Kaons on the Lattice

The XIII International Conference on Heavy Quarks and Leptons

May 24, 2016

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RBC and UKQCD Collaborations

Kaons on the lattice

- Lattice QCD – 2016
  - Kaons from Euclidean space
    - Multi-particle final states
    - 2^{nd} order long distance effects
- $K \rightarrow \pi\,\pi$ decay: $\Delta I = ½$, ε'
- $K_L - K_S$ mass difference
- Long distance contribution to ε_K
- Long distance contribution to rare kaon decay: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Lattice QCD – 2016

- Physical quark masses (ChPT not needed)
- Chiral quarks (doubling problem solved)
- Large physical volumes: $(6\ \mathrm{fm})^3$
- Small lattice spacing: $1/a$ = 2.4 GeV
  - $(\Lambda_{\mathrm{QCD}}\, a)^2$ effects < 1% ☺
  - $(m_{\mathrm{charm}}\, a)^2$ effects ~ 20% ☹

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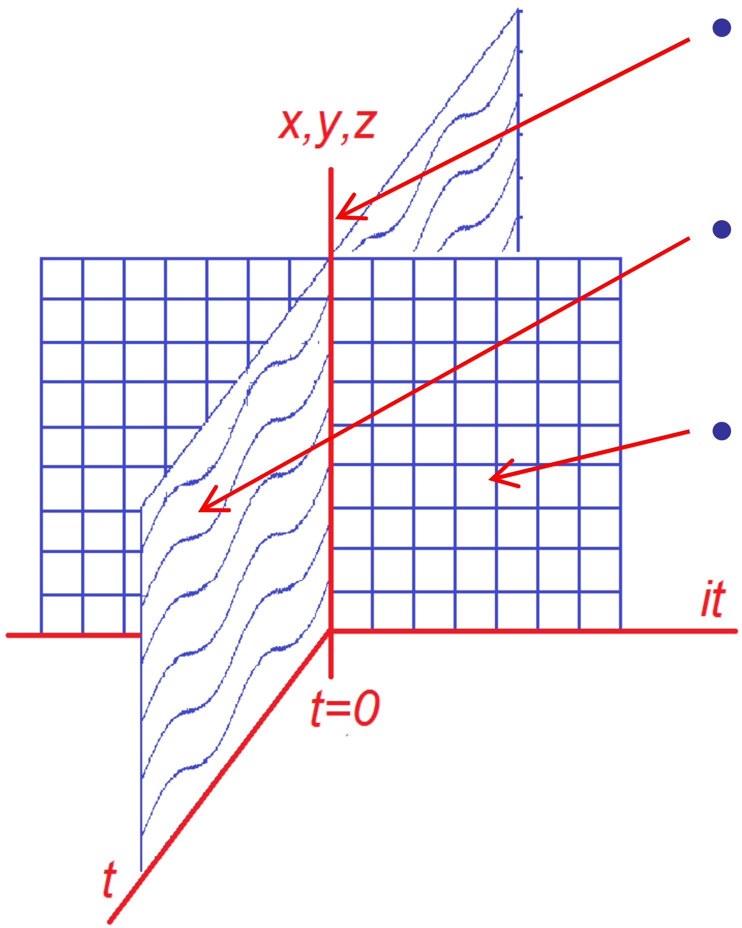
Kaons from Euclidean Space

Kaons from Euclidean space



- Begin with standard Hilbert space QM at $t = 0$
- Use e^{-iHt} for physical time development.
- Lattice QCD requires the use of e^{-Ht} without the `i'
  - Low lying eigenvalues and eigenstates of H are easily accessible
  - More massive unstable states are increasingly difficult

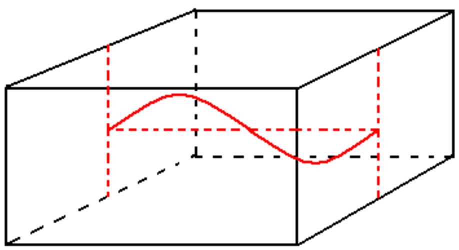
Kaons from Euclidean space

- Matrix elements between stable states are easy: [JHEP 1506 (2015) 164]

$$\langle \pi(p_\pi)|V_\mu(\vec{0})|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K - p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K + p_\pi)_\mu$$

$$f_+^{K\pi}(0) = 0.9685(34)(14) \qquad |V_{us}| = 0.2233(5)(9)$$

- Final two-pion state more difficult:
  - e^{-Ht} projects onto the ground state
  - can obtain excited states by using finite volume quantization.
  - Lellouch-Luscher finite volume correction.



