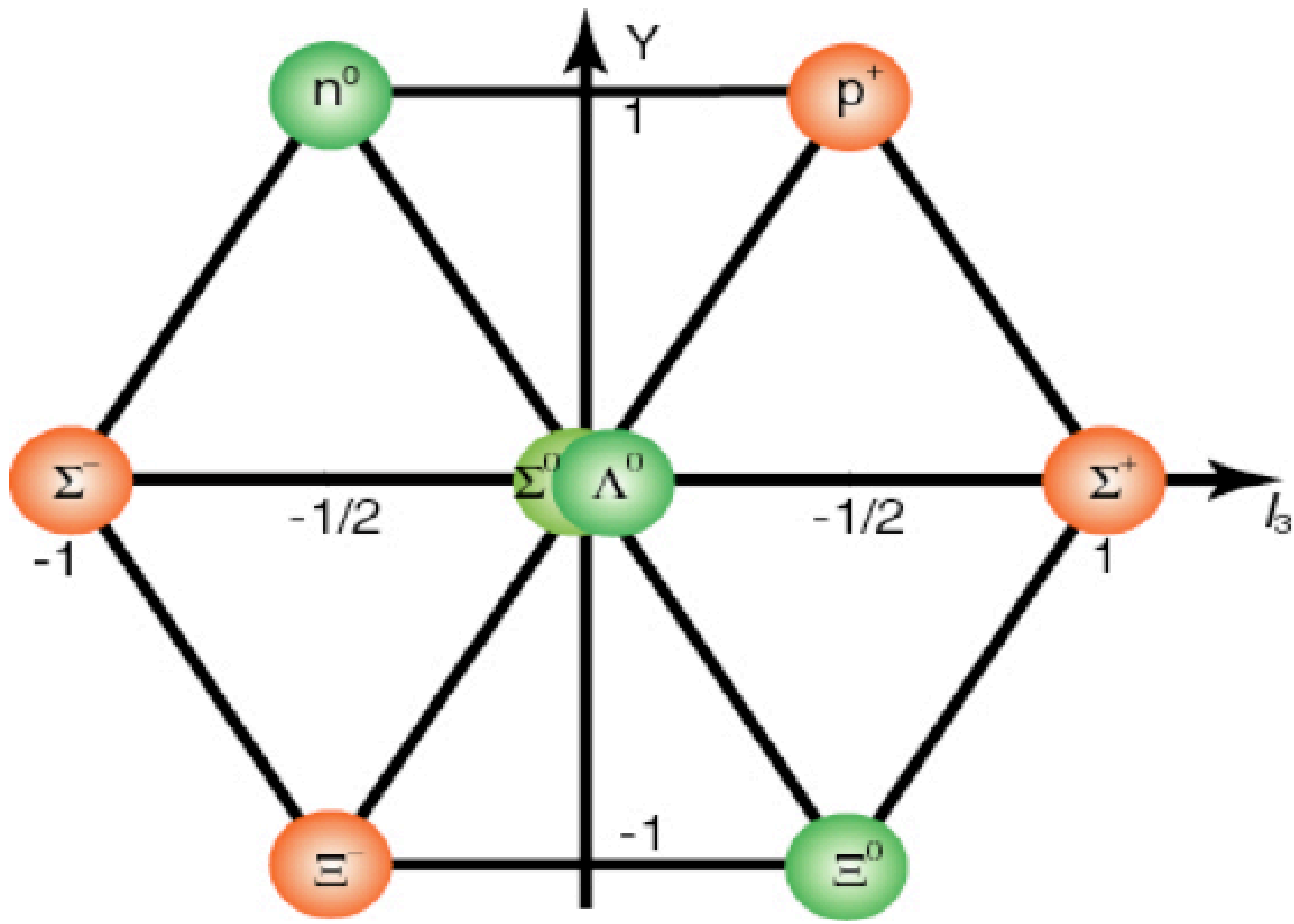
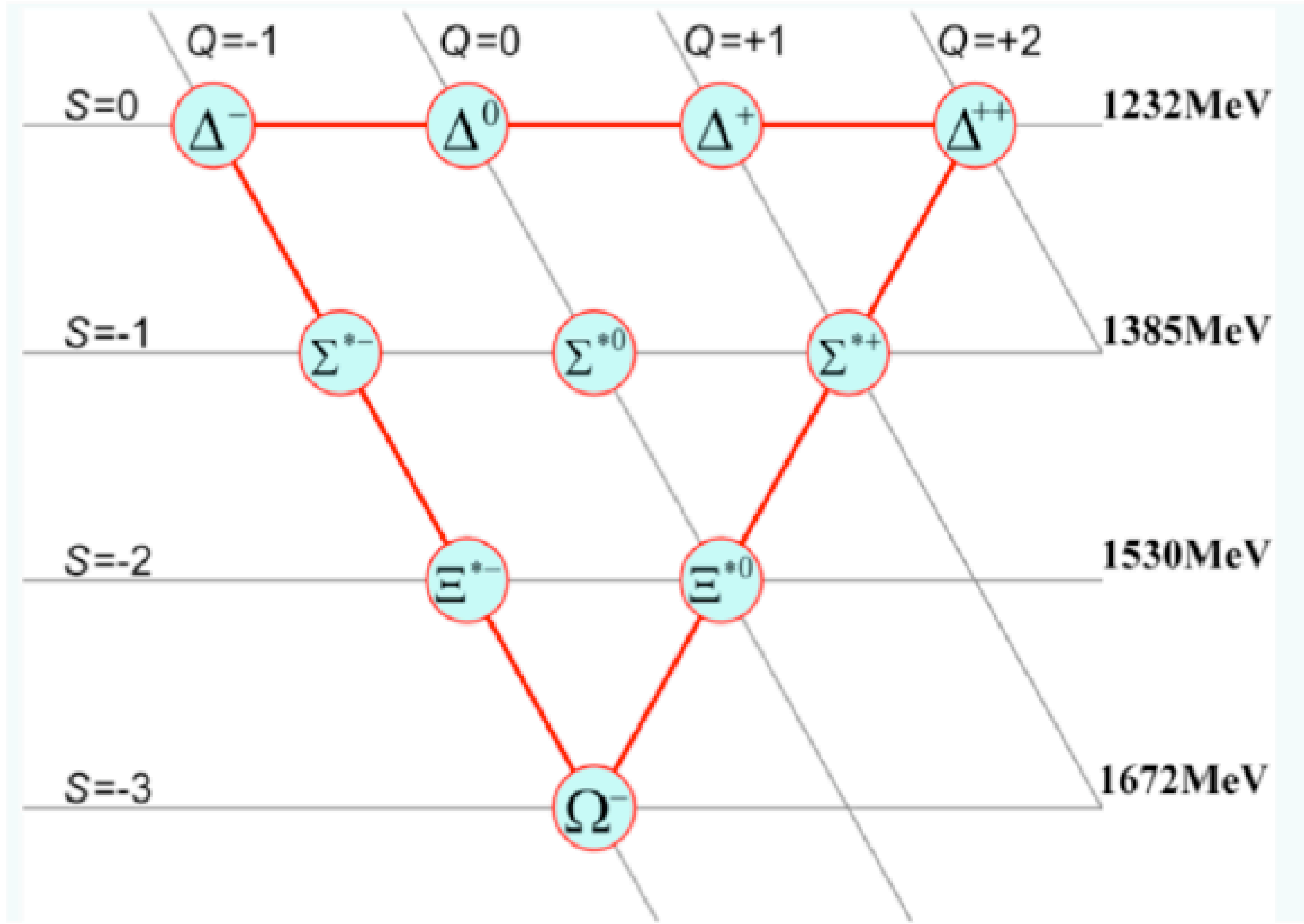
Mathematical Surprises From Off-Shell SUSY Representation Theory



