Rare B decays: theory overview

Enrico Lunghi
Indiana University

General considerations

- Long standing issues
  - Experimental:
    dark matter, baryon-antibaryon asymmetry, strong CP, neutrino masses,…
  - Theoretical:
    naturalness (hierarchies, fine tuning, fundamental scalars, …), gravity, …

- (So) many possible (more or less) elegant solutions
  - Supersymmetry, extra dimensions, extended Higgs sectors, composite Higgs models, vectorlike fermions, axions, …

- More recent experimental issues:
  $B \to D^{(*)} \tau \nu$, $B \to K^{(*)} ll$ anomalies, Unitarity Triangle fits ($B \to \tau \nu$, angle γ, V_{ub}, V_{cb}), Lepton Flavor Universality Violation ($B \to K \mu\mu$, Kee), muon g-2, LFV ($h \to \tau \mu$), …

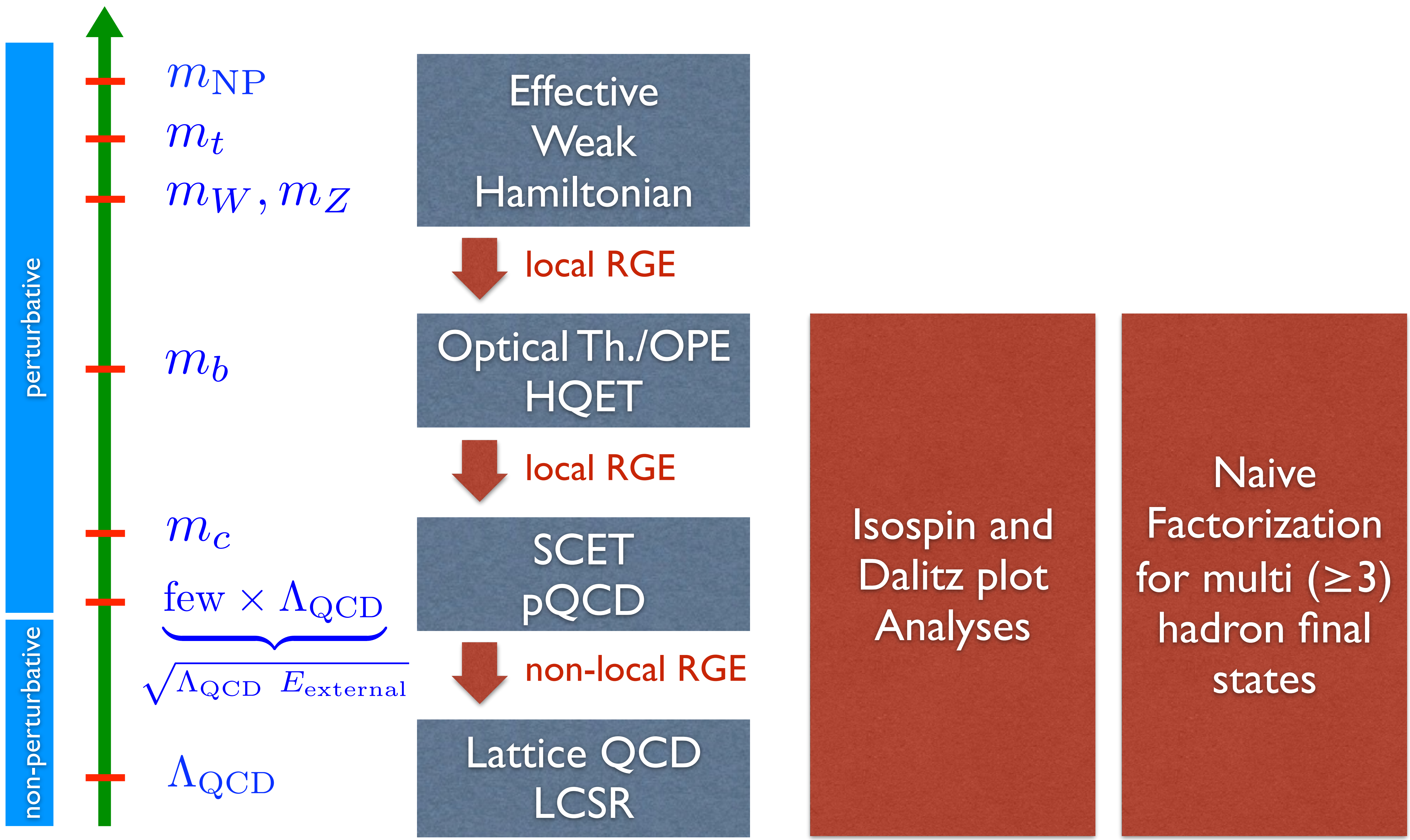
- *** No <u>compelling</u> BSM models that address <u>all issues</u> ***
  - Every anomaly seems to point in different directions
  - Proposed solutions involve leptoquarks, variants of 2HDM, new scalars, light Z' bosons, gauging lepton number, …

General considerations

- The experimental landscape is bright!
  - **LHCb**: focus on exclusive modes ($B \to K^{(*)} ll$, $B \to D^{(*)} D^{(*)}$, Baryonic modes, …)
  - **Belle II**: clean environment allows (in addition to what LHCb does) inclusive modes ($B \to X_{s/d} \gamma$, $B \to X_{s/d} ll$), inclusive and exclusive semileptonic ($B \to \pi l \nu$, $B \to D^{(*)} l \nu$, $B \to X_{u/c} l \nu$, …), $B \to \tau \nu$, LFUV, LFV, …
- Among the current tensions
  - some rely on measurements that are only possible at Belle II (e.g. $B \to \tau \nu$)
  - some require non-perturbative results (form factors, hadronic/LBL contributions to g-2, ε'/ε from lattice QCD)
  - some are at a level comparable to possible non-perturbative effects whose size there is no universal agreement on (i.e. power corrections, quark-hadron duality violation)

Interplay between exclusive (LHCb) and inclusive (Belle II) measurements with orthogonal theory systematics is ABSOLUTELY CRUCIAL to establish a breakdown of the SM.

A tale of multiple scales



Effective Hamiltonian

- The effective Hamiltonian responsible for $b \to q$ (q=d,s) transitions in the SM is:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[\sum_{i=1}^{10} C_i Q_i + \underbrace{\frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*}}_{\lambda_q} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b + C_{\nu\nu} Q_{\nu\nu} \right]$$

- λ_q contributions are relevant only for b→d transitions and yield large CP asymmetries ($\lambda_s = -0.0074 + 0.020\, i$, $\lambda_d = -0.036 - 0.43\, i$)

- Phenomenologically important operators are:

$$Q_7 = \frac{e}{16\pi^2} (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$B \to (K^*, K_1, \rho, X_s, X_d, \ldots)\gamma$

$$Q_{\nu\nu} = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\nu}_\ell \gamma_\mu (1-\gamma_5) \nu_\ell)$$

$B \to (K^*, X_s, \ldots)\nu\bar{\nu}$

$$Q_9 = \frac{\alpha_{\rm em}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \ell)$$

$$Q_{10} = \frac{\alpha_{\rm em}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$B \to (K^{(*)}, \pi, X_s, X_d, \ldots)\ell\ell$

$$Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L)$$

charmonium resonances:

$B \to (K^{(*)}, \pi, X_s, X_d, \ldots)(\psi_{cc} \to \ell\ell)$

Theoretical Tools

The central problem is the calculation of hadronic matrix elements of the type
$\langle M|T\, O_i(x) J_{\rm em}(y)|B\rangle$, $\langle M_1 M_2|O_i(x)|B\rangle$ or $\langle B|T\, O_i(x) O_j(y)|B\rangle$

- **HQET/SCET$_{\rm II}$** : Scales are systematically isolated in an effective theory framework. Power expansion in $1/m_b$. Inputs are form factors and LCDA's.

$$A(B \to M_1 M_2) \sim F_{B\to M_1} \otimes H_4 \otimes \phi_{M_2} + \phi_B \otimes H_6 \otimes \phi_{M_1} \otimes \phi_{M_2} + O\left(\frac{\Lambda}{m_b}\right)$$

- **pQCD** : Endpoint singularities are smeared by integrating over parton transverse momenta. Sudakov double logs can be resummed (S)

$$A(B \to M_1 M_2) \sim \phi_B \otimes H_6 \otimes \phi_{M_1} \otimes \phi_{M_2} \otimes S + O\left(\frac{\Lambda}{m_b}\right)$$

non-local

- **Lattice QCD** : direct calculation of matrix elements, decay constants, form factors and some LCDA moments from first principles.
Note that form factors are calculable at **large-q^2**.

- **LCSR** : Calculation of form factors at **low-q^2** in terms of LCDA's. Uncertainties are related to quark-hadron duality, elastic threshold and Borel parameter.

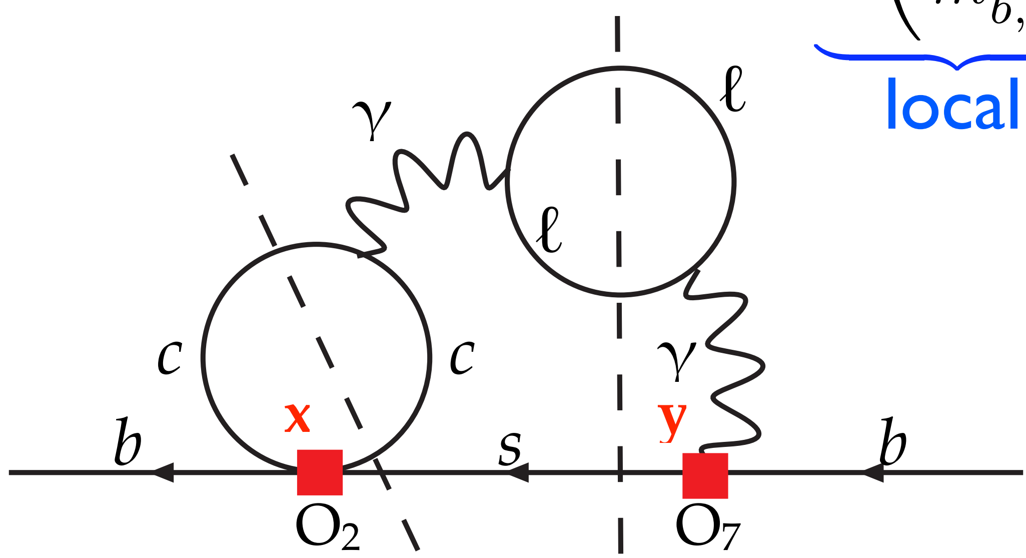
Theoretical Tools

- **OPE:** $T\,O_i(x)O_j(y) \xrightarrow{y\to x} \sum_i C_i(x-y)\,Q_i(x)$

  We usually have $(x-y)^2 \sim 0$ instead of $x^\mu - y^\mu \sim 0$: quark-hadron duality

$B \to X_s \ell\ell$

$$\Gamma[B \to X_s \ell\ell] = \Gamma[b \to X_s \ell\ell] + \underbrace{O\left(\frac{\Lambda^2}{m_{b,c}^2}\right)}_{\text{local}}$$

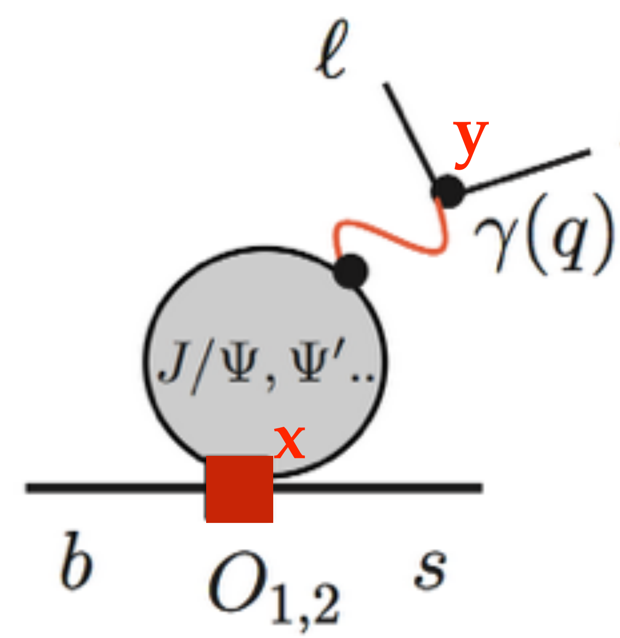


$$(x-y)^2 \sim p_{X_s}^{-2} \sim \left(m_b - \sqrt{q^2}\right)^{-2}$$

The OPE breaks down at large q^2. Charmonium resonances can be included using $ee \to \text{hadrons}$

[hep-ph/9603237; Krüger, Sehgal]
[0902.4446; Beneke, Buchalla, Neubert, Sachrajda]

$B \to K^{(*)}\ell\ell$



$$(x-y)^2 \sim q^{-2}$$

The OPE breaks down at small q^2. Charmonium resonances correspond to large $x^\mu - y^\mu$ and must be dealt with invoking quark-hadron duality

[1101.5118; Beylich, Buchalla, Feldmann]

Non-perturbative inputs

- f_B and f_{B_s} from lattice [ALPHA2, ETM2+1+1, FNAL/MILC2+1, HPQCD2+1+1, RBC/UKQCD2+1]
- Form Factors. LQCD: $B\to(\pi,K)$ [FNAL/MILC2+1, HPQCD2+1, RBC/UKQCD2+1]

  $B_s\to K$ [HPQCD2+1, RBC/UKQCD2+1]

  $B\to K^*$ and $B_s\to\phi$ [(Horgan, Liu, Meinel, Wingate)2+1]

  $B\to D$ [HPQCD2+1, FNAL/MILC2+1, (Atoui et al.)2]

  $B\to D^*$ [FNAL/MILC2+1]

  $\Lambda_b\to p$ and $\Lambda_b\to\Lambda_c$ [(Detmold, Lehner, Meinel)2+1]

  $\Lambda_b\to\Lambda$ [(Detmold, Meinel)2+1]

  LCSR: $B\to(\pi,K,\eta,\varrho,\omega,K^*)$ [Ball, Zwicky; Bharucha, Straub, Zwicky]

  $B_s\to(K^*,\phi)$ [Ball, Zwicky; Bharucha, Straub, Zwicky]

- modified z-expansion
- non-optimal treatment of the unstable K*

(← $B\to K^*$)

- modified z-expansion

(← $\Lambda_b\to p$)

- LCDA. ϕ_π and ϕ_K: Lattice QCD gives the first few moments [Arthur et al.]
  ϕ_B: Guesstimates! But see [1404.1343; Feldmann, Lange, Wang]
- Power Corrections
  - Inclusive: calculable in terms of local matrix elements.
  - Exclusive: within SCET they can be expressed in terms of new non-local matrix element but there are no current estimates beyond naive scaling

Recent Results (last year)

- Multi-hadron (n>2) exclusive decays
  - $B^\pm \to K^+ p\bar{p}$ [1505.07439; Di Salvo, Fontanelli]: naive factorization, some modeling, CP asymmetry
  - $B \to D^-\pi^+\pi^+\pi^-$ [1506.03996; Talebtash, Mehraban]: naive factorization, HHχPT
  - $B^\pm \to K^+K^-K^\pm$ [1509.06979; Lesniak, Zenczykowski]: QCD factorization, KK rescattering in S,P and D wave, large strong phases, large CP asymmetry
- Two-hadron exclusive decays
  - $B_s \to \pi^+\pi^-\ell\ell$ [1502.05104; Wang, Zhou]: non-resonant ($\rho \to \pi\pi$) study
  - $B_{(s)} \to D_{(s)}D_{(s)}$ [1505.01361; Bel, De Bruyn, Fleischer, Mulder, Tuning]
  - $B \to \pi\pi\ell\nu$ [1511.02509; Hambrock, Khodjamirian]: LCSR, di-pion form factor
  - Asymmetries in $B \to K\pi$ [1510.05910; Liu, Li, Xiao]: pQCD and Glauber modes
  - $\Lambda_b \to \Lambda(\phi,\eta,\eta')$ [1603.06682; Geng, Hsiao, Lin, Yu]: QCD factorization
  - $B \to J/\psi K_1 \to J/\psi K\pi\pi$ [1604.07708; Kou, Le Yaouanc, Tayduganov]: Dalitz plot analysis to extract details of the K_1 decay. This can be used to extract the photon polarization in $B \to K_1\gamma$

Recent Results (last year)

- Single hadron exclusive decays
  - $B \to (\pi, D, D^*)\ell\nu$ and $B_s \to K\ell\nu$

    A comment on the R_D and R_{D^*} anomaly: the measured $B \to (D, D^*)\tau\nu_\tau$ rates are (with reasonable estimates of higher resonances contributions) in disagreement with $B \to X_c\tau\nu_\tau$ predictions [1506.08896; Freytsis, Ligeti, Ruderman.
  - $\Lambda_b \to p\ell\nu$ [1503.01421; Detmold, Lehner, Meinel]: lattice QCD

    $\Lambda_b \to (p\ell\nu, \Lambda\ell\ell)$ [1511.03540; Kozachuk, Melnikov, Nikitin]: Bethe-Salpeter equation approach

    $\Lambda_b \to \Lambda\ell\ell$ [1602.01399; Detmold, Meinel. 1603.02974; Meinel, van Dyk]: lattice QCD
  - $B \to D_s^*\gamma$ and $B_s \to J/\psi\gamma$ [1511.03540; Kozachuk, Melnikov, Nikitin]: annihilation topologies