- Final three-pion state is hard [Hansen and Sharpe]

Kaons from Euclidean space

- Long-distance parts of second-order processes: ΔM_K, ε_K, and $K^+ \to \pi^+ \nu \nu$ are possible
- Exploit connection between time-dependent and time-independent perturbation theory:

$$E_n^{(2)} = \sum_{n'} \frac{|V_{n'n}|^2}{E_n - E_{n'}}$$

$$\begin{aligned} 1 - i E_n^{(2)}(T_a - T_b) &= \langle n | T \left\{ e^{-i \int_{T_b}^{T_a} V_I(t) dt} \right\} | n \rangle \\ &= \langle n | \left(1 - \frac{1}{2} \int_{T_b}^{T_a} \int_{T_b}^{T_a} dt_1 dt_2 T \left\{ V_I(t_2) V_I(t_1) \right\} \right) | n \rangle \end{aligned}$$

- Can be used in Euclidean space if unphysical $e^{+(E_n - E_{n'})(T_a - T_b)}$ are removed

$K \rightarrow \pi\,\pi$ Decay

$K \rightarrow \pi\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

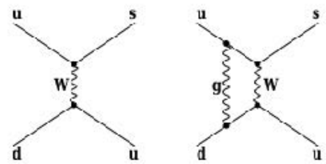
- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \right)$$

Direct CP violation

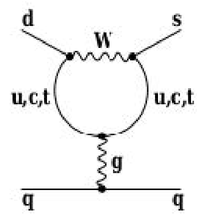
Local four quark operators

- **Current-current operators**



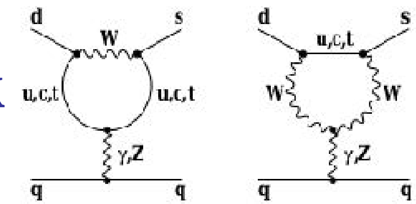
$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**



$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

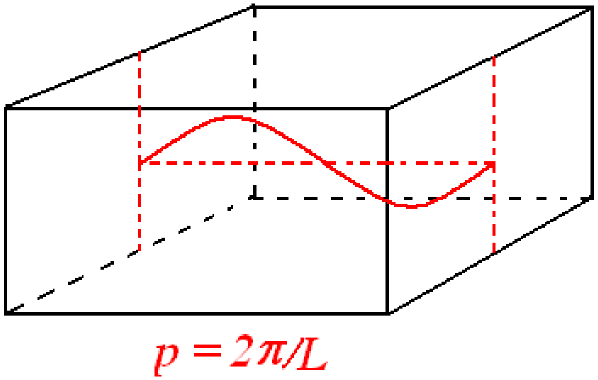
- **Electro-Weak Penguins**



$$Q_7 \equiv \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A}$$

Physical $\pi\,\pi$ states
Lellouch-Luscher

- Euclidean $e^{-H_{QCD}\,t}$ projects onto $|\pi\pi(\vec{p}=0)\rangle$
- Exploit finite-volume quantization.



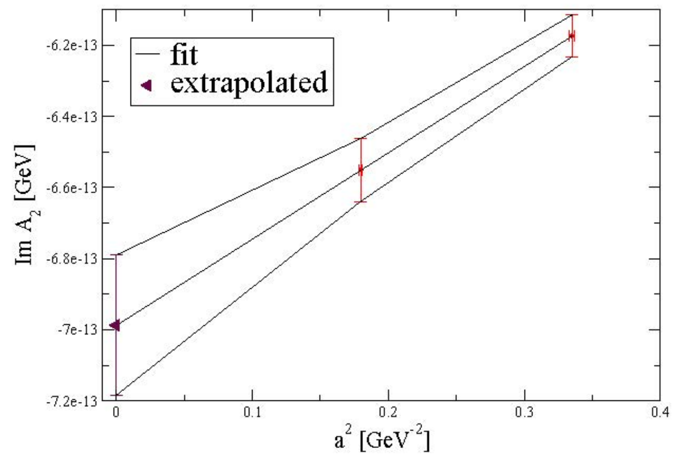
- Boundary conditions give ground state has physical $\vec{p}$
  - $\Delta I = 3/2$: impose anti-periodic BC on *d* quark
  - $\Delta I = 1/2$: impose G-parity BC
- Correctly include π - π interactions, including normalization

Calculation of A_2

Δ I = 3/2 – Continuum Results

(M. Lightman, E. Goode T. Janowski)

