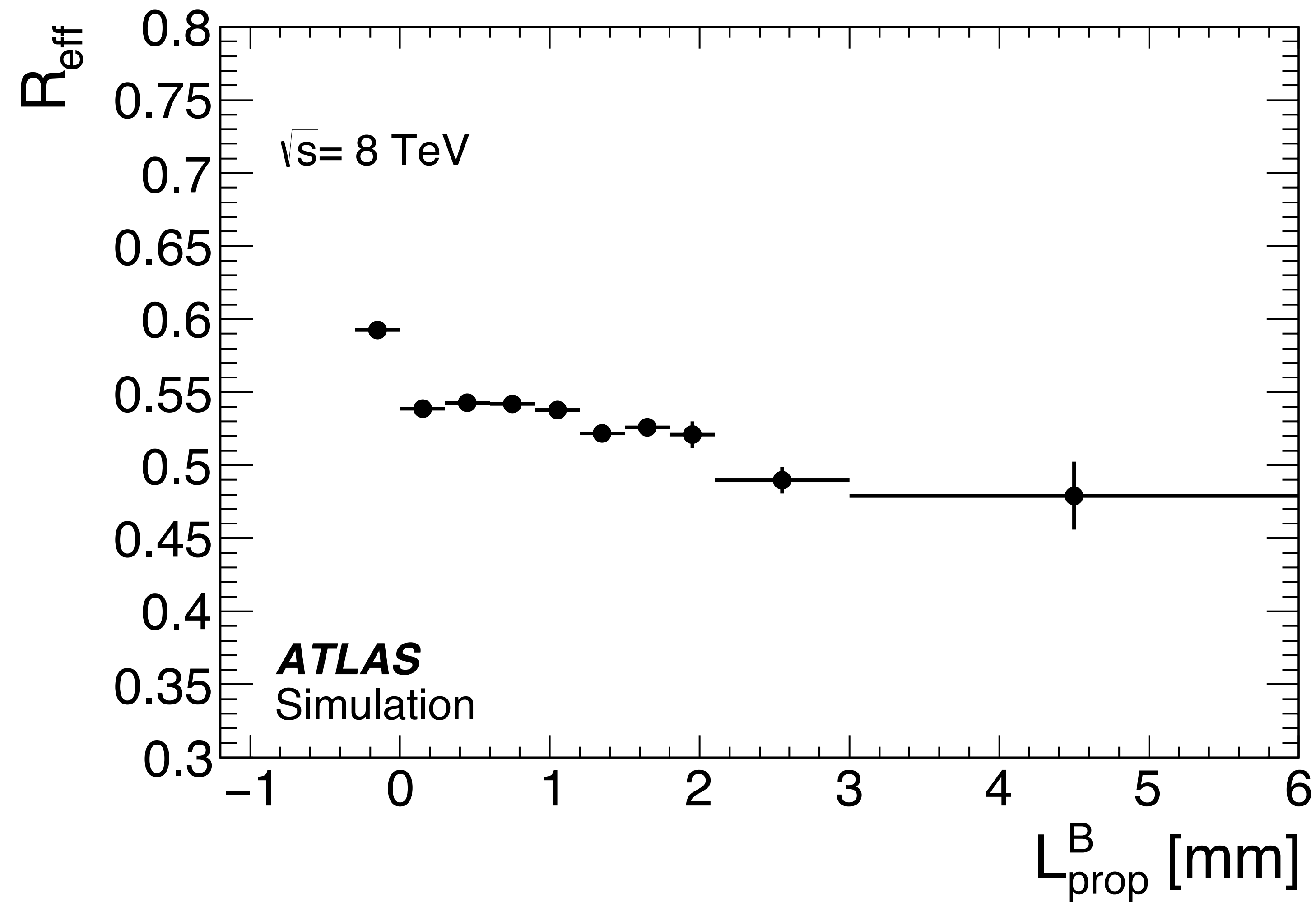
$\Delta\Gamma_d$ Ratio of Efficiencies

The numbers of $B^0 \to J/\psi K_S$ and $B^0 \to J/\psi K^{*0}$ decays in each interval are used to obtain the ratio of the L^B_{prop} distributions of the two decays modes