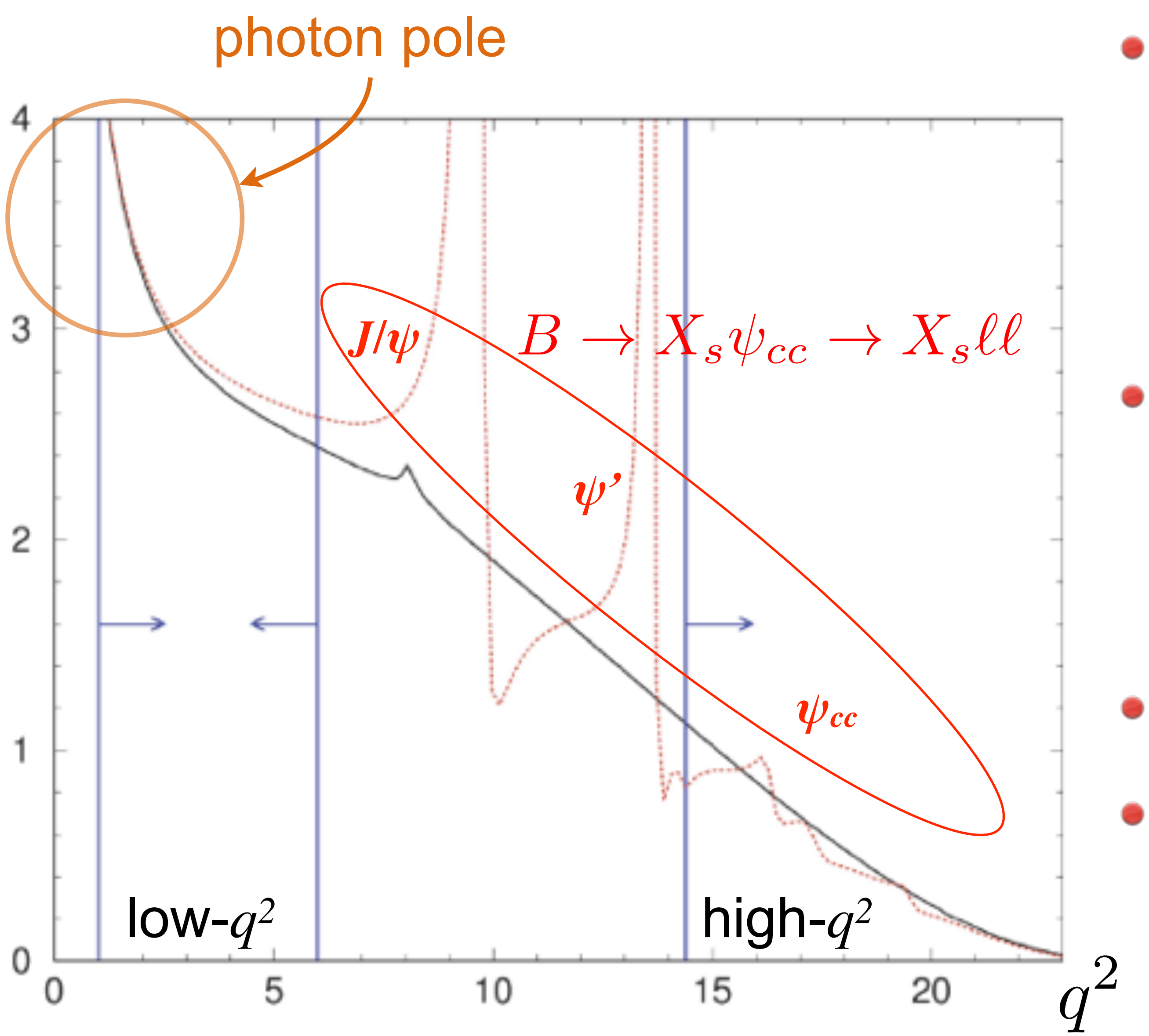
    $B \to K_1\gamma$ [1604.07708; Kou, Le Yaouanc, Tayduganov]: photon polarization
  - $B \to (\pi, K, K^*)\ell\ell$
- Inclusive decays:
  - $B \to X_{s,d}\gamma$ and $B \to X_s\ell\ell$

see upcoming slides

- Leptonic decays
  - $B_q \to \ell\ell$: for reviews see [1405.4907, Bobeth. 1407.0916, Fleischer. 1407.2771, Knegjens]
  - $B \to \gamma\ell\nu$ [1604.08300; Yang, Yang]: Factorization at 1-loop, attempt to discuss soft photons

$B \to X_s \ell\ell$

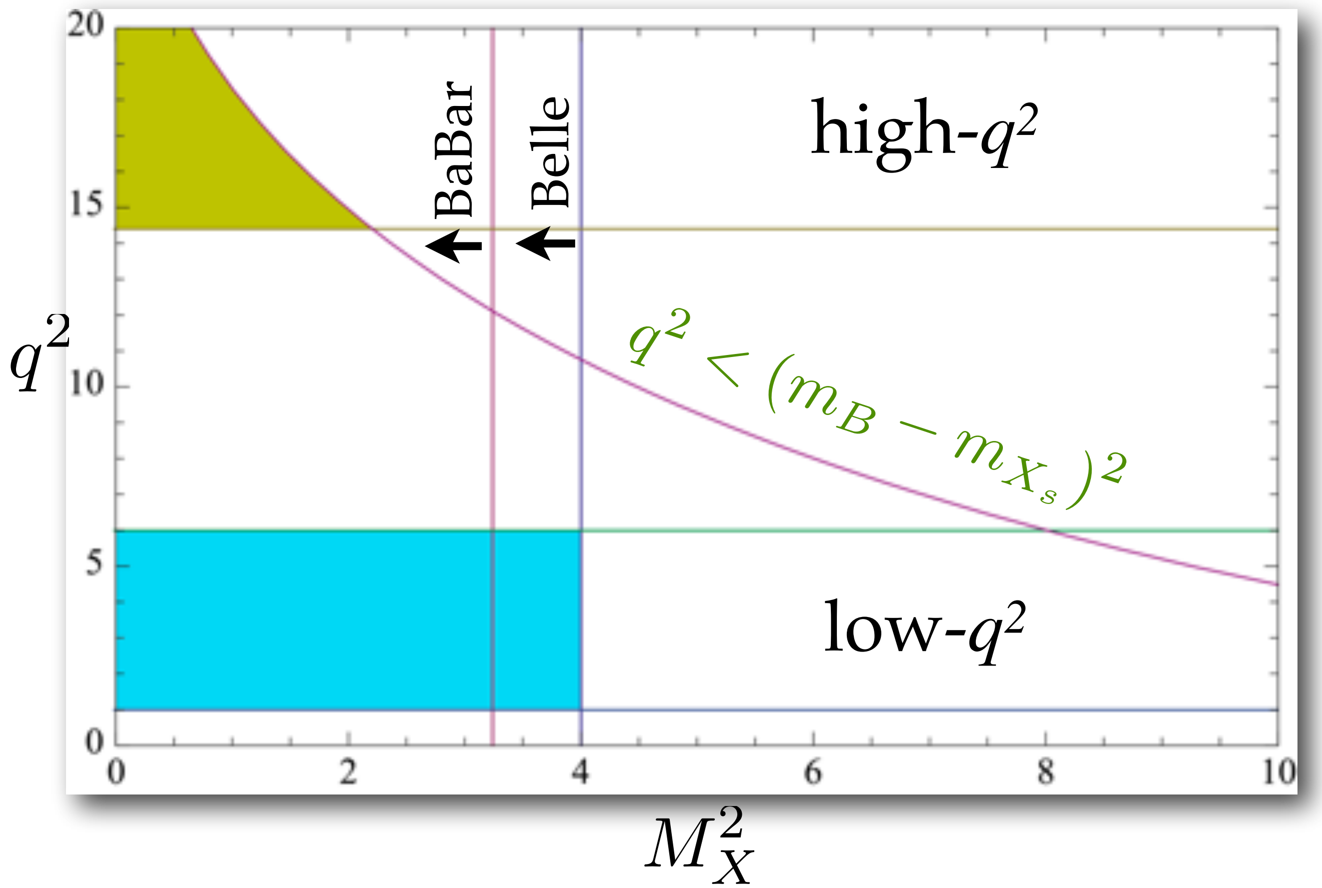
- A collaborative effort:
  Asatryan, Asatrian, Bobeth, Buras, Gambino, Ghinculov, Gorbahn, Greub, Haisch, Huber, Hurth, Isidori, Lee, Ligeti, Lunghi, Misiak, Munz, Stewart, Tackmann, Urban, Walker, Wyler, Yao …



- The effect of each intermediate resonance is proportional to the inverse of the resonance width and is non-perturbative. [0902.4446; Beneke, Buchalla, Neubert, Sachrajda]
- The J/ψ and ψ' lie below the open charm threshold, have very narrow width and yield a contribution that is two orders of magnitude larger than everything else.
- Need to remove the J/ψ and ψ' with q^2 cuts.
- Still we need to take into account non-perturbative effects due to the broad resonances at high-q^2 and to the tail of the J/ψ at low-q^2.

$B \to X_s \ell\ell$

- A $M_{X_s} \lesssim 2\ \mathrm{GeV}$ cut to remove double semileptonic decay background is necessary
  This phase space cut introduces sensitivity to new scales, the rate becomes less inclusive and new non-perturbative effects appear



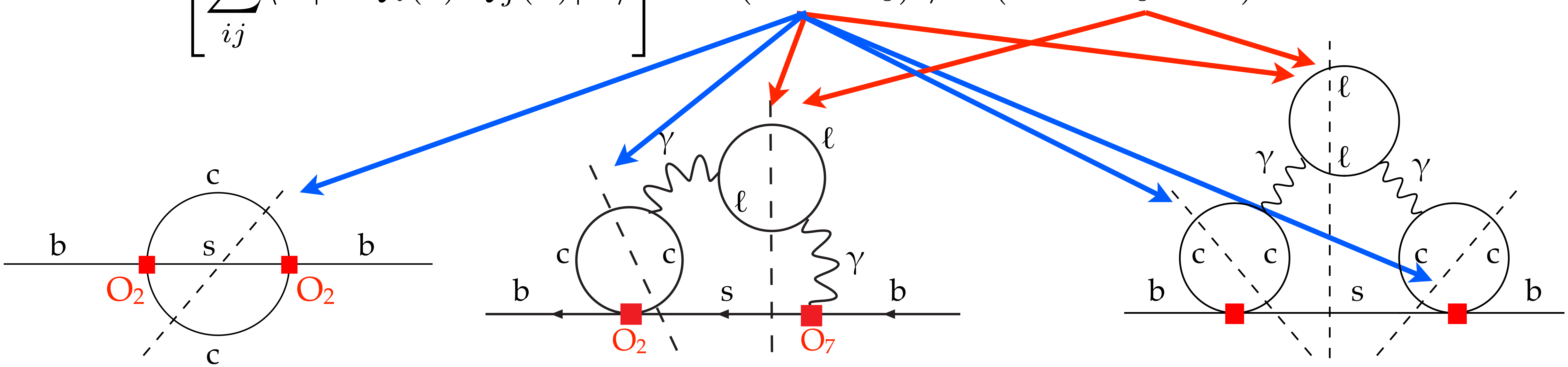
- High-q^2 region unaffected
- **Experiments correct using Fermi motion model**
- M_X cuts can be calculated in SCET$_I$ [0512191; Lee, Ligeti, Stewart, Tackmann]
  - effect of cuts on $b \to sll$ is universal (a change in overall normalization)
  - effects of cuts on $b \to sll$ and $b \to ul\nu$ is the same $\Rightarrow \Gamma_{cut}(B \to X_s ll)/\Gamma_{cut}(B \to X_u l\nu)$

- At Belle II level of precision:
  - move beyond the Fermi motion model
  - attempt the simultaneous study of $B \to X_s ll$ and $B \to X_u l\nu$
  - probably need new Monte Carlo

$B \to X_s \ell\ell$: breakdown of the OPE?

- Optical theorem: [0902.4446; Beneke, Buchalla, Neubert, Sachrajda]

$$\mathrm{Im}\left[\sum_{ij} \langle \bar{B}| T\, Q_i(0)\, Q_j(x) |\bar{B}\rangle\right] \sim \Gamma(\bar{B} \to X_s) \neq \Gamma(\bar{B} \to X_s \ell^+\ell^-)$$



$|A(b \to s c\bar{c})|^2$

$A(b \to s c\bar{c})^* A(b \to s\ell\ell \to s c\bar{c})$

$A(b \to s c\bar{c} \to s\ell\ell)^* A(b \to s\ell\ell)$

$A(b \to s c\bar{c})^* A(b \to s c\bar{c} \to s\ell\ell \to s c\bar{c})$

$|A(b \to s c\bar{c} \to s\ell\ell)|^2$

- Rates are very different: $\mathcal{B}(B \to X_s) \sim 10^{-2}$ $\mathcal{B}(B \to X_s \ell\ell) \sim 10^{-6}$

  $\mathcal{B}(B \to X_s \psi_{cc} \to X_s \ell\ell) \sim 10^{-4}$

- Dropping the hadronic cuts in O_2O_2, O_2O_7 and O_2O_9 implies a $1/\Gamma_\psi$ scaling of the integrated rate: **we cannot ignore long distance effects**

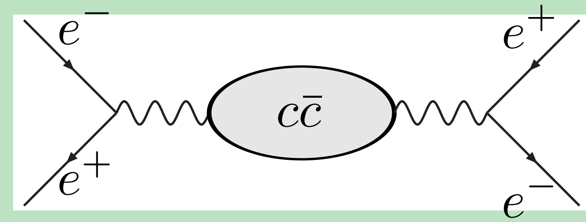
$B \to X_s \ell\ell$: breakdown of the OPE?

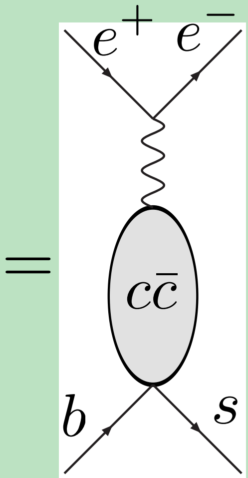
- The problem is a failure of the perturbative quark calculation to reproduce the full hadronic contribution upon integration ($\int \mathrm{Im}\Pi$ vs $\int |\Pi|^2$)
- Small effects for $q^2 < 7\mathrm{GeV}^2$ and at high-q^2 (where resonances have $\Gamma_\psi \gtrsim 10^{-2}$ GeV)
- The use of e^+e^- data to reconstruct the full non-perturbative charm blob (KS method) reproduces the correction branching ratio without introducing extra phenomenological parameters:

Resonances (color singlet):

The factorizable contribution using NNLO Wilson Coefficients is in good agreement with data ("fudge factor" ≈1)

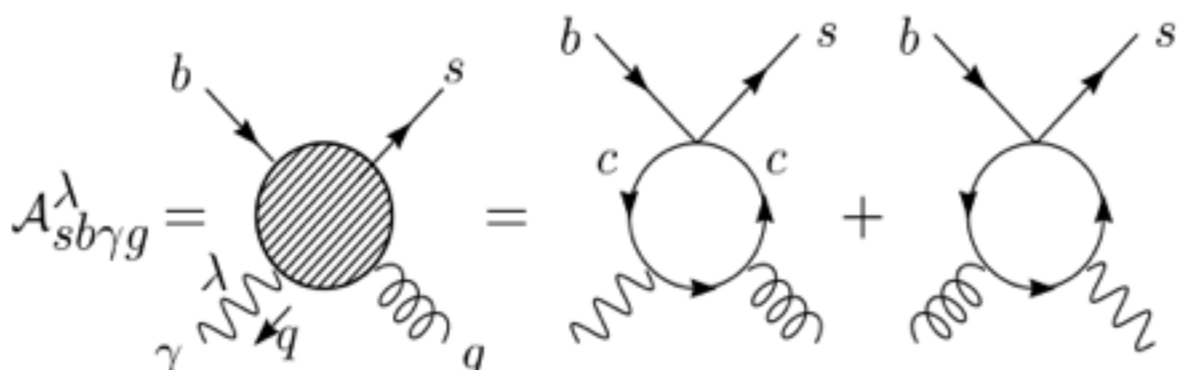
$$R_{\mathrm{had}}^{c\bar{c}} = \frac{\sigma(e^+e^- \to c\bar{c}\ \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$=$ 

⟺ $\langle O_2 \rangle =$ 

[hep-ph/9603237; Krüger, Sehgal]

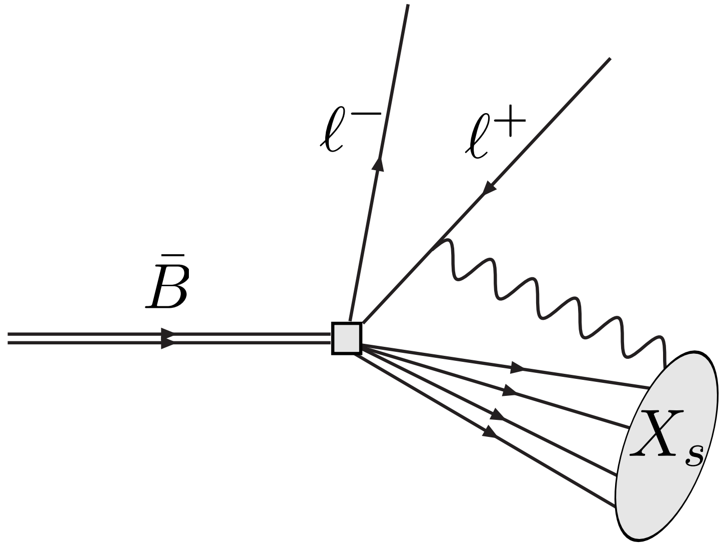
Resonances (color octet):



[hep-ph/9705253; Buchalla, Isidori, Rey]

$B \to X_s \ell\ell$: QED effects

- Photons emitted by the final state leptons (especially electrons) should be technically included in the Xs system:



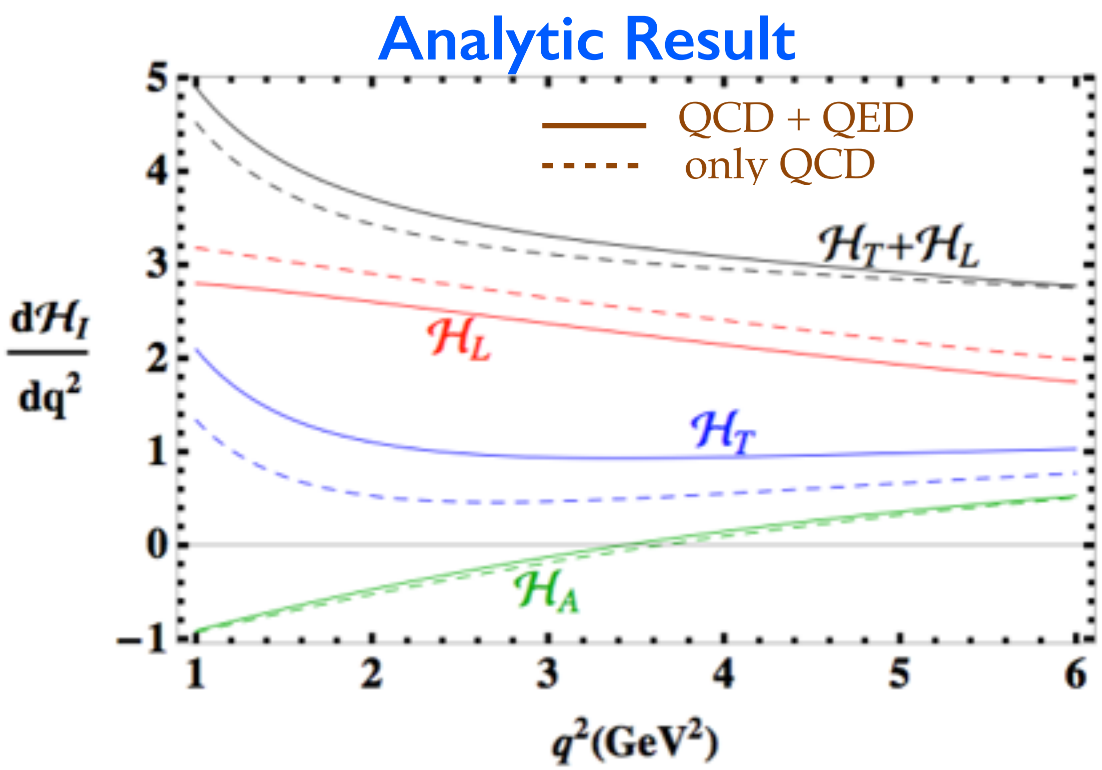
- This implies very large $\alpha_{em} \log(m_e/m_b)$ at low and high-q^2
- Cannot escape the logs: if all photons are included in the dilepton system we get $\log(m_s/m_b)$ effects
- At B-factories most but not all of these photons are included in the Xs system
- Need Monte Carlo studies (EVTGEN+PHOTOS) to find the correction factor:

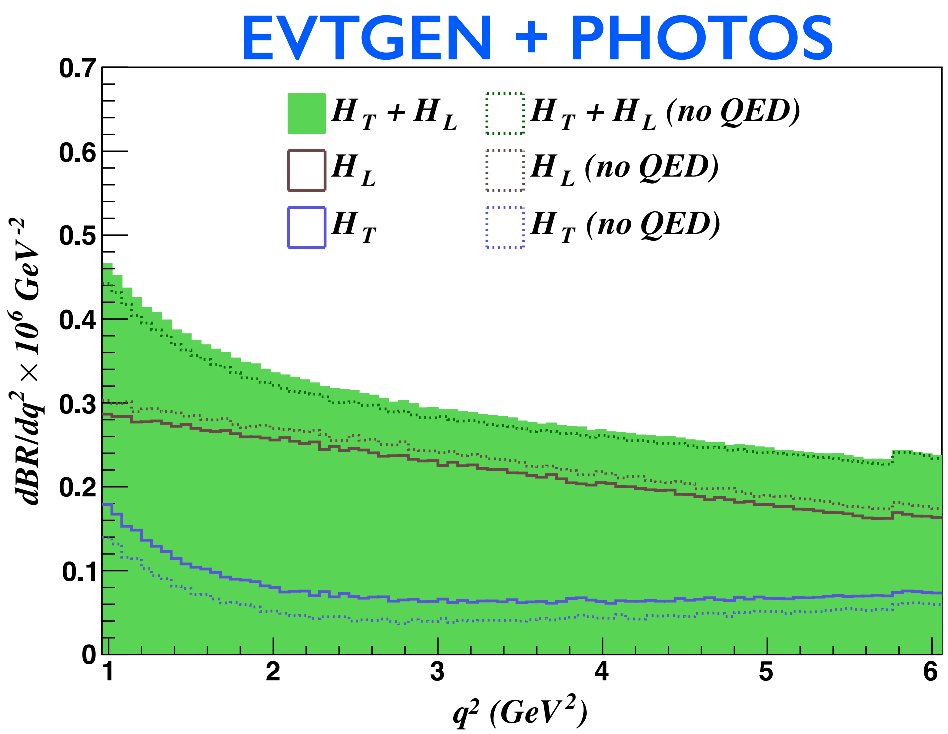
$$\frac{\left[\mathcal{B}_{ee}^{\text{low}}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\text{coll}}}}}{\left[\mathcal{B}_{ee}^{\text{low}}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\% \qquad \frac{\left[\mathcal{B}_{ee}^{\text{high}}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\text{coll}}}}}{\left[\mathcal{B}_{ee}^{\text{high}}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

$B \to X_s \ell\ell$: QED effects

$$\frac{d^2\Gamma^{X_s}}{dq^2\, d\cos\theta_\ell} = \frac{3}{8}\left[(1+\cos^2\theta_\ell)\, H_T + 2(1-\cos^2\theta_\ell)\, H_L + 2\cos\theta_\ell\, H_A\right]$$

$$H_T \sim 2\hat{s}(1-\hat{s})^2\left[\left|C_9 + \frac{2}{\hat{s}}C_7\right|^2 + |C_{10}|^2\right] \qquad H_L \sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$





agreement between analytical and MC results

	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

[0612156; Lee, Ligeti, Stewart, Tackmann]

[0712.3009; Huber, Hurth, EL]

[1503.04849; Huber, Hurth, EL]

$B \to X_s \ell\ell$: Current Status

BaBar: 471x10[6] BB pairs (424 fb^{-1})
Belle: 152x10[6] BB pairs (140 fb^{-1})
711 fb^{-1} on tape!!

- World averages (Babar, Belle):

$$\mathrm{BR}^{\mathrm{exp}} = (1.58 \pm 0.37) \times 10^{-6} \qquad q^2 \in [1,6] \qquad \delta_{\mathrm{exp}} \approx 23\%$$

$$\mathrm{BR}^{\mathrm{exp}} = (0.48 \pm 0.10) \times 10^{-6} \qquad q^2 > 14.4 \qquad \delta_{\mathrm{exp}} \approx 21\%$$

$$\overline{A}_{\mathrm{FB}}^{\mathrm{exp}} = \begin{cases} 0.34 \pm 0.24 & q^2 \in [0.2, 4.3] \\ 0.04 \pm 0.31 & q^2 \in [4.3, 7.3(8.1)] \end{cases}$$

non-optimal binning

- Theory:

$$\mathrm{BR}^{\mathrm{th}} = (1.65 \pm 0.10) \times 10^{-6} \qquad q^2 \in [1,6] \qquad \delta_{\mathrm{th}} \approx 6\%$$

$$\mathrm{BR}^{\mathrm{th}} = (0.237 \pm 0.070) \times 10^{-6} \qquad q^2 > 14.4 \qquad \delta_{\mathrm{th}} \approx 30\%$$

$$\overline{A}_{\mathrm{FB}}^{\mathrm{th}} = \begin{cases} -0.077 \pm 0.006 & q^2 \in [0.2, 4.3] \\ 0.05 \pm 0.02 & q^2 \in [4.3, 7.3(8.1)] \end{cases}$$

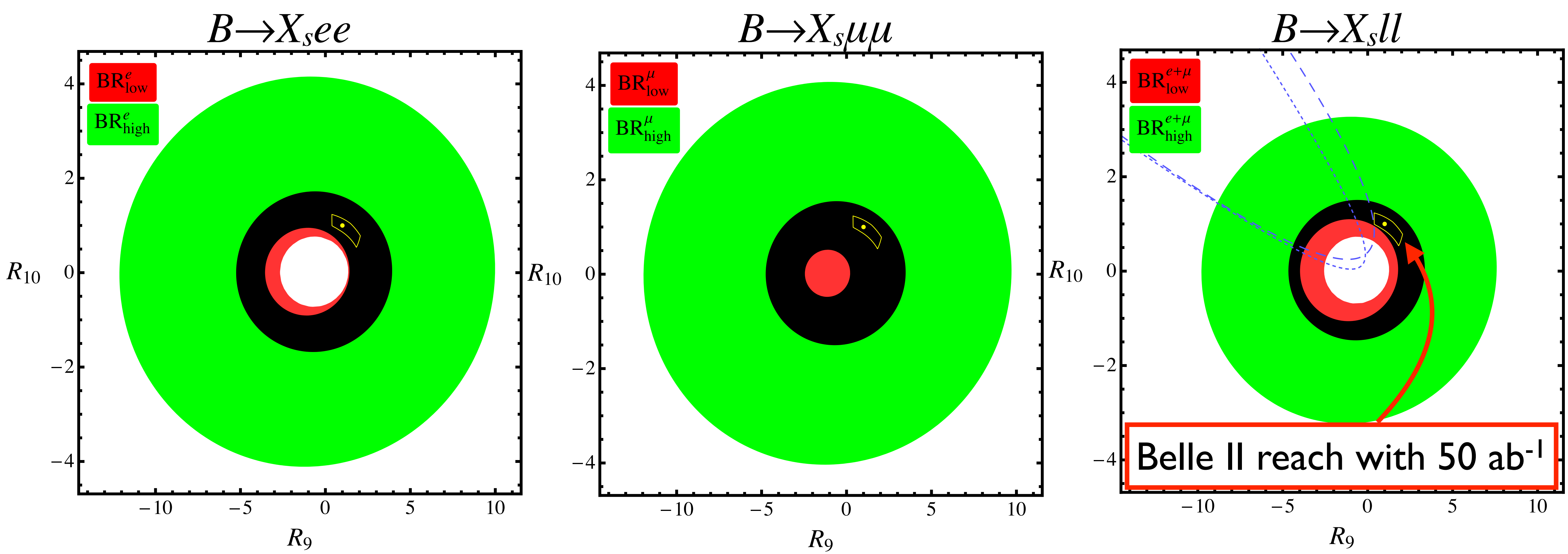
non-optimal binning

$$\mathrm{BR} = H_T + H_L \qquad \overline{A}_{\mathrm{FB}} = \frac{3}{4}\frac{H_A}{H_T + H_L}$$

- Scale uncertainties dominate at low-q^2
- Power corrections and scale uncertainties dominate at high-q^2

$B \to X_s \ell\ell$: Current Status

- $95\%C.L.$ constraints in the [R_9,R_{10}] plane ($R_i = C_i(\mu_0)/C_i^{\rm SM}(\mu_0)$):



- Note that $C_9^{\rm SM}(\mu_0) = 1.61$ and $C_{10}^{\rm SM}(\mu_0) = -4.26$
- Best fits from the exclusive anomaly translate in R_9 ~ 0.3 (for the single WC fit) or R_9 ~ 0.65 and R_{10} ~ 0.9 (for the $C_9^{\rm NP} = -C_{10}^{\rm NP}$ scenario)

$B \to X_s \ell\ell$: reducing errors at high-q^2

- Normalize the decay width to the semileptonic $B \to X_u l\nu$ rate with the **same dilepton invariant mass cut**:

$$\mathcal{R}(s_0) = \frac{\displaystyle\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\, \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+\ell^-)}{\mathrm{d}\hat{s}}}{\displaystyle\int_{\hat{s}_0}^{1} \mathrm{d}\hat{s}\, \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell\nu)}{\mathrm{d}\hat{s}}}$$

[0707.1694; Ligeti, Tackmann]

- Impact of $1/m_b^2$ and $1/m_b^3$ power corrections drastically reduced:

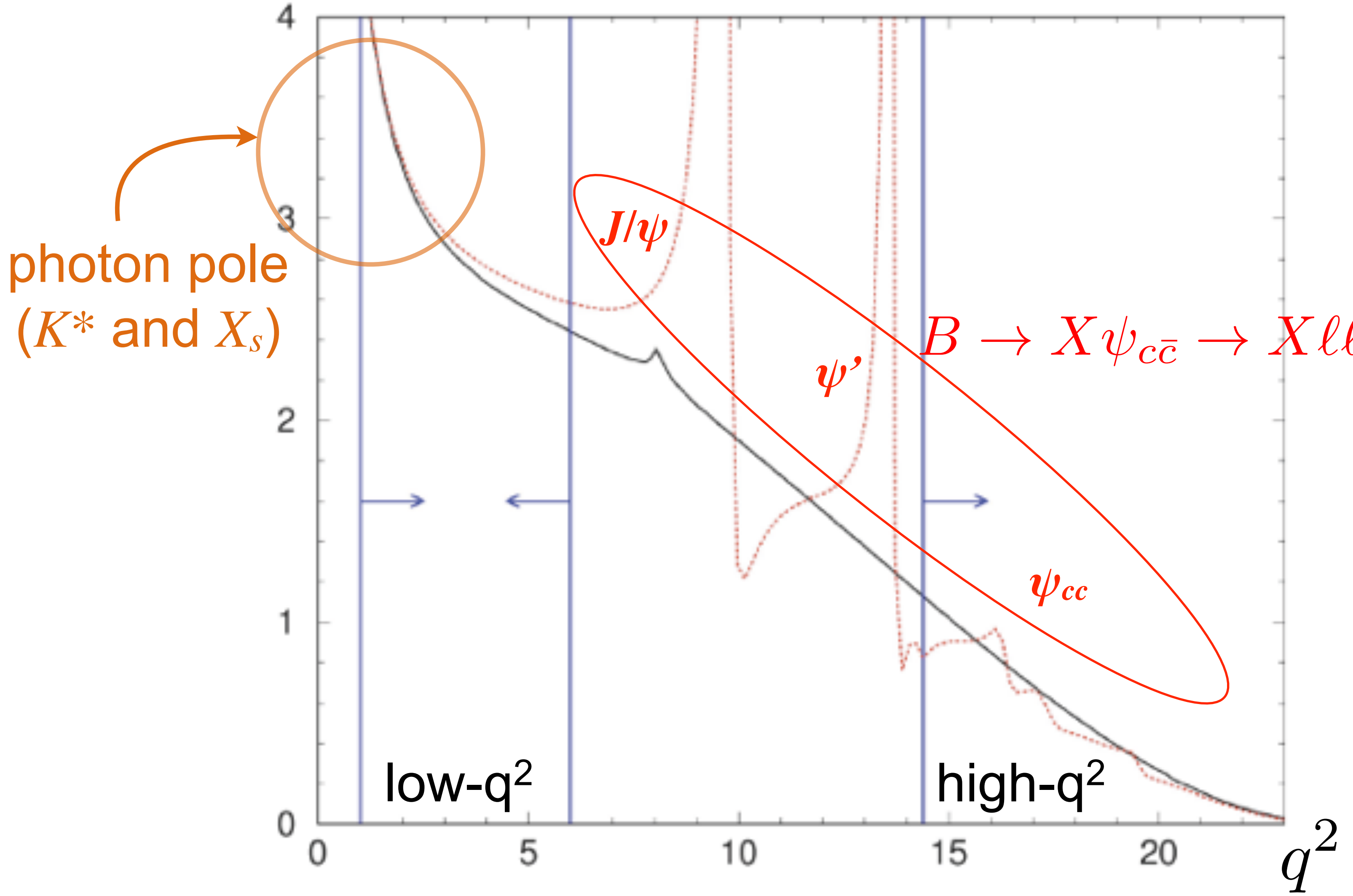
$$\begin{aligned}\mathcal{R}(14.4)_{\mu\mu} =& (2.62 \pm 0.09_{\rm scale} \pm 0.03_{m_t} \pm 0.01_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.23_{\rm CKM} \\ & \pm 0.0002_{\lambda_2} \pm 0.09_{\rho_1} \pm 0.04_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\ =& (2.62 \pm 0.30) \cdot 10^{-3}\end{aligned}$$

$$\begin{aligned}\mathcal{R}(14.4)_{ee} =& (2.25 \pm 0.12_{\rm scale} \pm 0.03_{m_t} \pm 0.02_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.20_{\rm CKM} \\ & \pm 0.02_{\lambda_2} \pm 0.14_{\rho_1} \pm 0.08_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\ =& (2.25 \pm 0.31) \cdot 10^{-3}\end{aligned}$$

- ***The largest source of uncertainty is V_{ub}***

$B \to (\pi, K, K^*)\ell\ell$: general considerations

- Typical spectrum :



SCET (1/E$_K$):
LCDA (π,K,K^*)
Form Factors

OPE (1/q^2):
Form Factors

LCSR: low-q^2
LQCD: high-q^2
+ z-parametrization

- **Low-q^2**
  - non-local power corrections
  - need more inputs (LCDA)
  - LQCD form factors need to be extrapolated from high-q²
- **High-q^2**
  - need to integrate over several broad charmonium resonances

- ***K** vs *K***

  Disadvantages:
  - larger power corrections
  - $K^*\to K\pi$ decay (S vs P wave)
  - status of LQCD form factors

  Advantages:
  - $K^*\to K\pi$ decay (angular observables)

$B \to (\pi, K, K^*)\ell\ell$: references

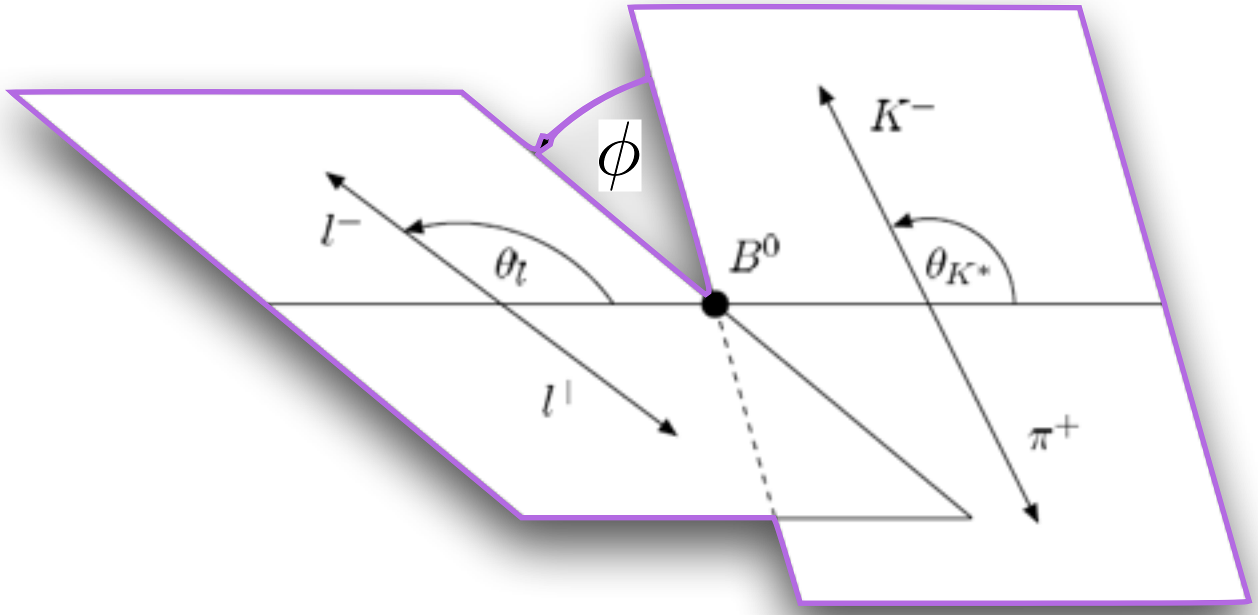
- Some references (last year):

  1502.05509; Descotes-Genon, Virto
  1502.00920; Hofer, Matias
  1503.03328; Descotes-Genon, Hofer, Matias, Virto
  1503.05534; Barucha, Straub, Zwicky
  1503.06199; Altmannshofer, Straub
  1503.09024; Becirevic, Fajfer, Kosnik
  1506.02661; Cabibbi, Crivellin, Ota
  1506.04535; Mandal, Sinha
  1506.06699; Das, Hiller, Jung
  1507.01618; Fermilab/MILC, EL
  1510.02349; Fermilab/MILC, EL
  1510.04239; Descotes-Genon, Matias, Virto
  1511.04015; Crivellin
  1511.04887; Dubnicka et al.
  1512.01560; Barbieri, Isidori, Pattori, Senia
  1512.07157; Ciuchini et al.
  1602.01372; Colangelo, de Fazio, Santorelli
  1603.00865; Hurth, Mahmoudi, Neshatpour
  1603.04355; Karan, Nayak, Sinha, Browder
  1605.02934; Crivellin
  1605.03156; Capdevila, Descotes-Genon, Matias, Virto

- find title 750 and GeV and date after 2014: 195 records found

$B \to (\pi, K, K^*)\ell\ell$: differential rate

- $B \to K^* ll \to K\pi ll$ events can be described in terms of three angles: $(\theta_\ell,\ \phi,\ \theta_{K^*})$



- $B \to K^* ll$ fully differential rate:

$$\frac{1}{\mathrm{d}(\Gamma+\bar\Gamma)/dq^2}\frac{\mathrm{d}^3(\Gamma+\bar\Gamma)}{\mathrm{d}\vec\Omega} = W_P + W_S$$

