Holographic 3-Point Functions & Their Applications
(Real-Time, Finite-Temperature)

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- ***Real-time Finite-temperature Holographic 3-Point Functions***

Arnold, Barnes, Vaman & CW : Phys.Rev.D82:025019,2010; arXiv:1004.1179

- ***Application (I) : 2nd Order Conformal Relativistic Hydrodynamics***

Arnold, Vaman, CW & Xiao : JHEP 1110 (2011) 033; arXiv:1105.4645

- ***Application (II) : Jet Quenching with Finite Chemical Potential***

Barnes, Vaman & CW : Work in process

A Brief Review of Holography

- Quark-Gluon Plasma at RHIC & LHC
  - Strong-interacting: Strongly-coupled QCD
  - Nearly-perfect fluid: Hydrodynamics
- Holography (Gravity/Gauge Duality, AdS/CFT duality…)
  - Feynman diagram method breaks down at strong coupling
  - Alterative approach to compute strongly-coupled QFT
  - Not known exactly for QCD
  - Simplest example:

    AdS_5 (IIB Supergravity, weakly-coupled) / CFT_4 ($\mathcal{N}=4$ SYM, strongly-coupled)
  - Extended to non-conformal FTs
- Real-Time & Finite-Temperature
  - Finite-temperature: AdS + Black Holes (Schwarzschild, RN…)
  - Real-time: various types of correlators (causal, time-ordered, non-time-ordered…)
    - Skenderis & van Rees *(arXiv:0805.0150 & 0812.2909)*
    - Starinets, Son & Herzog (2-pt, *2002-2003*) + Ours (3-pt ↑, *2010*)
- 2-pt correlators were well studied (e.g. η, σ…, *2002-*)

  What higher n-pt correlators can do? 3-pt? (e.g. τ, λ_i…, *not known*)

2^{nd} Order Conformal Relativistic Hydrodynamics – Review

- Early litertature:
  - *Baier, Romatschke, Son, Starinets & Stephanov, arXiv:0712.2451*
  - *Bhattacharyya, Hubeny, Minwalla & Rangamani, arXiv:0712.2456*

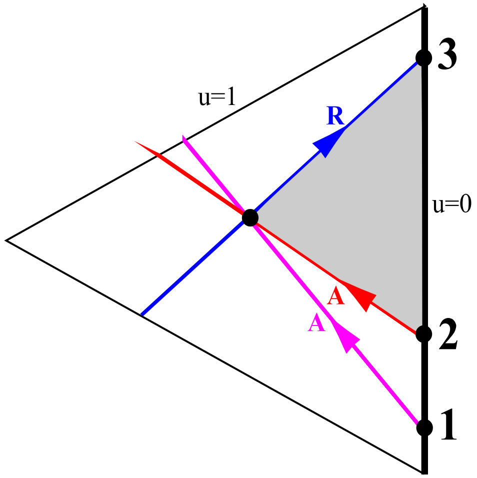
- Our approach: inspired by

$$\eta = \lim_{\omega,k\to 0} i\partial_\omega < T^{xy}T^{xy}(\omega,k) >_{ra}$$

2^{nd} order hydro coefficients = lim 3-point functions <TTT>

- Ingredient (1): **Kubo formulae for 2^{nd} order hydro coefficients**
  - *Moore & Sohrabi, arXiv:1007.5333*

- Ingredient (2): **Holographic 3-pt functions**
  - *Arnold, Barnes, Vaman & CW, arXiv:1004.1179*



$$G^{(3)}_{aar} \sim \delta^{(4)}(p_1+p_2+p_3)\int_0^1 du\sqrt{-g}\,\Delta_A(p_1,u)\Delta_A(p_2,u)\Delta_R(p_3,u)$$

2^{nd} Order Conformal Relativistic Hydrodynamics – Hydro Side

Deriving 2^{nd} order Kubo formulae (Moore & Sohrabi, arXiv:1007.5333)

