Kaon Theory

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Outline

1. Introduction and Motivation
2. Leptonic decays: lepton universality and R_K
3. Semileptonic decays
   - CKM Unitarity and Callan-Treiman
   - Determinations of LECs
4. Non-leptonic Decays: ε'/ε
5. Rare and Radiative Decays.
6. Conclusion and outlook

See the review by Cirigliano, Ecker, Neufeld, Pich, Portoles'12,
NA62 handbook workshop, Mainz

1. Introduction and Motivation

1.1 Why Kaon physics is interesting?

Goals:

- Test of the Standard Model:
  - Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Test of lepton universality
- Probe QCD at low energy
- Indirect searches of new physics, several possible high-precision tests

Tools: ChPT, OPE, … & Lattice

Data: KLOE, KTeV, NA48 ⇨ KLOE2, NA62, KOTO, ORKA, TREK

1.2 Theoretical framework

- Multi scale theoretical description

Pich@NA62 handbook workshop'16

Scale	Fields	Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model + *New Physics?*
↓ OPE		
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{QCD}^{(n_f=3)}$, $\mathcal{L}_{eff}^{\Delta S=1,2}$
↓		
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

1.2 Theoretical framework

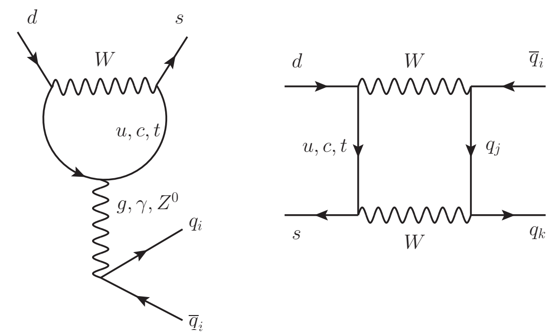
- Multi scale theoretical description

Pich@NA62 handbook workshop'16

M_W

W, Z, γ, g
τ, μ, e, ν_i
t, b, c, s, d, u

Short distance physics



OPE

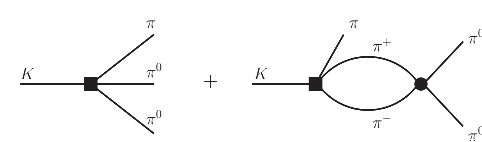
$\lesssim m_c$

$\gamma, g\ ;\ \mu, e, \nu_i$
s, d, u

M_K

$\gamma\ ;\ \mu, e, \nu_i$
π, K, η

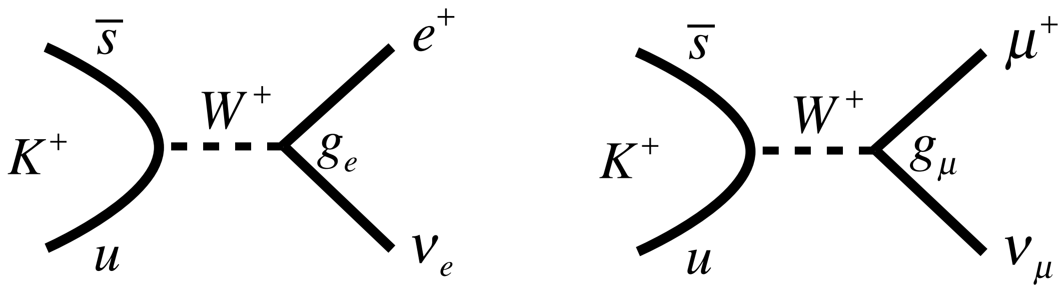
Long distance physics



2. Leptonic decays

2.1 K_{l2} decays

- K_{l2} decays



Only the *axial current* contributes in the SM

- The branching ratio in the SM:

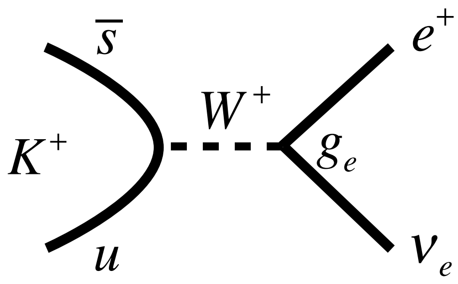
$$B(K \to \ell\nu) = \frac{G_F^2 |V_{us}|^2}{8\pi} f_K^2 m_K m_\ell^2 \left(1 - \frac{m_\ell^2}{M_K^2}\right)^2 \left(1 + 2\frac{\alpha}{\pi} \log \frac{M_Z}{M_\rho}\right) \left(1 + \frac{\alpha}{\pi} F(m_\ell/m_K)\right)(1 + O(\alpha))$$

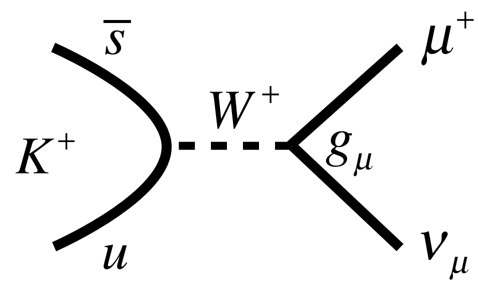
Marciano & Sirlin'93, Finkemeier'96, Cirigliano & Rosell'07

   - *Short distance* effects (universal)
   - Long distance effects (universal)
   - Structure dependent effects (process dependent)

2.2 R_K

- K_{l2} decays





- Define the RK ratio to reduce the theoretical uncertainties: most of the hadronic and radiative contributions cancel

Cirigliano, Rosell'07

$$R_K^{SM} = \frac{\Gamma(K^+ \to e^+\nu_e[\gamma])}{\Gamma(K^+ \to \mu^+\nu_\mu[\gamma])} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_K^2 - m_e^2}{m_K^2 - m_\mu^2}\right)^2 (1+\delta R_{QED}) = 2.477(1)\times 10^{-5}$$

$$g_e / g_\mu = 1$$

in the standard model

Experimental result:

NA62-R_K:
R_K = (2.488 ± 0.007$_{stat}$ ± 0.007$_{syst}$) × 10^{-5}
R_K = (2.488 ± 0.010) × 10^{-5}

- Compatible with SM but experimental uncertainty one order of magnitude higher than theory ⇨ *NA62*

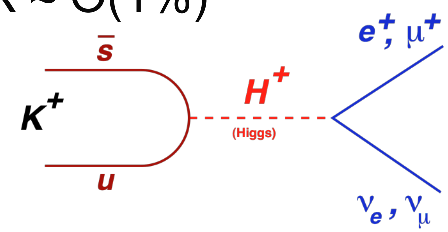
2.3 Test of Lepton Universality

- Compare to other measurements: $R_{e/\mu}^{(P)} \equiv \dfrac{\Gamma(P^- \to e^- \bar{\nu}_e)}{\Gamma(P^- \to \mu^- \bar{\nu}_\mu)}$

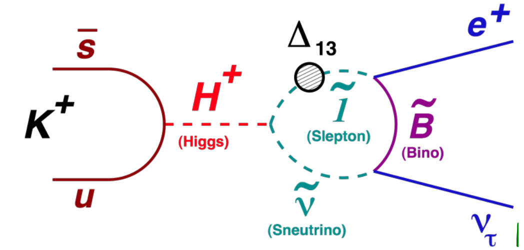
$$\frac{|g_\mu|}{|g_e|} = \begin{cases} 1.0021 \pm 0.0016 & {\scriptstyle \pi \to \mu/e} \\ 0.9978 \pm 0.0024 & {\scriptstyle K \to \mu/e} \\ 1.0010 \pm 0.0025 & {\scriptstyle K \to \pi\mu/e} \\ 1.0018 \pm 0.0014 & {\scriptstyle \tau \to \mu/e} \end{cases}$$

2.3 Test of New Physics in R_K

- R_K sensitive to *lepton flavour violating effects*, $\Delta R/R \approx O(1\%)$



- 2HDM – tree level: ⇨ additional contribution due to charged Higgs, does not contribute to R_K



- Possibility to constrain LFV at one loop in MSSM

  Masiero, Paradisi, Petronzio'06,'08

- Update and extension by *Girrbach & Nierste'12*

  – *LFV:* $R_K^{LFV} \approx R_K^{SM}(1+0.013)$

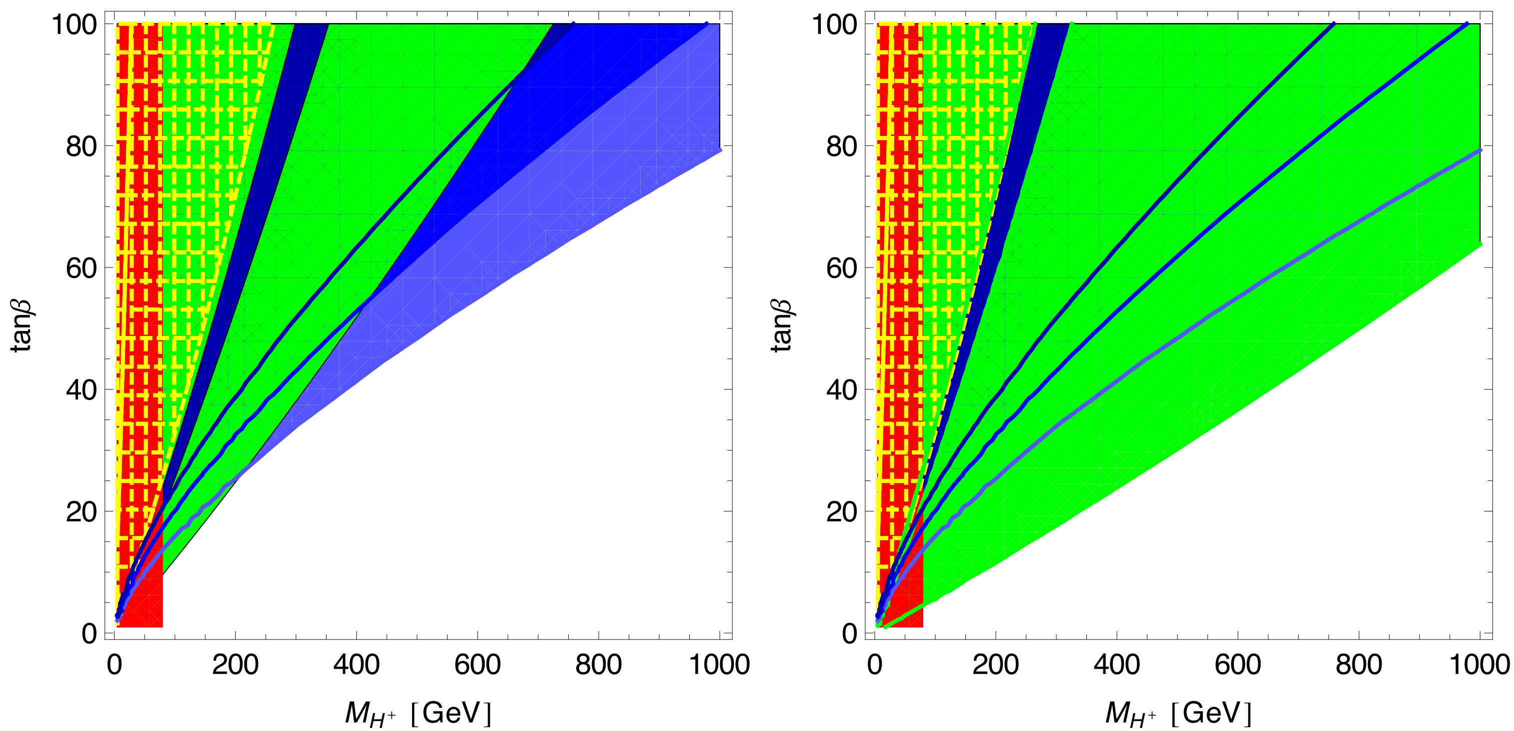
  – Can become negative if interference with LFC effects:

$$R_K^{LFV} \approx R_K^{SM}(1-0.032)$$

Ex : $\tan\beta = 40$, M_H = 500 GeV, $\Delta^{31}{}_R = 5\times10^{-4}$.