P-wave: $K^* \to K\pi$

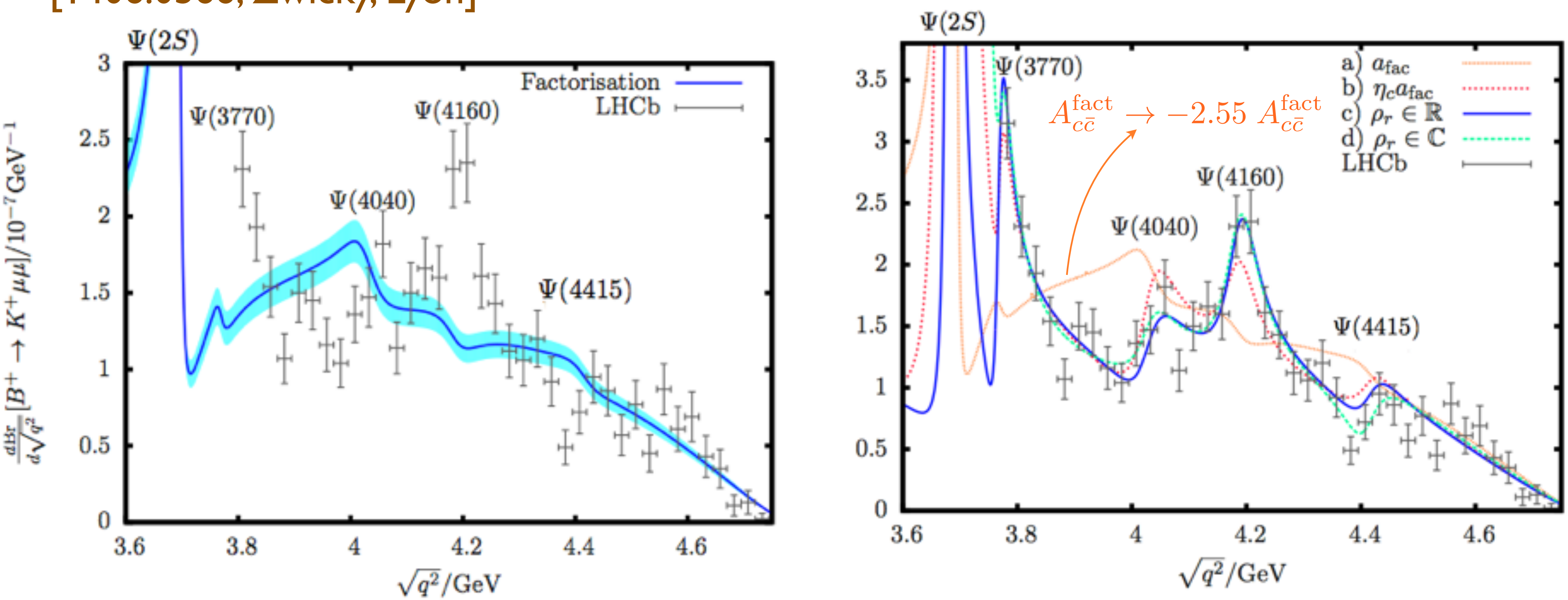
$$W_P = \frac{32}{\pi}\left[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l \right.$$
$$+ J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+ (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi$$
$$\left. + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\right]$$

S/P-wave: $K_0^* \to K\pi$, non-res

$$W_S = \frac{1}{4\pi}\left[\tilde J_{1a}^c + \tilde J_{1b}^c\cos\theta_K + (\tilde J_{2a}^c + \tilde J_{2b}^c\cos\theta_K)\cos 2\theta_\ell + \tilde J_4\sin\theta_K\sin 2\theta_\ell\cos\phi \right.$$
$$\left. + \tilde J_5\sin\theta_K\sin\theta_\ell\cos\phi + \tilde J_7\sin\theta_K\sin\theta_\ell\sin\phi + \tilde J_8\sin\theta_K\sin 2\theta_\ell\sin\phi\right]$$

$B \to (\pi, K, K^*)\ell\ell$: on charmonium and the high-q^2 OPE

- An attempt to use naive factorization with no relative strong phases to describe the resonant structure at high-q^2 fails:
[1406.0566; Zwicky, Lyon]



- This should be interpreted as a failure of QCD factorization to describe the hadronic $B \to \psi_{cc} K$ process (e.g. color octet contributions might be important) and, most of all, its interference with the non-resonant rate
[1101.5118; Beylich, Buchalla, Feldmann]

$B \to (\pi, K, K^*)\ell\ell$: on power corrections

- The very schematic expression for the amplitude is ($B \to Kll$ at low-q^2):

$$A(B \to K\ell\ell) \sim C_7\, f_T + C_9\, f_+ + C_{10}\, f_+ + \sum_{i \neq 7,9,10} C_i \langle K | T J_{\rm em} Q_i | B \rangle$$

exact

$$\sim C_7\, f_T + C_9\, f_+ + C_{10}\, f_+ + \sum_{i \neq 7,9,10} \left[C_i \left[A_i^T\, f_T + A_i^+\, f_+ \right] + \phi_B \otimes H_i \otimes \phi_K + O\left(\frac{\Lambda_{\rm QCD}}{m_b}\right) \right]$$

$C_{7,9}^{\rm eff}(q^2)$ — non-factorizable corrections — power corrections

- Prescription: every term in the amplitude not proportional to $C_{7,9,10}$ receives a O(10%) power correction
[1507.01618; Fermilab/MILC, EL]
[1510.02349; Fermilab/MILC, EL]
- One can parametrize and fit power corrections to data
[1006.4945; Khodjamirian, Mannel, Pivovarov, Wang] → Estimate using LCSR and dispers. relations
[1212.2263; Jäger, Camalich] → q^2 dependent parametrization of power corrections.
[1512.07157; Ciuchini et al.] → Ascribe b→K*ll tensions to q^2 dependent power corrections.
The fit points to PC's of order (20-50)% of the whole amplitude [too large?]

$B \to (\pi, K, K^*)\ell\ell$: on power corrections

- Factorizable power corrections are a self inflicted wound:

$$F_i(B \to K^*) = C_{i\perp}\, \xi_\perp + C_{i\|}\, \xi_\| + \sum_{a=\pm} \phi_B^a \otimes H_i^a \otimes \phi_{K^*} + \text{factorizable PC's}$$

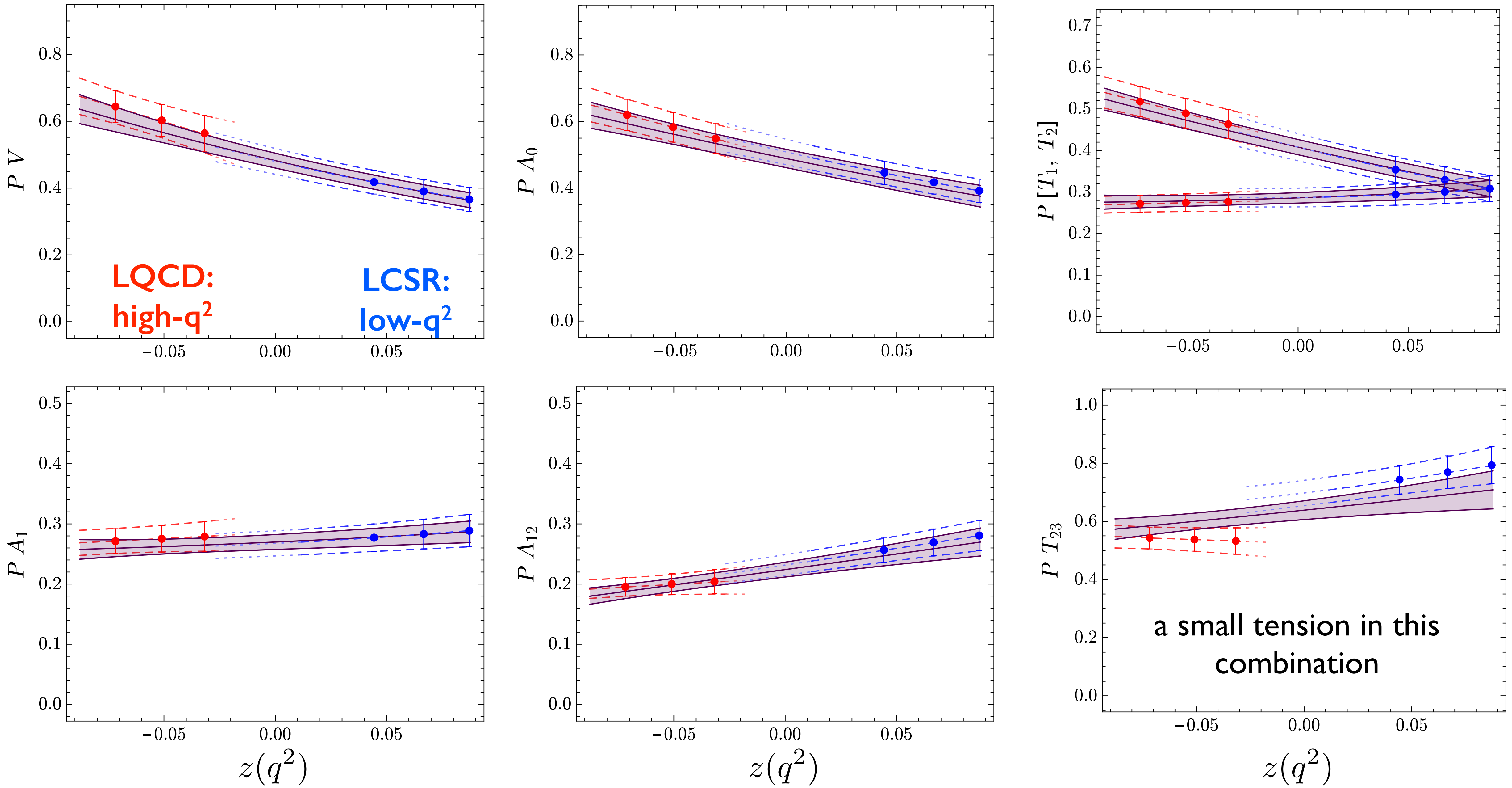
  - Use the full form factors: LQCD (high-q^2) and LCSR (low-q^2)
  - Using z-expansions (e.g. 3 params) for the 7 form factors one can perform a LQCD+LCSR fit that gives the 19 z-fit parameters with a 19x19 correlation matrix
  - Personally I prefer potential systematic issues in the LCSR approach to unknowable factorizable PC's (that are many and enter everywhere)

- Clean observables (e.g. P_5') are defined in such a way that form factors drop out up to <u>factorizable and non-factorizable</u> power corrections
  - If QCD factorization at leading power is a good description at low-q^2, uncertainties on these observables will be small once correlations between form factors uncertainties are taken into account

$B \to (\pi, K, K^*)\ell\ell$: on power corrections

- LQCD vs LCSR $B \to K^*$ form factors:

[1310.3722; Horgan, Liu, Meinel, Wingate]
[hep-ph/0412079; Ball, Zwicky]
[1503.05534; Barucha, Straub, Zwicky]



$B \to (\pi, K, K^*)\ell\ell$: some recent results

- Using the most recent Fermilab/MILC form factors:

$$\Delta\mathcal{B}(B^+ \to K^+\mu^+\mu^-)^{\rm SM} \times 10^9 = \begin{cases} 174.7(9.5)(29.1)(3.2)(2.2), & 1.1\ {\rm GeV}^2 \le q^2 \le 6\ {\rm GeV}^2, \\ 106.8(5.8)(5.2)(1.7)(3.1), & 15\ {\rm GeV}^2 \le q^2 \le 22\ {\rm GeV}^2, \end{cases}$$

$$\Delta\mathcal{B}(B^0 \to K^0\mu^+\mu^-)^{\rm SM} \times 10^9 = \begin{cases} 160.8(8.8)(26.6)(3.0)(1.9), & 1.1\ {\rm GeV}^2 \le q^2 \le 6\ {\rm GeV}^2, \\ 98.5(5.4)(4.8)(1.6)(2.8), & 15\ {\rm GeV}^2 \le q^2 \le 22\ {\rm GeV}^2, \end{cases}$$

[1510.02349; Fermilab/MILC, EL]

  - Errors are (CKM elements)(Form Factors)(matching scale)(everything else)
  - Power correction error is about 1%

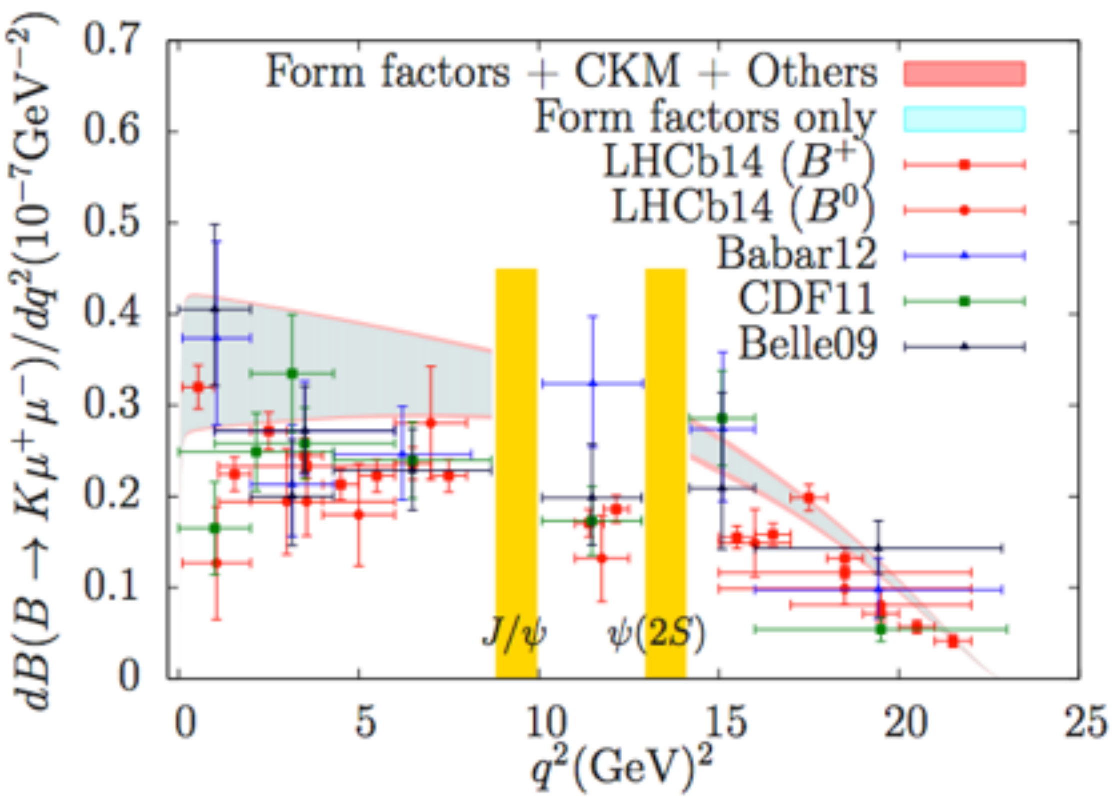
- Experimental LHCb results are:

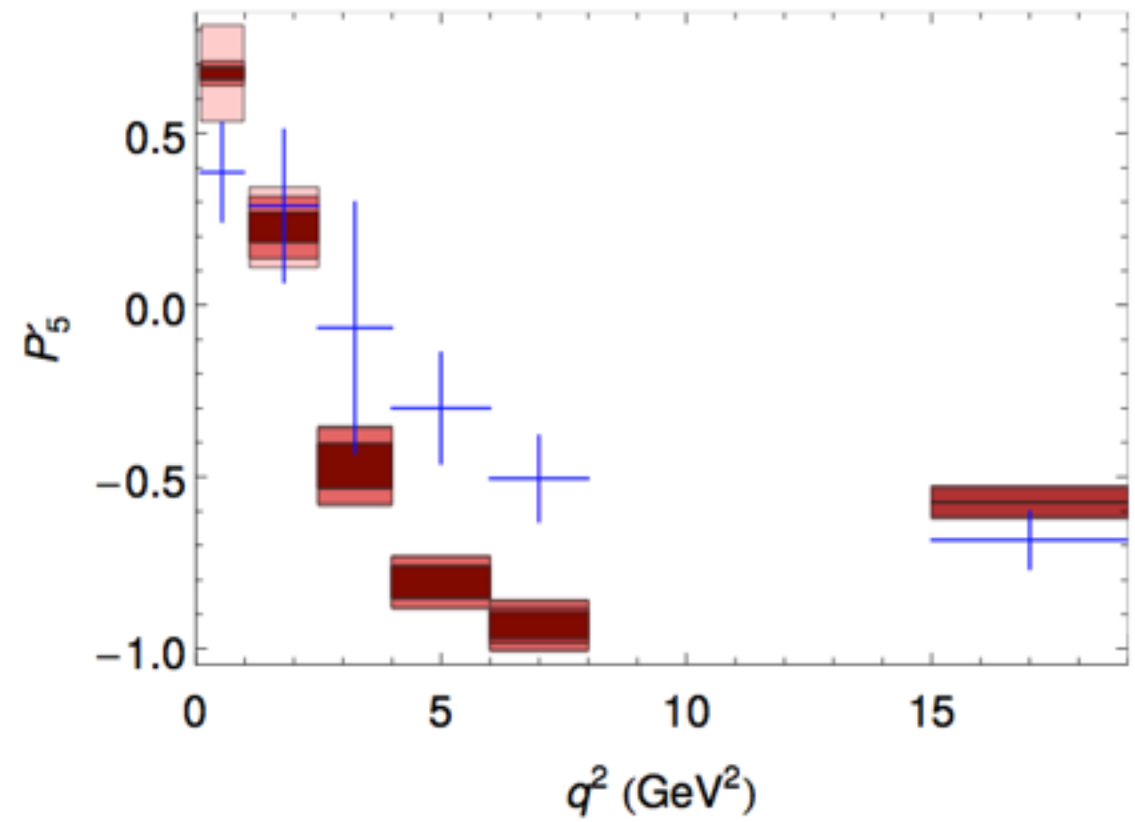
$$\Delta\mathcal{B}(B^+ \to K^+\mu^+\mu^-)^{\rm exp} \times 10^9 = \begin{cases} 118.6(3.4)(5.9) & 1.1\ {\rm GeV}^2 \le q^2 \le 6\ {\rm GeV}^2, \\ 84.7(2.8)(4.2) & 15\ {\rm GeV}^2 \le q^2 \le 22\ {\rm GeV}^2, \end{cases}$$