The proper decay length ratio is corrected by the ratio of the reconstruction efficiencies of the two decay modes

The deviation of this ratio from the average value does not exceed 5% for $L^B_{\text{prop}} < 2\ \text{mm}$

Ratio of reconstruction efficiencies of of $B^0 \to J/\psi K_S$ and $B^0 \to J/\psi K^{*0}$ decays
The normalisation is arbitrary



B^0 Production Asymmetry A_P

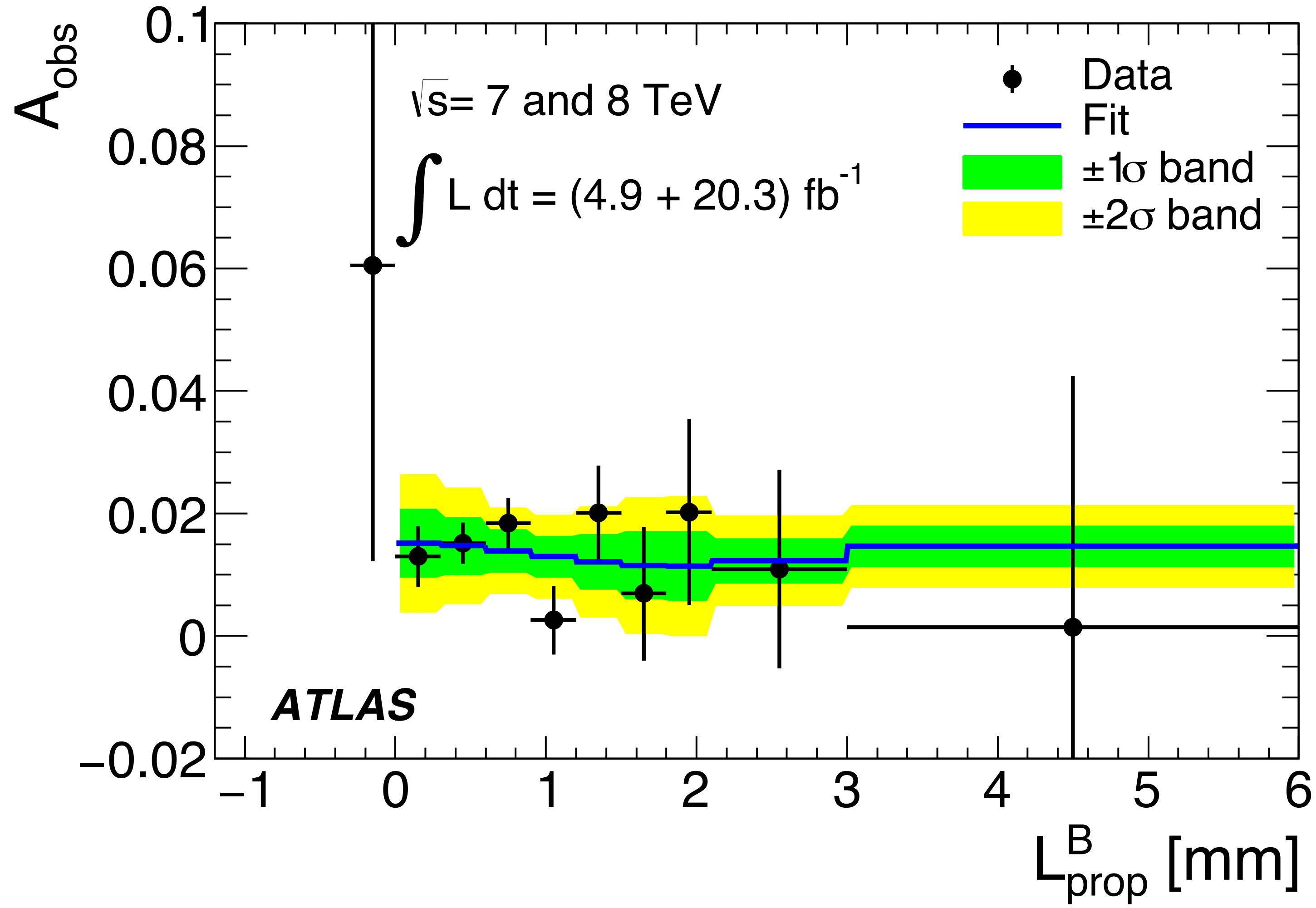
Measured from the charge asymmetry $A_{\rm obs}$ of the $B^0 \to J/\psi K^{*0}$ decay as a function of $L^B_{\rm prop}$