$$p = 1 \quad , \quad q = 1$$



$$p = 3 \quad , \quad q = 0$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

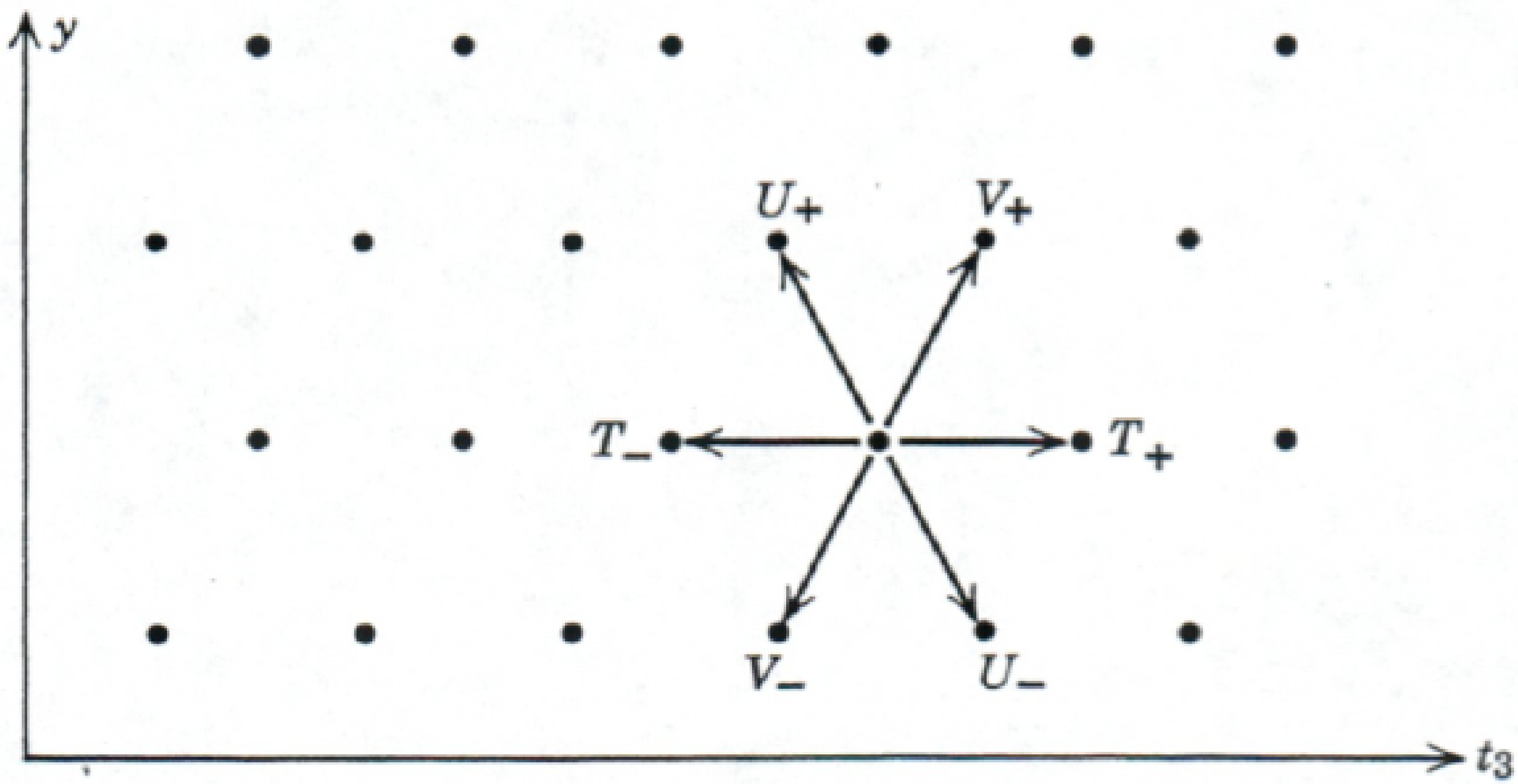
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$$F_i = \tfrac{1}{2}\lambda_i$$

$$[F_i, F_j] = i f_{ijk} F_k$$

$$\begin{aligned}
T_\pm &= F_1 \pm iF_2 \\
U_\pm &= F_6 \pm iF_7 \\
V_\pm &= F_4 \pm iF_5 \\
T_3 &= F_3 \\
Y &= \frac{2}{\sqrt{3}} F_8
\end{aligned}$$

