Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary

# Antineutrinos From Neutron Capture

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Spectral Corrections ●○○	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary		
Correction Introduction						
Sources of Corrections						



VS.



- Used 'single' fissile spectra
- Operated at high flux:  $\mathcal{O}(10^{14} \,\mathrm{n/cm^2/sec})$
- Low irradiation time:  $\leq 2 \, d$
- Measured beta-spectrum

- Mixture of all fissiles
- Operate at lower fluxes:  $\mathcal{O}(10^{13}\,\mathrm{n/cm^2/sec})$
- Much higher irradiation times:  $\sim 1 \, yr$

Spectral Corrections ○●○	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Known Corrections				
Non-equilib	rium Co	rection		

- Mueller *et al.* conducted study in 2011 (PRC 83 054615) √
  - Long-lived nuclides building up in reactor environment
  - Primarily from <sup>90</sup>Sr, <sup>106</sup>Ru, and <sup>144</sup>Ce



- Low-E neutrino correction
- $\bullet~\sim 1-4\%$  correction for PWRs and typical irradiation periods

Spectral Corrections ○○●	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Known Corrections				

## Spent Nuclear Fuel Correction

- ② Zhou Bin *et al.* conducted study in 2012 (Chin.Phys. C **36** 1-5) and indepedently verified by PJ √
  - Long-lived nuclides in nearby spent fuel pools
  - Primarily from <sup>90</sup>Sr, <sup>106</sup>Ru, and <sup>144</sup>Ce (again)



•  $\sim 1 - 3\%$  low-E neutrino correction for typical PWRs

Spectral Corrections	Non-linear ●0000	Analytical Solution to NL 00	Computing the NL Correction	Summary
NL Intro				
Non-linear	Correctic	on		

#### **③** Correction arising from neutron-capture in the reactor

- Neutron capture usually small contribution (decay dominates)
- However, some nuclides are dominantly produced by neutron capture!

# Goals:

- Overall size of the non-linear (NL) correction
- Spectral shape of the correction
- Opendence on the reactor

Spectral Corrections	Non-linear ○●○○○	Analytical Solution to NL 00	Computing the NL Correction	Summary
NL Intro				
Linear Nucl	lides			

 Most nuclide production (destruction) dominated by fission or beta-decays (beta-decays)



Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
NL Intro				
Non-linear	(NL) Nu	clides		

 Non-linear effect arises from nuclides with dominating neutron-capture components (φσ<sub>P</sub>N<sub>P</sub>)



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Spectral Corrections	Non-linear 000●0	Analytical Solution to NL 00	Computing the NL Correction	Summary
NL Intro				
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## Non-linear Candidates

About 30 nuclides satisfy the format; shorten via...

- Large precursor cumulative fission yield  $\sum_{f=1}^{N_f} Z_P^f \ge 0.025$
- 2 Large neutron capture cross-section  $\sigma_P^c$
- Son-linear nuclide N decays sufficiently quickly
- Beta-decay of N has  $E_0 \ge 1.8 \,\mathrm{MeV}$

		<sup>100</sup> Tc	<sup>104</sup> Rh	<sup>110</sup> Ag	<sup>142</sup> Pr
$N = E_0 $ (MeV)		3.2	2.45	2.9	2.15
$N = \tau_{1/2}$ (sec)		15.54	42.3	24.6	68830
P Cumul.	<sup>235</sup> U(522b)	0.061	0.031	0.00029	0.059
Fission Yields	<sup>239</sup> Pu(698b)	0.062	0.069	0.017	0.052
(atoms/fiss.)	<sup>241</sup> Pu (950b)	0.056	0.065	0.030	0.049
$P \sigma_P^c$ (b)		17.0	127	80.9	6.53
$L \tau_{1/2}^{L}$ (d)		2.75	39.3	0.57	32.5
$L \sigma_L^c$ (b)		1.57	7.08	18.2	26.7

Spectral Corrections	Non-linear ○○○○●	Analytical Solution to NL 00	Computing the NL Correction	Summary
Limiting Behavior				
Non-linear	Limits			

- First, *L* must be in equilibrium to produce *P*, so  $T_{irr} > \tau_{L1/2}$
- Next,  $\phi$  must be large enough to promote  $P(n, \gamma)N$ , but not too large such that  $N_L \lambda_L < N_L \phi \sigma_L^c$

• Critical flux given by 
$$\tilde{\phi} = \frac{\ln 2}{\tau_{L_{1/2}}} \frac{1}{\sigma_L^c}$$

 $\, \hookrightarrow \, \tilde{\phi} = 9 \times 10^{15} - 2 \times 10^{18} \, \mathrm{n/cm^2/sec} \, \, (\text{unphysical})$ 