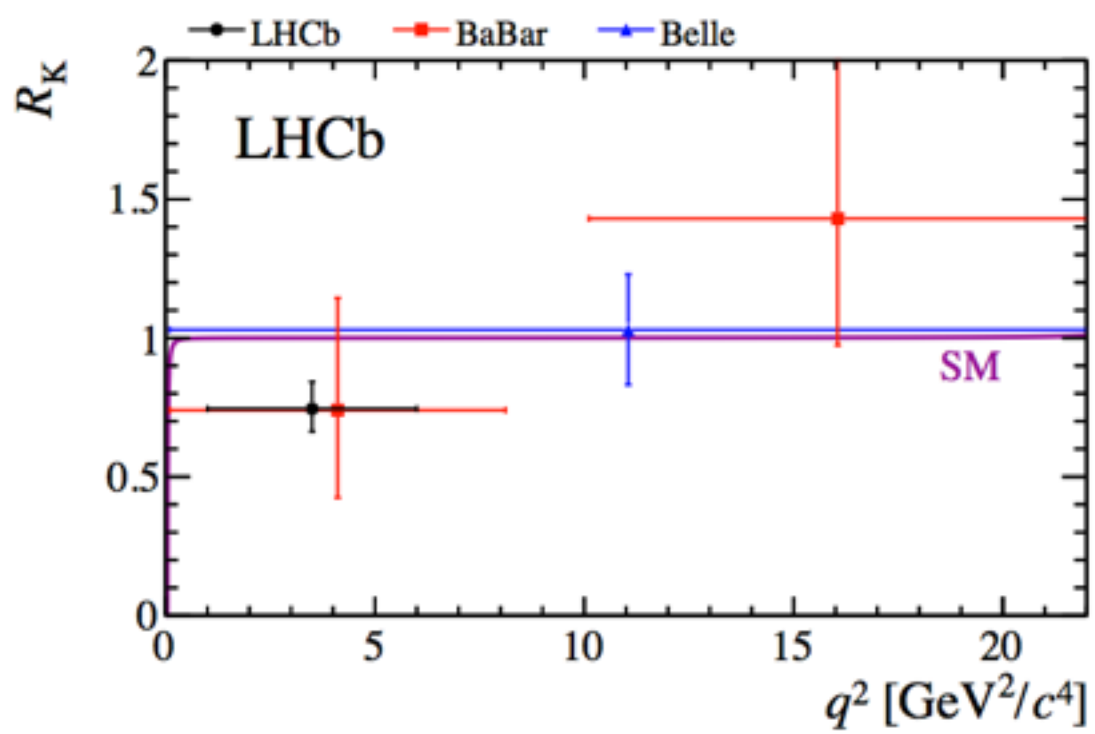
$$\Delta\mathcal{B}(B^0 \to K^0\mu^+\mu^-)^{\rm exp} \times 10^9 = \begin{cases} 91.6\left({}^{+17.2}_{-15.7}\right)(4.4) & 1.1\ {\rm GeV}^2 \le q^2 \le 6\ {\rm GeV}^2, \\ 66.5\left({}^{+11.2}_{-10.5}\right)(3.5) & 15\ {\rm GeV}^2 \le q^2 \le 22\ {\rm GeV}^2, \end{cases}$$

- We observe a 2σ tension

$B \to (\pi, K, K^*)\ell\ell$: anomalies







- P_5' is an "optimized observable". At leading power is independent of FF's:

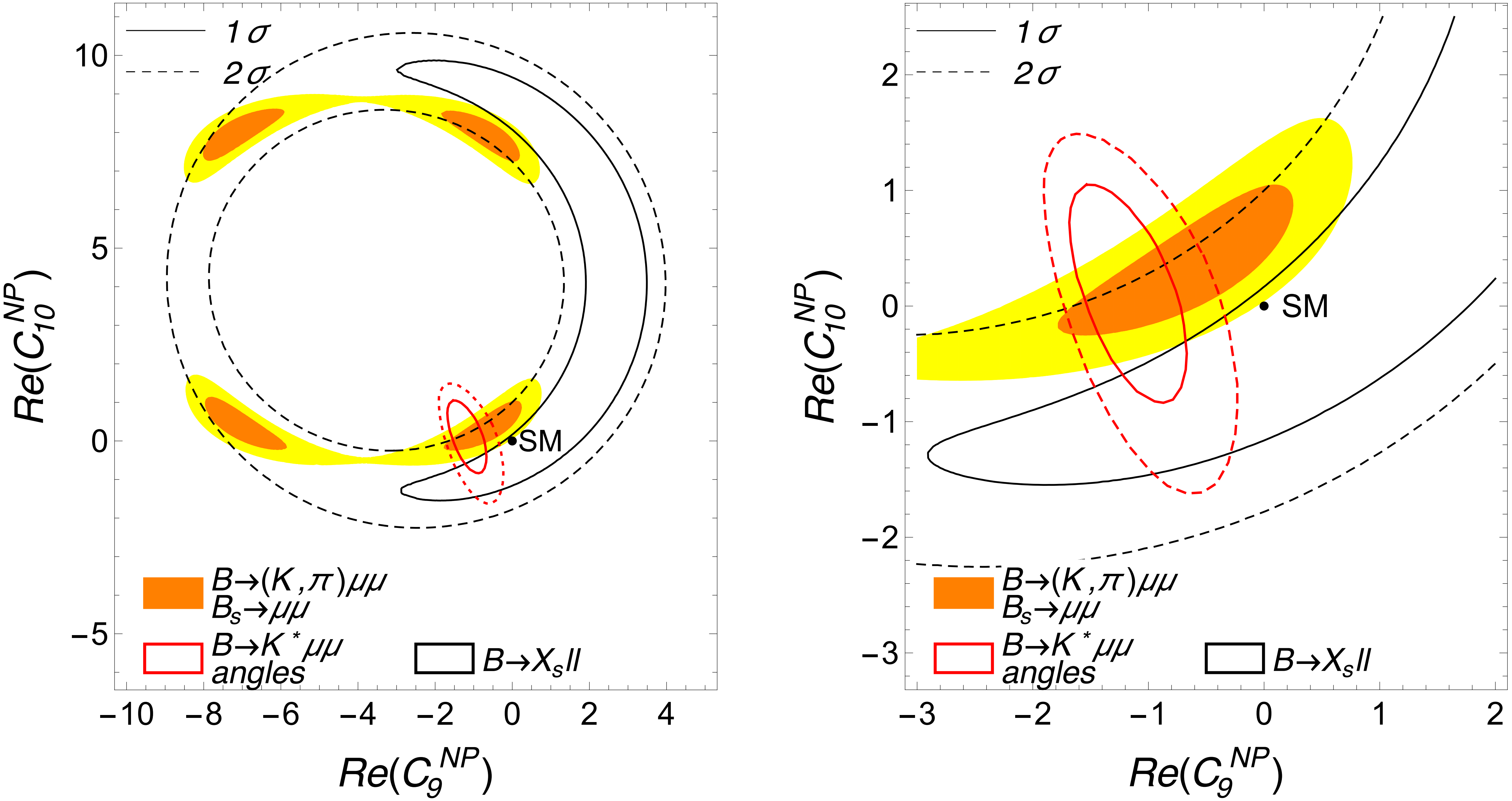
$$P_5' = \frac{C_{10}(C_{9\perp} + C_{9\parallel})}{\sqrt{(C_{9\parallel}^2 + C_{10}^2)(C_{9\perp}^2 + C_{10}^2)}} + O(\alpha_s, \Lambda/m_b)$$

factorizable and non-factorizable power corrections

- $$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu\mu)}{\mathcal{B}(B^+ \to K^+ee)} = 1 + \mathcal{O}(10^{-4})$$

$B \to (\pi, K, K^*)\ell\ell$: some recent results

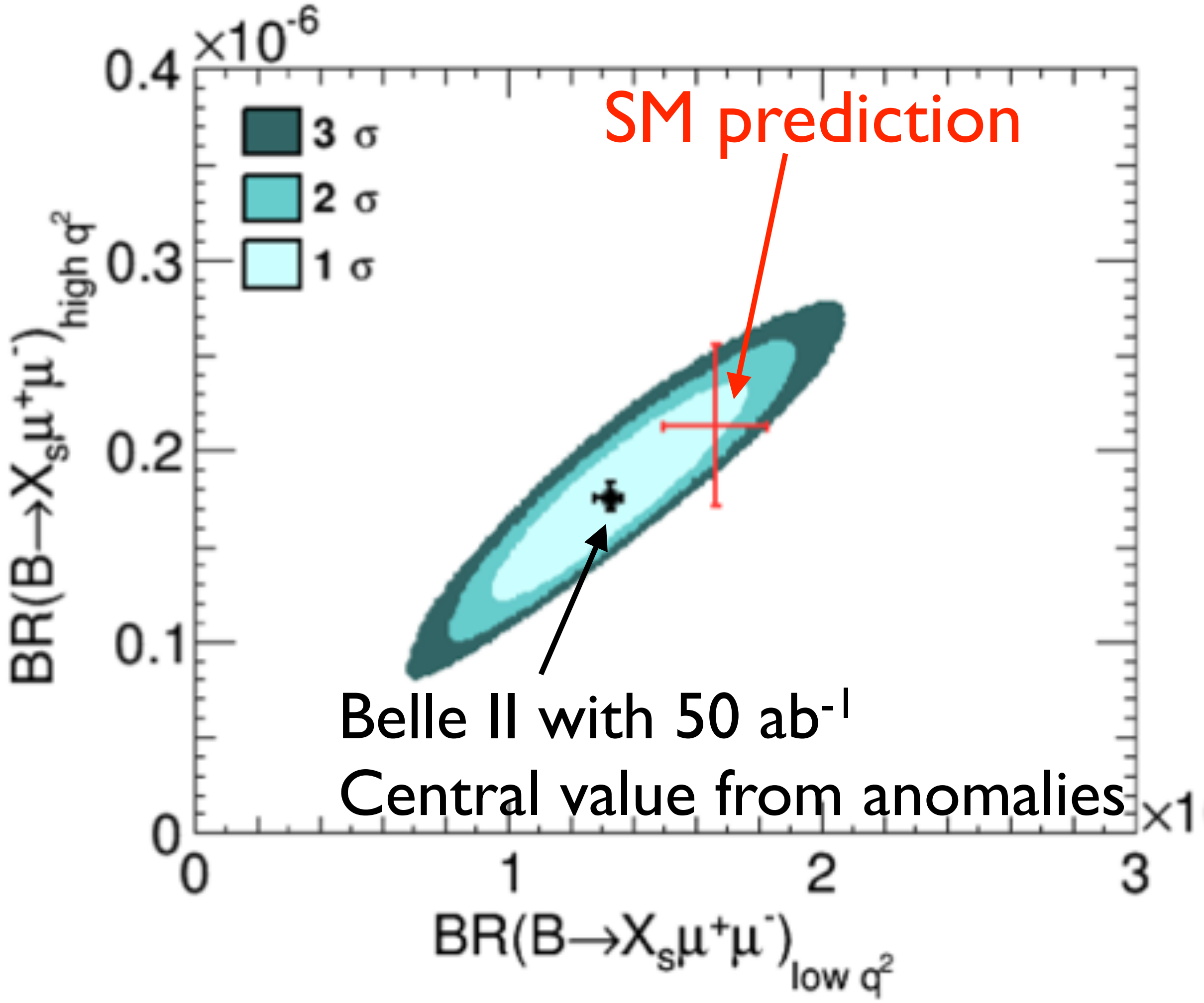
[1510.02349; Fermilab/MILC, EL]

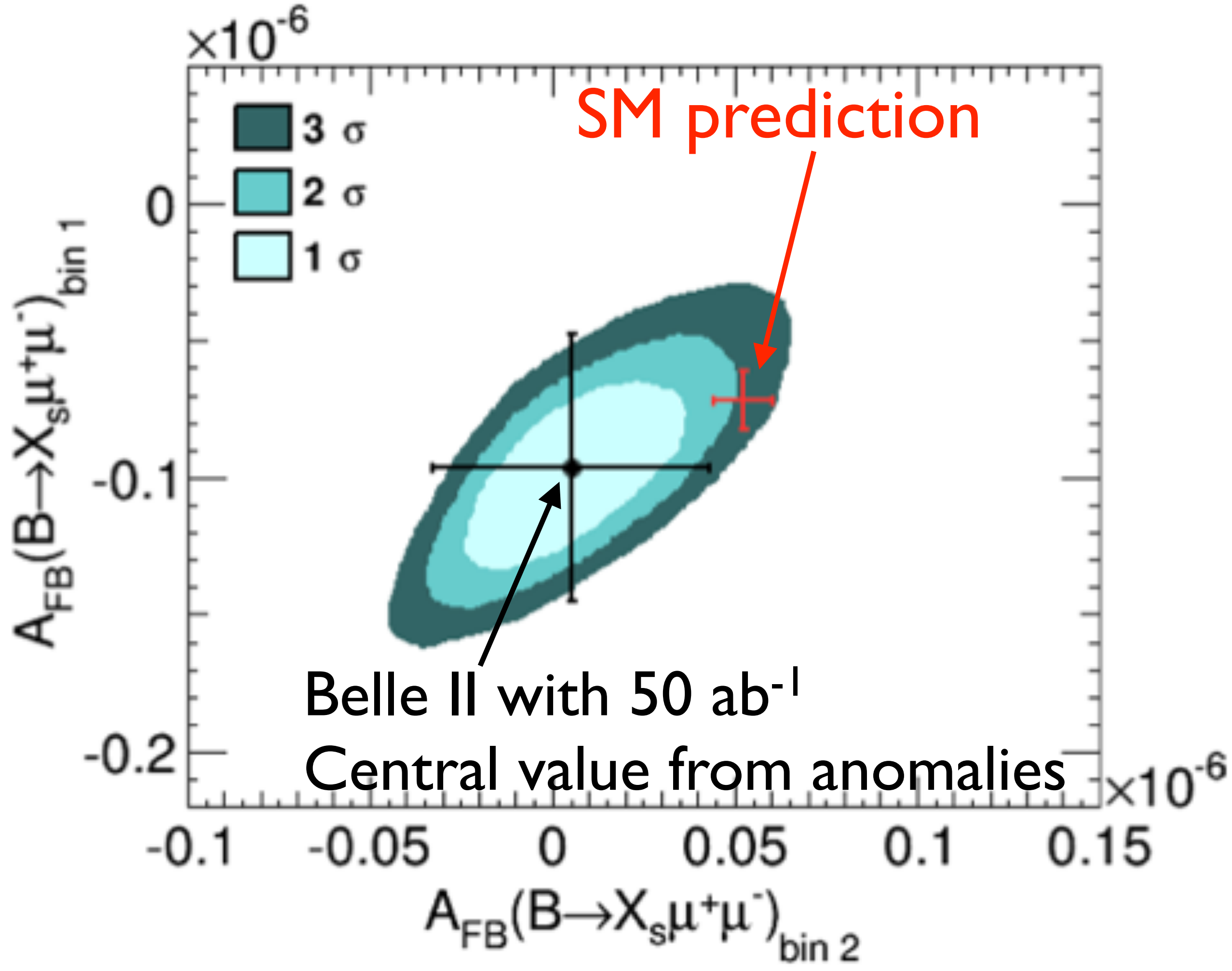


- $B \to K^* ll$ fit results taken from [1503.06199; Altmannshofer, Straub]

$B \to K^{(*)}\ell\ell$ vs $B \to X_s\ell\ell$: LHCb anomalies at Belle-II

- The effects on C_9 and C_9' are large enough to be easily checked at Belle II with inclusive decays [1410.4545 ; Hurth, Mahmoudi, Neshatpour]





Let’s Weasel proof all future HEP experiments!

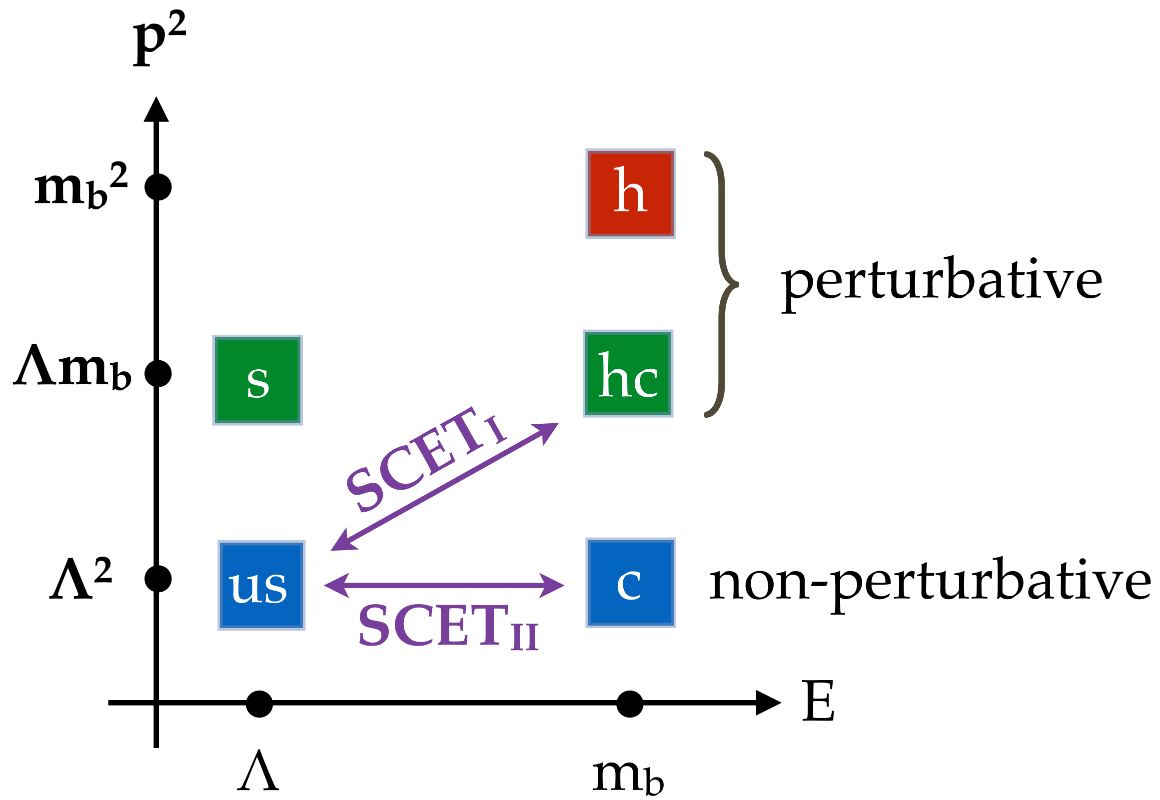
backup…

“Jim was told that he could back up his data by making an image of his computer.”

Theoretical Tools

The central problem is the calculation of hadronic matrix elements of the type $\langle M|T\, O_i(x) J_{\rm em}(y)|B\rangle$, $\langle M_1 M_2|O_i(x)|B\rangle$ or $\langle B|T\, O_i(x) O_j(y)|B\rangle$

- **HQET/SCET$_{\rm II}$**



- Modes are systematically isolated in an effective theory framework
- Power expansion in $1/m_b$
- Factorization proofs at all orders in perturbation theory and at leading power
- Matrix elements are expressed in terms of mesons Light Cone Distribution Amplitudes and Form Factors:

$$A(B\to M_1 M_2) \sim F_{B\to M_1} \otimes H_4 \otimes \phi_{M_2} + \phi_B \otimes H_6 \otimes \phi_{M_1} \otimes \phi_{M_2}$$

- **pQCD**

Endpoint singularities are smeared by integrating over parton transverse momenta resulting in a Sudakov double log that can be resummed (S):
$A(B\to M_1 M_2) \sim \phi_B \otimes H_6 \otimes \phi_{M_1} \otimes \phi_{M_2} \otimes S$

Theoretical Tools

- **Lattice QCD**: direct calculation of matrix elements, decay constants, form factors and some LCDA moments from first principles.
  Note that form factors are calculable at **large q^2**.

- **LCSR**. The calculation of form factors starts from the following correlator

$$\langle M|TO_iJ_B|\Omega\rangle \sim \begin{cases} \sum_{\text{twist n}} H_i^{(n)} \otimes \phi_M^{(n)} \equiv \Pi^{\text{LCE}} & \text{at small } q^2 \\ \sum_X \langle M|O_i|X\rangle\langle X|J_B|\Omega\rangle \sim F_{B\to M}\frac{f_B}{m_B^2-p_B^2} + \text{non pole} \end{cases}$$

  Introduce a Borel transformation, use a dispersion relation to describe the non-pole terms and assume quark-hadron duality:

$$\hat{B}\Pi^{\text{LCE}} \sim F_{B\to M} f_B\, e^{-m_B^2/M^2} + \int_{s_0}^{\infty} \text{Im}\left[\Pi^{\text{LCE}}\right] e^{-t^2/M^2}$$

  Ingredients: light-cone expansion at **small q^2**, Borel parameter M, continuum threshold s_0, quark-hadron duality, decay constants and LCDA's.