2.3 Test of New Physics in R_K

- R_K sensitive to *lepton flavour violating effects*, $\Delta R/R \approx O(1\%)$
- If 0.05% effect on R_K found at NA62 (blue constraint): *Girrbach & Nierste'12*



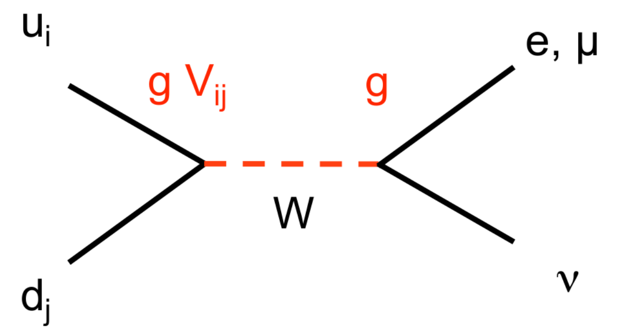
- R_K sensitive to neutrino mixing parameters within SM extensions involving sterile neutrinos. Depends on masses, hierarchy, and mixings of new neutrino states *Abada et al.'12*

3. CKM Unitarity from (semi)-leptonic decays

3.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \rightarrow 0^+$ $\pi^\pm \rightarrow \pi^0 e \nu_e$	$n \rightarrow p e \nu_e$	$\pi \rightarrow \ell \nu_\ell$
V_{us}	$K \rightarrow \pi \ell \nu_\ell$	$\Lambda \rightarrow p e \nu_e$	$K \rightarrow \ell \nu_\ell$



3.1 Paths to V_{ud} and V_{us}

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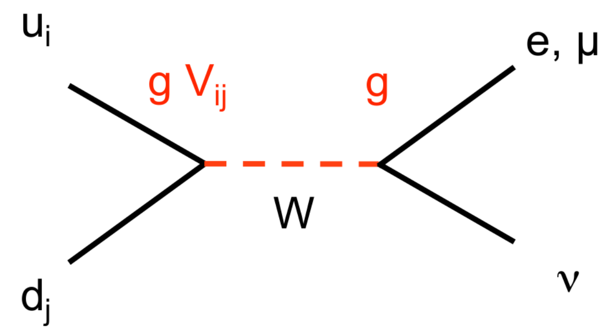
V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell \nu_\ell$



- These are the *golden modes* to extract V_{ud} and V_{us}
  - Only the *vector current* contributes
  - Normalization known in SU(2) [SU(3)] symmetry limit
  - Corrections start at 2nd order in SU(2) [SU(3)] breaking

Ademollo & Gato, Berhands & Sirlin

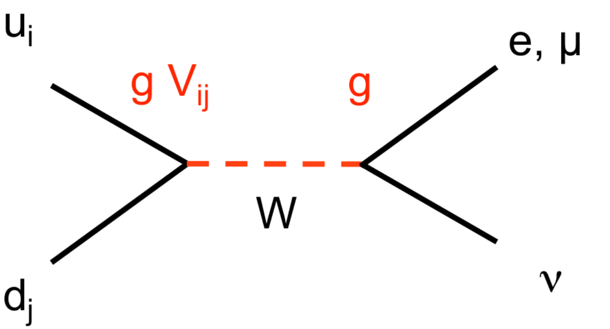
- Currently the most precise determination of V_{ud} and V_{us}

⇨ V_{ud} (*0.02 %*) and V_{us} (*0.5 %*)

3.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell \nu_\ell$



- K_{l2}/π_{l2}
  - Only the *axial current* contributes
  - Need to know the decay constants F_K, F_π
    ⇨ *Lattice QCD*
  - Probe different BSM operators than from the vector case
- Input on F_K/ F_π ⇨ V_{us}/V_{ud} very precisely

3.2 V_{us}/V_{ud} from K_{l2}/π_{l2}

- From K_{l2}/π_{l2}:

$$\frac{\Gamma\left(K \to \mu\nu[\gamma]\right)}{\Gamma\left(\pi \to \mu\nu[\gamma]\right)} = \frac{m_{K^\pm}}{m_{\pi^\pm}} \frac{\left(1 - m_\mu^2 / m_{K^\pm}^2\right)}{\left(1 - m_\mu^2 / m_{\pi^\pm}^2\right)} \frac{f_K^2}{f_\pi^2} \frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2} \left(1 + \delta_{EM}\right)$$

Inputs needed :

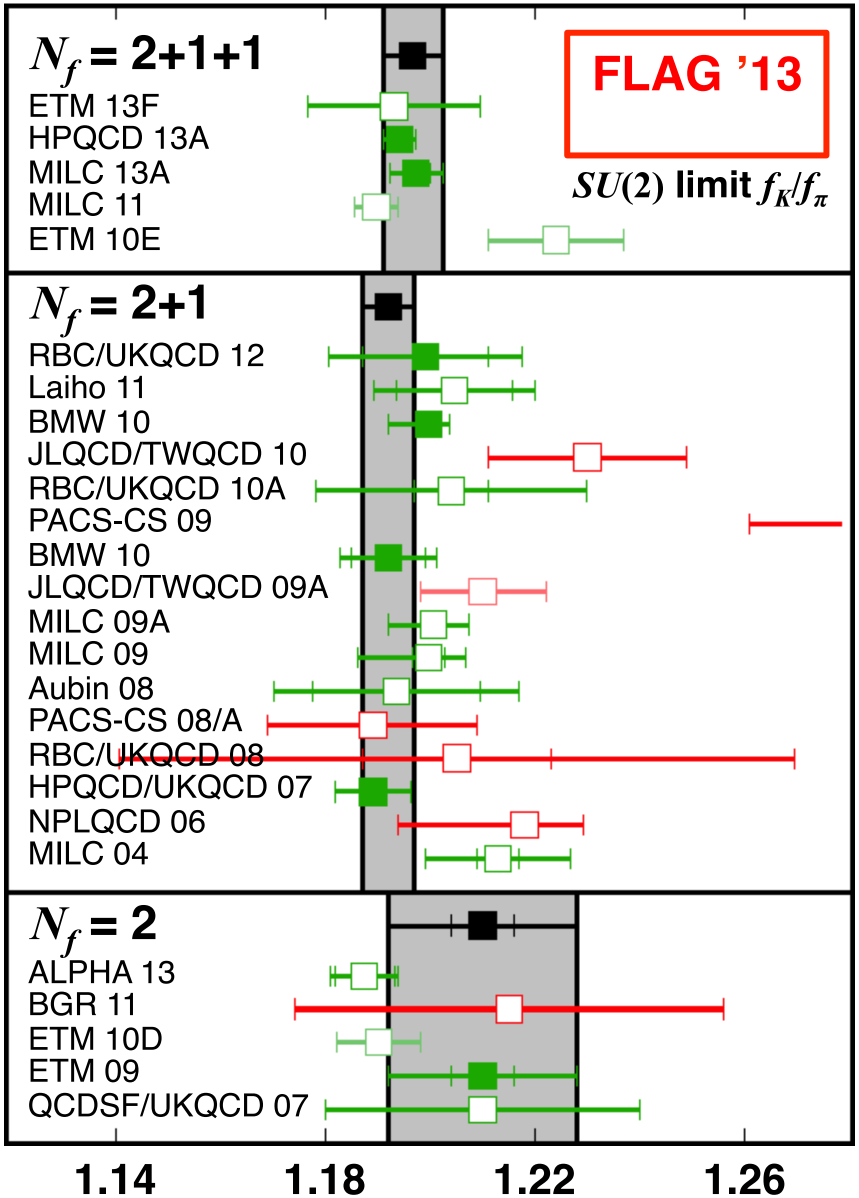
→ Experimental BRs from FlaviaNet kaon WG review *Antonelli et al.'10*

→ F_K/F_π *Lattice calculations*

→ Electromagnetic and isospin breaking corrections

Marciano'04, Knecht et al.'99

F_K/F_π from lattice QCD



- Corrections for IB taken into account in FLAG averages

FLAG'13

- N_f=2+1

$$\frac{F_K}{F_\pi} = 1.192 \pm 0.005$$

- N_f=2+1+1

$$\frac{F_K}{F_\pi} = 1.194 \pm 0.005$$

3.2 V_{us}/V_{ud} from K_{l2}/π_{l2}

- From K_{l2}/π_{l2}:

$$\frac{\Gamma(K\to\mu\nu[\gamma])}{\Gamma(\pi\to\mu\nu[\gamma])}=\frac{m_{K^\pm}}{m_{\pi^\pm}}\frac{\left(1-m_\mu^2/m_{K^\pm}^2\right)}{\left(1-m_\mu^2/m_{\pi^\pm}^2\right)}\frac{f_K^2}{f_\pi^2}\frac{|V_{us}|^2}{|V_{ud}|^2}\left(1+\delta_{EM}\right)$$

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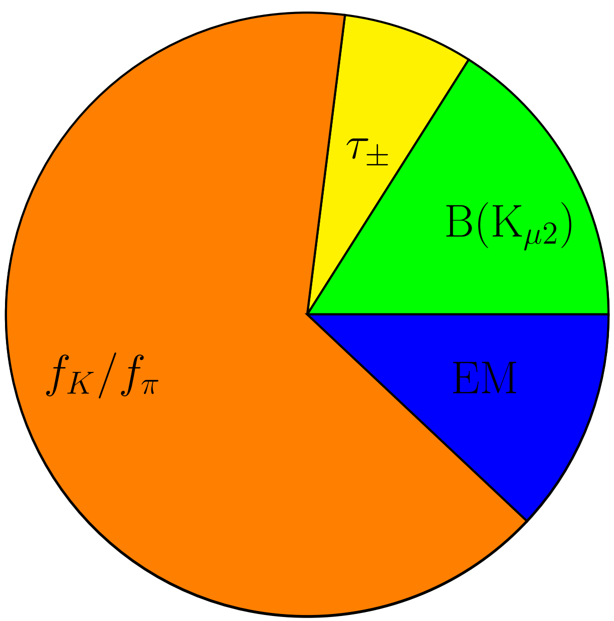
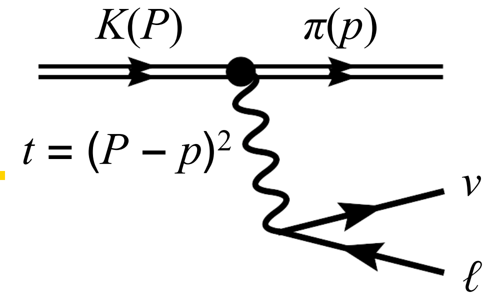


Choice of $f_{K\pm}/f_{\pi\pm}$		V_{us}/V_{ud}
N_f = 2+1	1.192(5)	**0.2315(10)**
N_f = 2+1+1	1.1960(25)	**0.2308(6)**

3.3 V_{us} from K_{l3} decays



- Master formula for $K \to \pi l \nu_l$: $K = \{K^+, K^0\}$, $l=\{e,\mu\}$

$$\Gamma\left(K \to \pi l \nu\left[\gamma\right]\right) = Br(K_{l3}) * \tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl} \left(1 + \delta_{\mathrm{EM}}^{Kl} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2$$

Experimental inputs:

$\Gamma\left(K_{l3}\right)$ Rates with well-determined treatment of radiative decays
- Branching ratios
- Kaon lifetimes

$I_{Kl}\left(\lambda_{Kl}\right)$ Integral of form factor over phase space: λs parametrize evolution in $t=q^2$

Inputs from theory:

S_{EW}^K Universal short distance EW corrections

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer (t=0)

$\delta_{\mathrm{EM}}^{Kl}$ Form-factor correction for long-distance EM effects

$\delta_{\mathrm{SU(2)}}^{K\pi}$ Form-factor correction for SU(2) breaking

$K_{\ell 3}$ form-factor parameterizations

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0}\left(\frac{t}{m^2_{\pi^+}}\right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0}\left(\frac{t}{m^2_{\pi^+}}\right) + \lambda''_{+,0}\left(\frac{t}{m^2_{\pi^+}}\right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Parameterizations incorporating physical constraints

