Boundary Conflicts and Cluster Coarsening: Waves of Life and Death in the Cyclic Competition of Four Species

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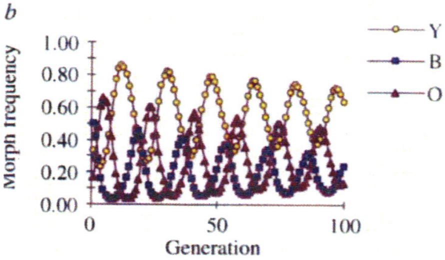
Friday, October 21

Motivation



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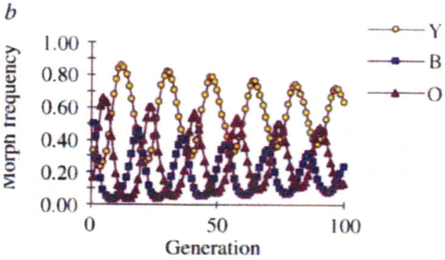

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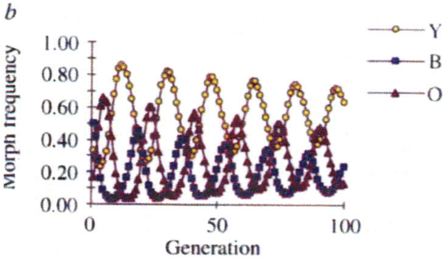

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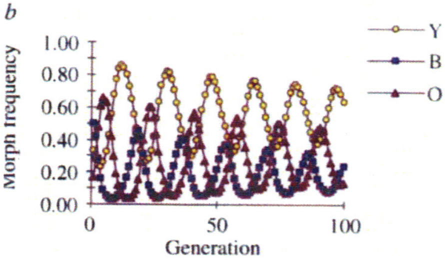

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- $D + A \xrightarrow{k_D} D + D$ else $D + A \xrightarrow{1-k_D} A + D$
- $A + C \xrightarrow{\mu_{AC}} C + A$ and $B + D \xrightarrow{\mu_{BD}} D + B$

Model Cont'd

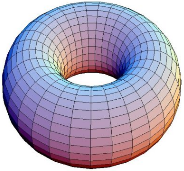

Figure: Periodic Boundary

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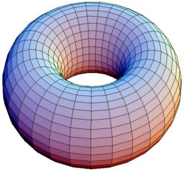

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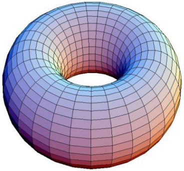

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The sum of the populations of the species is invariant and is equal to L^2 since the occupation number is 1 element per lattice site.

Three Species and Pattern Formation

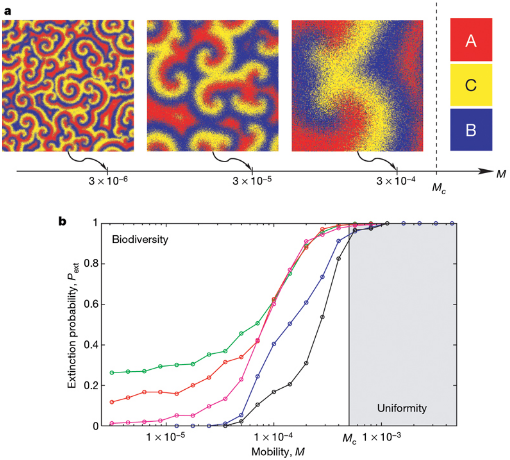

Figure: Pattern Formation in 3 Species Model [E. Frey Group]

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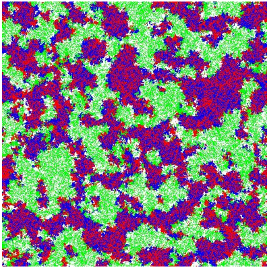

Figure: $t \sim 100$

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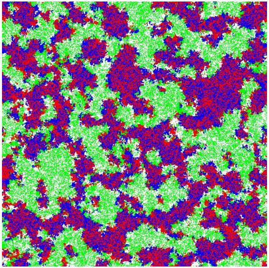

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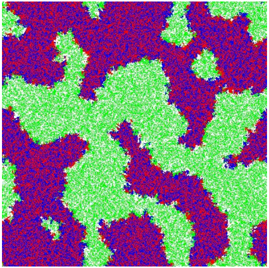


Figure: $t \sim 500$

Space Time Correlation function

Definition

$$\lambda_t^A(i,j) = \begin{cases} 1 & \text{if (i,j) contains A at time t} \\ -1 & \mathit{Otherwise}. \end{cases}$$

$$\gamma_t^A(i,j,r) = \frac{1}{4}\lambda_t^A(i,j)[\lambda_t^A(i+r,j) + \lambda_t^A(i-r,j) + \lambda_t^A(i,j+r) + \lambda_t^A(i,j-r)]$$

we similarly define $\gamma_t^\xi(i,j,r)$ and $\lambda_t^\xi(i,j)$ where $\xi = B, C$ or D. Then the space-time correlation function is defined as

$$C_t(r) = \sum_{r=1}^{\frac{L}{4}} \sum_{i=1}^{L} \sum_{j=1}^{L} [\gamma_t^A(i,j,r) + \gamma_t^B(i,j,r) + \gamma_t^C(i,j,r) + \gamma_t^D(i,j,r)]$$

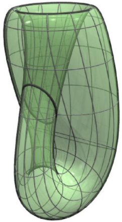
Correlation Function and Geometry

Measurement

We plot $\frac{1}{r}C_0(0) = \text{constant}$ *where* $r \in \mathbb{R}$ *chosen appropriately as well as* $C_t(r)$ *as function of time, then we plot the intersection of the two traces as a function of time yielding the length scale* $L \sim t^{1/z}$ *as a function of time.*

Problem

Are there "better" geometries to observe the length scale?



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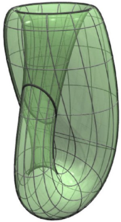

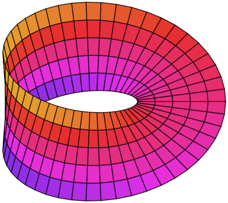

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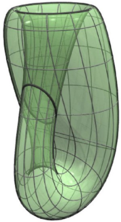

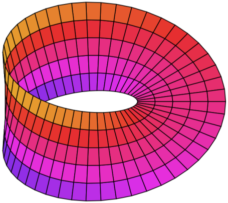

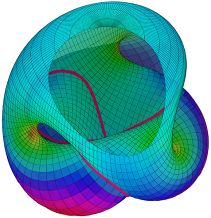

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- *Which confined geometry provides the largest time regime where the dynamical exponent can be observed free of early time effects or finite-size effects? (Work In Progress.)*

Questions

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Do you have a question? Ask Away...