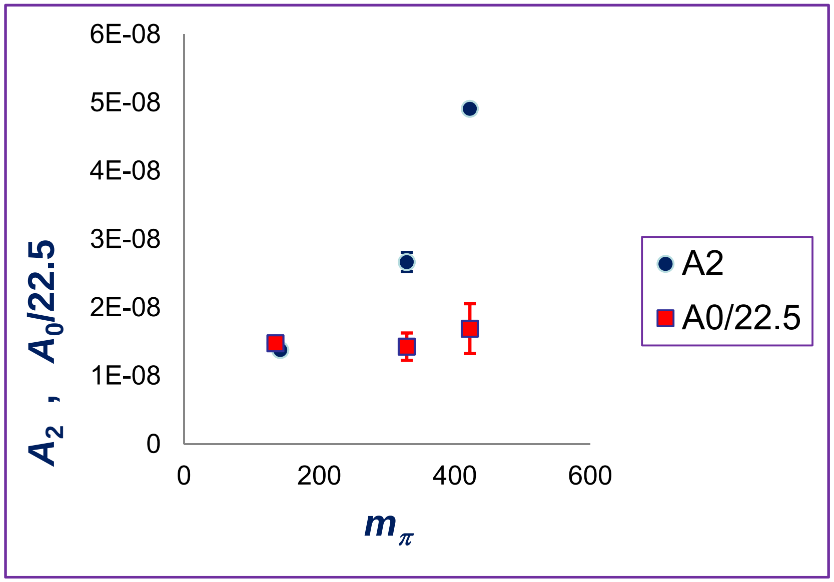
- Use two new large ensembles to remove a^2 error (m_p=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV



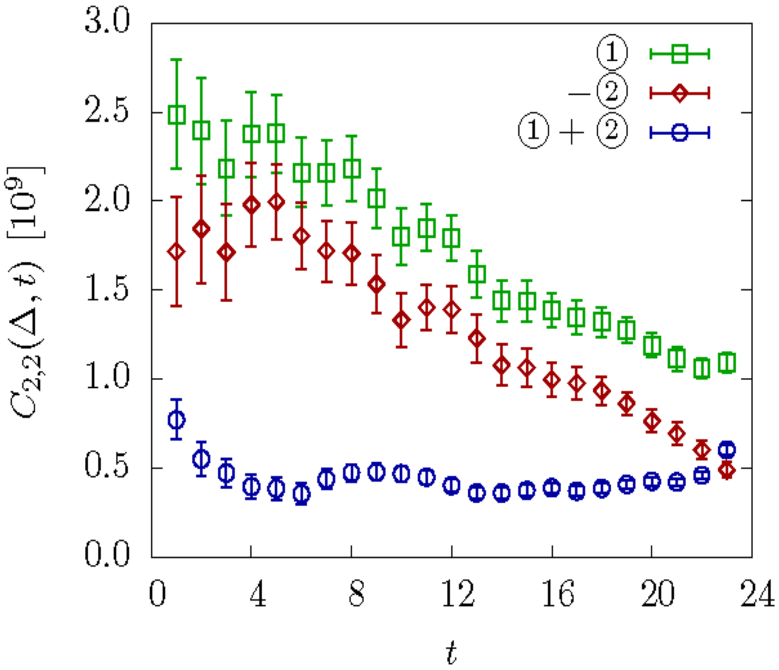
- Continuum results:
  - $\mathrm{Re}(A_2) = 1.50(0.04_{\mathrm{stat}})\,(0.14)_{\mathrm{syst}} \times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -6.99(0.20)_{\mathrm{stat}}\,(0.84)_{\mathrm{syst}} \times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV
- $E_{\pi\pi} \rightarrow \delta_2 = -11.6(2.5)(1.2)^o$
- Phys.Rev. **D91**, 074502 (2015)