- Hydrodynamic **expansion** of energy-stress tensor:

$$T^{\mu\nu}[h] = (\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \eta\sigma^{\mu\nu}$$
$$+\eta\tau_\Pi\left({}^{<}u\cdot\nabla\sigma^{\mu\nu>} + \frac{\nabla\cdot u}{3}\sigma^{\mu\nu} \right) + \kappa\left(R^{<\mu\nu>} - 2u_\alpha u_\beta R^{\alpha<\mu\nu>\beta}\right)$$
$$+\lambda_1\sigma_\rho^{<\mu}\sigma^{\nu>\rho} + \lambda_2\sigma_\rho^{<\mu}\Omega^{\nu>\rho} + \lambda_3\Omega_\rho^{<\mu}\Omega^{\nu>\rho}$$

- **EOMs** & **Constraints** (on flat background):

$$\begin{cases} \nabla_\mu T^{\mu\nu} = 0 \\ T_\mu^\mu = 0 \\ u^2 = -1 \end{cases} \Rightarrow \begin{cases} \varepsilon(x) \\ P(x) \\ u^\mu(x) \end{cases} \Rightarrow T^{\mu\nu}$$

- **Solve**: double expansion of $h_{\mu\nu}$ & $(\omega, \mathbf{k})$

$$\varepsilon = \bar{\varepsilon} + \frac{4\bar{\varepsilon}}{3\omega^2 - k^2}\left(-\omega k h_{tz} - \frac{1}{2}k^2 h_{tt} - \frac{1}{2}\omega^2 h_{ii} \right) + O\left(h^2, \omega, k\right)$$

- **Compare** with correlator expansion

$$T^{\mu\nu}[h] = G_r^{\mu\nu}(x) - \frac{1}{2}\int G_{ra}^{\mu\nu,\alpha\beta}(x,y)h_{\alpha\beta}(y)d^4y + \frac{1}{8}\int G_{raa}^{\mu\nu,\alpha\beta,\gamma\delta}(x,y,z)h_{\alpha\beta}(y)h_{\gamma\delta}(z)d^4y d^4z$$

2^{nd} Order Conformal Relativistic Hydrodynamics – Hydro Side

- Hydrodynamic expansion of correlators:

$$G_{aar}^{xz,tx,xx}(1,2,3)\Big|_{\substack{k_1=0\\ \omega_2=0}} = -i2\pi T\eta \frac{\omega_1^2 k_2}{3\omega_1^2 - k_2^2} + 2\pi^2 T^2 \frac{\omega_1 k_2\left(\lambda_2(-9\omega_1^4 + 9\omega_1^2 k_2^2 - 2k_2^4) + \kappa\ldots + \eta\tau_\Pi\ldots + \eta^2/\bar{\varepsilon}\ldots\right)}{\left(3\omega_1^2 - k_2^2\right)^2}$$

- Kubo formula : **zoom into** a particular term by taking a certain limit:
- Order of limits matters sometimes due to the pole:

$$\begin{cases} \lim\limits_{k_2\to 0}\partial_{k_2} \lim\limits_{\omega_1\to 0}\partial_{\omega_1} G_{aar}^{xz,tx,xx}(1,2,3)\Big|_{\substack{k_1=0\\ \omega_2=0}} = -\lambda_2 - \kappa + 2\eta\tau_\Pi \\ \lim\limits_{\omega_1\to 0}\partial_{\omega_1} \lim\limits_{k_2\to 0}\partial_{k_2} G_{aar}^{xz,tx,xx}(1,2,3)\Big|_{\substack{k_1=0\\ \omega_2=0}} = -\frac{1}{2}\lambda_2 - \frac{1}{2}\kappa + \frac{4}{3}\eta\tau_\Pi + \frac{\eta^2}{2\bar{\varepsilon}} \end{cases}$$

- Can compare the **whole** 3-point functions from AdS with the hydrodynamic expansions **term by term** (fit into the templates)

  $\rightarrow$ A good consistency check on AdS calculation too !