expected to be ~ 1%

should oscillate if there is a difference in the production rate between B^0 and $\bar{B}^0$

offset from 0 due to $A_{\rm det}$

difference in the $K^+ - K^-$ reconstruction efficiencies, no lifetime independency



ATLAS result: $A_P = (+0.25 \pm 0.48 \pm 0.05) \times 10^{-2}$

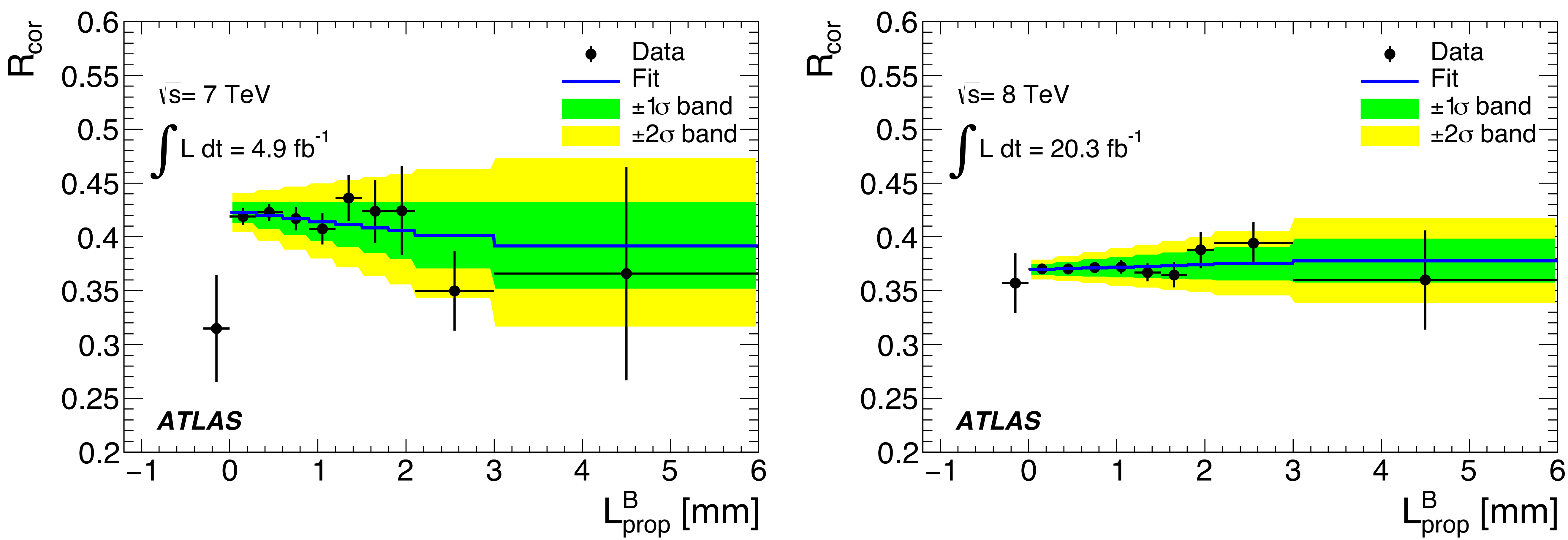
Measurement of $\Delta\Gamma_d$

The ratio of L^B_{prop} distributions, corrected by the ratio of reconstruction efficiencies is fitted to extract $\Delta\Gamma_d/\Gamma_d$