ΔI = 1/2 Rule

Compare A_2 and A_0 /22.5



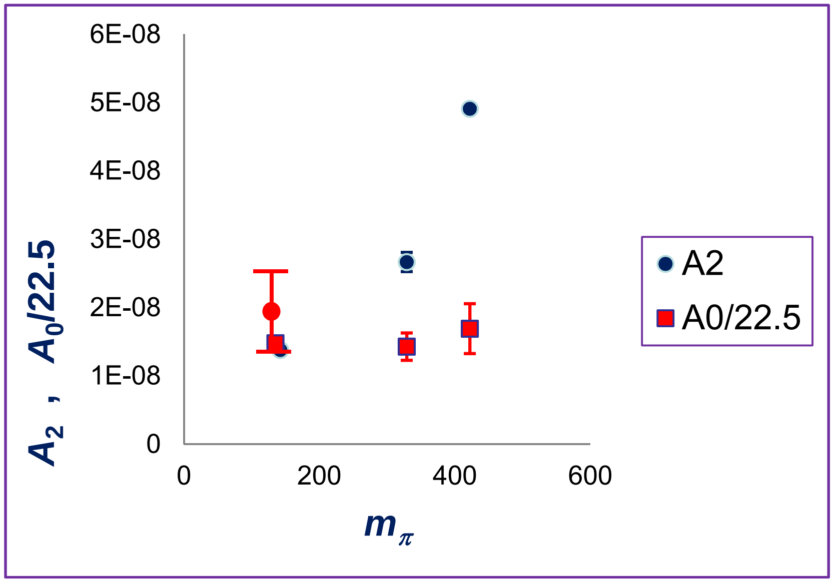
Cancellation in A_2



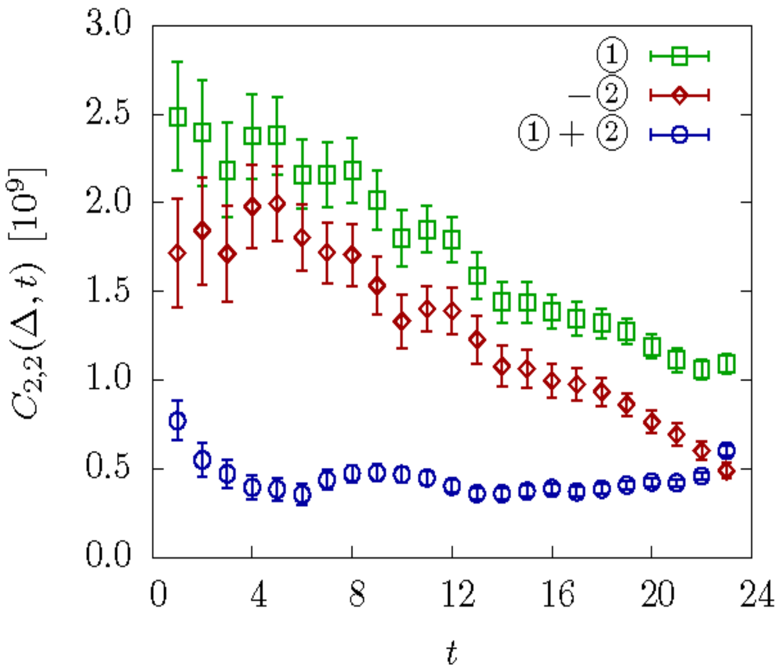
- 50 year puzzle resolved!
- A dynamical QCD effect – no more explanation needed?

$\Delta I = 1/2$ Rule

Compare A_2 and $A_0/22.5$



Cancellation in A_2



- 50 year puzzle resolved!
- A dynamical QCD effect – no more explanation needed?

Calculation of A_0 and ε'

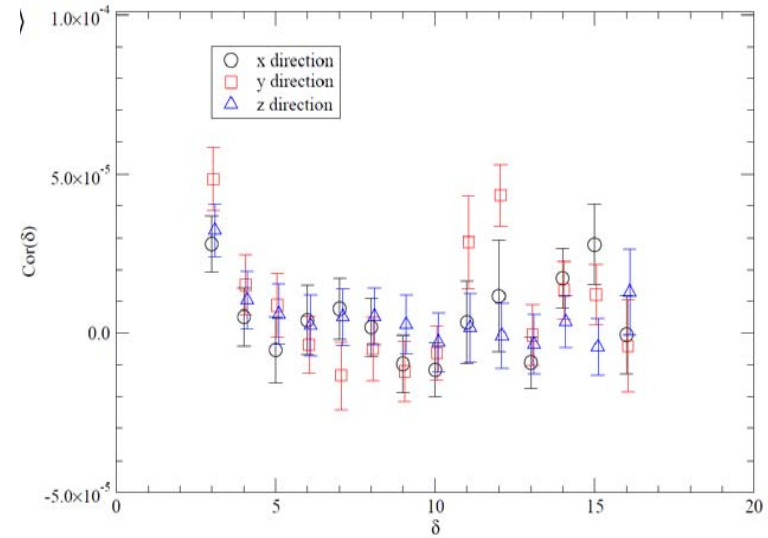
Overview of calculation
(Chris Kelly, Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - $1/a$ = 1.3784(68) GeV, L = 4.53 fm.
  - 216 configurations separated by 4 time units
  - 900 low modes for all-to-all propagators
  - Solve for $\pi\pi$ and kaon sources on each of 64 time slices

Overview of calculation

- Achieve essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV
  - m_{res} = 0.001842(7)

- Error in ensemble generation
  (*u* and *d* quark forces computed from the same random numbers after shift by 12 in y-direction)