Pole dominance:

$$\tilde{f}_{+,0}(t) = \frac{M^2_{V,S}}{M^2_{V,S} - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_+(t) = \exp\left[\frac{t}{m^2_\pi}(\Lambda_+ - H(t))\right]$$

$$\tilde{f}_0(t) = \exp\left[\frac{t}{m^2_K - m^2_\pi}(\ln C - G(t))\right]$$

Notes:

Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Fits to $K_{e3} + K_{\mu3}$ form-factor slopes: Update

KTeV	KLOE	ISTRA+	NA48/2 ’12 prel	2010 fit	Update

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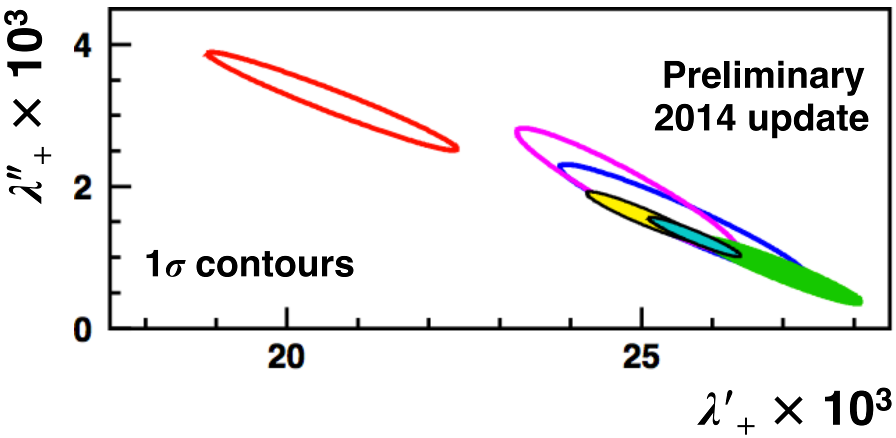

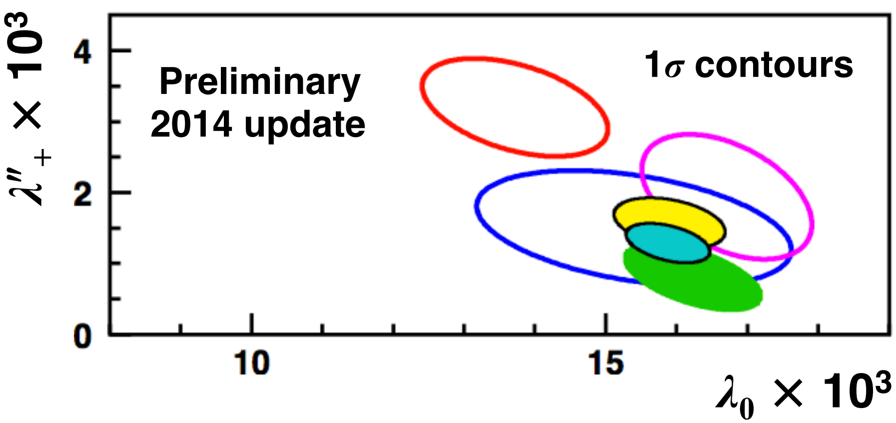

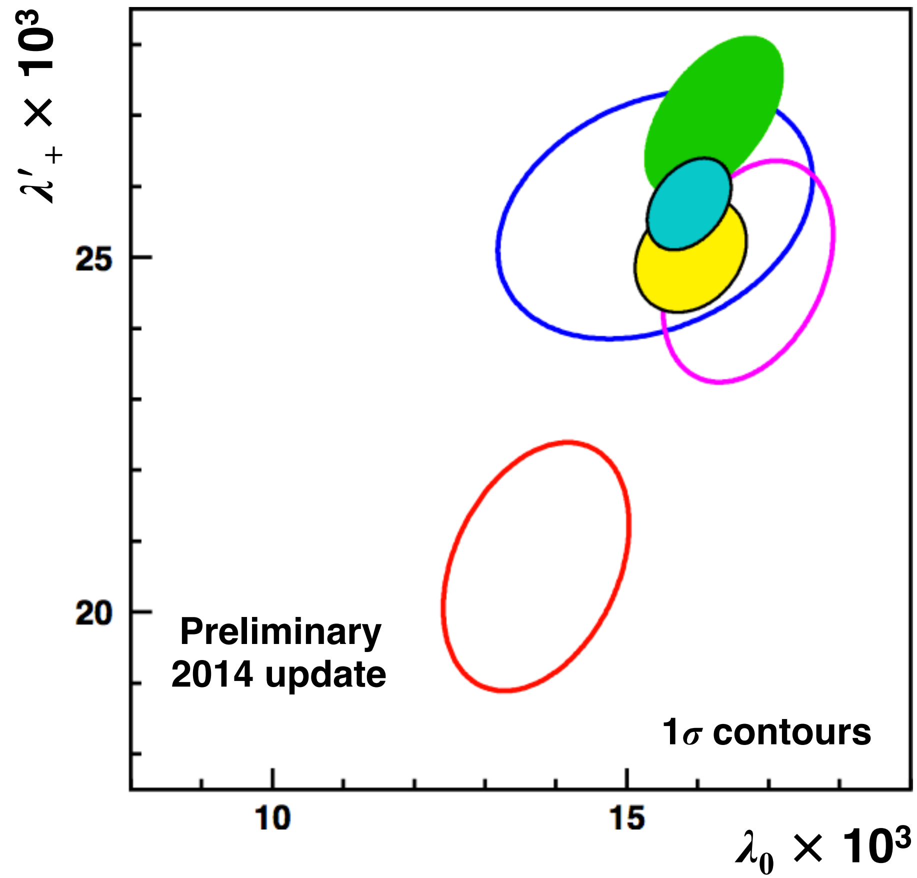

2010: χ^2 = 12.1/8 (P = 14.5%)	Update: χ^2 = 14.3/11 (P = 22.0%)

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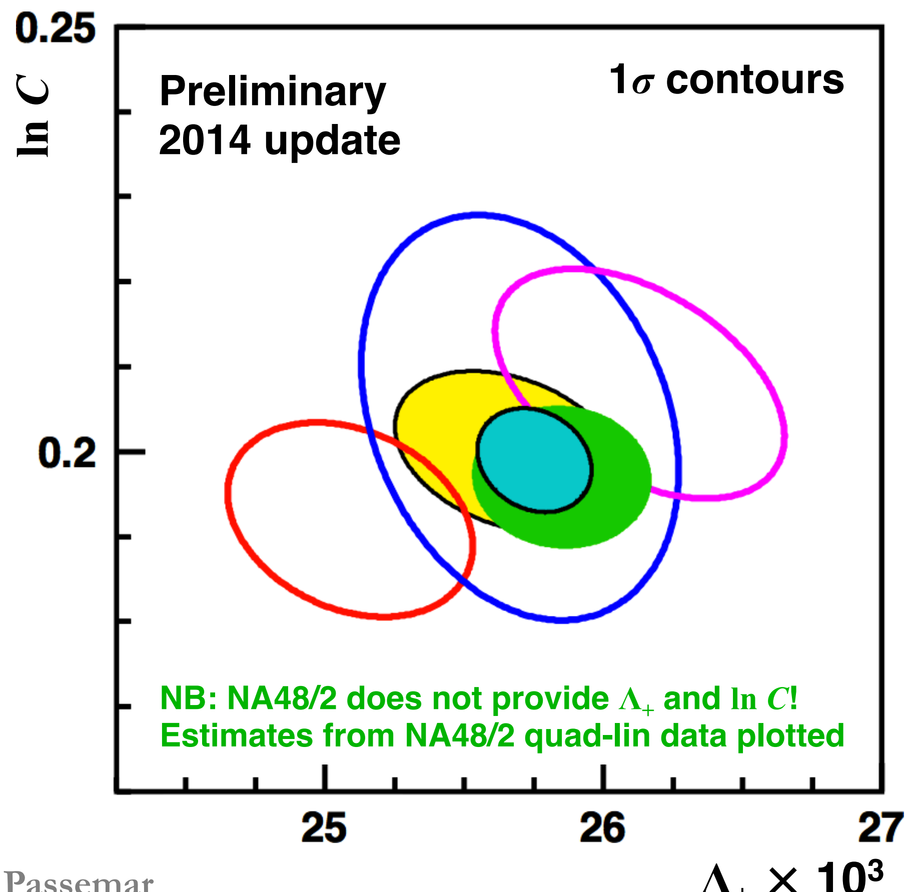
Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{\ell 3}$ avgs from KTeV | KLOE | ISTRA+ | NA48/2 '12 prel | 2010 fit | Update

For NA48, only K_{e3} data included in fits



$$\Lambda_+ \times 10^3 = \mathbf{25.75 \pm 0.36}$$
$$\ln C = \mathbf{0.1985(70)}$$
$$\rho(\Lambda_+, \ln C) = -0.202$$
$$\chi^2/\text{ndf} = 5.9/7\ (55\%)$$

Integrals

Mode	Update	2010
K^0_{e3}	**0.15481(14)**	0.15476(18)
K^+_{e3}	**0.15927(14)**	0.15922(18)
$K^0_{\mu 3}$	**0.10253(13)**	0.10253(16)
$K^+_{\mu 3}$	**0.10558(14)**	0.10559(17)

Only tiny changes in central values

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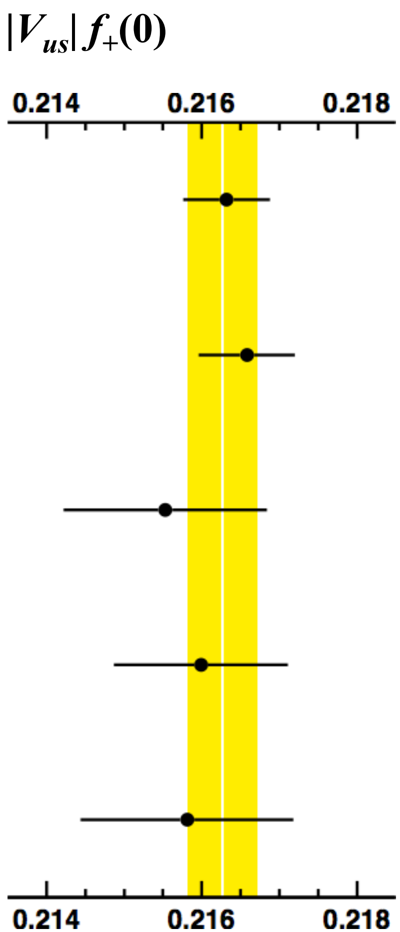
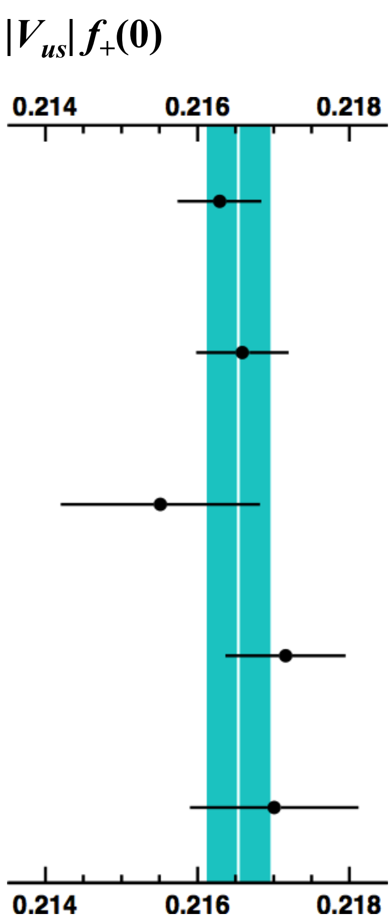
$|V_{us}|\, f_+(0)$ from world data: 2010