$$\Delta\Gamma_d/\Gamma_d = (-2.8 \pm 2.2\ (\mathrm{stat.}) \pm 1.5\ (\mathrm{MC\ stat.})) \times 10^{-2} \qquad (2011)$$

$$\Delta\Gamma_d/\Gamma_d = (+0.8 \pm 1.3\ (\mathrm{stat.}) \pm 0.5\ (\mathrm{MC\ stat.})) \times 10^{-2} \qquad (2012)$$

arbitrary normalisation



Combination of the 7 TeV and 8 TeV results:

$$\Delta\Gamma_d/\Gamma_d = (-0.1 \pm 1.1\ (\mathrm{stat.}) \pm 0.9\ (\mathrm{syst.})) \times 10^{-2}$$

Comparison with Other Experiments

The ATLAS result is consistent with other measurements of $\Delta\Gamma_d/\Gamma_d$

$$\Delta\Gamma_d/\Gamma_d = (-0.1 \pm 1.1 \pm 0.9) \times 10^{-2} \quad \text{(ATLAS)}$$

$$\Delta\Gamma_d/\Gamma_d = (-4.4 \pm 2.5 \pm 1.1) \times 10^{-2} \quad \text{(LHCb)}$$

$$\Delta\Gamma_d/\Gamma_d = (+1.7 \pm 1.8 \pm 1.1) \times 10^{-2} \quad \text{(Belle)}$$

$$\Delta\Gamma_d/\Gamma_d = (+0.8 \pm 3.7 \pm 1.8) \times 10^{-2} \quad \text{(Babar)}$$

Currently it is the most precise single measurement

It is consistent with the SM prediction

$$\Delta\Gamma_d/\Gamma_d = (0.42 \pm 0.08) \times 10^{-2}$$

Summary

The analysis of Run-1 ATLAS data reveals no anomalies in the $B_s \to J/\psi\phi$ decay