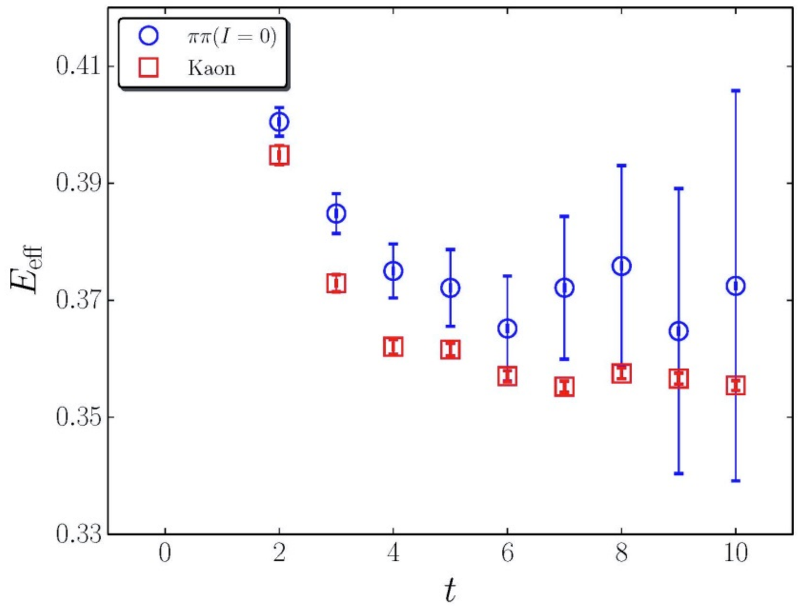
Average plaquette

Correct ensemble 0.512239(3)(7)

Incorrect ensemble 0.512239(6)

$I = 0$, $\pi\pi - \pi\pi$ correlator

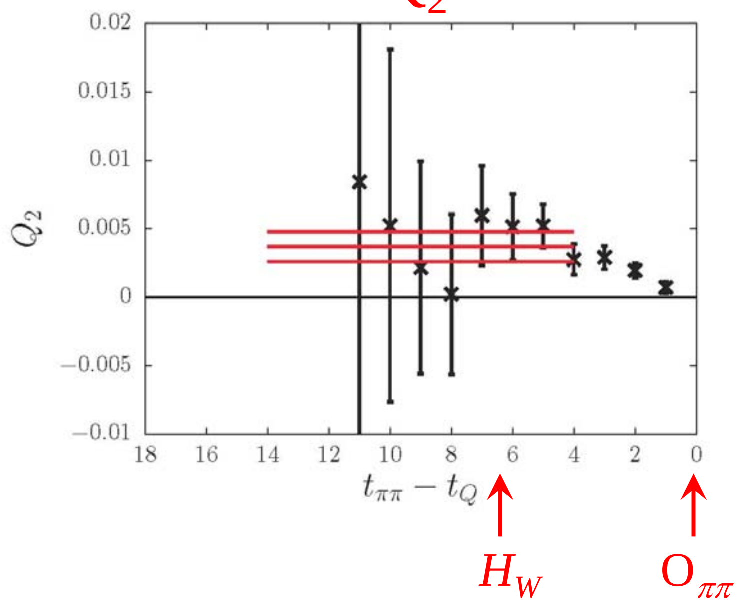
- Determine normalization of $\pi\pi$ interpolating operator
- Determine energy of finite volume, $I = 0$, $\pi\pi$ state: $E_{\pi\pi} =$ 498(11) MeV
- Determine $I = 0$ $\pi\pi$ phase shift: $\delta_0 = 23.8(4.9)(2.2)^o$
- Phenomenological result: $\delta_0 = 38.0(1.3)^o$ [G. Colangelo]



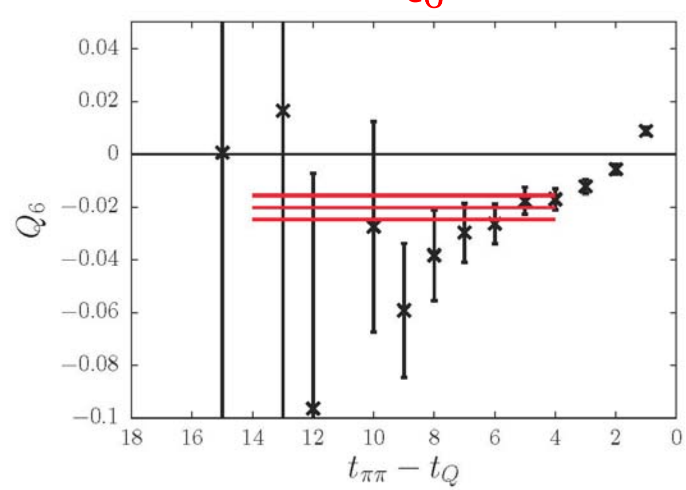
$\Delta I = ½ \;\; K \rightarrow \pi\,\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations t_Q - $t_K \geq 6$ and $t_{\pi\pi}$ - t_K = 10, 12, 14, 16 and 18.
- Fit correlators with $t_{\pi\pi}$ - $t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi}$ - $t_Q \geq 3$ or 5