			Approx. contrib. to % err from:			
		% err	**BR**	τ	Δ	**Int**
$K_L e3$	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.06
$K_L \mu 3$	**0.2166(6)**	**0.29**	0.15	0.18	0.11	0.08
$K_S e3$	**0.2155(13)**	**0.61**	0.60	0.03	0.11	0.06
$K^\pm e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.40	0.06
$K^\pm \mu 3$	**0.2158(14)**	**0.63**	0.47	0.08	0.39	0.08

Average: $|V_{us}|\, f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.77/4 (94%)**

$|V_{us}| f_+(0)$ from world data: Update



Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.05
$K_L \mu 3$	**0.2166(6)**	**0.28**	0.15	0.18	0.11	0.06
$K_S e3$	**0.2155(13)**	**0.61**	0.60	0.02	0.11	0.05
$K^\pm e3$	**0.2172(8)**	**0.36**	0.27	0.06	0.23	0.05
$K^\pm \mu 3$	**0.2170(11)**	**0.51**	0.45	0.06	0.23	0.06

Average: $|V_{us}| f_+(0)$ = 0.2165(4) **χ^2/ndf = 1.61/4 (81%)**

3.2 V_{us} from K_{l3}

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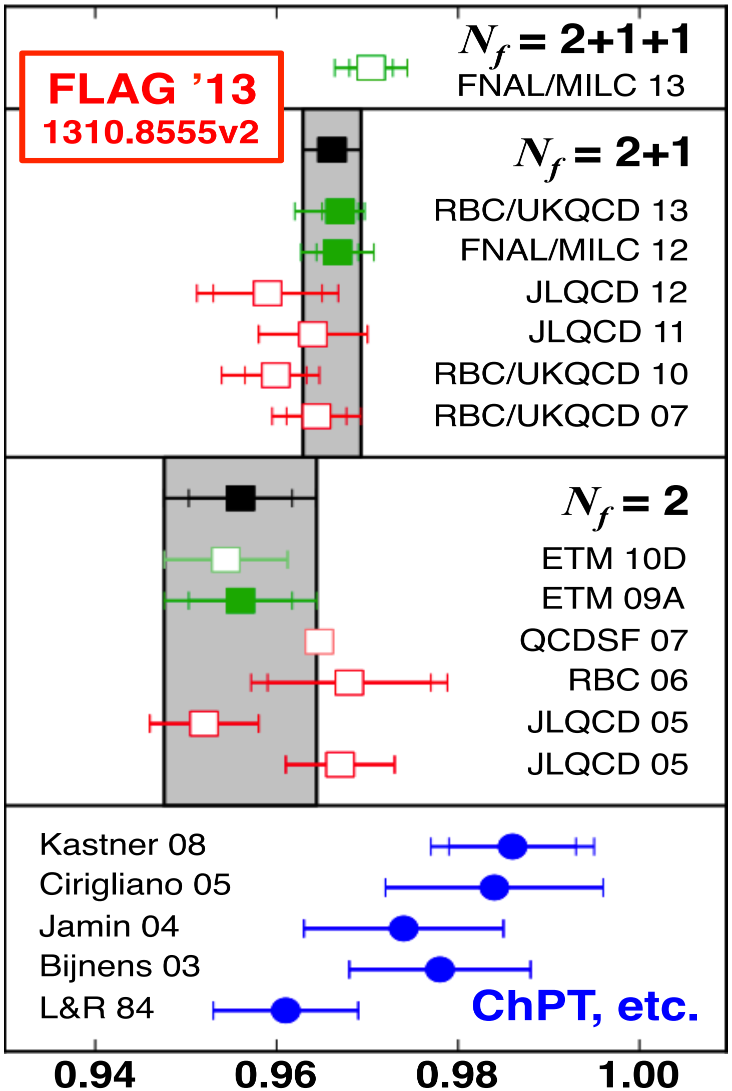

Choice of $f_+(0)$		V_{us}
N_f = 2+1	0.9661(32)	**0.2241(9)**
N_f = 2+1+1	0.9704(32)	**0.2232(9)**

3.4 Global fit to V_{us} & V_{ud}

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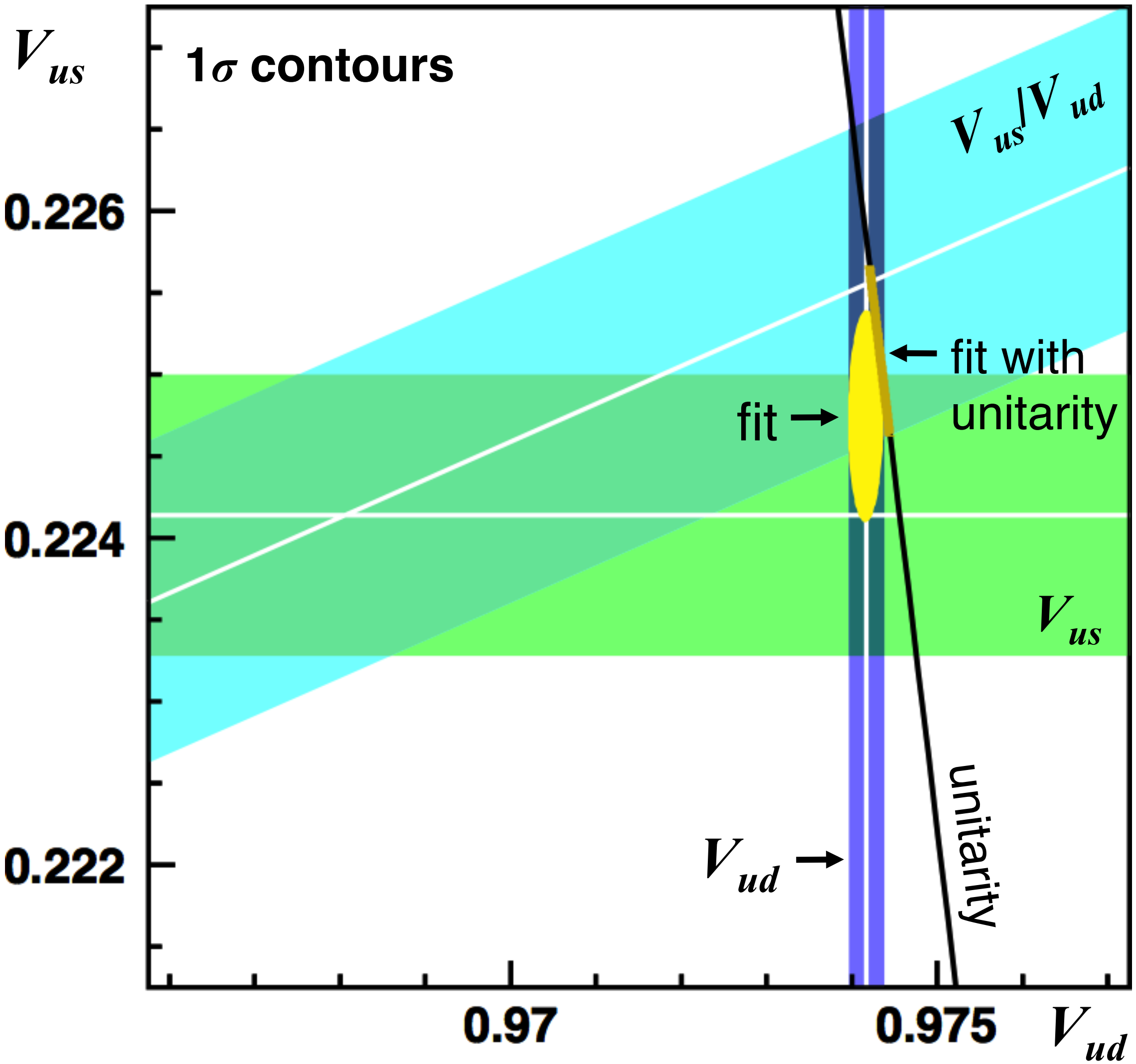


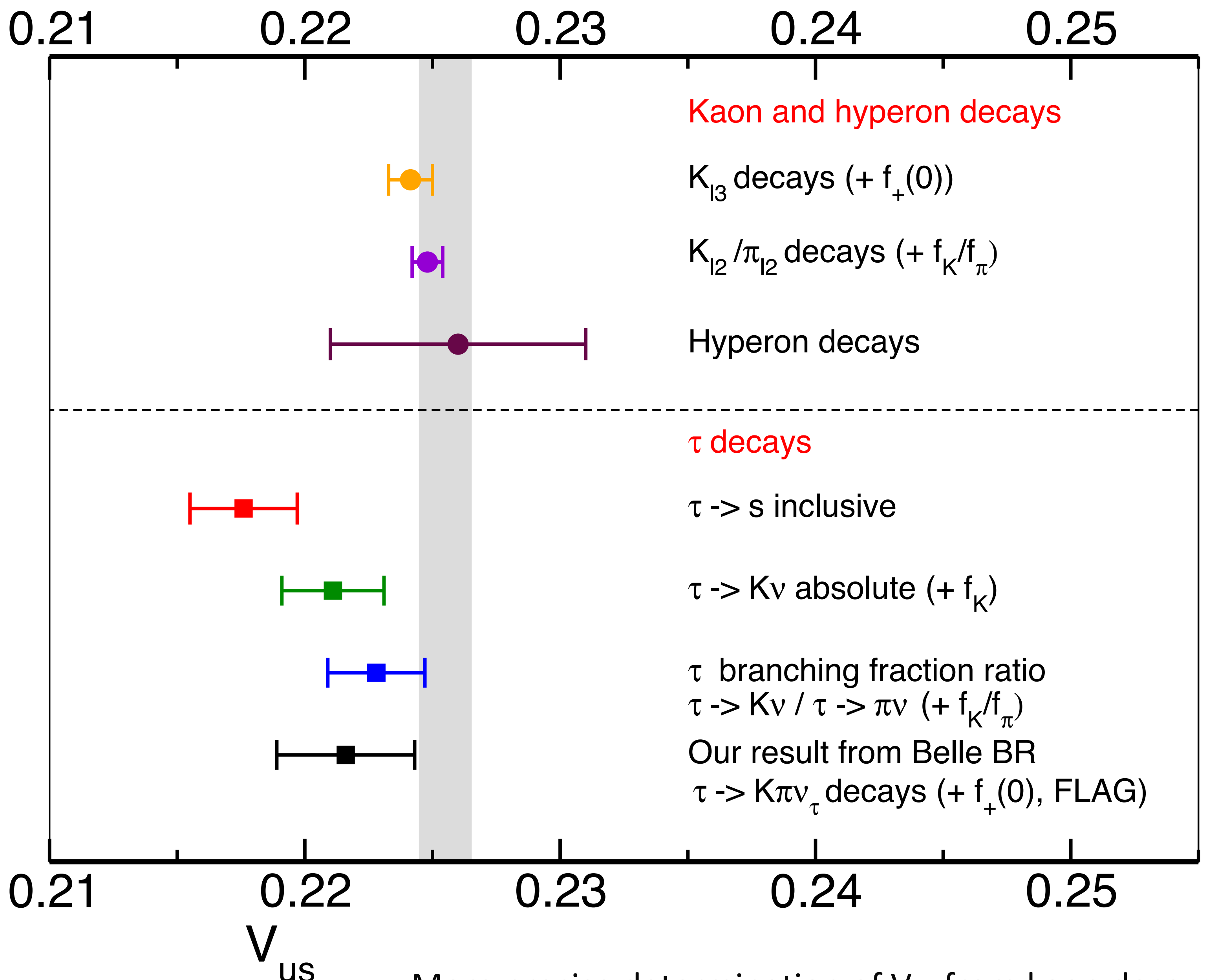
V_{ud} = **0.97416(21)**

V_{us} = **0.2248(7)**

χ^2/ndf = 1.16/1 (28.1%)

Δ_{CKM} = **–0.0005(5)**

–1.0σ



More precise determination of V_{us} from kaon decays!