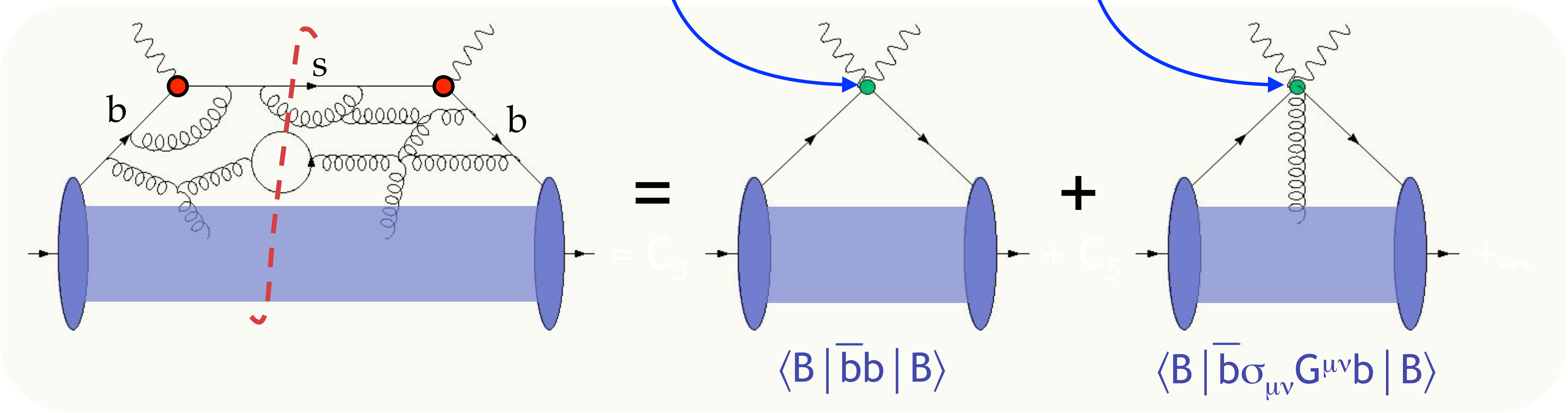
Theoretical Tools

- **OPE:** the T-product of operators evaluated at $x^\mu \sim y^\mu$ is given in terms of a sum over local operators

$$T\; O_i(x) O_j(y) \xrightarrow[y^\mu \to x^\mu]{} \sum_i C_i(x-y)\; Q_i(x)$$

In inclusive ($B \to X_s \ell\ell$) and exclusive ($B \to K^{(*)}\ell\ell$ at high-q^2) decays we have $(x-y)^2 \sim 0$ instead of $x^\mu - y^\mu \sim 0$: quark-hadron duality.

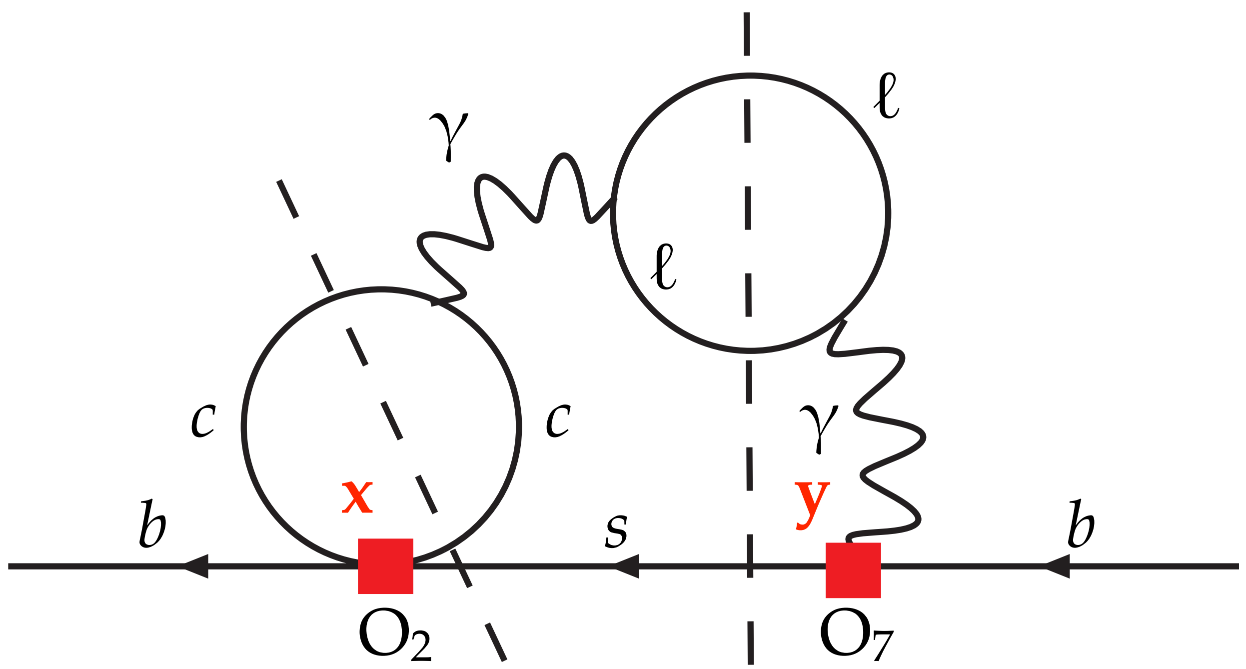
$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right] + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2}, \dots\right)$$



Theoretical Tools

- **OPE in inclusive vs exclusive decays:**

Inclusive

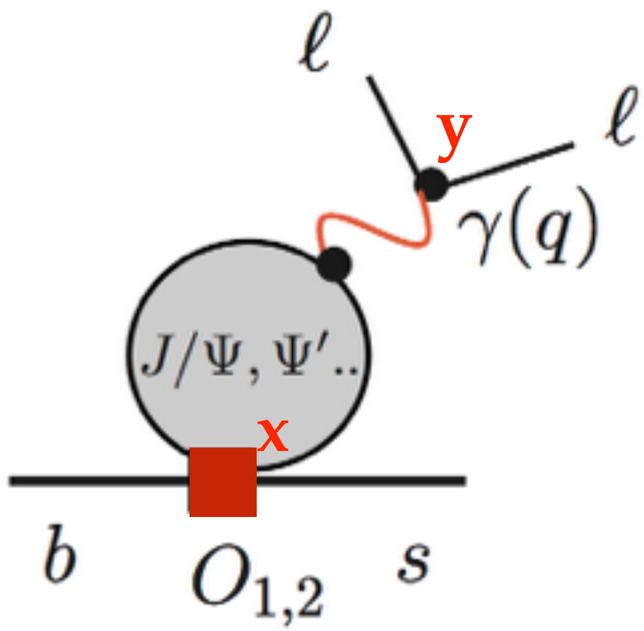


$$(x-y)^2 \sim \frac{1}{p^2_{X_s}} \sim \frac{1}{\left(m_b - \sqrt{q^2}\right)^2}$$

The OPE breaks down at large q[2].
Charmonium resonances can be included using $e^+e^- \to \mathrm{hadrons}$

[hep-ph/9603237; Krüger, Sehgal]

Exclusive



$$(x-y)^2 \sim \frac{1}{q^2}$$

Charmonium resonances correspond to large $x^\mu - y^\mu$ and must be dealt with invoking quark-hadron duality

[1101.5118; Beylich, Buchalla, Feldmann]

$B \to X_{s,d}\gamma$

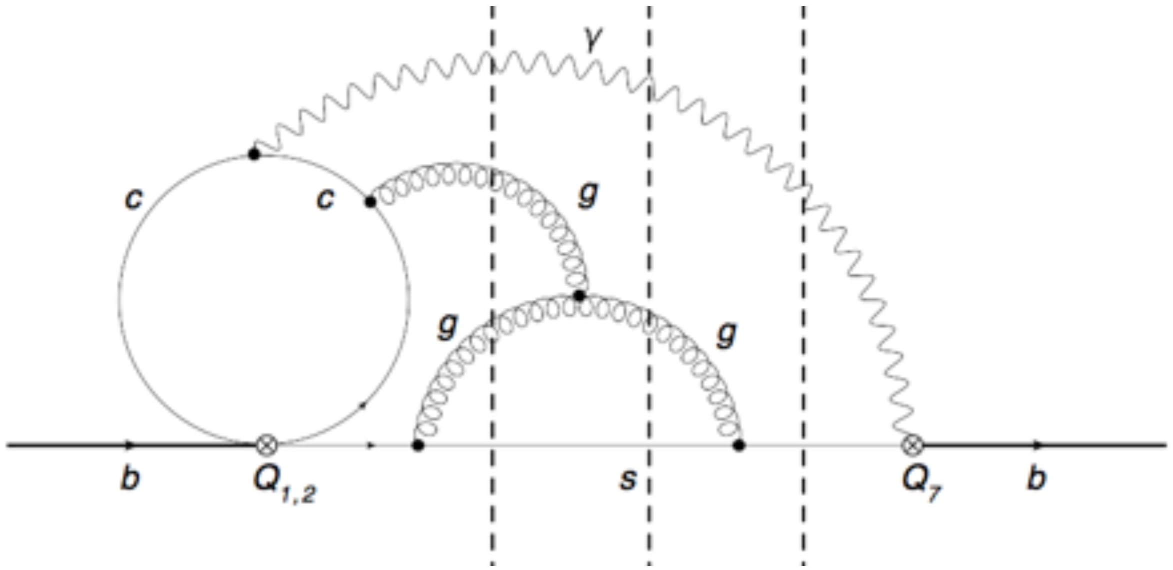
- This calculation is one of the greatest perturbative feats in flavor physics: [hep-ph/0609232 and 1503.01789; Misiak, Asatrian, Bieri, Boughezal, Czakon, Czarnecki, Ewerth, Ferroglia, Fiedler, Gambino, Gorbahn, Greub, Haisch, Hovhannisyan, Huber, Hurth, Kaminski, Mitov, Ossola, Poghosyan, Poradzinski, Rehman, Schutzmeier, Slusarczyk, Steinhauser, Virto]

- Using the optical theorem and a local OPE the rate can be written as:

$$\Gamma(B \to X_q\gamma) = \Gamma(b \to X_q\gamma) + \delta\Gamma_{\rm nonp}$$

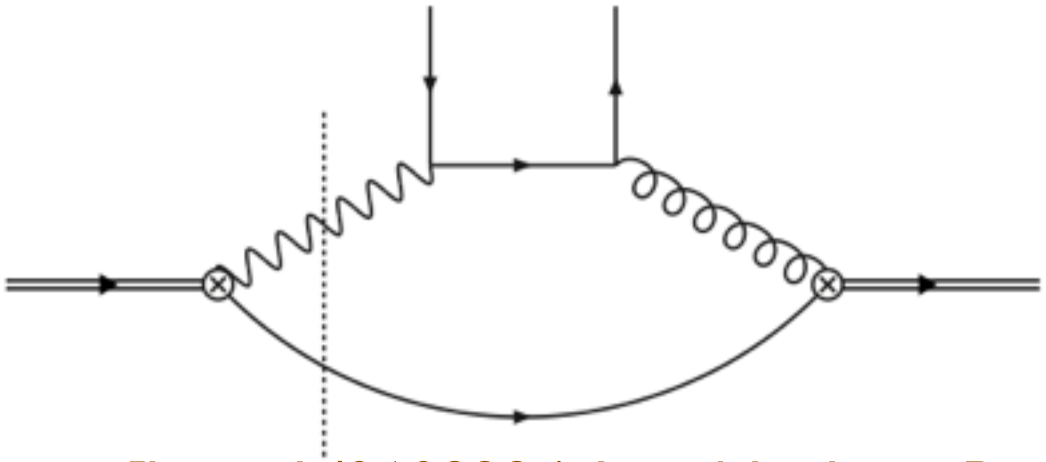
$\pm 4.7\%$ $\pm 5\%$

Known at NNLO (4 loops)



[1503.01791; Czakon, Fiedler, Huber, Misiak, Schutzmeier, Steinhauser]

Some non-local power corrections scale as $\Lambda_{\rm QCD}/m_b$:



[hep-ph/0609224; Lee, Neubert, Paz]
[1003.5012; Benzke, Lee, Neubert, Paz]

$B \to X_{s,d}\gamma$

- Current SM predictions (with $E_\gamma > 1.6\ \mathrm{GeV}$):

$$\mathcal{B}^{\rm SM}_{s\gamma} = (3.36 \pm 0.23) \times 10^{-4}$$
$$\mathcal{B}^{\rm SM}_{d\gamma} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5}$$

larger error because of collinear photons in $b \to u\bar{u}d\gamma$

$$\frac{\mathcal{B}^{\rm SM}_{s\gamma} + \mathcal{B}^{\rm SM}_{d\gamma}}{\mathcal{B}_{c\ell\nu}} = (3.31 \pm 0.22) \times 10^{-3}$$

untagged rate

- Current experimental world average (with $E_\gamma > 1.6\ \mathrm{GeV}$):

$$\mathcal{B}^{\rm exp}_{s\gamma} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$
$$\mathcal{B}^{\rm exp}_{d\gamma} = (1.41 \pm 0.57) \times 10^{-5}$$

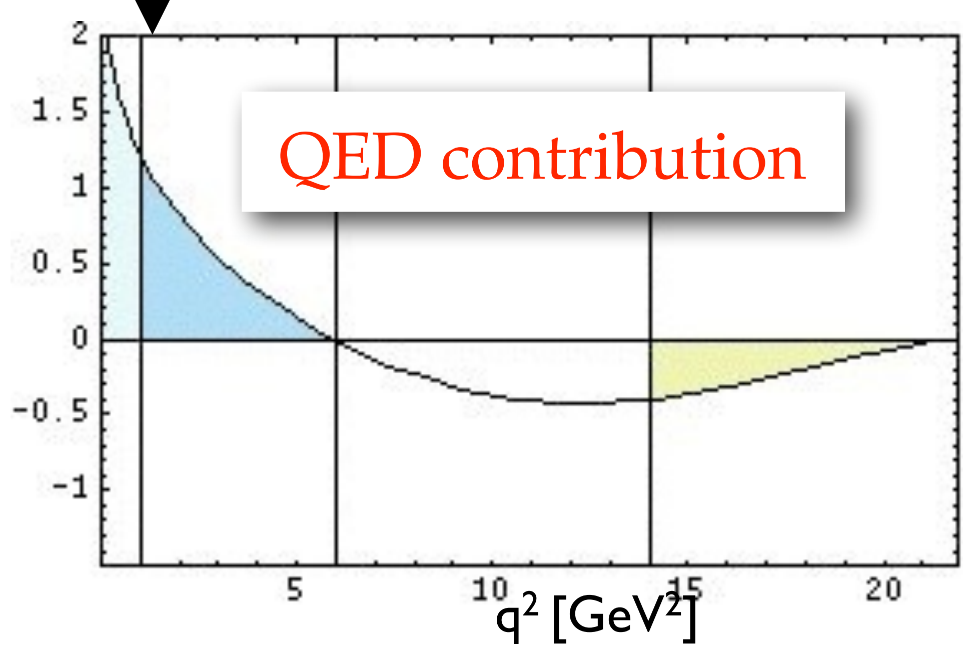
⇐ perfect agreement!

- In the context of a Type-II Two-Higgs Doublet Model:

$$m_{H^\pm} > 480\ \mathrm{GeV}\ \ \text{at } 95\%\ \text{C.L.}$$

$B \to X_s \ell\ell$: QED effects

- Known at NNLO in QCD and NLO in QED
- In particular QED effects are large: virtual effects due to the overall $\alpha_{em}^2(\mu)$ normalization and real effects due to real photon emission:
  - RGE for WC's: $\alpha_{em} \log(m_W/m_b)$ [hep-ph/0312090 ;Bobeth,Gambino,Gorbahn,Haisch]
  - Matrix elements: $\alpha_{em} \log(m_\ell/m_b)$ [hep-ph/0512066 ; Huber, EL, Misiak, Wyler]



$$\text{virtual} = \frac{A_{\text{soft+collinear}}}{\epsilon^2} + \frac{B_{\text{collinear}} + B_{\text{soft}}}{\epsilon} + C$$

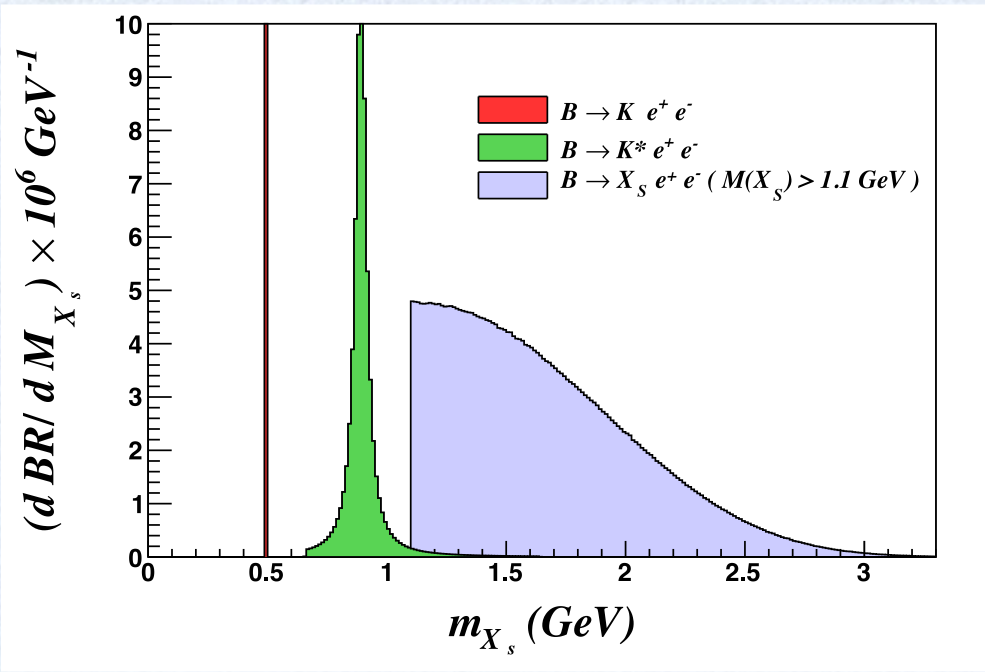
$$\text{real} = -\frac{A_{\text{soft+collinear}}}{\epsilon^2} - \frac{B'_{\text{collinear}} + B_{\text{soft}}}{\epsilon} + C'$$