$$\phi_s = -0.098 \pm 0.084(\text{stat}) \pm 0.040(\text{syst})$$

$$\Delta\Gamma_s = -0.083 \pm 0.011(\text{stat}) \pm 0.007(\text{syst})$$

ATLAS has obtained the most precise single measurement of $\Delta\Gamma_d$

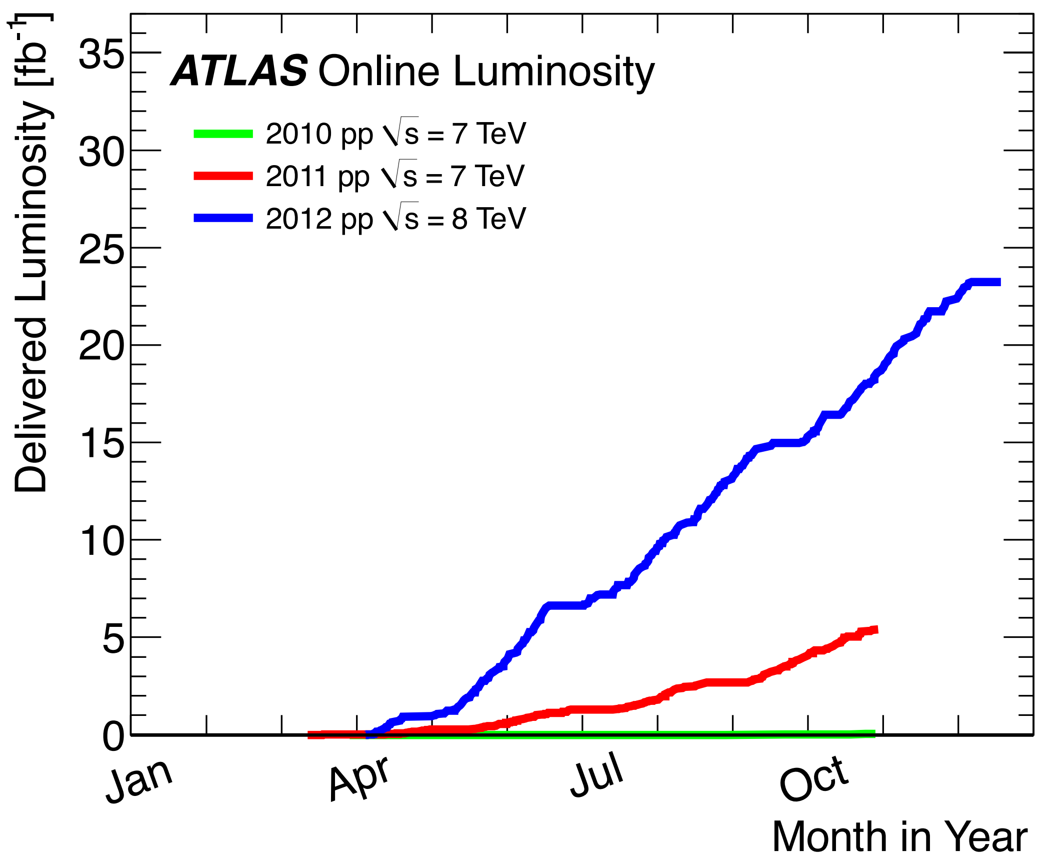
$$\Delta\Gamma_d/\Gamma_d = (-0.1 \pm 1.1\ (\text{stat}) \pm 0.9\ (\text{syst})) \times 10^{-2}$$

LHC Run-II data should allow the precision of these measurements to be considerably improved

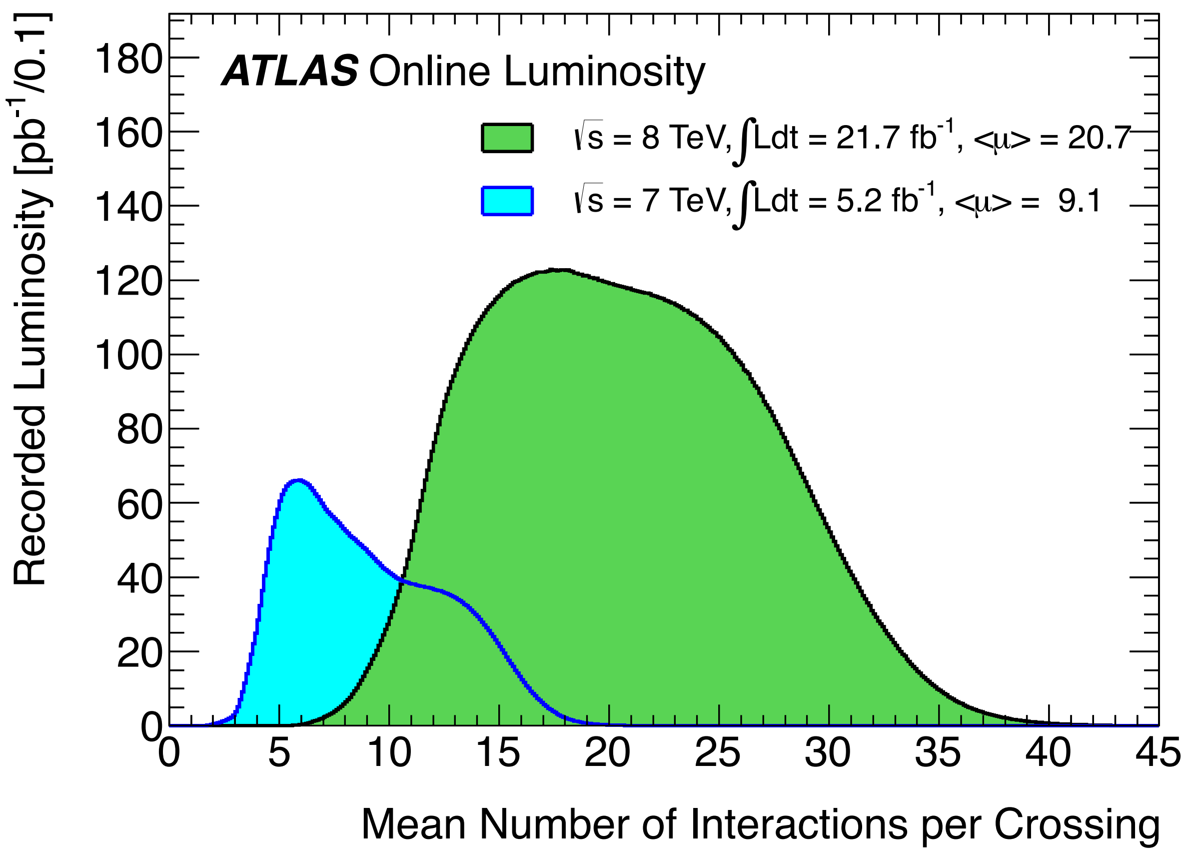
IBL providing improved vertex resolution: σ_τ reduced by 25-30%