• If  $T_{irr} > \tau_{L1/2}$  and  $\phi < \tilde{\phi}$  the non-linear decay rate is given by  $\Gamma_{non-linear} = \underbrace{\sigma^{\text{fiss}} \phi Z_P T_{\text{irr}}}_{P} \sigma_P^c \phi \propto T_{\text{irr}} \phi^2$ 

atoms  $N_P$ 

• Decay rate proportional to  $\phi^2 T_{irr}$  (non-linear) instead of  $\phi$  (linear)

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Spectral Corrections	Non-linear 00000	Analytical Solution to NL ●○	Computing the NL Correction	Summary
Bateman Equations				
Set-up				

- Start with the famous Bateman equations
- Originally used simple decay-only chain
- Generalize with inclusion of fission and neutron capture

Pa



$$\begin{aligned} \frac{dN_{\alpha,i}}{dt} &= \vec{Y}_{\alpha,i} \cdot \vec{\mathcal{F}} + \lambda_{\alpha,i-1} N_{\alpha,i-1} + \sigma_{\alpha-1,j} \phi N_{\alpha-1,j} - \tilde{\lambda}_{\alpha,i} N_{\alpha,i} \\ \tilde{\lambda} &= \lambda + \sigma \phi \qquad \vec{\mathcal{F}} = \{\mathcal{F}_{U235}, \mathcal{F}_{Pu239}, \mathcal{F}_{Pu241}\} \qquad \vec{Y}_{\alpha,i} = \{Y_{U235}^{\alpha,i}, Y_{U235}^{\alpha,i}, Y_{U235}^{\alpha,i}\} \\ \text{trick Jaffke} \qquad \qquad \text{Applied Antineutrino Physics 2015} \qquad 10 \end{aligned}$$

Spectral Corrections	Non-linear 00000	Analytical Solution to NL ⊙●	Computing the NL Correction	Summary
Bateman Equations				
Analytical S	Solution			

• Need to solve 3 linearly coupled non-homogeneous diff. eqn.s

$$\frac{dN_L}{dt} = \vec{Z}_L \cdot \vec{F} - \tilde{\lambda}_L N_L \qquad \frac{dN_P}{dt} = \vec{Y}_P \cdot \vec{F} + \lambda_L N_L - \phi \sigma_P^c N_P$$
$$\frac{dN_N}{dt} = \phi \sigma_P^c N_P - \lambda_N N_N$$
  
• Analytical solution yields expected 1000  
limits (turn-off at high flux and increase with  $T_{irr}$ )  
• Abundance peaks for a  $\sum_{l=1}^{10}$  100  
"sweet-spot" flux

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 $\phi_t [n/cm^2/sec]$ 

Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction ●○○○	Summary
SCALE				
SCALE Intro	О			

- SCALE converts core structure into 0d cross-section object
- Depletes core along power history supplied

$$\frac{dN_i}{dt} = \sum_{j=1}^m I_{ij}\lambda_j N_j + \bar{\Phi} \sum_{k=1}^m f_{ik}\sigma_k N_k - (\lambda_i + \bar{\Phi}\sigma_i)N_i$$



- w17 × 17 very similar to Daya Bay/Ling Ao cores!
- SCALE provides nuclide abundance and fission rates at all time intervals

Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction ○●○○	Summary
Calculating NL				
Calculating	Non-line	ar Contribution		

# Use SCALE to compute the non-linear correction:

- Irradiate various reactors to their typical power history
- Pull non-linear abundances and compute non-linear neutrino spectrum: Φ<sub>NL</sub>(t, E)
- Pull fission rates and compute the reactor spectrum:  $\Phi_{Rx}(t, E)$
- Compute time-averaged non-linear correction:  $\langle \Phi_{NL}(t, E) / \Phi_{RX}(t, E) \rangle_{T_{irr}}$

Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction ○○●○	Summary		
Calculating NL						
Computational Results						