Q_2



Q_6



Results

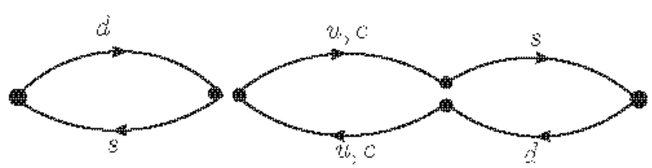
- Determine the complex ΔI=1/2 amplitude A_0
  - $\mathrm{Re}(A_0) = (4.66 \pm 1.00_{stat} \pm 1.26_{sys}) \times 10^{-7}$ GeV
  - Expt: $(3.3201 \pm 0.0018) \times 10^{-7}$ GeV
  - $\mathrm{Im}(A_0) = (-1.90 \pm 1.23_{stat} \pm 1.08_{sys}) \times 10^{-11}$ GeV
- Calculate $\mathrm{Re}(\varepsilon'/\varepsilon)$:
- $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15_{stat} \pm 4.59_{sys}) \times 10^{-4}$
  - Expt.: $(16.6 \pm 2.3) \times 10^{-4}$
  - 2.1 σ difference

$K_L - K_S$ mass difference

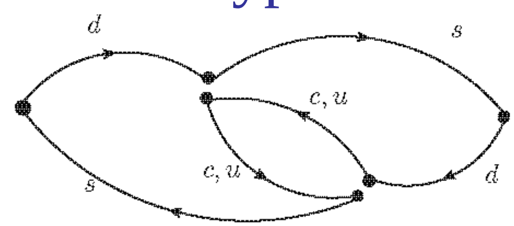
$K_L - K_S$ mass difference

- Perturbative result integrates out charm and shows poor convergence (Brod and Gorbahn)
  - NNLO is 36% of LO
  - Large μ_c dependence
- Lattice must include charm quark (GIM)

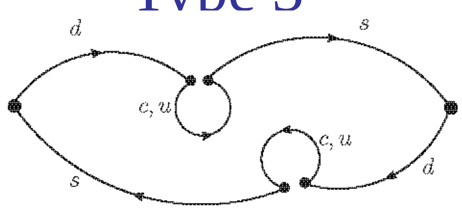
Type 1



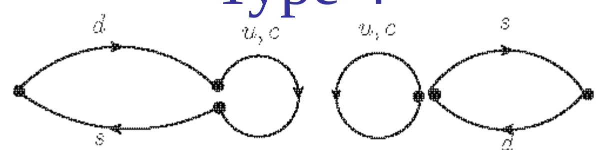
Type 2



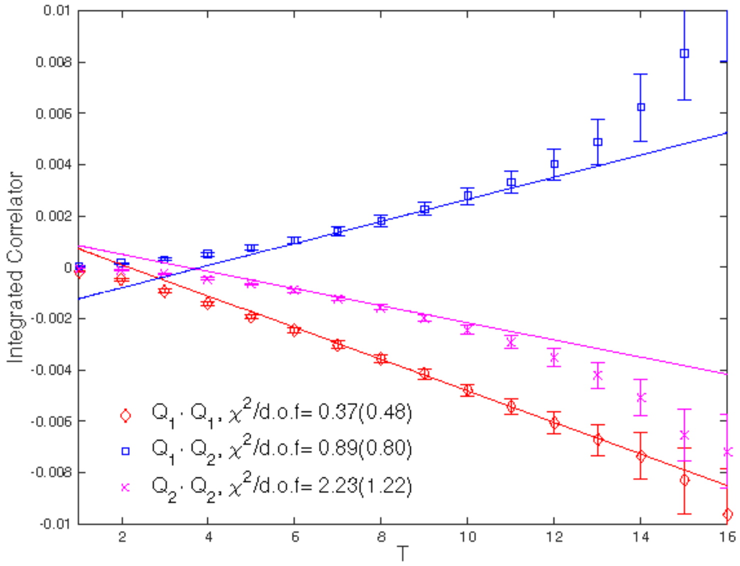
Type 3



Type 4



ΔM_K Present Results
(Ziyuan Bai)



	ΔM_K **x 10**$^{+12}$ MeV
Types 1-4	5.76(73)
Types 1-2	4.19(15)
η	0
π	0.27(14)
$\pi\pi$, I=0	-0.097(49)
$\pi\pi$, I=2	-6.56(6) x 10^{-4}
Δ_{FV}	0.029(19)
Expt.	3.483(6)