2^{nd} Order Conformal Relativistic Hydrodynamics – AdS Side

- Energy-stress tensor $T^{\mu\nu} \leftrightarrow$ metric fluctuation $\mathbf{h}_{\mu\nu}$
- **Solve** linearized **Einstein equation** on AdS-Schwarzschild BH background
  small ω, k to 2^{nd} order → Boundary-to-bulk propagators

  (Policastro, Son & Starinets, hep-th/0205052 & 0210220)

- Perturbatively **expand** Einstein-Hilbert & Gibbons-Hawking **action** to $O(h^3)$ to get cubic vertices

  (Arutyunov & Frolov, hep-th/9901121, need to restore boundary terms)

- **Do** the bulk **integral**

$$G_{aar}^{\bar{\alpha}\bar{\beta},\bar{\gamma}\bar{\delta},\bar{\mu}\bar{\nu}}(1,2,3) \sim \delta^{(4)}(p_1+p_2+p_3)\int_0^1 du\sqrt{-g}V^{\alpha\beta,\gamma\delta,\mu\nu}(u)h_{A,\alpha\beta}^{\bar{\alpha}\bar{\beta}}(p_1,u)h_{A,\gamma\delta}^{\bar{\gamma}\bar{\delta}}(p_2,u)h_{R,\mu\nu}^{\bar{\mu}\bar{\nu}}(p_3,u)$$

Gauge Condition:

$$h_{\mu 5} = 0$$

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- **Divergent at both boundaries !!!**
  - Boundary divergence (at u=0):

    Add gravitational counter terms: volume + curvature + …

    (Balasubramanian et al, hep-th/9902121, 9903238, 9906127 & 0002230 et al)

    Completely removed ☺

2^{nd} Order Conformal Relativistic Hydrodynamics – AdS Side

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- **Divergent at both boundaries !!!**
  - Horizon divergence (at u=1):

    Due to **gauge** freedom (purely-gauge solutions near horizon)

    Form gauge-invariant combinations or limits

    Completely removed ☺

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- **Finite results** ☺

2^{nd} Order Conformal Relativistic Hydrodynamics – AdS Side

Results: (Arnold, Vaman, CW & Xiao, arXiv:1105.4645)

- Gauge-invariant limit:

$$G_{raa}^{xy,tx,ty}\left(k_3=-k_1-k_2;k_1,k_2\right)\Big|_{\omega_1,\omega_2=0}$$

$$=-P+\pi^2T^2\left(2\kappa(k_1^2+k_2^2)-\lambda_3k_1k_2\right)+O\left(k^4\right) \qquad \textit{... hydro}$$

$$=-\frac{\pi^2N^2T^4}{8}+\frac{\pi^2N^2T^4}{4}\left(k_1^2+k_2^2\right)+O\left(k^4\right) \qquad \textit{... AdS}$$

- Gauge-invariant combination:

$$\left(G_{raa}^{xx,tx,tx}+G_{raa}^{yy,tx,tx}-2(1-k_3^2)G_{raa}^{zz,tx,tx}+\frac{2}{3}k_3^2G_{raa}^{tt,tx,tx}-2ik_3G_{raa}^{tz,tx,tx}\right)\Bigg|_{\omega_1,\omega_2=0}$$

$$=-2P+\left(\frac{4}{3}P-4\pi^2T^2\kappa\right)\left(k_1^2+k_2^2\right)+\left(\frac{8}{3}P+2\pi^2T^2\lambda_3\right)k_1k_2+O\left(k^4\right) \qquad \textit{... hydro}$$

$$=-\frac{\pi^2N^2T^4}{4}-\frac{\pi^2N^2T^4}{3}\left(k_1^2+k_2^2-k_1k_2\right)+O\left(k^4\right) \qquad \textit{... AdS}$$

- Consistent results:

$$\lambda_1=\frac{1}{16}N^2T^2,\lambda_2=-\frac{\ln 2}{8}N^2T^2,\lambda_3=0,\kappa=\frac{1}{8}N^2T^2,\tau_\Pi=\frac{2-\ln 2}{2\pi T}$$

Summary

1) All 2nd order hydrodynamic coefficients (for conformal QGP) were derived via Kubo formulae (related to causal 3-pt correlators).

2) Prescriptions to compute real-time higher n-pt correlators via AdS/CFT had been built up.

3) Causal 3-pt correlators in (1) were computed via (2), thus all 2nd order hydrodynamic coefficients were obtained.

   – Extend & complete *Baier et al*'s approach;
   – An independent check to *Bhattacharyya et al*'s results.

4) Can be generalized to include higher-derivative corrections, to non-conformal cases…

Thanks!