SHOULD WE TALK ABOUT CHARGED LEPTON OSCILLATION?

Group 11

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- We know that neutrinos oscillate. What about charged leptons? Can they oscillate?
- Can we create certain linear superposition of charged leptons and detect this superposition instead of individual leptons?
- Let's find out...
Can charged leptons oscillate?

- Not so immediate answer: **YES!**
- Let's analyse the quantum-mechanical uncertainty \( \sigma_m^2 \) for processes where we can produce more than one lepton:
  
  **CASE 1**
  
  \[ \sigma_m^2 < m_\mu^2 - m_e^2 \]
  
  - In this case we would know (exactly!) which lepton was produced.
  
  Incoherent mixture.
  
  \[ \pi^\pm \rightarrow l^\pm \nu \]

  **CASE 2**
  
  \[ \sigma_m^2 > m_\mu^2 - m_e^2 \]
  
  - In this case it is (in principle) impossible to determine which lepton was produced.
  
  Coherent superposition (condition for charged lepton oscillation).
  
  \[ W^\pm \rightarrow l_a^\pm \nu \quad (l_a = e, \mu, \tau) \]
Estimating the mass uncertainty

- From $m^2 = E^2 - p^2$ we have:
  
  $$
  \sigma_{m^2} = \left[ (2E\sigma_E)^2 + (2p\sigma_p)^2 \right]^{1/2}
  $$

  $\sigma_p \simeq [p/E \tau_\pi]^{-1} = (E/p)\Gamma_\pi$

  $\sigma_E \simeq \Gamma_\pi = \frac{\Gamma_\pi^0}{\gamma}$

  Lorentz's factor for the parent particle

- Therefore:
  
  $$
  \sigma_{m^2} \simeq 2\sqrt{2} E \sigma_E
  $$

  Parent's decay width
Estimating the mass uncertainty in $W^\pm$ decay

- For $W^\pm$ decays:

$$\sigma_m^2 \sim 2\sqrt{2} E \sigma_E \sim 2\sqrt{2} \cdot 40 \text{ GeV} \cdot 230 \text{ MeV}$$

$$\sigma_m^2 \sim (5 \text{ GeV})^2$$

$$\sigma_m^2 \gg m_\mu^2 - m_e^2$$

$$\sigma_m^2 > m_\tau^2 - m_\mu^2 \simeq (1.77 \text{ GeV})^2$$

- All three leptons are produced coherently in $W^\pm$ decays.

- $\sigma_m^2$ is Lorentz-invariant → so all of this also applies for $W^\pm$ decays in flight.
Can we observe charged lepton oscillations?

- Observability → emitted state should preserve its coherence until detection.

- Coherence loss → due to the different group velocities of each mass-eigenstates, $v_g = \frac{\partial E}{\partial p}$, in the mixed state.

- For $W^\pm$ decays:  
  
  \begin{align*}
    \text{In rest:} & \quad \left(x_{\text{coh}}\right)_{\text{max}} \approx 2.5 \times 10^{-8} \text{ cm} \\
    \text{In flight:} & \quad \left(x_{\text{coh}}\right)_{\text{max}} > 1\text{m} \quad \Rightarrow \quad E_W \gtrsim 130 \text{ TeV}, \quad E_{\text{leptons}} > 4.8 \text{ TeV}
  \end{align*}

- Lets compare with the observability of the $\nu$ oscillation:

  \begin{align*}
    L \gtrsim 1\text{ km} \quad \Rightarrow \quad E_\nu \gtrsim 20 \text{ eV}
  \end{align*}

- Meaning: the charged lepton oscillations are “a little bit difficult to observe”.
FOR ALL THESE REASONS PEOPLE DON'T USUALLY TALK ABOUT CHARGED LEPTON OSCILLATIONS.
But... Should we talk about it?

- Using similar calculation and we can show that: charged lepton states produced in the decays of heavy sterile neutrinos can be coherent superpositions of e, μ and τ.

\[ N_i \rightarrow e_i^- + \Phi^+ \quad \Phi^\pm \rightarrow N_i + e_i^{\pm} \]

- They can maintain their coherence over macroscopic distances provided that their energies exceed a few hundred TeV.

- Sterile neutrinos, being very heavy, are either emitted incoherently or lose their coherence almost immediately, providing a measurement of the “flavor” of the charged lepton.

SO, YES! MAYBE WE SHOULD BE TALKING ABOUT IT.

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Should we talk about charged lepton oscillations?

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Conclusions

- If we consider the quantum-mechanical mass uncertainty we can find a condition for having a superposition of charged leptons, and, therefore, they would oscillate.

- This happens in $W^\pm$ decays.

- However, the oscillation is short. We would need very high energies for observing it. And that's why we don't talk about charged lepton oscillation... 