# Results:

	$5{\rm MW_e}$	IR40	PV	VR	IDT		ILL		LIEID
		11(40	1-batch	3-batch		<sup>235</sup> U	<sup>239</sup> Pu	<sup>241</sup> Pu	
Fuel/Moderator	NU+C	NU+D <sub>2</sub> O	LEU-	+H <sub>2</sub> O	HEU+H <sub>2</sub> O		$HEU{+}D_2O$		$HEU+H_2O$
Burn-up [MWd/t]	32380	31200	31510	1890000	2230	$7.3  imes 10^{-5}$	$1.1  imes 10^{-4}$	$1.7  imes 10^{-4}$	2550
$\phi [n/cm^2/sec]$	$1.6\times10^{12}$	$3.6  imes 10^{13}$	$4.4\times10^{13}$	$4.4  imes 10^{13}$	$1.5  imes 10^{14}$	$3.3\times10^{14}$	$3.3  imes 10^{14}$	$3.3\times10^{14}$	$2.5  imes 10^{15}$
$Max[\langle \Phi_{\rm NL} / \Phi_{\rm R} \rangle_T]$ [%]	0.027	0.15	0.25	0.93	0.11	$3.1  imes 10^{-5}$	$2.6  imes 10^{-3}$	$4.7\times10^{-3}$	0.10

Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction ○○○●	Summary
Calculating NL				
Non-linear	Results			



- Larger corrections for "sweet-spot" flux and high *T<sub>irr</sub>* (i.e. high burnup)
- Small corrections for *T<sub>irr</sub>* ≪ 30 d (ILL measurements are safe)
- Commercial reactors can see a maximum non-linear effect ~ 1%
- Results in arXiv:1510.08948

Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary

### **Correction Summary**



- Three corrections all with  $\mathcal{O}(1\%)$  effects
- All corrections require detailed reactor simulations (and could be more!)
- Directly impacts geoneutrino searches

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Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Thank you!				

Spectral Corrections	Non-linear	Analytical Solution to NL	Computing the NL Correction	Summary

# Producing a Reactor Spectrum



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Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Other Corre	ctions?			

#### Shape/Forbiddenness Corrections (Hayes + Dwyer)?



# Neutron-capture (i.e. Non-linear) Correction? ???

Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Non-linear	Nuclides			

- Test all nuclides in JEFF yields (900) for their linearity
- Those with  $\lambda\sim \tilde{\phi}\sigma_c$  for  $\tilde{\phi}\leq 10^{17}\,{\rm n/cm^2/sec}$  given in black



Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Analytical S	olution			

• Solutions for L and P are...

$$\begin{split} N_L(t) &= \left[\frac{Z_L \cdot \vec{\mathcal{F}}}{\tilde{\lambda}_L}\right] \left(1 - e^{-\tilde{\lambda}_L t}\right) \\ N_P(t) &= \left[\frac{\lambda_L \vec{Z}_L \cdot \vec{\mathcal{F}} + \tilde{\lambda}_L \vec{Y}_P \cdot \vec{\mathcal{F}}}{\phi \sigma_P \tilde{\lambda}_L}\right] \left(1 - e^{-\phi \sigma_P t} - \left[\frac{\lambda_L \vec{Z}_L \cdot \vec{\mathcal{F}}}{\tilde{\lambda}_L (\phi \sigma_P - \tilde{\lambda}_L)}\right] \left(e^{-\tilde{\lambda}_L t} - e^{-\phi \sigma_P t}\right) \end{split}$$

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Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Analvtical S	olution			

• Solution for N is...

$$\begin{split} N_{N}(t) &= \left[ \frac{\tilde{\lambda}_{L} \vec{Y}_{F} \cdot \vec{\mathcal{F}} + \lambda_{L} \vec{Z}_{L} \cdot \vec{\mathcal{F}}}{\tilde{\lambda}_{L} \tilde{\lambda}_{N}} \right] \left( 1 - e^{-\tilde{\lambda}_{N} t} \right) \\ &+ \left[ \frac{\lambda_{L} \phi \sigma_{F} \vec{Z}_{L} \cdot \vec{\mathcal{F}}}{\tilde{\lambda}_{L} (\tilde{\lambda}_{L} - \phi \sigma_{F}) (\tilde{\lambda}_{N} - \tilde{\lambda}_{L})} \right] \left( e^{-\tilde{\lambda}_{L} t} - e^{-\tilde{\lambda}_{N} t} \right) \\ &- \left[ \frac{(\tilde{\lambda}_{L} - \phi \sigma_{F}) \vec{Y}_{F} \cdot \vec{\mathcal{F}} + \lambda_{L} \vec{Z}_{L} \cdot \vec{\mathcal{F}}}{(\tilde{\lambda}_{L} - \phi \sigma_{F}) (\tilde{\lambda}_{N} - \phi \sigma_{F})} \right] \left( e^{-\phi \sigma_{F} t} - e^{-\tilde{\lambda}_{N} t} \right) \end{split}$$

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### Analytical vs. Computational



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Spectral Corrections	Non-linear 00000	Analytical Solution to NL 00	Computing the NL Correction	Summary
Correction	Result			

• 1-8% correction for low-E reactor  $\bar{\nu}_e$  could swamp geoneutrino signal



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