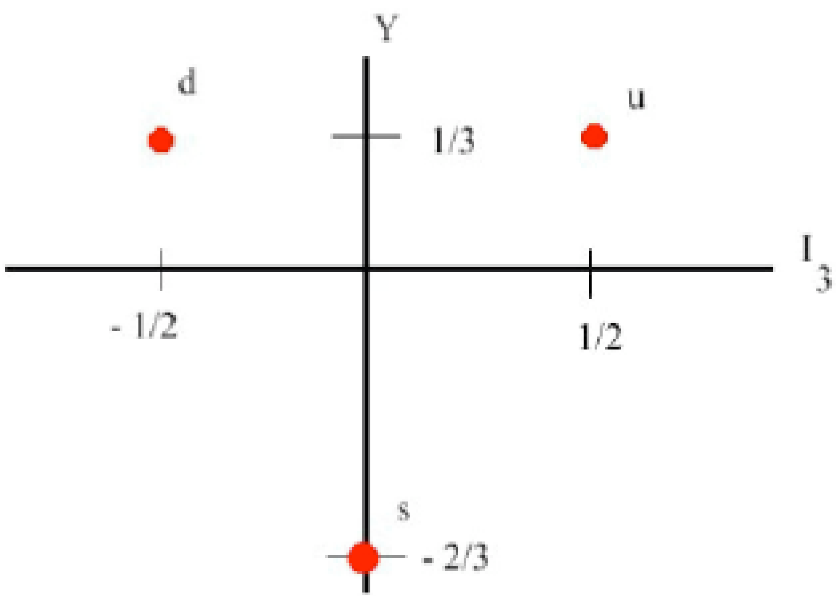
$$\boldsymbol{T_\pm = F_1 \pm i\,F_2 \quad , \quad U_\pm = F_4 \pm i\,F_5 \quad , \quad V_\pm = F_6 \pm i\,F_7}$$



$$|\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}\rangle \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|-\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}\rangle \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|0, -\tfrac{1}{\sqrt{3}}\rangle \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$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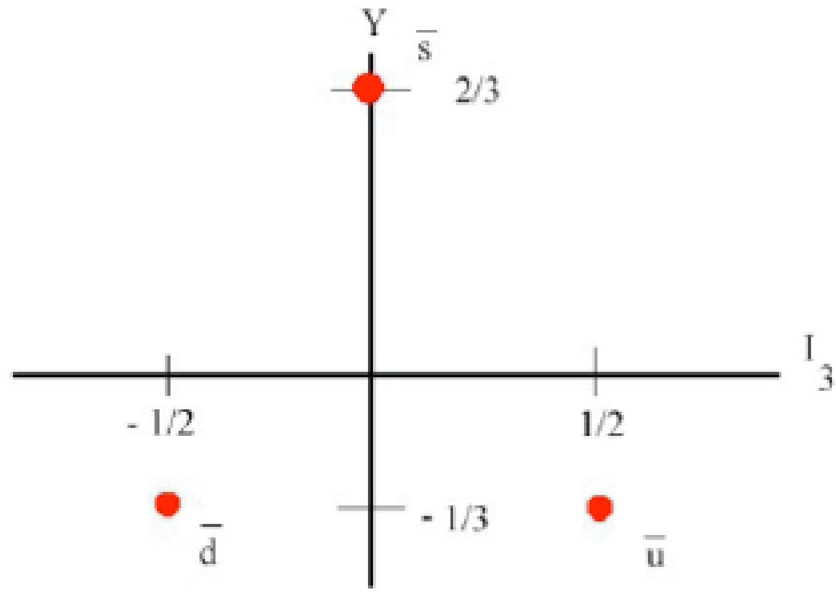
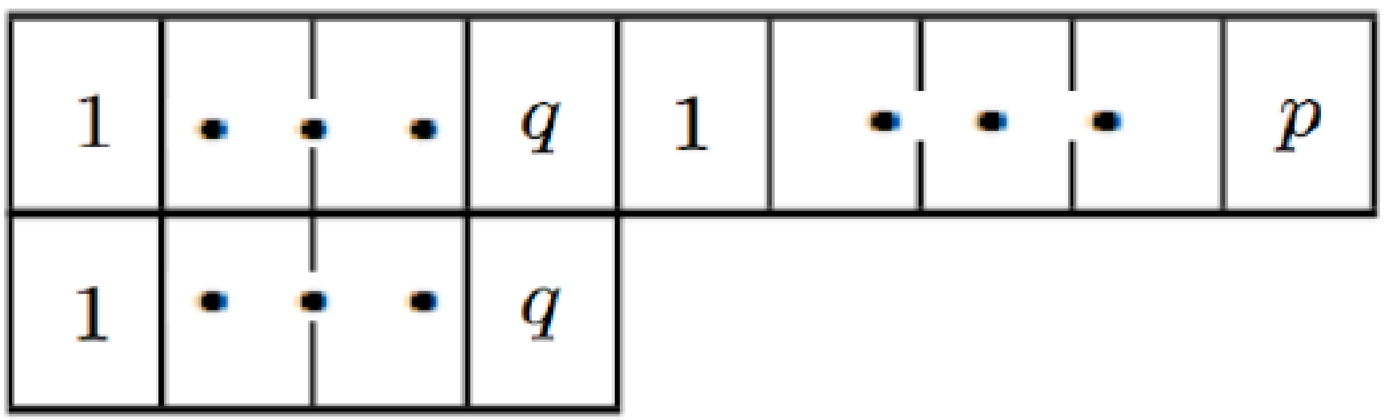