- We could also have charged lepton oscillation in the decays of heavy sterile neutrinos, which may be detected in future experiments. So maybe we should talk about charged lepton oscillations...
THANK YOU! Should we talk about charged lepton oscillations?
Attempt in measuring the sterile $\nu$ mass eigenstate
• WTF Experiment
(WTF - Willis Tower Flight)
SHOULD WE TALK ABOUT CHARGED LEPTON OSCILLATION?

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• We know that neutrinos oscillate. What about charged leptons? Can they oscillate?

• Can we create certain linear superposition of charged leptons and detect this superposition instead of individual leptons?

• Let's find out...
Can charged leptons oscillate?

- Immediate answer: **NO!**

**Why? →**

**Charged leptons are mass eigenstates (states of definite mass) and mass eigenstates do not oscillate!**

- Lets think about process like beta and muon decays:

\[ \beta^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \]

- The production of $\mu^\pm$ and $\tau^\pm$ is kinematically forbidden.

- Hence, there are no charged lepton oscillations associated with these process.
Can charged leptons oscillate?

- Not so immediate answer: **YES!**

- Lagrangian: \[ \mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} (\bar{e}_a L \gamma^\mu U_{ai} \nu_i L) W^-_\mu + h.c. \]
  \[ (a = e, \mu, \tau, \quad i = 1, 2, 3) \]

- What if only \( l \) could oscillate?
  
  \[ |e_1\rangle = U_{1e} |e\rangle + U_{1\mu} |\mu\rangle + U_{1\tau} |\tau\rangle \]
  
  \[ |e_2\rangle = U_{2e} |e\rangle + U_{2\mu} |\mu\rangle + U_{2\tau} |\tau\rangle \]
  
  \[ |e_3\rangle = U_{3e} |e\rangle + U_{3\mu} |\mu\rangle + U_{3\tau} |\tau\rangle \]

- What if \( \nu \) and \( l \) could oscillate?
  
  \[ |e_\beta\rangle = \sum_a W_{\beta a}^* |e_a\rangle, \quad |\nu_\beta\rangle = \sum_i V_{\beta i}^* |\nu_i\rangle \]
  
  \[ e_a = e, \mu, \tau, \quad i = 1, 2, 3 \]

  \[ W^\dagger V = U \]
Can charged leptons oscillate? - YES!

- Lets consider processes where more than one charged lepton can be produced:

\[ \pi^\pm \rightarrow l^\pm \nu \quad W^\pm \rightarrow l_a^\pm \nu \quad (l_a = e, \mu, \tau) \]

- For these processes \( \rightarrow \sigma_E \) and \( \sigma_p \)

(quantum-mechanical uncertainties)

particle should be described by wave packets of spatial size

\[ \sigma_x \sim 1/\sigma_p \]

- If we know \( E \) and \( p \) \( \rightarrow \) we can determine the the \( m^2 \) of the particle with an uncertainty \( \sigma_m^2 \).
Conditions for charged lepton oscillations

CASE 1

\[ \sigma m^2 < m^2_\mu - m^2_e \]

- In this case we would know (exactly!) which lepton was produced.

Incoherent mixture.

CASE 2

\[ \sigma m^2 > m^2_\mu - m^2_e \]

- In this case it is (in principle) impossible to determine which lepton was produced.

Coherent superposition.

Condition for charged lepton oscillation.
Estimating the mass uncertainty

- From \( m^2 = E^2 - p^2 \) we have:

\[
\sigma_m^2 = \left[ (2E\sigma_E)^2 + (2p\sigma_p)^2 \right]^{1/2}
\]

\[
\sigma_p \simeq \left[ (p/E)\tau_\pi \right]^{-1} = (E/p)\Gamma_\pi
\]

\[
\sigma_E \simeq \Gamma_\pi = \Gamma_\pi^0 / \gamma
\]

Lorentz's factor for the parent particle

Parent's decay width

- Therefore:

\[
\sigma_m^2 \simeq 2\sqrt{2} E \sigma_E
\]
Estimating the mass uncertainty in pion decay

- Let's consider pion decays: $\pi^\pm \rightarrow l^\pm \nu$

- From the pion rest-frame decay width, $\Gamma_\pi^0 = 2.5 \cdot 10^{-8}$ eV

  $$\sigma_{m^2} \simeq 2\sqrt{2} E \Gamma_\pi^0 \simeq 2\sqrt{2} \cdot 90 \text{ MeV} \cdot 2.5 \cdot 10^{-8} \text{ eV}$$

  $$\sigma_{m^2} \simeq 6.4 \text{ eV}^2$$

  $\sigma_{m^2} < m_{\mu}^2 - m_e^2 \simeq (106 \text{ MeV})^2$ \quad CASE 1

  (for neutrinos $\rightarrow 7.6 \times 10^{-5}$ eV)

- Similar approach can be applied to kaon decays, leading to the same results.