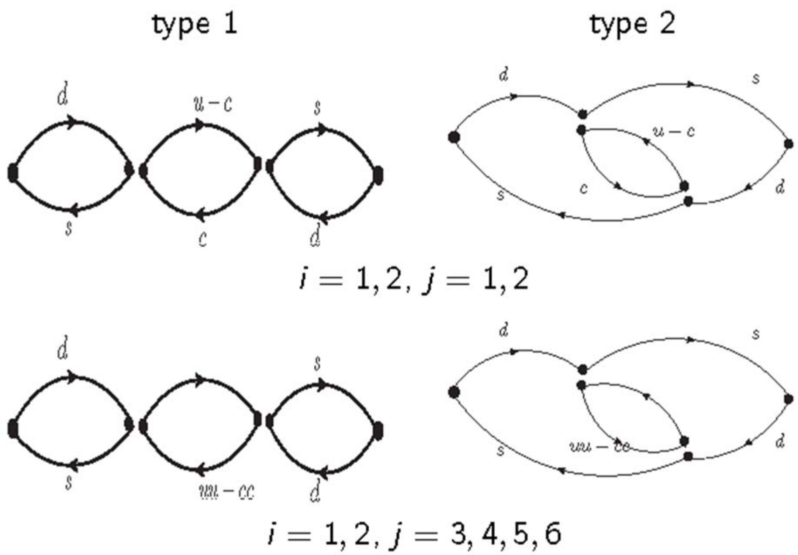
- m_c = 750 MeV, M_π = 170 MeV
- Disconnected contribution small
- $\pi\pi$ contribution ~2% and FV correction ~0.5%
- New 64^3x128, 1/a=2.38 GeV, m_c=1.2 GeV, M_π = 140 MeV
  26 configs: ΔM_K = 0.6(3.3) x 10^{-12} MeV (26 → 400?)

Long distance part of ε_K

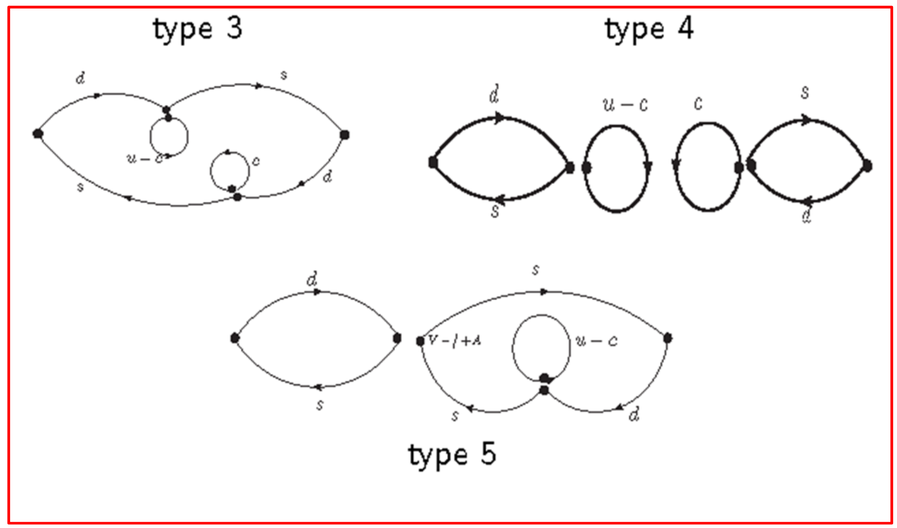
Diagrams for $\lambda_t \lambda_u$ contribution to ε_K

(Ziyuan Bai)

- Identify five types of diagrams

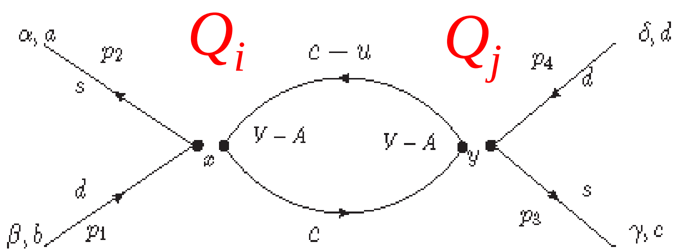


Omit from 1[st] study

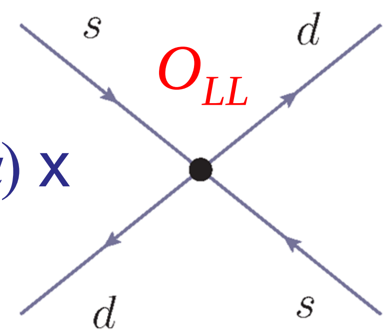


New ΔS = 2 counter term

(Ziyuan Bai)



$-E_{ij}(\mu)$ x



- Subtract $E_{ij}(\mu)\,(\bar{s}\gamma^{\nu}(1-\gamma^5)d)\,(\bar{s}\gamma^{\nu}(1-\gamma^5)d)$ to make off-shell Greens function vanish at $p_i^2 = \mu_{RI}{}^2$
- Define infrared-safe Rome-Southampon normaliztion for bi-local operator.