Backup slides

Luminosity



ATLAS Online Luminosity
2010 pp √s = 7 TeV
2011 pp √s = 7 TeV
2012 pp √s = 8 TeV
Delivered Luminosity [fb^-1]
0
5
10
15
20
25
30
35
Jan
Apr
Jul
Oct
Month in Year



ATLAS Online Luminosity
√s = 8 TeV, ∫Ldt = 21.7 fb^-1, <μ> = 20.7
√s = 7 TeV, ∫Ldt = 5.2 fb^-1, <μ> = 9.1
Recorded Luminosity [pb^-1/0.1]
0
20
40
60
80
100
120
140
160
180
0
5
10
15
20
25
30
35
40
45
Mean Number of Interactions per Crossing

$B_s^0 \to J/\psi\phi$

Table showing the ten time-dependent functions and the functions of the transversity angles. The amplitudes |A0(0)|2 and |A‖(0)|2 are for the CP-even components of the B0s→J/ψϕ decay, |A⊥(0)|2 is the CP-odd amplitude; they have corresponding strong phases δ0, δ‖ and δ⊥. By convention δ0 is set to be zero. The S-wave amplitude |AS(0)|2 gives the fraction of B0s→J/ψK+K−(f0) and has a related strong phase δS. The ± and ∓ terms denote two cases: the upper sign describes the decay of a meson that was initially a B0s meson, while the lower sign describes the decays of a meson that was initially B¯0s.