3.5 Looking for New Physics with K_{l2} and K_{l3}

- Effective Theory approach:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

- Δ_{CKM} a constraining quantity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$$

Negligible (B decays)

3.5 Looking for New Physics with K_{l2} and K_{l3}

from: SJ, talk at NA62 Handbook workshop 2009

Operator	Observable	$K^+ \to \pi^+ \nu\bar{\nu}$	$K_L \to \pi^0 \nu\bar{\nu}$	$K_L \to \pi^0 \ell^+\ell^-$	$K_L \to \ell^+\ell^-$	$K^+ \to \ell^+\nu$	$P_T(K^+ \to \pi^0\mu^+\nu)$	$\Delta_{\rm CKM}$	ϵ'/ϵ	ϵ_K	in MSSM?
$O_{lq}^{(1)}$	$(\bar{D}_L\gamma^\mu S_L)(\bar{L}_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	✓
$O_{lq}^{(3)}$	$(\bar{D}_L\gamma^\mu\sigma^i S_L)(\bar{L}_L\gamma_\mu\sigma^i L_L)$	✓	✓	✓	hs	hs	✓	✓	–	–	✓
O_{qe}	$(\bar{D}_L\gamma^\mu S_L)(\bar{l}_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
O_{ld}	$(\bar{d}_R\gamma^\mu s_R)(\bar{L}_L\gamma_\mu L_L)$	✓	✓	✓	hs	–	–	–	–	–	small
O_{ed}	$(\bar{d}_R\gamma^\mu s_R)(\bar{l}_R\gamma_\mu l_R)$	–	–	✓	hs	–	–	–	–	–	small
$O_{lq}^{\dagger}$	$(\bar{u}_R S_L)\cdot(\bar{l}_R L_L)$	–	–	–	–	✓	✓	✓	–	–	tiny
$(O_{lq}^{t})^\dagger$	$(\bar{u}_R\sigma_{\mu\nu} S_L)\cdot(\bar{l}_R\sigma^{\mu\nu} L_L)$	–	–	–	–	–	?	?	–	–	tiny
O_{qde}	$(\bar{d}_R S_L)(\bar{L}_L l_R)$	–	–	✓	✓	–	–	–	–	–	tiny
$O_{qde}^{\dagger}$	$(\bar{D}_L s_R)(\bar{l}_R L_L)$	–	–	✓	✓	✓	✓	✓	–	–	large $\tan\beta$
$O_{\varphi q}^{(1)}$	$(\bar{D}_L\gamma^\mu S_L)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	✓
$O_{\varphi q}^{(3)}$	$(\bar{D}_L\gamma^\mu\sigma^i S_L)(H^\dagger D_\mu\sigma^i H)$	✓	✓	✓	hs	hs	✓	✓	✓	(✓)	✓
$O_{\varphi d}$	$(\bar{d}_R\gamma^\mu s_R)(H^\dagger D_\mu H)$	✓	✓	✓	hs	–	–	–	✓	(✓)	large $\tan\beta$ (non-MFV)

3.5 Looking for New Physics with K_{l2} and K_{l3}

- Callan-Treiman theorem:

Bernard, Oertel, E.P., Stern'06, '08

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V_{us}|}{F_\pi |V_{ud}|} \frac{1}{f_+(0)|V_{us}|} |V_{ud}|}_{\text{Very precisely known}} r + \Delta_{CT}$$

$\Delta_{K\pi} = m_K^2 - m_\pi^2$

Very precisely known from Br(Kl2/πl2), Γ(Ke3) and $|V_{ud}|$ ⇨ $B_{\exp} = 1.2446(41)$

- In the Standard Model : $r = 1$ $\left(\ln C_{SM} = 0.2141(73)\right)$ $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

- In presence of new physics, new couplings : $r \neq 1$

NLO value + large error bars in agreement with

Bijnens&Ghorbani'07
Kastner & Neufeld'08

Experiment K_{e3}+$K_{\mu 3}$	ln C
NA48'07 (**$K_{\mu 3}$ alone**)	**0.144(14)**
KLOE'08	**0.204(25)**
KTeV'10	**0.192(12)**
NA48 **(preliminary)**	**?**

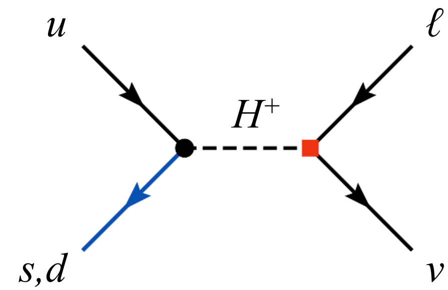
3.5 Looking for New Physics with K_{l2} and K_{l3}

- Ex: Constraints on the aligned 2-Higgs-doublet model:

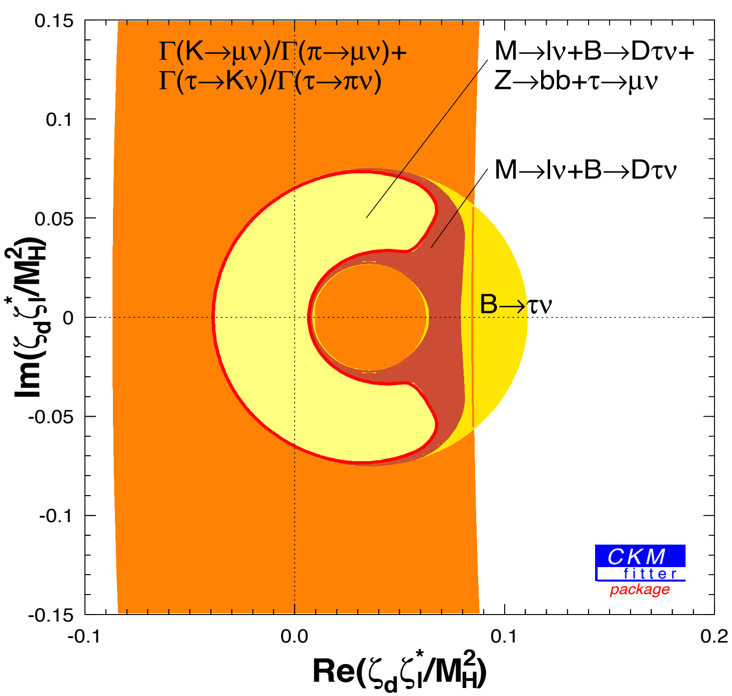
Jung, Pich, Tuzon'10

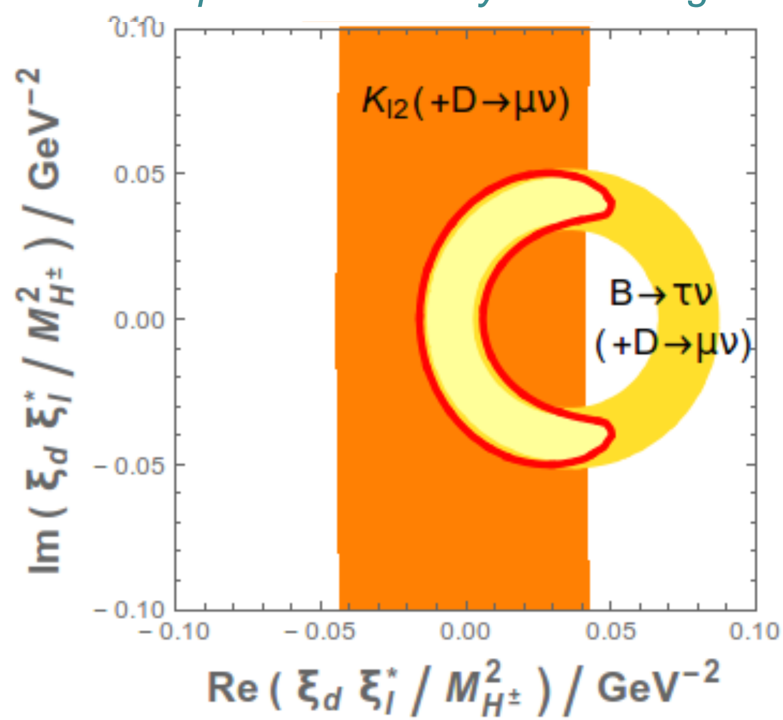
Pich@HQL'12

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} H^+ \left\{ \bar{u} \left[\varsigma_d \, V_{\rm CKM} M_d \mathcal{P}_R - \varsigma_u \, M_u^\dagger V_{\rm CKM} \mathcal{P}_L \right] d + \varsigma_l \, \left(\bar{\nu} \, M_l \, \mathcal{P}_R \, l \right) \right\} + \text{h.c.}$$



Update: Courtesy of M. Jung

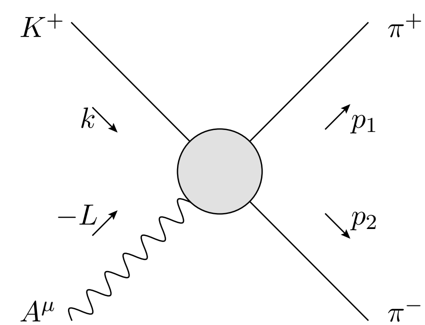




3.6 Test of Low-Energy QCD with K_{l4} Decays

3.6 Test of low energy QCD with K_{l4} decays

- Main interest:
  - access to ππ threshold region ⇨ ππ scattering lengths
  - form factors, LECs, . . .



- Standard problem of the NNLO treatment
  ⇨ strong final state rescattering

  Amoros, Bijnens, Talavera'00

- Use dispersion relations:
  ⇨ matching to CHPT at both one- and two-loop levels: LECs

$$\mathcal{L}_{eff} = \sum_{d\geq 2} \mathcal{L}_d \ , \mathcal{L}_d = \mathcal{O}\left(p^d\right) \ , p \equiv \left\{q, m_q\right\}$$

$$\mathbf{p} \ll \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}$$

- Isospin breaking and radiative corrections have been computed

Chiral expansion

- $$\mathcal{L}_{ChPT} = \underbrace{\mathcal{L}_2}_{\text{LO}:\ \mathcal{O}(p^2)} + \underbrace{\mathcal{L}_4}_{\text{NLO}:\ \mathcal{O}(p^4)} + \underbrace{\mathcal{L}_6}_{\text{NNLO}:\ \mathcal{O}(p^6)} + \ldots.$$

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

- The method has been rigorously established and can be formulated as a set of calculational rules:

LO : tree level diagrams with $\mathcal{L}_2$

NLO: tree level diagrams with $\mathcal{L}_4$
1-loop diagrams with $\mathcal{L}_2$