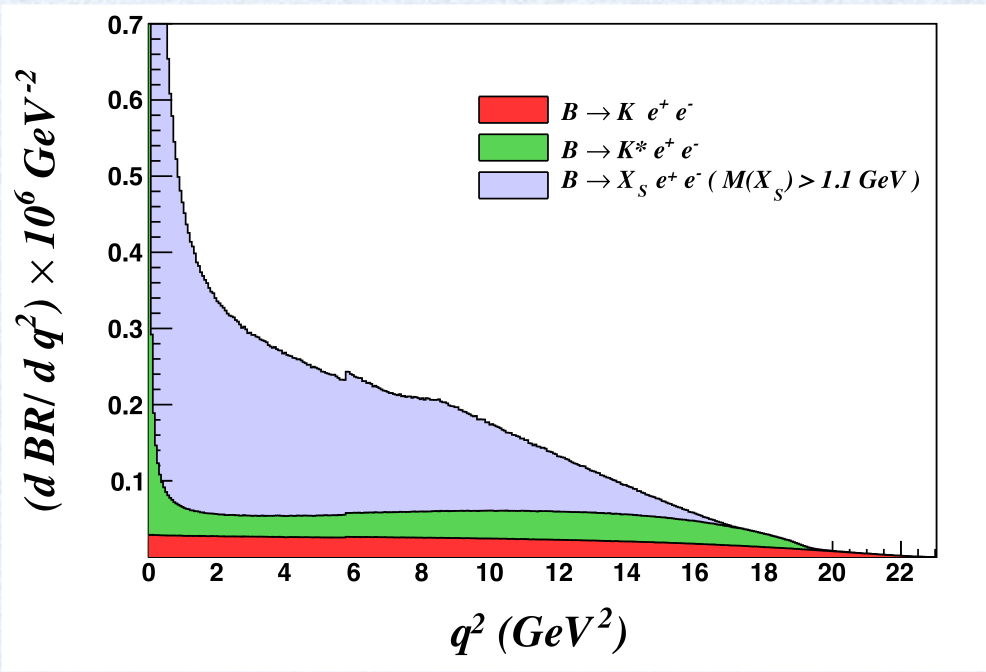
$$\int dq^2 \, (B_{\text{collinear}} - B'_{\text{collinear}}) = 0$$

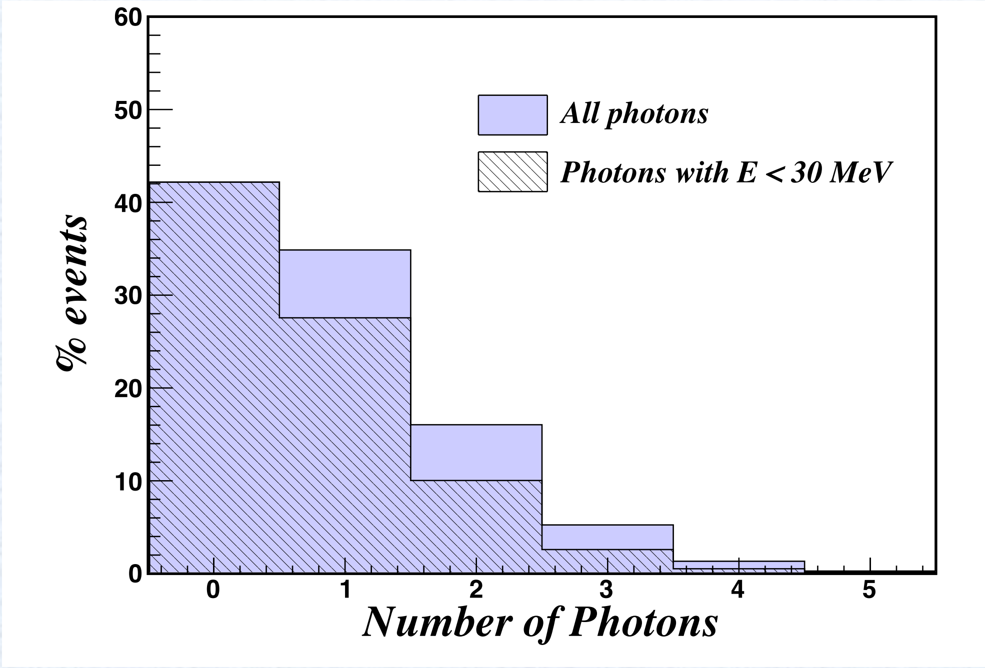
The sign of the contribution changes (accidentally) at $q^2 = 6$ GeV^2
This effect is numerically important.

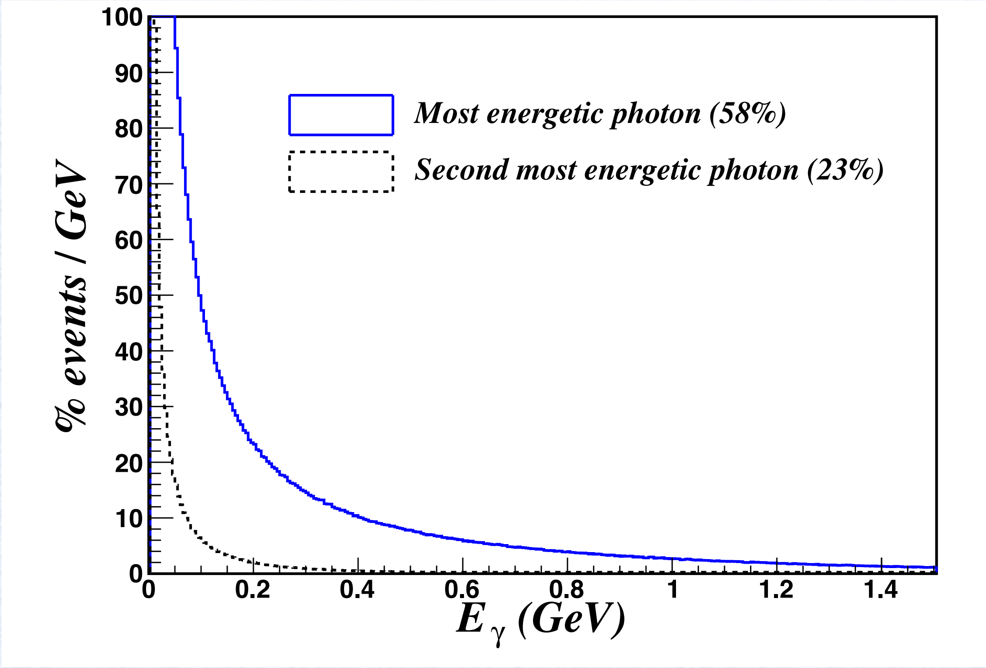
MONTE CARLO CHECK

- EM effects have been calculated analytically and cross checked against Monte Carlo generated events (EVTGEN + PHOTOS)



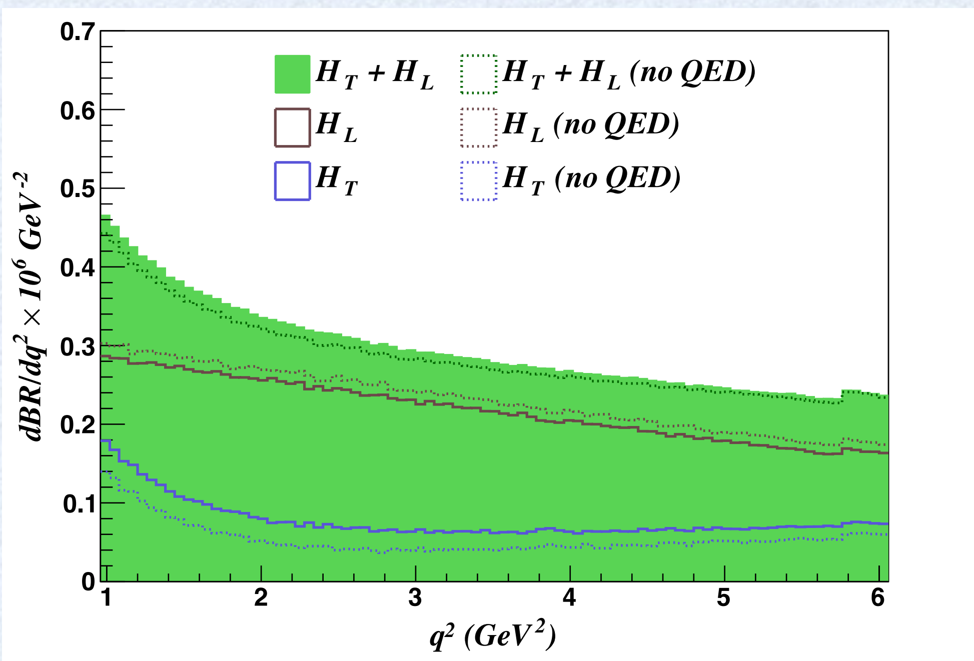


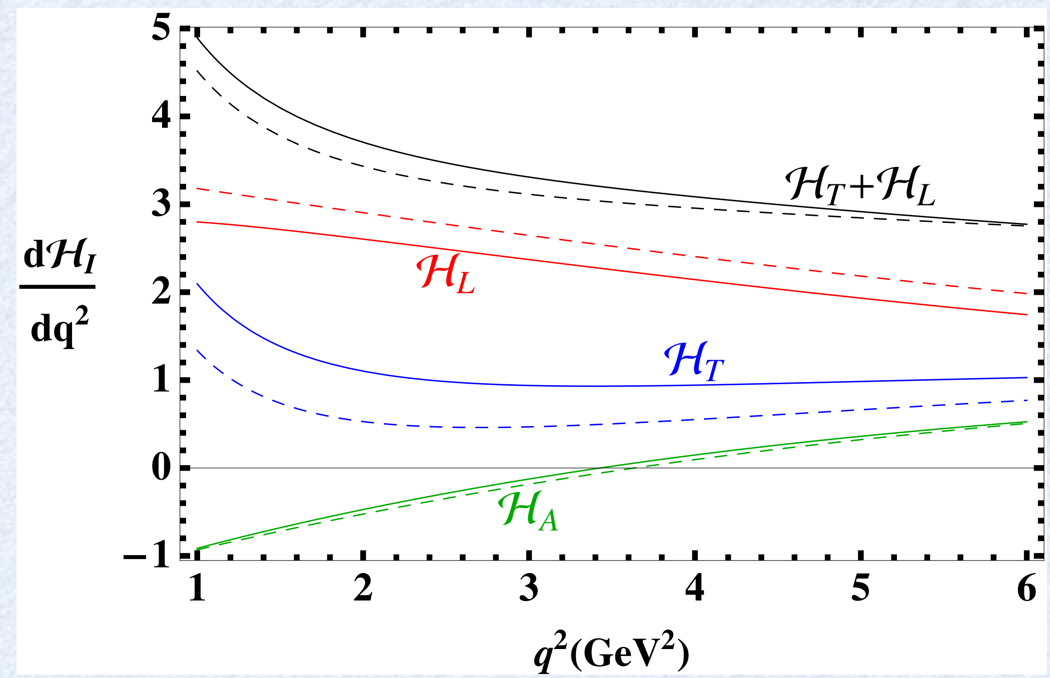


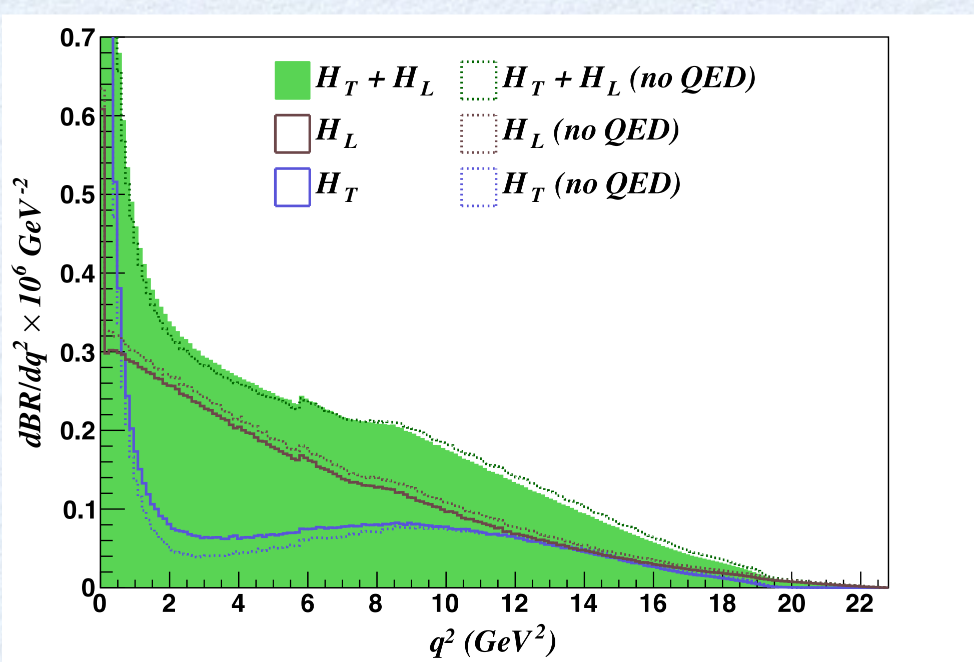


Monte Carlo check

- The Monte Carlo study reproduces the main features of the analytical results







Monte Carlo:

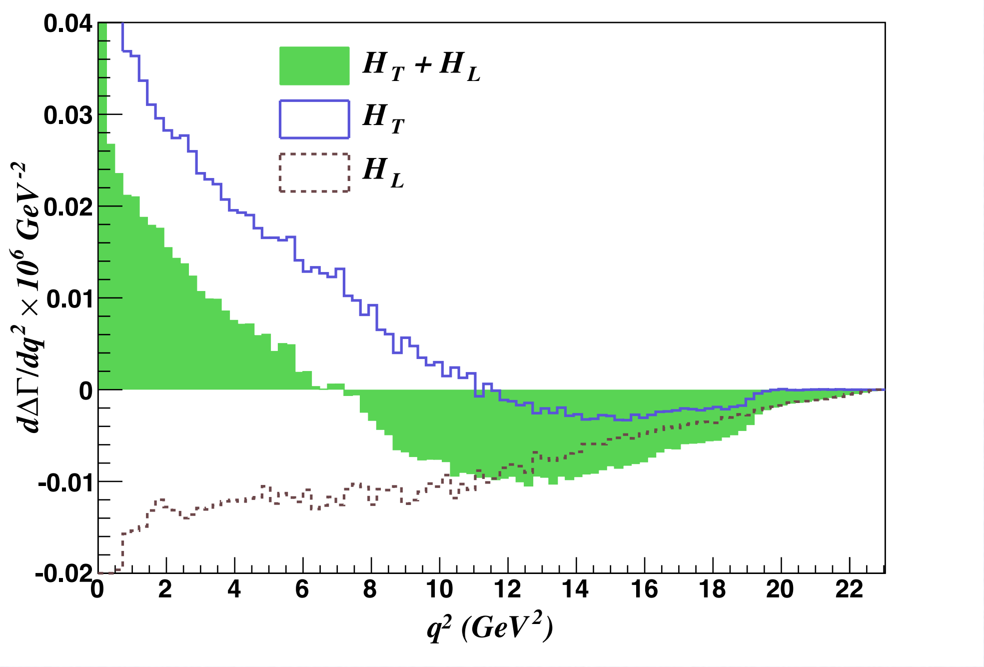
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	3.5	3.5
$\mathcal{H}_T$	19.0	8.0	43.0
$\mathcal{H}_L$	81.0	-4.5	-5.5

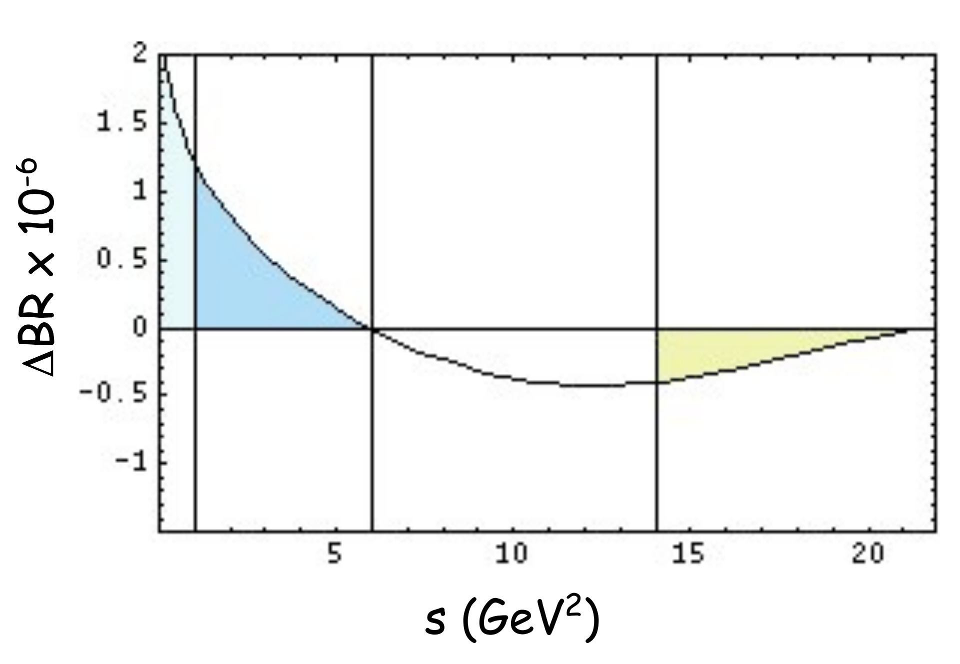
Analytical:

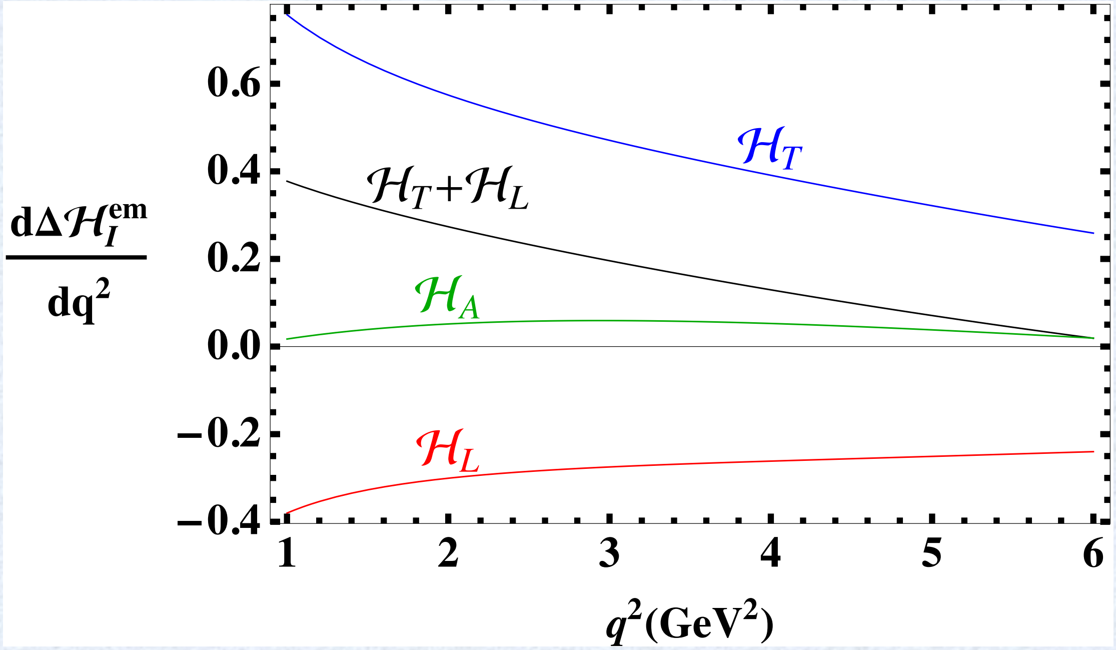
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	5.1	5.1
$\mathcal{H}_T$	19.5	14.1	72.5
$\mathcal{H}_L$	80.0	-8.7	-10.9