k	$\mathcal{O}^{(k)}(t)$	$g^{(k)}(\theta_T,\psi_T,\phi_T)$
1	$\frac{1}{2}\lvert A_0(0)\rvert^2\left[(1+\cos\phi_s)\,e^{-\Gamma_{\mathrm{L}}^{(s)}t}+(1-\cos\phi_s)\,e^{-\Gamma_{\mathrm{H}}^{(s)}t}\pm 2e^{-\Gamma_s t}\sin(\Delta m_s t)\sin\phi_s\right]$	$2\cos^2\psi_T(1-\sin^2\theta_T\cos^2\phi_T)$
2	$\frac{1}{2}\lvert A_\parallel(0)\rvert^2\left[(1+\cos\phi_s)\,e^{-\Gamma_{\mathrm{L}}^{(s)}t}+(1-\cos\phi_s)\,e^{-\Gamma_{\mathrm{H}}^{(s)}t}\pm 2e^{-\Gamma_s t}\sin(\Delta m_s t)\sin\phi_s\right]$	$\sin^2\psi_T(1-\sin^2\theta_T\sin^2\phi_T)$
3	$\frac{1}{2}\lvert A_\perp(0)\rvert^2\left[(1-\cos\phi_s)\,e^{-\Gamma_{\mathrm{L}}^{(s)}t}+(1+\cos\phi_s)\,e^{-\Gamma_{\mathrm{H}}^{(s)}t}\mp 2e^{-\Gamma_s t}\sin(\Delta m_s t)\sin\phi_s\right]$	$\sin^2\psi_T\sin^2\theta_T$
4	$\frac{1}{2}\lvert A_0(0)\rvert\lvert A_\parallel(0)\rvert\cos\delta_\parallel$ $\left[(1+\cos\phi_s)\,e^{-\Gamma_{\mathrm{L}}^{(s)}t}+(1-\cos\phi_s)\,e^{-\Gamma_{\mathrm{H}}^{(s)}t}\pm 2e^{-\Gamma_s t}\sin(\Delta m_s t)\sin\phi_s\right]$	$\frac{1}{\sqrt{2}}\sin 2\psi_T\sin^2\theta_T\sin 2\phi_T$
5	$\lvert A_\parallel(0)\rvert\lvert A_\perp(0)\rvert[\frac{1}{2}(e^{-\Gamma_{\mathrm{L}}^{(s)}t}-e^{-\Gamma_{\mathrm{H}}^{(s)}t})\cos(\delta_\perp-\delta_\parallel)\sin\phi_s$ $\pm e^{-\Gamma_s t}(\sin(\delta_\perp-\delta_\parallel)\cos(\Delta m_s t)-\cos(\delta_\perp-\delta_\parallel)\cos\phi_s\sin(\Delta m_s t))]$	$-\sin^2\psi_T\sin 2\theta_T\sin\phi_T$
6	$\lvert A_0(0)\rvert\lvert A_\perp(0)\rvert[\frac{1}{2}(e^{-\Gamma_{\mathrm{L}}^{(s)}t}-e^{-\Gamma_{\mathrm{H}}^{(s)}t})\cos\delta_\perp\sin\phi_s$ $\pm e^{-\Gamma_s t}(\sin\delta_\perp\cos(\Delta m_s t)-\cos\delta_\perp\cos\phi_s\sin(\Delta m_s t))]$	$\frac{1}{\sqrt{2}}\sin 2\psi_T\sin 2\theta_T\cos\phi_T$
7	$\frac{1}{2}\lvert A_S(0)\rvert^2\left[(1-\cos\phi_s)\,e^{-\Gamma_{\mathrm{L}}^{(s)}t}+(1+\cos\phi_s)\,e^{-\Gamma_{\mathrm{H}}^{(s)}t}\mp 2e^{-\Gamma_s t}\sin(\Delta m_s t)\sin\phi_s\right]$	$\frac{2}{3}\left(1-\sin^2\theta_T\cos^2\phi_T\right)$
8	$\lvert A_S(0)\rvert\lvert A_\parallel(0)\rvert[\frac{1}{2}(e^{-\Gamma_{\mathrm{L}}^{(s)}t}-e^{-\Gamma_{\mathrm{H}}^{(s)}t})\sin(\delta_\parallel-\delta_S)\sin\phi_s$ $\pm e^{-\Gamma_s t}(\cos(\delta_\parallel-\delta_S)\cos(\Delta m_s t)-\sin(\delta_\parallel-\delta_S)\cos\phi_s\sin(\Delta m_s t))]$	$\frac{1}{3}\sqrt{6}\sin\psi_T\sin^2\theta_T\sin 2\phi_T$
9	$\frac{1}{2}\lvert A_S(0)\rvert\lvert A_\perp(0)\rvert\sin(\delta_\perp-\delta_S)$ $\left[(1-\cos\phi_s)\,e^{-\Gamma_{\mathrm{L}}^{(s)}t}+(1+\cos\phi_s)\,e^{-\Gamma_{\mathrm{H}}^{(s)}t}\mp 2e^{-\Gamma_s t}\sin(\Delta m_s t)\sin\phi_s\right]$	$\frac{1}{3}\sqrt{6}\sin\psi_T\sin 2\theta_T\cos\phi_T$
10	$\lvert A_0(0)\rvert\lvert A_S(0)\rvert[\frac{1}{2}(e^{-\Gamma_{\mathrm{H}}^{(s)}t}-e^{-\Gamma_{\mathrm{L}}^{(s)}t})\sin\delta_S\sin\phi_s$ $\pm e^{-\Gamma_s t}(\cos\delta_S\cos(\Delta m_s t)+\sin\delta_S\cos\phi_s\sin(\Delta m_s t))]$	$\frac{4}{3}\sqrt{3}\cos\psi_T\left(1-\sin^2\theta_T\cos^2\phi_T\right)$