Y
$\bar{s}$
2/3
I
3
- 1/2
1/2
- 1/3
$\bar{d}$
$\bar{u}$

SU(3) Young Tableaux & The integers p and q

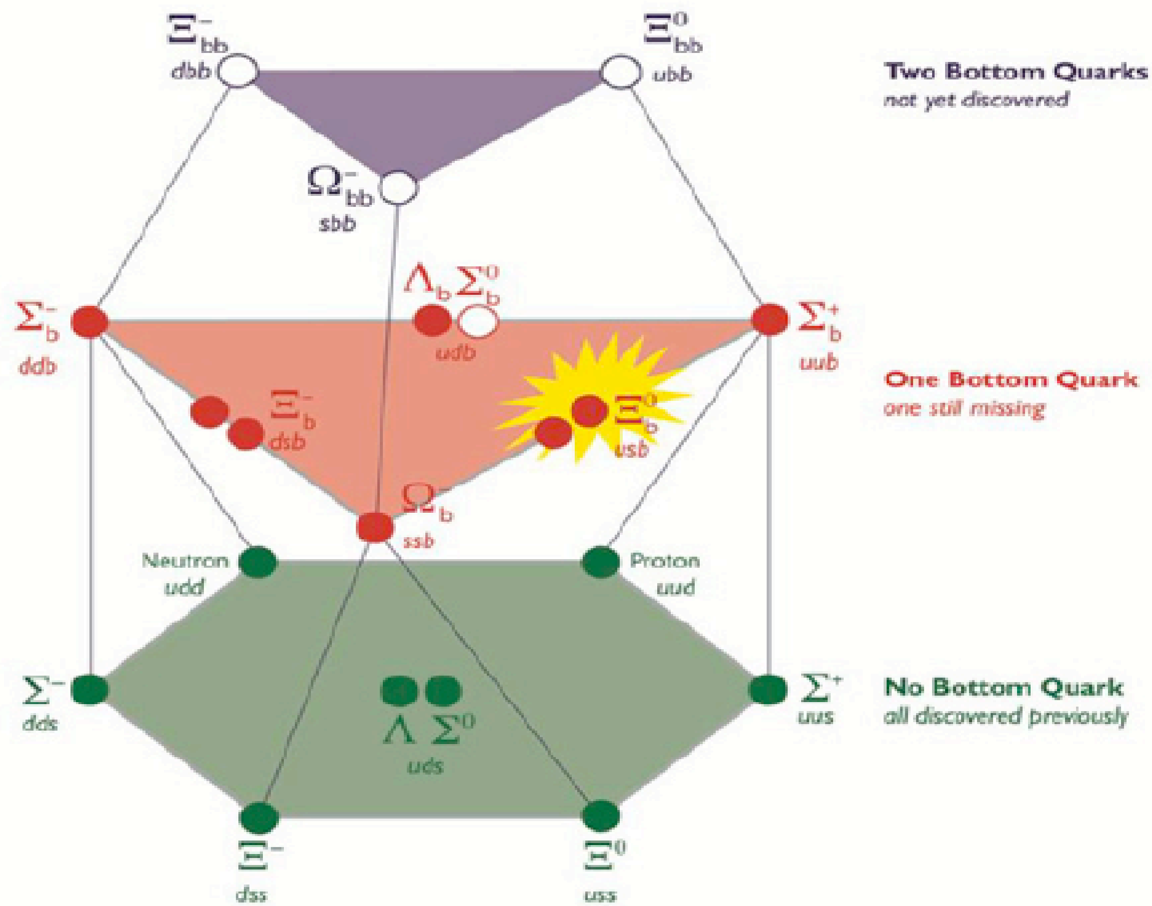


Counting Representations

$$\boldsymbol{d}_{SU(2)} = (2j+1)$$

$$\boldsymbol{d}_{SU(3)} = \frac{1}{2}(p+1)(q+1)(p+q+2)$$

Baryons with Up, Down, Strange and Bottom Quarks and Spin J=1/2



FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

BOSONS

force carriers
spin = 0, 1, 2, ...