No charged lepton oscillation.
Estimating the mass uncertainty in $W^\pm$ decay

- Lets consider $W^\pm$ decays: $W^\pm \rightarrow l_\alpha^\pm \nu \ (l_\alpha = e, \mu, \tau)$

- From the $W^\pm$ rest-frame decay width,

$$\Gamma_{W \rightarrow l_\alpha \nu}^0 \simeq \frac{G_F m_W^3}{6 \sqrt{2} \pi} \simeq 230 \text{ MeV}$$

$$\sigma_m^2 \sim 2 \sqrt{2} E \sigma_E \simeq 2 \sqrt{2} \cdot 40 \text{ GeV} \cdot 230 \text{ MeV}$$

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$$\sigma_m^2 \gg m_\mu^2 - m_e^2$$

$$\sigma_m^2 > m_\tau^2 - m_\mu^2 \simeq (1.77 \text{ GeV})^2$$

CASE 2

Charged lepton oscillation!!!

(for neutrinos $\rightarrow 2.4 \times 10^{-3} \text{ eV}$)

- All three leptons are produced coherently in $W^\pm$ decays.

- $\sigma_m^2$ is Lorentz-invariant $\rightarrow$ so all of this also applies for $W^\pm$ decays in flight.

Should we talk about charged lepton oscillations?
Can we observe charged lepton oscillations?

- Observability → emitted state should preserve its coherence until detection.

- Coherence loss → due to the different group velocities of each mass-eigenstates, $v_g = \partial E / \partial p$, in the mixed state.

- Coherence length for $W^\pm$ decays:

  **In rest:**
  
  \[
  (\Delta v_g)_{\text{min}} \simeq 2 \frac{m_\mu^2 - m_e^2}{m_W^2} 
  \]

  \[
  (x_{\text{coh}})_{\text{max}} \simeq [\Gamma_{W \rightarrow l_a \nu} (\Delta v_g)_{\text{min}}]^{-1} \simeq 2.5 \times 10^{-8} \text{ cm}
  \]

  **In flight:**
  
  \[
  (x_{\text{coh}})_{\text{max}} \rightarrow \gamma^3 (x_{\text{coh}})_{\text{max}}
  \]

  for \((x_{\text{coh}})_{\text{max}} > 1 \text{m}\) → \(\gamma \gtrsim 1600\) and \(E_W \gtrsim 130 \text{ TeV}\)
Can we observe charged lepton oscillations?

- If the coherence length doesn't depend on the size of the wave packet, we have:

\[
L < \frac{4\sqrt{2} E^3}{(\Delta m^2_{\mu e})^2} \simeq 8.9 \times 10^{-10} \left( \frac{E}{\text{GeV}} \right)^3 \text{ cm}
\]

for \( L \gtrsim 1 \text{ m} \quad \Rightarrow \quad E_{\text{leptons}} > 4.8 \text{ TeV} \)

- Let's compare with the observability of the neutrinos oscillation:

for \( L \gtrsim 1 \text{ km} \quad \Rightarrow \quad E_\nu \gtrsim 20 \text{ eV} \)

- Meaning: the charged lepton oscillations are “a little bit difficult to observe”.

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Other considerations

- We have assumed that the size of the wave packet doesn't change with time.

- Further calculation show that we would have the same results if the wave packet spreading was considered.

- The macroscopic sizes of the source and detector would wash out the effects of the oscillations of charged leptons unless the corresponding oscillation length exceeds the source and detector sizes in the direction of the beam.

- The requirement of no washout for 1 m puts lower bound on the energy of the decaying parent particle: $E > 10^{13}$ GeV.

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Conclusions

- Charged leptons $e$, $\mu$ and $\tau$ do not oscillate into each other because they are mass eigenstates.

- Charged leptons born in $\pi^\pm$ and $K^\pm$ decays are produced incoherently. Therefore they do not oscillate.

- For charged leptons produced in $W^\pm$ decays the coherence production condition is satisfied. However, for $W^\pm$ decays at rest the coherence is lost over microscopic distances because of the wave packet separation. For decays in flight with $E_w$ $100$ TeV the coherence lengths can formally take macroscopic values.

- Charged lepton states produced in the decays of heavy sterile neutrinos can be coherent superpositions of $e$, $\mu$ and $\tau$ and may be detected in future experiments.
Attempt in measuring the sterile \( v \) mass eigenstate
Heavy Sterile Neutrinos
Keep away from children!
• Y.N. Srivastava and A. Widom, Of course muons can oscillate, hep-ph/9707268.