NNLO: tree level diagrams with $\mathcal{L}_6$
2-loop diagrams with $\mathcal{L}_2$
1-loop diagrams with one vertex from $\mathcal{L}_4$

$$\mathcal{L}_2 : F_0, B_0$$

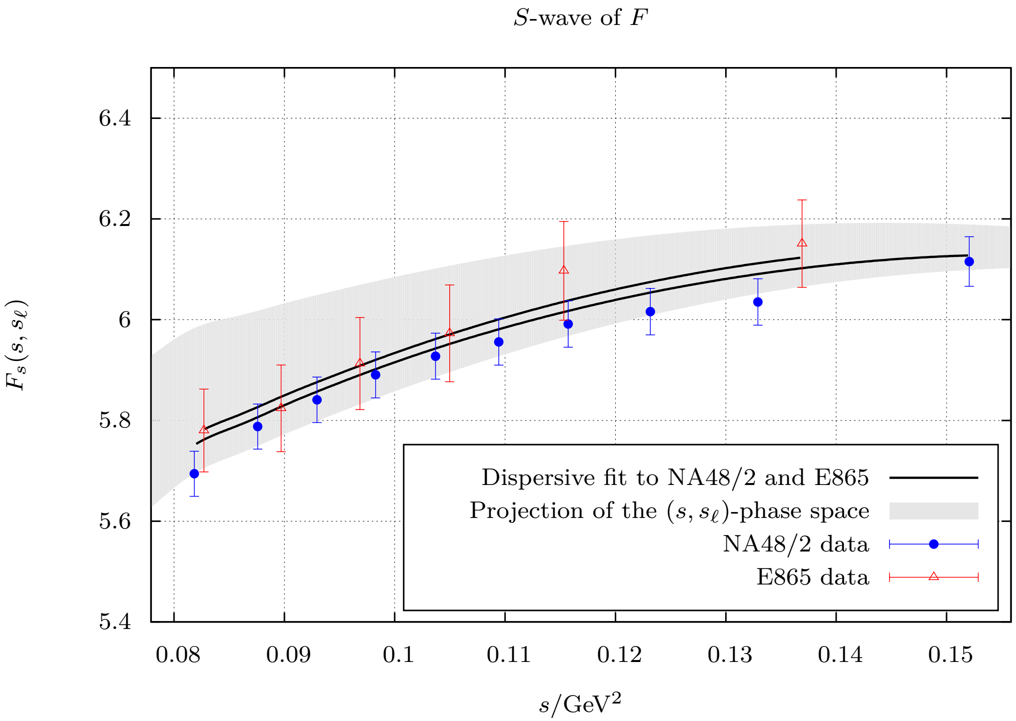
$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i\, O_4^i,$$

$$\mathcal{L}_6 = \sum_{i=1}^{90} C_i\, O_6^i$$

- Renormalizable and unitary order by order in the expansion

3.6 Test of low energy QCD with K_{l4} decays

Colangelo, E.P., Stoffer'15



Contrary to ChPT, the dispersive measurement allows to take into account for the curvature in the form factor

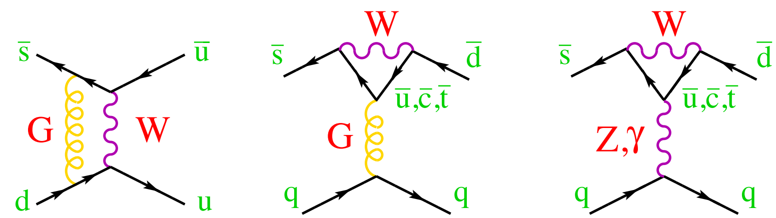
	NLO	NNLO	Bijnens, Ecker (2014)
$10^3 \cdot L_1^r$	$0.51(2)(6)$	$0.69(16)(8)$	$0.53(6)$
$10^3 \cdot L_2^r$	$0.89(5)(7)$	$0.63(9)(10)$	$0.81(4)$
$10^3 \cdot L_3^r$	$-2.82(10)(7)$	$-2.63(39)(24)$	$-3.07(20)$
χ^2/dof	$141/116 = 1.2$	$124/122 = 1.0$	

4. Non-leptonic Decays: ε'/ε

4.1 ε'/ ε

- Octet Enhancement: $\frac{A(K \to \pi\pi)_{I=0}}{A(K \to \pi\pi)_{I=2}} \approx 22$

  – Short-distance: gluonic corrections, penguins



  – Long-distance: large ChPT corrections (FSI) : ππ rescattering large

  – On-going lattice effort ⇨ very recent result from RBC-UKQCD!

4.1 ε'/ ε

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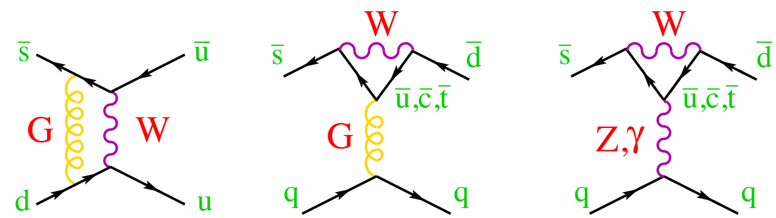


  – Long-distance: large ChPT corrections (FSI) : ππ rescattering large

  – On-going lattice effort ⇨ very recent result from RBC-UKQCD!

- Direct CP Violation:

$$\eta_{ij} \equiv \frac{A(K_L \to \pi^i \pi^j)}{A(K_S \to \pi^i \pi^j)}$$

$$\mathrm{Re}\,(\epsilon'/\epsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.8 \pm 1.4)\cdot 10^{-4}$$

- Lattice result: $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.43)\cdot 10^{-4}$ *RBC-UKQCD'15*

4.1 ε'/ ε

- Important discrepancy! 2.9σ

- Analytical result: Normalise to K^+ decay (ω_+, a) and ε_K, expand in A_2/A_0 and CP violation:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\frac{\omega_+}{\sqrt{2}\,|\epsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left(1-\hat{\Omega}_{\mathrm{eff}}\right) - \frac{1}{a}\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

Adjusted to keep electroweak penguins in Im A_0 *Cirigliano, et.al.`11*

- Challenge: compute : $A_I = \langle(\pi\pi)_I|\mathcal{H}_{\mathrm{eff}}|K\rangle$

- $$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{B_6^{(1/2)}\left(1-\Omega_{\mathrm{eff}}\right) - 0.4\, B_8^{(3/2)}\right\}$$

Pich@NA62 handbook workshop'16

Based on *Pallante, Pich, Scimemi'02*

⇨ $$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = \left(19 \pm 2_{\mu}\ {}^{+9}_{-6}{}_{m_s} \pm 6_{1/N_C}\right) \times 10^{-4}$$

4.1 ε'/ ε

- Analytical result: $\mathrm{Re}\,(\varepsilon'/\varepsilon) \;=\; \left(19 \pm 2_{\mu}\, {}^{+9}_{-6}{}_{m_s} \pm 6_{1/N_C}\right) \times 10^{-4}$

Pallante, Pich, Scimemi'02

  - O(p^4) χPT Loops: Large correction (FSI)
  - O(p^4) LECs fixed at $N_C \to \infty$: Small correction
  - Isospin Breaking O[(mu − md) p^2, e^2p^2]: Sizeable corrections
  - O(p^4) LECs [Re(g_8), Re(g_{27})] and phase-shifts fitted to data
  - m_s(2 GeV) = 110 ± 20 MeV

  ⇨ To be updated

- Challenge: Control of subleading $1/N_C$ corrections to χPT couplings

- Work from *Buras & Gerard'15, Buras et al.'16* relying on $1/N_C$ arguments
  ⇨ supports lattice result

New physics?

4.2 Comparison

- RBC/UKQCD results in isospin limit:

Pich@NA62 handbook workshop'16

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_2 = (1.50 \pm 0.04 \pm 0.14)\cdot 10^{-8}\ \mathrm{GeV} \qquad \mathrm{exp}: 1.482\,(2)\cdot 10^{-8}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_2 = -(6.99 \pm 0.20 \pm 0.84)\cdot 10^{-13}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_0 = (4.66 \pm 1.00 \pm 1.21)\cdot 10^{-7}\ \mathrm{GeV} \qquad \mathrm{exp}: 3.112\,(1)\cdot 10^{-7}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_0 = -(1.90 \pm 1.23 \pm 1.04)\cdot 10^{-11}\ \mathrm{GeV}$$

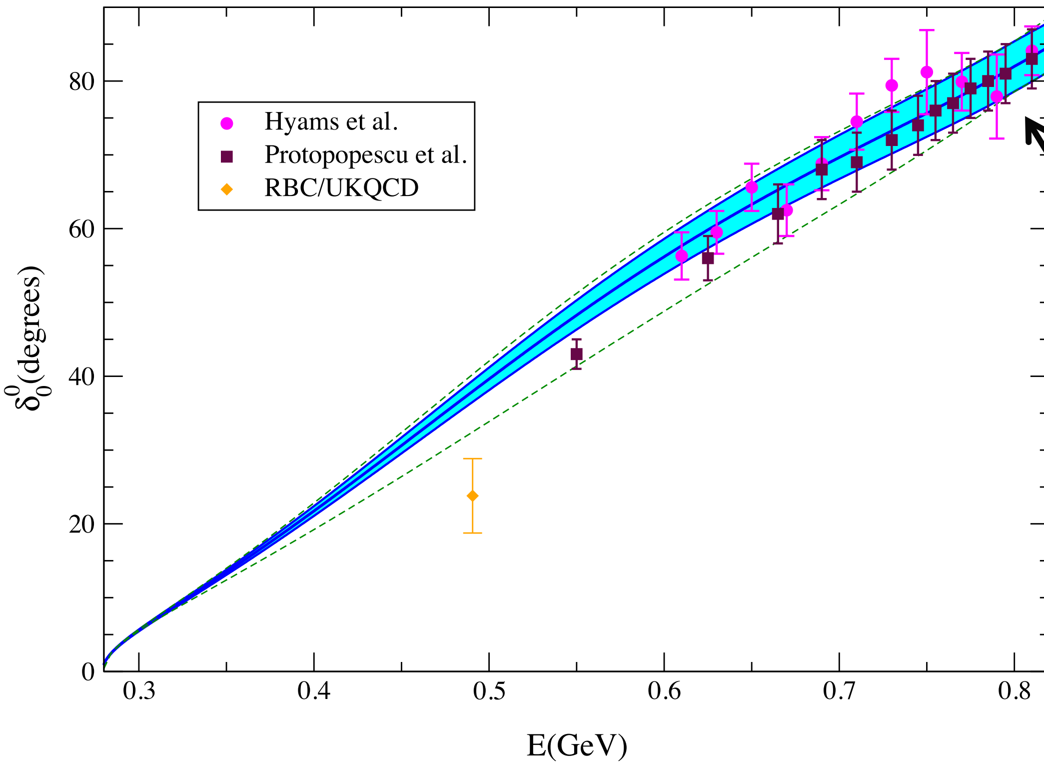
$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.43)\cdot 10^{-4} \qquad \mathrm{exp}: (16.8 \pm 1.4)\cdot 10^{-4}$$

$$\delta_0 = (23.8 \pm 4.9 \pm 1.2)^\circ \qquad \mathrm{exp}: (39.2 \pm 1.5)^\circ$$

$$\delta_2 = -(11.6 \pm 2.5 \pm 1.2)^\circ \qquad \mathrm{exp}: -(8.5 \pm 1.5)^\circ$$

4.2 Comparison

- ππ phase shift: *Colangelo@NA62 handbook workshop'16*



RBC/UKQCD result in diasgreement with experimental results and Roy Steiner solutions

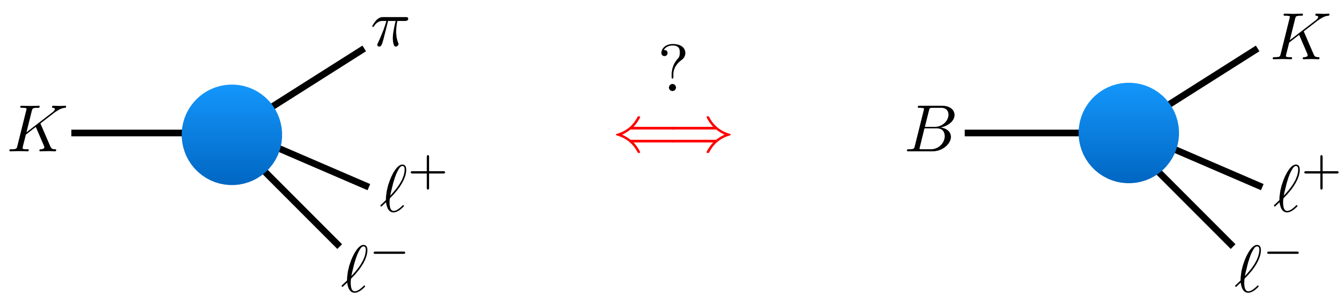
- Result from the fit in isospin limit: $[\delta_0 - \delta_2]_{K\to\pi\pi} = (52.5 \pm 0.8_{exp} \pm 2.8_{th})^\circ$

- Result from Roy-Steiner : $\delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$