$B_s^0 \to J/\psi\phi$

Current measurement using data from 8 TeV pp collisions, the previous measurement using data taken at centre of mass energy of 7 TeV and the values for the parameters of the two measurements, statistically combined.

Par	8 TeV data			7 TeV data			Run1 combined		
	Value	Stat	Syst	Value	Stat	Syst	Value	Stat	Syst
ϕ_s[rad]	-0.123	0.089	0.041	0.12	0.25	0.05	-0.098	0.084	0.040
$\Delta\Gamma_s$[ps^{-1}]	0.096	0.013	0.007	0.053	0.021	0.010	0.083	0.011	0.007
Γ_s[ps^{-1}]	0.678	0.004	0.004	0.677	0.007	0.004	0.677	0.003	0.003
$\|A_\parallel(0)\|^2$	0.230	0.005	0.006	0.220	0.008	0.009	0.227	0.004	0.006
$\|A_0(0)\|^2$	0.514	0.004	0.002	0.529	0.006	0.012	0.514	0.004	0.003
$\|A_S\|^2$	0.090	0.008	0.020	0.024	0.014	0.028	0.071	0.007	0.017
$\delta_\perp$ [rad]	4.46	0.48	0.29	3.89	0.47	0.11	4.13	0.33	0.16
$\delta_\parallel$ [rad]	3.15	0.13	0.05	[3.04, 3.23]		0.09	3.15	0.13	0.05
$\delta_\perp - \delta_S$ [rad]	-0.08	0.04	0.01	[3.02, 3.25]		0.04	-0.08	0.04	0.01

$\Delta\Gamma_d$ Systematic Uncertainties

Source	$\delta(\Delta\Gamma_d/\Gamma_d)$, 2011	$\delta(\Delta\Gamma_d/\Gamma_d)$, 2012
K_S decay length	0.21×10^{-2}	0.16×10^{-2}
K_S pseudorapidity	0.14×10^{-2}	0.01×10^{-2}
$B_d^0 \to J/\psi K_S$ mass range	0.47×10^{-2}	0.59×10^{-2}
$B_d^0 \to J/\psi K^{*0}$ mass range	0.30×10^{-2}	0.15×10^{-2}
Background description	0.16×10^{-2}	0.09×10^{-2}
$B_s^0 \to J/\psi K_S$ contribution	0.11×10^{-2}	0.08×10^{-2}
L_{prop}^B resolution	0.29×10^{-2}	0.29×10^{-2}
Fit bias (Toy MC)	0.07×10^{-2}	0.07×10^{-2}
B_d^0 production asymmetry	0.01×10^{-2}	0.01×10^{-2}
MC sample	1.54×10^{-2}	0.45×10^{-2}
Total uncertainty	1.69×10^{-2}	0.84×10^{-2}