Monte Carlo check

- The Monte Carlo study reproduces the main features of the analytical results







Results

	δ_{th}		R(μ/e)
$\mathcal{H}_T[1,6]_{\mu\mu} = (4.03 \pm 0.28) \cdot 10^{-7}$	±7%	$\mathcal{H}_T[1,6]_{ee} = (5.34 \pm 0.38) \cdot 10^{-7}$	0.75
$\mathcal{H}_L[1,6]_{\mu\mu} = (1.21 \pm 0.07) \cdot 10^{-6}$	±6%	$\mathcal{H}_L[1,6]_{ee} = (1.13 \pm 0.06) \cdot 10^{-6}$	1.07
$\mathcal{H}_A[1,3.5]_{\mu\mu} = (-1.10 \pm 0.05) \cdot 10^{-7}$	±5%	$\mathcal{H}_A[1,3.5]_{ee} = (-1.03 \pm 0.05) \cdot 10^{-7}$	1.07
$\mathcal{H}_A[3.5,6]_{\mu\mu} = (+0.67 \pm 0.12) \cdot 10^{-7}$	±18%	$\mathcal{H}_A[3.5,6]_{ee} = (+0.73 \pm 0.12) \cdot 10^{-7}$	0.92
$\mathcal{H}_3[1,6]_{\mu\mu} = (3.71 \pm 0.50) \cdot 10^{-9}$	±13%	$\mathcal{H}_3[1,6]_{ee} = (8.92 \pm 1.20) \cdot 10^{-9}$	0.42
$\mathcal{H}_4[1,6]_{\mu\mu} = (3.50 \pm 0.32) \cdot 10^{-9}$	±9%	$\mathcal{H}_4[1,6]_{ee} = (8.41 \pm 0.78) \cdot 10^{-9}$	0.42
$\mathcal{B}[1,6]_{\mu\mu} = (1.62 \pm 0.09) \cdot 10^{-7}$	±5%	$\mathcal{B}[1,6]_{ee} = (1.67 \pm 0.10) \cdot 10^{-7}$	0.97
$\mathcal{B}[> 14.4]_{\mu\mu} = (2.53 \pm 0.70) \cdot 10^{-7}$	±28%	$\mathcal{B}[> 14.4]_{ee} = (2.20 \pm 0.70) \cdot 10^{-7}$	1.15

- Scale uncertainties dominate at low-q^2
- Power corrections and scale uncertainties dominate at high-q^2
- Log-enhanced QED corrections at low and high q^2 are correlated

$B \to X_s \ell\ell$: new observables

- At leading order in QED and at all orders in QCD, the double differential width is a quadratic polynomial: Γ~a cos2θ+b cosθ+c.
- Γ receives non polynomial log-enhanced QED corrections
- Best strategy: **measure individual observables (BR, A_{FB}) and use Legendre polynomial as projectors**

$$H_I(q^2) = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} W_I(z) dz$$

$$W_T = \frac{2}{3} P_0(z) + \frac{10}{3} P_2(z)\,,$$
$$W_L = \frac{1}{3} P_0(z) - \frac{10}{3} P_2(z)\,,$$
$$W_A = \frac{4}{3}\mathrm{sign}(z)\,.$$

$$W_3 = P_3(z)$$
$$W_4 = P_4(z)$$

new observables

$$\frac{d\Gamma}{dq^2} = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} dz = H_T + H_L$$

$$\frac{dA_{\rm FB}}{dq^2} = \int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} \mathrm{sign}(z) dz = \frac{3}{4} H_A$$

$$\frac{d\overline{A}_{\rm FB}}{dq^2} = \frac{\int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} \mathrm{sign} dz}{\int_{-1}^{+1} \frac{d^2\Gamma}{dq^2 dz} dz} = \frac{3}{4} \frac{H_A}{H_T + H_L}$$

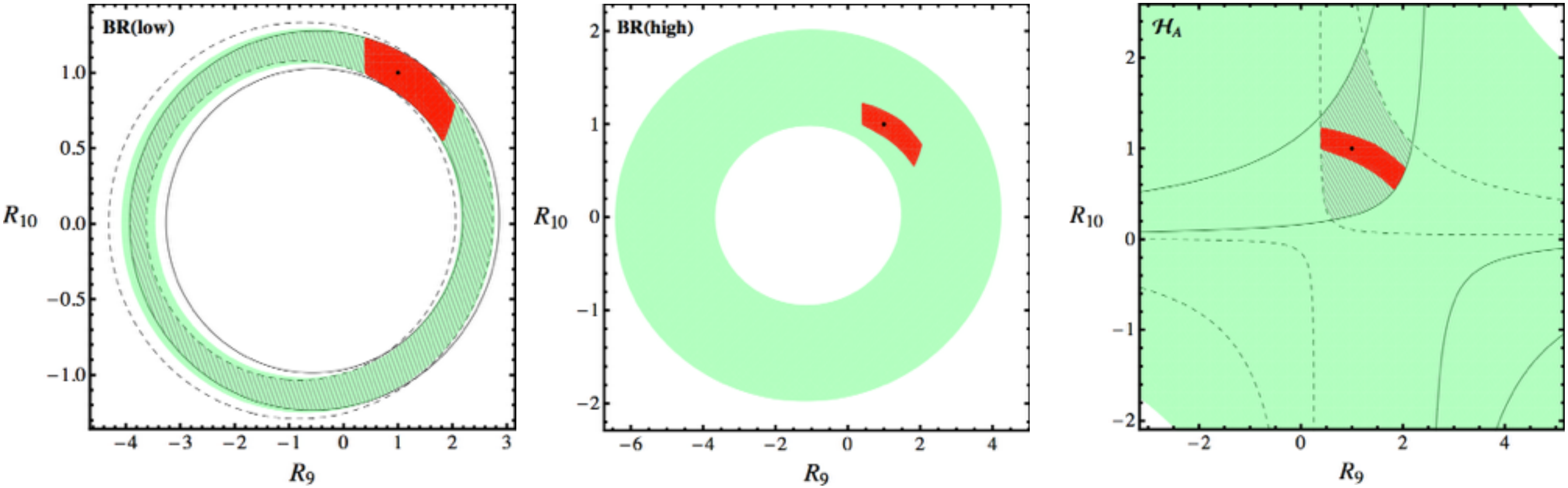
$B \to X_s \ell\ell$: Belle II expectations

- Projected reach with 50 ab^{-1} of integrated luminosity

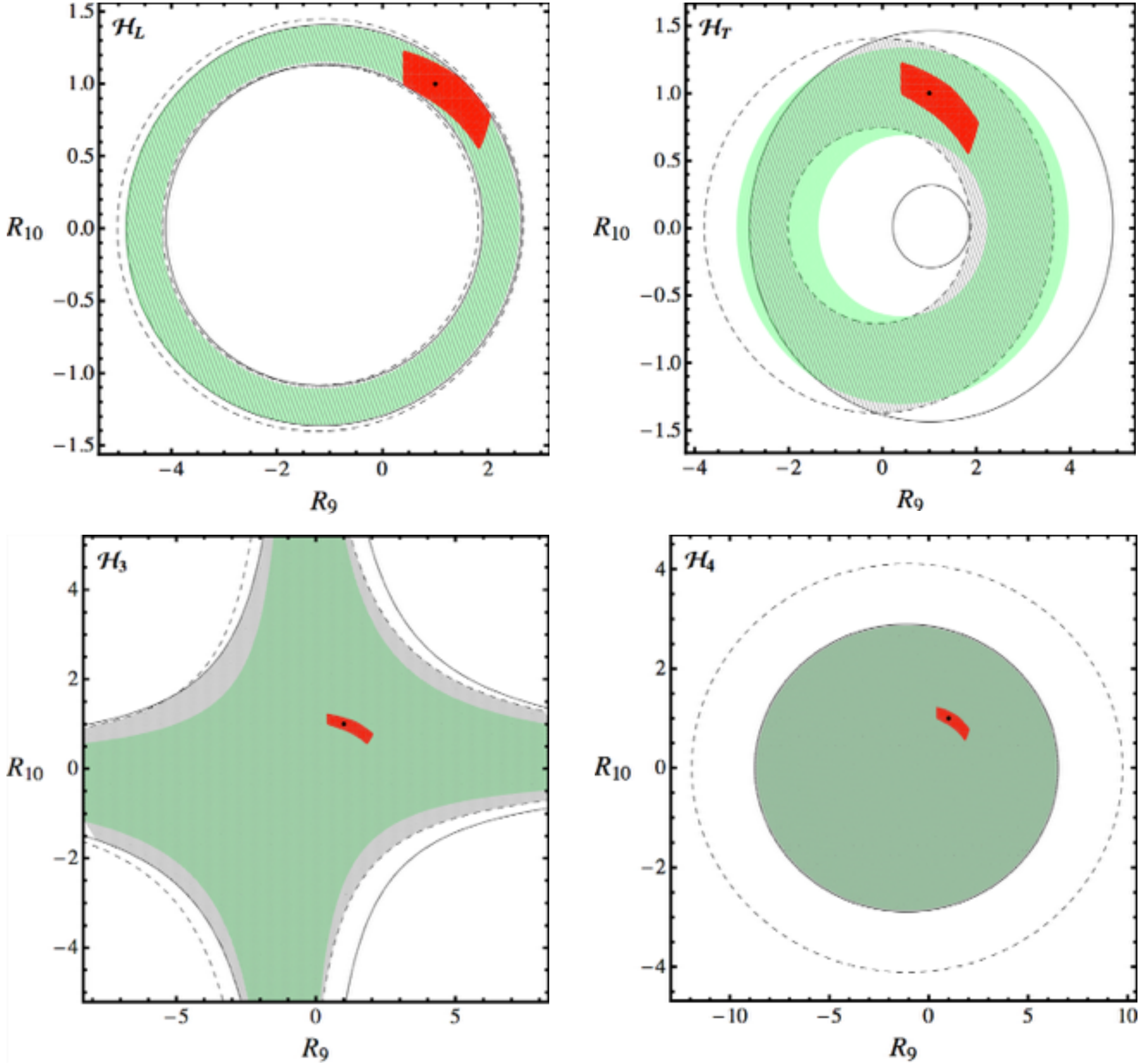
$$\mathcal{O}_{\rm exp} = \int \frac{d^2\mathcal{N}}{d\hat{s}dz}\, W[\hat{s}, z]\, d\hat{s}\, dz\,,$$

$$\delta\mathcal{O}_{\rm exp} = \left[\int \frac{d^2\mathcal{N}}{d\hat{s}dz}\, W[\hat{s}, z]^2\, d\hat{s}\, dz\right]^{\frac{1}{2}}$$

	$[1, 3.5]$	$[3.5, 6]$	$[1, 6]$	> 14.4
$\mathcal{B}$	3.7 %	4.0 %	3.0 %	4.1%
$\mathcal{H}_T$	24 %	21 %	16 %	-
$\mathcal{H}_L$	5.8 %	6.8 %	4.6 %	-
$\mathcal{H}_A$	37 %	44 %	200 %	-
$\mathcal{H}_3$	240 %	180 %	150 %	-
$\mathcal{H}_4$	140 %	360 %	140 %	-



$B \to X_s \ell\ell$: Belle II expectations



$B \to (\pi, K, K^*)\ell\ell$: differential rate

- B→Kll rate at low-q2:

$$\frac{d\Gamma}{dq^2} \sim \left|f_+(q^2)\, C_{10}\right|^2 + \left| f_+(q^2)\, C_9^{\rm eff}(q^2) + \frac{2m_b}{m_B+m_K} f_T(q^2)\, C_7^{\rm eff}(q^2) \right.$$

$$\underbrace{\left. + \frac{2m_b}{m_B}\frac{\pi^2}{N_c}\frac{f_B f_K}{m_B}\sum_\pm \int \frac{d\omega}{\omega}\Phi_{B,\pm}(\omega)\int_0^1 du\, \Phi_K(u)\left[T^{(0)}_{P,\pm} + \tilde\alpha_s C_F\, T^{(\rm nf)}_{P,\pm}\right]\right|^2}_{\text{absent at high-}q^2}$$

- The form factor f_T can be expressed in terms of f_+:

$$\frac{m_B}{m_B+m_K} f_T = f_+\left[1 + \tilde\alpha_s C_F\left(\log\frac{m_b^2}{\mu^2} + 2L\right)\right]$$

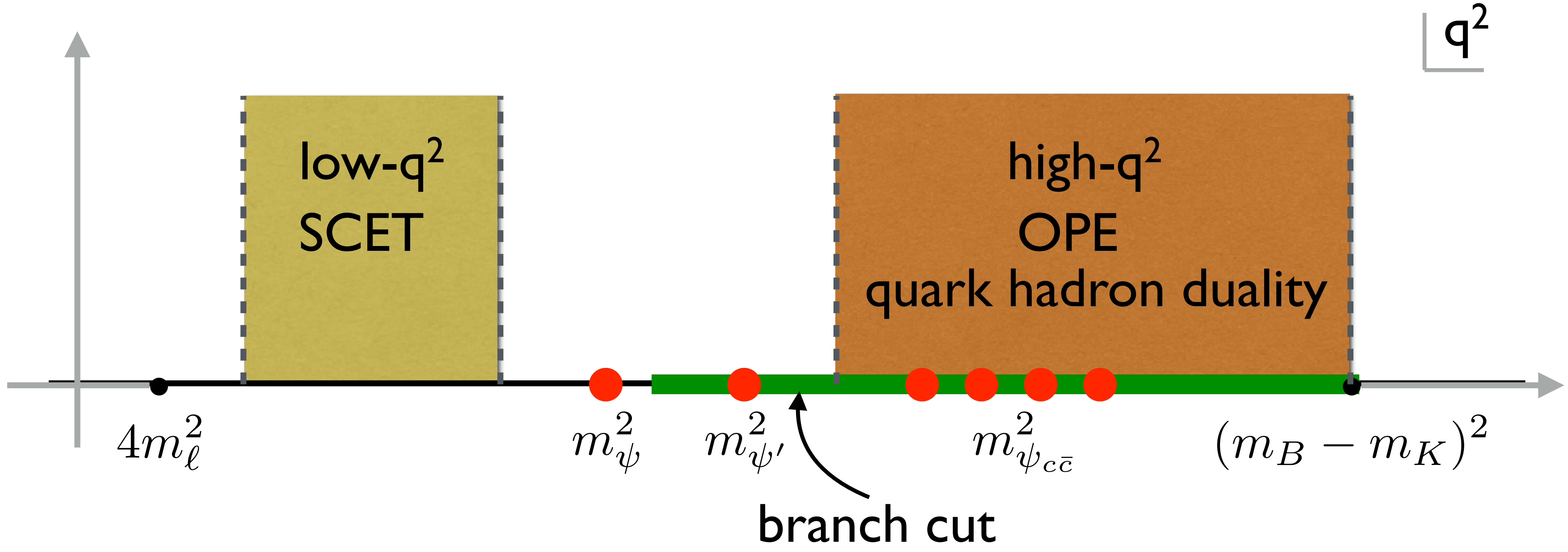
$$- \frac{\pi}{N_c}\frac{f_B f_K}{E}\alpha_s C_F \overbrace{\int\frac{d\omega}{\omega}\Phi_{B,+}(\omega)}^{\lambda_{B,+}^{-1}}\int_0^1\frac{du}{\bar u}\,\Phi_K(u)$$

$B \to (\pi, K, K^*)\ell\ell$: S/P wave pollution

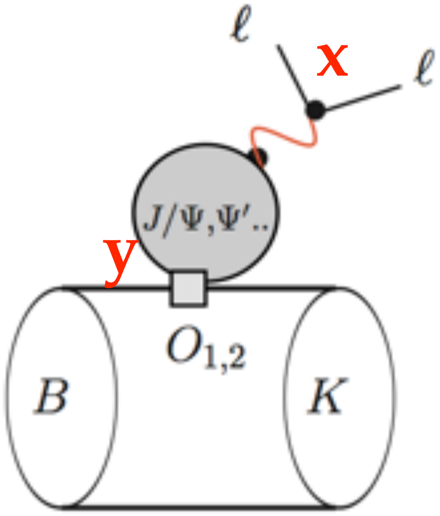
- In $B\to K^*ll\to K\pi ll$, the $K\pi$ system is produced in P wave ($J^P(K^*)=1^-$)
- The $K_0^*(800)$ resonance ($J^P=0^+$) generates a $K\pi$ pairs in S wave. This background can be removed by studying the $(\theta_\ell,\ \phi,\ \theta_{K^*})$ dependence of the differential width
  [1207.4004; Becirevic, Tayduganov
  1209.1525; Matias
  1210.5279; Blake, Egede, Shires
  1303.5794; Descotes-Genon, Hurth, Matias, Virto]
- Non-resonant $K\pi$ decays are more problematic because their P-wave channel is an irreducible background to $B\to K^*ll\to K\pi ll$
  - At high-q^2 this background can be estimated using HHχPT but it is simpler to remove it using sideband subtraction
    [1406.6681; Das, Hiller, Jung, Shires]
  - At low-q^2 the situation is similar. See [1307.0947; Doering, Meissner, Wang] for a discussion based on pQCD.

$B \to (\pi, K, K^*)\ell\ell$: on charmonium and the high-q² OPE

- Analytic structure of the q² plane:



- Diagrammatically:



$$\langle K^{(*)} | T J^\mu(x) O_{1,2}(y) | B \rangle \sim h(q^2)\ f_+(q^2)$$ highly non-local

Need to integrate over a large enough q² range

- $P_5' = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2 + |A_\parallel|^2)}}$

- SCET/QCD factorization at low-q^2:

$$A_\perp^{L,R} = \mathcal{N}_\perp \left[\mathcal{C}^+_{9\mp10} V(q^2) + \mathcal{C}^+_7 T_1(q^2)\right] + \mathcal{O}(\alpha_s, \Lambda/m_b \cdots)$$

$$A_\parallel^{L,R} = \mathcal{N}_\parallel \left[\mathcal{C}^-_{9\mp10} A_1(q^2) + \mathcal{C}^-_7 T_2(q^2)\right] + \mathcal{O}(\alpha_s, \Lambda/m_b \cdots)$$

$$A_0^{L,R} = \mathcal{N}_0 \left[\mathcal{C}^-_{9\mp10} A_{12}(q^2) + \mathcal{C}^-_7 T_{23}(q^2)\right] + \mathcal{O}(\alpha_s, \Lambda/m_b \cdots)$$

non-factorizable PC's

factorizable PC's

- Factorization of the form factors (up to order α_s and Λ/m_b):

$$\frac{m_B}{m_B + m_{K^*}} V(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) = T_1(q^2) = \frac{m_B}{2E} T_2(q^2) = \xi_\perp(E),$$

$$\frac{m_{K^*}}{E} A_0(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) - \frac{m_B - m_{K^*}}{m_B} A_2(q^2) = \frac{m_B}{2E} T_2(q^2) - T_3(q^2) = \xi_\parallel(E)$$

- $P_5' = \frac{C_{10}(C_{9\perp} + C_{9\parallel})}{\sqrt{(C_{9\parallel}^2 + C_{10}^2)(C_{9\perp}^2 + C_{10}^2)}} + O(\alpha_s, \Lambda/m_b)$

factorizable and non-factorizable PC's