5. Rare and Radiative Decays

5.1 LFV in Rare kaon decays

Crivellin, D'Ambrosio, Hoferichter, Tunstall'16



- Anomalies in the B physics sector
  - 2-3σ from SM in $B \to K^* \mu^+ \mu^-$ *Descotes-Genon et al.'13*
  - 2.6σ evidence of LFUV

$$R(K) = \frac{\mathrm{Br}[B \to K \mu^+ \mu^-]}{\mathrm{Br}[B \to K e^+ e^-]} = 0.745^{+0.090}_{-0.074} \pm 0.036$$ *LHCb.'14*

$$R_{\mathrm{SM}}(K) = 1.003 \pm 0.0001$$ *Bobeth, Hiller, Piranishvili'07*

  - Combined 3.9σ evidence of LFUV in

$$R(D)_{\mathrm{exp}} = 0.391 \pm 0.041 \pm 0.028$$
$$R(D^*)_{\mathrm{exp}} = 0.322 \pm 0.018 \pm 0.012$$

HFAG

$$R_{\mathrm{SM}}(D) = 0.297 \pm 0.017$$
$$R_{\mathrm{SM}}(D^*) = 0.252 \pm 0.003$$

Fajfer, Kamenik, Nisandzic'12

5.1 LFV in Rare kaon decays

Crivellin, D'Ambrosio, Hoferichter, Tunstall'16

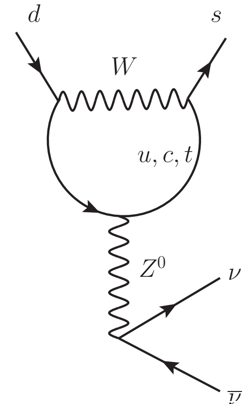
- Analogous process for kaon decays: $K^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$
- Translate bounds of B physics in K physics assuming MFV
  ⇨ Prediction for LFV modes because of correlations

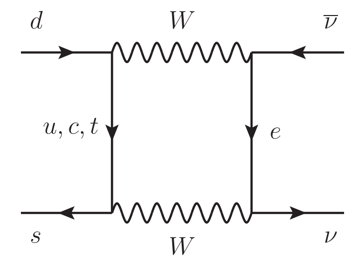
	$K_L \to \mu^{\pm}e^{\mp}$	$K^+ \to \pi^+\mu^{\pm}e^{\mp}$	$K_L \to \pi^0\mu^{\pm}e^{\mp}$	$K^+ \to \pi^+\mu^{\pm}e^{\mp}$ (NA62 projection)
$\left(\lvert C_{7V}^{\mu e}\rvert^2 + \lvert C_{7A}^{\mu e}\rvert^2\right)^{1/2}$	$< 1.3 \times 10^{-6}$	$< 2.2 \times 10^{-5}$		$< 5.1 \times 10^{-6}$
$\left(\lvert y_{7V}^{\mu e}\rvert^2 + \lvert y_{7A}^{\mu e}\rvert^2\right)^{1/2}$			< 0.040	
$\left(\lvert C_9^{B,\mu e}\rvert^2 + \lvert C_{10}^{B,\mu e}\rvert^2\right)^{1/2}$	< 0.71	< 12	< 35	< 2.7

- 3 possibilities:
  - New Physics explanations for B-anomalies + MFV
    ⇨ signal at NA62 sensitivies
  - Negative searches at NA62 ⇨ rule out MFV solutions
  - signal seen near current sensitivities ⇨ also rule out MFV

5.2 $K \to \pi\nu\nu$

- $$T \sim F\left(V_{is}^* V_{id}, \frac{m_i^2}{M_W^2}\right) \left(\bar{\nu}_L \gamma_\mu \nu_L\right) \langle \pi | \bar{s}_L \gamma^\mu d_L | K \rangle$$





- Very clean prediction in the SM: negligible long-distance contribution
- SM prediction very small:

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (7.8 \pm 0.8) \cdot 10^{-11} \sim A^4 \left[\eta^2 + (1.4 - \rho)^2\right]$$

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = (2.4 \pm 0.4) \cdot 10^{-11} \sim A^4 \eta^2$$

Buras et al.'15

- Clear signature of BSM physics ⇒ direct CPV

BNL-E949: few events! → $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (1.73^{+1.15}_{-1.05}) \cdot 10^{-10}$

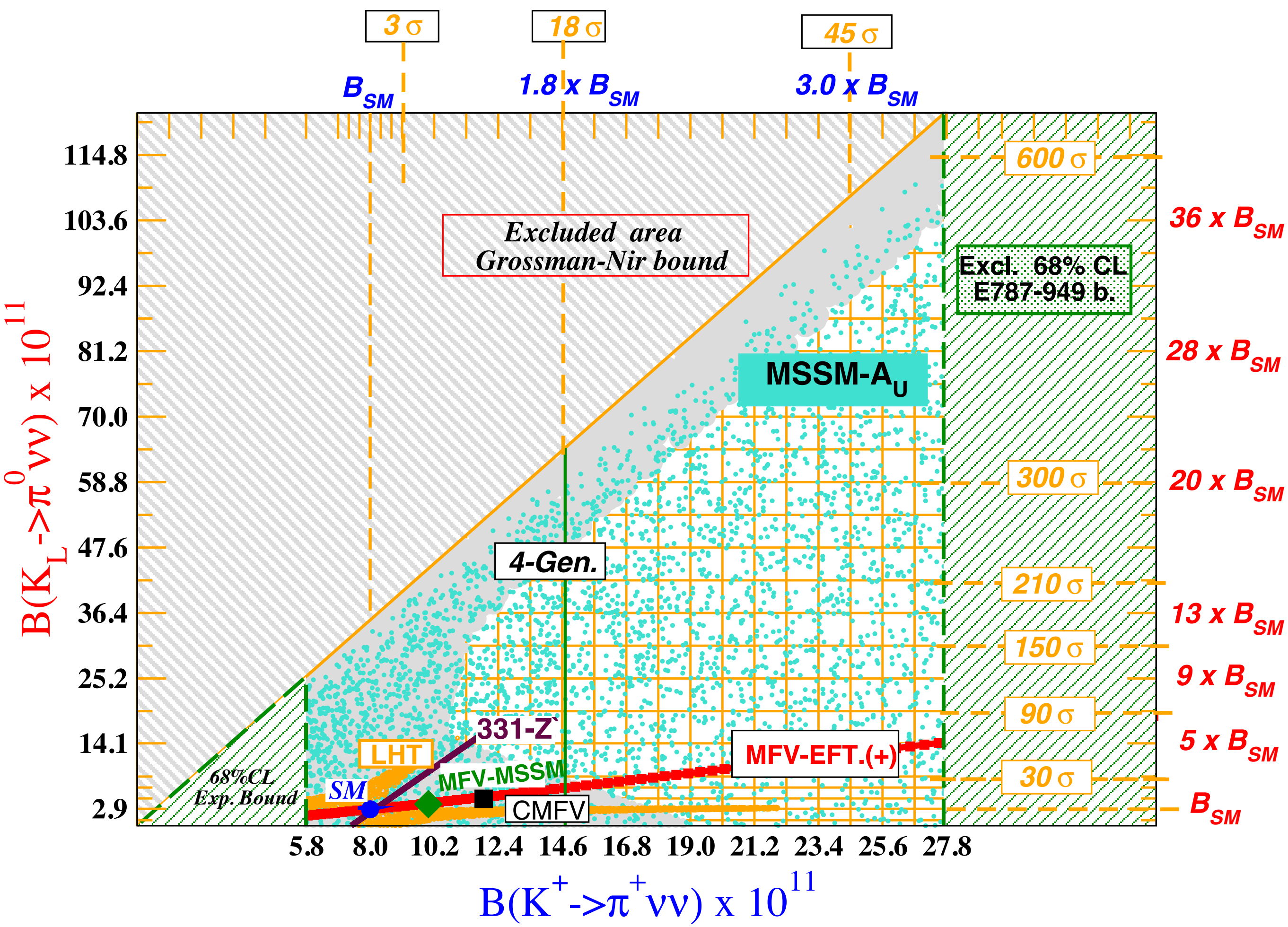
KEK-E391a: $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) < 2.6 \cdot 10^{-8}$ (90% CL)

- On going experiment: NA62, KOTO

5.2 $K \to \pi\nu\nu$

Mescia, Smith'08

- Stringent test of BSM scenarios



6. Conclusion and Outlook

Conclusion and Outlook

- Kaon decays very interesting to study: very rich phenomenology
  - Excellent testing ground of chiral dynamics
  - Interesting interplay of short and long-distances
  - Probe of flavour dynamics and violation of CP
  - Allow for tests of New Physics

- We have entered an era of precision:
  - Very impressive experimental sensitivities ($K \to \pi\nu\nu$)
  - Theoretical challenge: Precise control of QCD effects

    ⇒ Impressive progress in lattice QCD

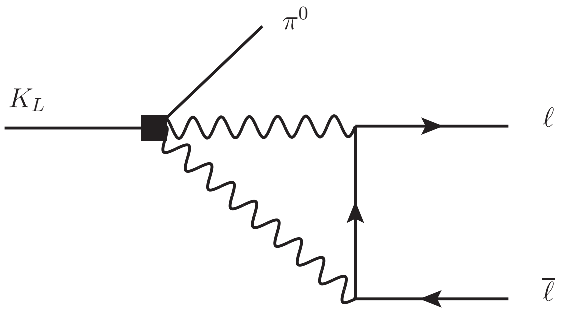
7. Back-up

5.3 $K_{L,S} \to \pi^0 \ell\ell$

- Experimental results:

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$$

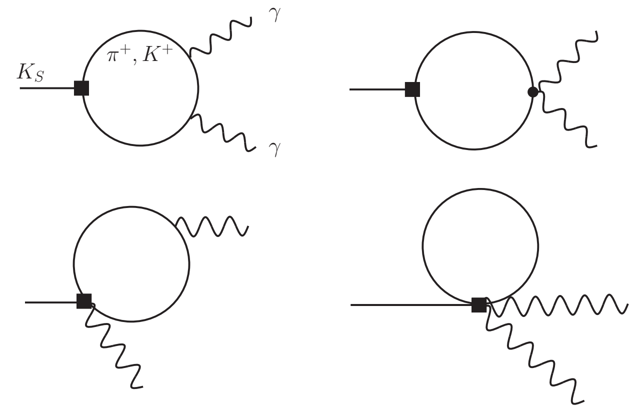


- 3 contributions:
  - Direct CP violation
  - Indirect CP violation
  - CP conserving (2γ)

- CP violation dominates for e+e-

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.1\ (0.9) \cdot 10^{-11}$$

5.1 $K^0 \to \gamma\gamma$



- Prediction at LO for K_S: Finite loop

$$\mathrm{Br}_{\mathrm{LO}} = 2.0 \cdot 10^{-6}$$

- Measurement:

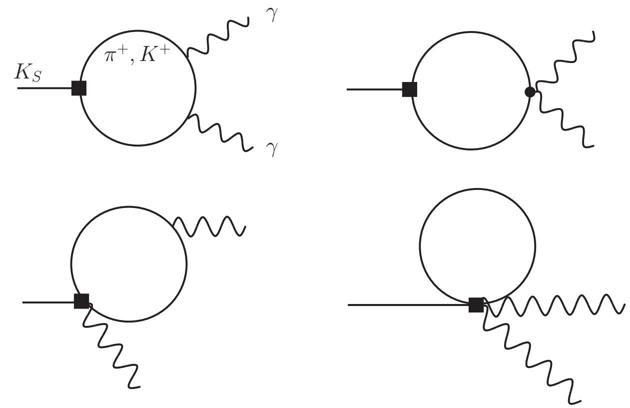
$$\mathrm{Br}(K_S \to \gamma\gamma) = (2.63 \pm 0.17) \cdot 10^{-6}$$