Progress toward long-distance part of ε_K

(Ziyuan Bai)

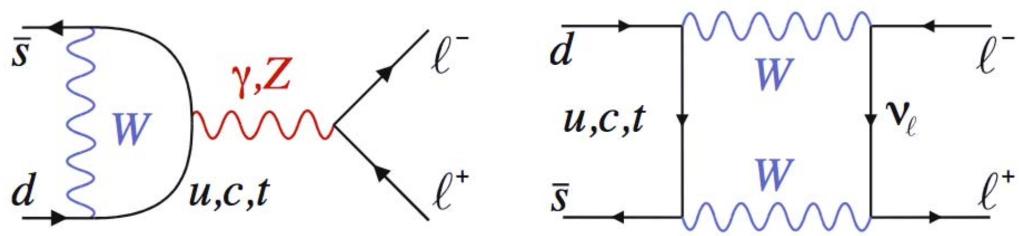
- Examine only type 1 and 2 diagrams
- Only current-current operators
- Compute NLO (one-loop) conversion from bilocal RI to $\overline{\text{MS}}$
- Dependence on μ_{RI}
- Preliminary
- $|\varepsilon_K|$ = 2.228(11) x 10^{-3} expt.

μ_{RI}	$\delta\|\varepsilon_K\|$
1.54	0.1384 x 10^{-3}
1.92	0.1483 x 10^{-3}
2.11	0.1473 x 10^{-3}
2.31	0.1405 x 10^{-3}
2.56	0.1246 x 10^{-3}

Rare Kaon Decays

$$K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$$

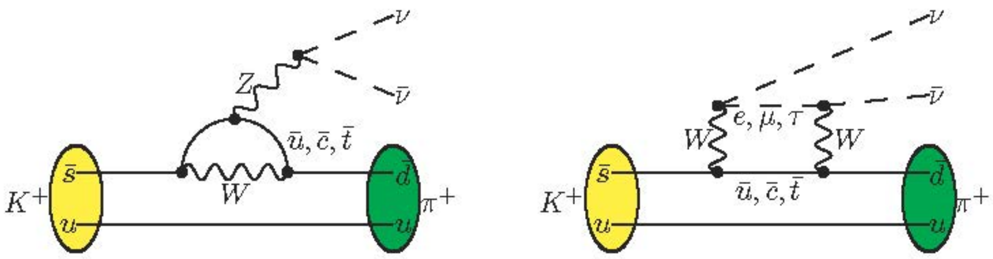
Rare Kaon Decays
(Xu Feng, Andrew Lawson, Antonin Portelli)



- $K_L \rightarrow \pi^0 + l^+ + \bar{l}^-$: determine the sign of the indirect CP violating amplitude.
- $K^+ \rightarrow \pi^+ + \nu + \bar{\nu}$: calculate the long distance ($l \geq 1/m_c$) part of charm contribution. Estimated to be small (≈4%) but should be verified

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$

(Xu Feng)



Both are bilocal products with short-distance singularity

- Estimate 3 contributions: top : charm-*sd* : charm-*ld*
  [Cirigliano et.al. Rev. Mod. Phys.]

$$\lambda_t \frac{m_t^2}{M_W^2} : \lambda_c \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} : \lambda_u \frac{\Lambda_{\mathrm{QCD}}^2}{M_W^2} = 68\% : 29\% : 3\%$$

- However
  - Charm contributes 50% of $K^+ \rightarrow \pi^+ + \nu \bar{\nu}$ BR
  - 50% charm contribution comes form $p \sim m_c$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ - results

(Xu Feng)

- Present phenomenological treatment [Buras, et al. hep-ph/1503.02693]
  - Integrate out charm & represent by local $(\bar{s}d)_L(\bar{\nu}\nu)_L$ operator
  - Add correction for up quark
  - Add $(\Lambda_{QCD}/m_c)^2$ correction for dim-8 in OPE
  - $\delta P_{cu} = 0.04(2)$ [Isidori et.al, hep-ph/0503107]
- Lattice result
  - $\delta P_{cu} = -0.007(2)$ (large systematic errors)
  - Includes all physics at energies below $\mu_{RI} = 2$ GeV

Outlook

- Lattice QCD now reaches sufficient far into Minkowski space to allow 1st-principles calculation of:
  - $K \rightarrow \pi\pi$, ΔI = 3/2 and 1/2, ε'/ε
  - $M_{K_L} - M_{K_S}$, long dist. contribution to ε
  - Long distance parts of $K \rightarrow \pi l \bar{l}$, $K \rightarrow \pi \nu\bar{\nu}$
- First realistic calculation of ΔM_K underway
- Must wait for next generation of computers to accurately include charm