- Understood because of FSI: ⇨ agreement at $O(p^6)$

5.1 $K^0 \to \gamma\gamma$

- Prediction at LO for K_S: Finite loop

  $$\mathrm{Br}_{\mathrm{LO}} = 2.0 \cdot 10^{-6}$$

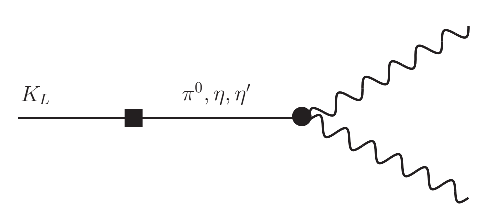


- Measurement:

  $$\mathrm{Br}(K_S \to \gamma\gamma) = (2.63 \pm 0.17) \cdot 10^{-6}$$

- Understood because of FSI: ⇨ agreement at $O(p^6)$

- For K_L: WZW anomaly



- Measurement : $\mathrm{Br}(K_L \to \gamma\gamma) = (5.47 \pm 0.04) \cdot 10^{-4}$

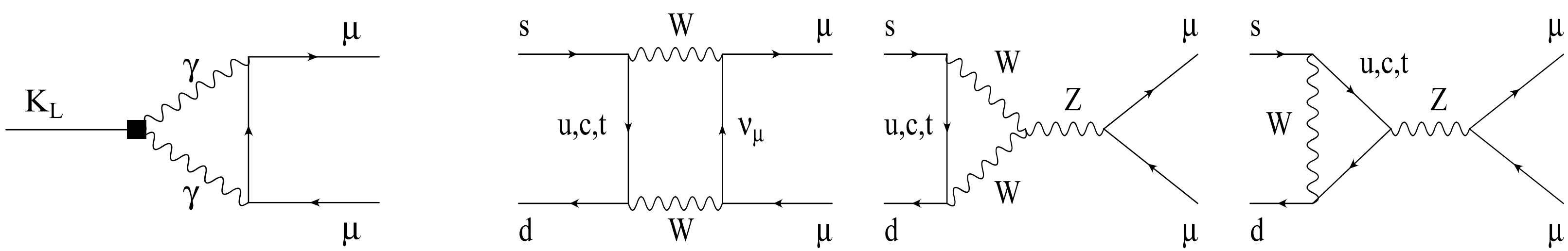
- T_{LO}=0: At $O(p^4)$ GMO cancellation

- $O(p^6)$: SU(3) breaking, η–η′ mixing well understood

5.1 $K_{L,S} \rightarrow \ell\ell$

- Very usefull source of information on the structure of ΔS = 1, FCNC transitions

- Both long distance and short distance components: *Isidori, Unterdorfer'03*

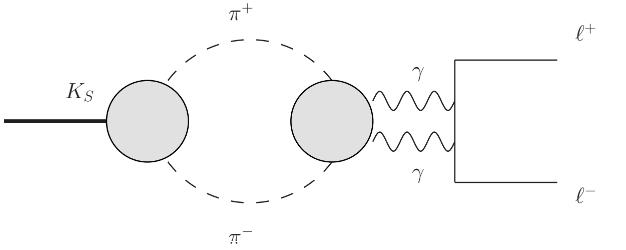


- K_L transition measured: $\mathrm{Br}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$

$$\mathrm{Br}(K_L \to e^+e^-) = (9\,^{+6}_{-4}) \cdot 10^{-12}$$

- Theoretically saturated by absorptive part, prediction in agreement with measurement:
  - Long distance extracted from $\pi^0, \eta \to \ell^+\ell^-$ *Gomez-Dumm & Pich*
  - Short distance contribution fitted

5.1 $K_{L,S} \to \ell\ell$



- K_S not measured yet only an upper bound

  $$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$

  $$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{exp}} < 3.2 \cdot 10^{-7}$$ *LHCb'13*

- Very interesting process
  ⇨ constrain the CP-violating part of the FCNC $s \to dl^+l^-$

  $$B(K_S \to \mu^+\mu^-)^{SM}_{short} = 10^{-5} \,|\, \mathrm{Im}(V^*_{ts} V_{td}) \,|^2 \simeq \mathcal{O}(10^{-13})$$

- Measurement of this mode :
  - New Physics
  - Bounds on CP-violating phase of $s \to dl^+l^-$

- Standard Model prediction: LO in ChPT is 2 loop diagram, finite

  $$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{LO}} = 2.1 \cdot 10^{-14}$$ *Ecker & Pich*

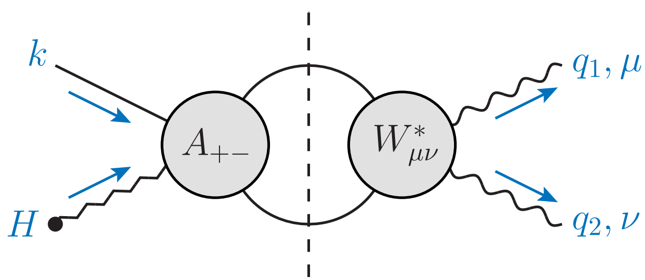
  $$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{LO}} = 5.1 \cdot 10^{-12}$$

5.1 $K_{L,S} \to \ell\ell$

- Higher order in ChPT ⇨ lots of unknown LECs

- Dispersive calculation in progress allows to take into account FSI

Colangelo, Stucki, Tunstall in progress



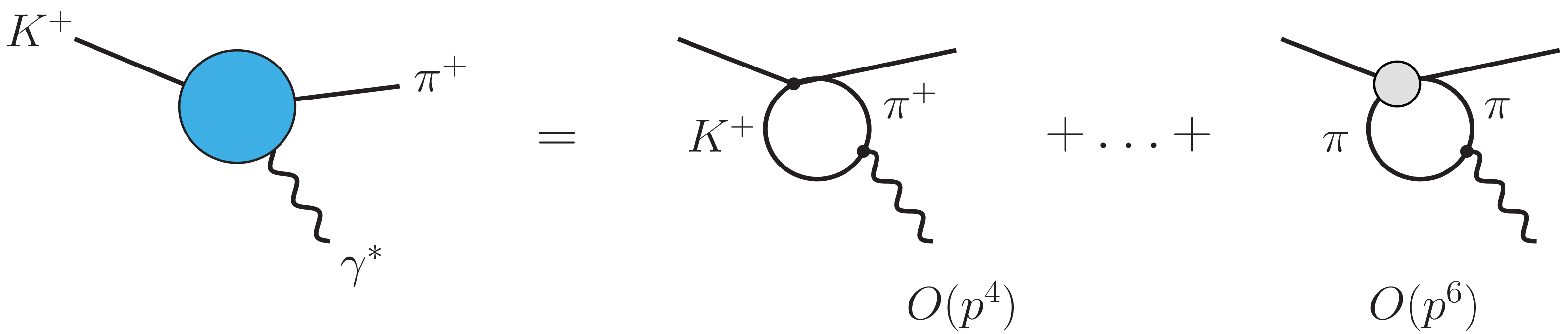
$$\mathrm{Im} A_{\mu\nu} = \frac{1}{2}\int d\phi_2\, A_{+-}(s)\, W^*_{\mu\nu}(s, q_1^2, q_2^2)$$

$$A_{+-} = \langle \pi^+\pi^- | \mathcal{H}_w | K_S \rangle$$

$$\epsilon^\mu \epsilon^\nu W_{\mu\nu} = \langle \gamma^*\gamma^* | \pi^+\pi^- \rangle$$

5.2 $K^+ \to \pi^+ \ell\ell$

- Computed in ChPT



- Chiral Dynamics contained in the electromagnetic vector FF

$$V_+(z) = a_+ + b_+ z + V_+^{\pi\pi}(z)\,, \qquad z = q^2/m_K^2$$

- Probe LECs in a_+ and b_+ via spectrum: $\dfrac{d\Gamma}{dz} \propto |V_+(z)|^2$

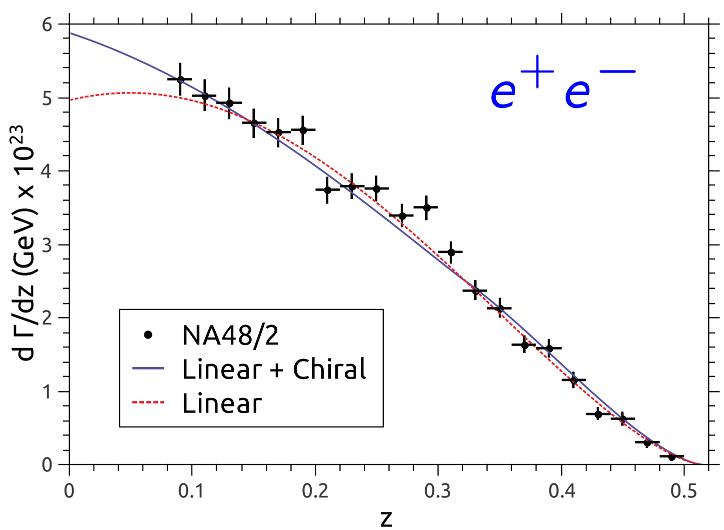
- Estimates using VMD *Coluccio Leskow et al.'16*

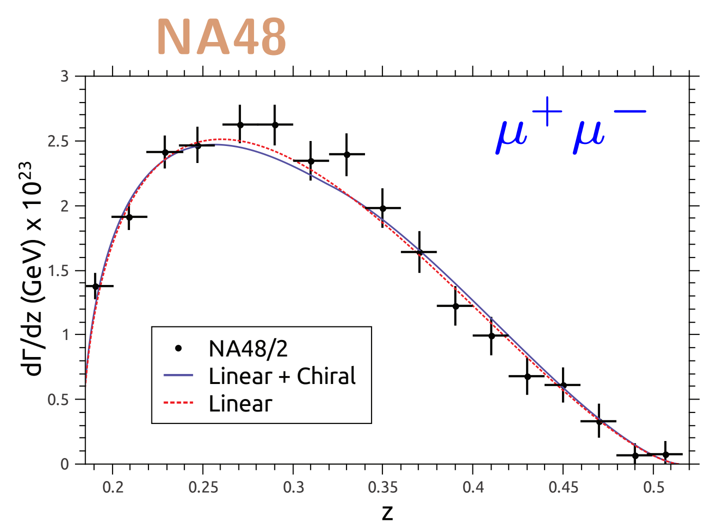
5.2 $K^+ \to \pi^+ \ell\ell$

- Experimental results:

$$\mathrm{Br}(K^\pm \to \pi^\pm e^+ e^-) = 3.14\,(10) \cdot 10^{-7}$$

$$\mathrm{Br}(K^\pm \to \pi^\pm \mu^+ \mu^-) = 9.62\,(25) \cdot 10^{-8}$$



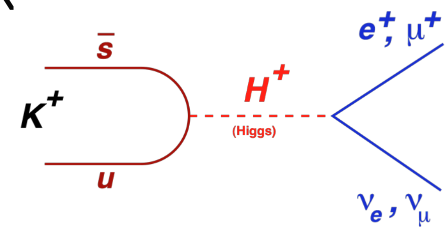


$$\mathrm{Br}[K_S \to \pi^0 e^+ e^-] = 3.0^{+1.5}_{-1.2} \times 10^{-9}$$

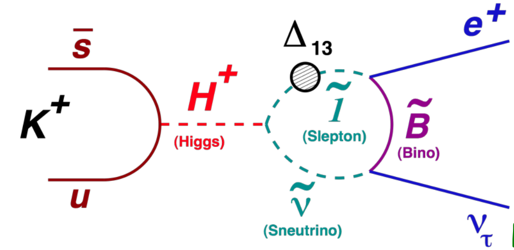
$$\mathrm{Br}[K_S \to \pi^0 \mu^+ \mu^-] = 2.9^{+1.5}_{-1.2} \times 10^{-9}$$

2.3 Test of New Physics in R_K

- R_K sensitive to *lepton flavour violating effects*, $\Delta R/R \approx O(1\%)$



- 2HDM – tree level: ⇨ additional contribution due to charged Higgs, does not contribute to R_K



- Possibility to constrain LFV at one loop in MSSM

  Masiero, Paradisi, Petronzio'06,'08

- Update and extension by *Girrbach & Nierste'12* ⇨ consider other constraints