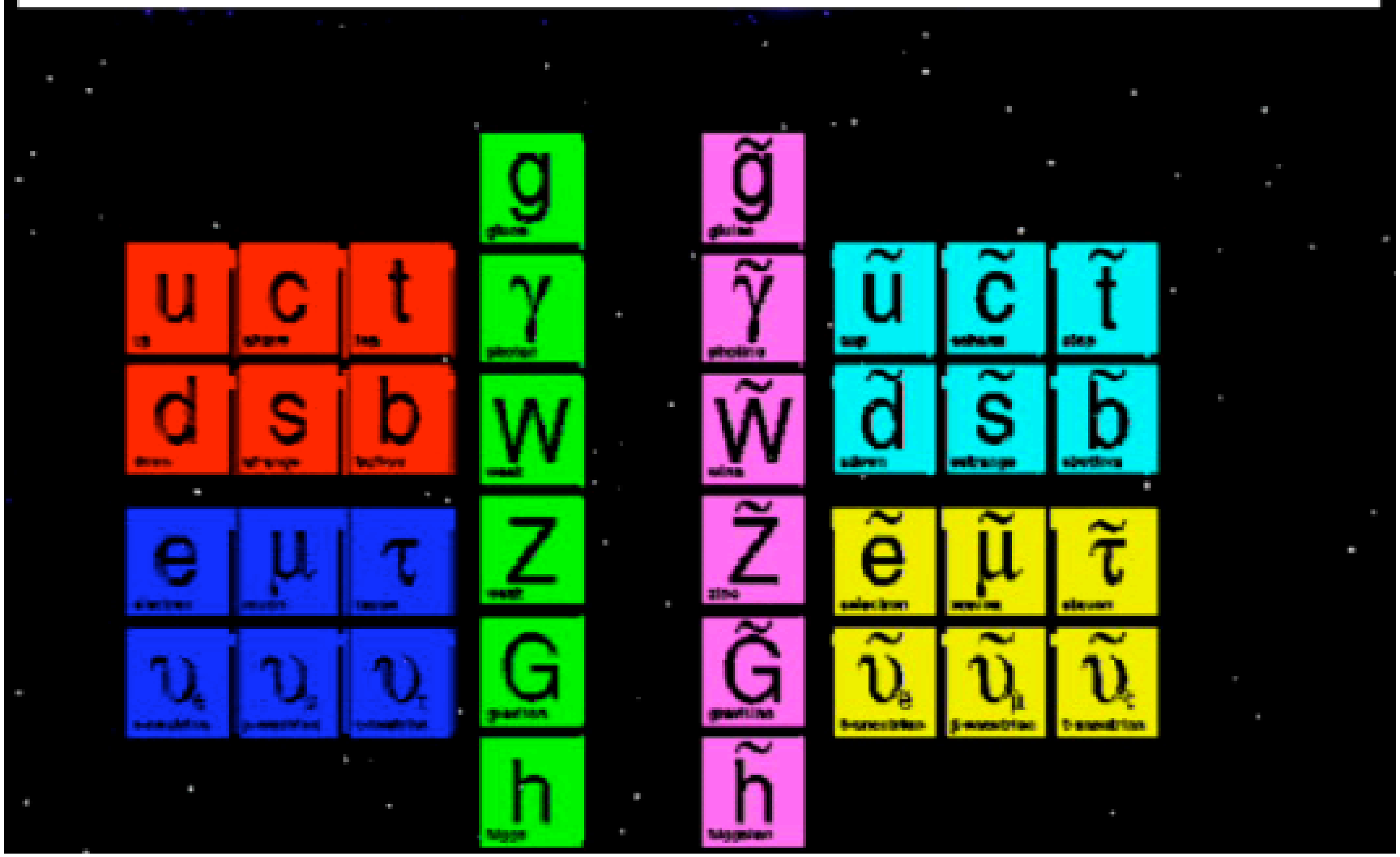
Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

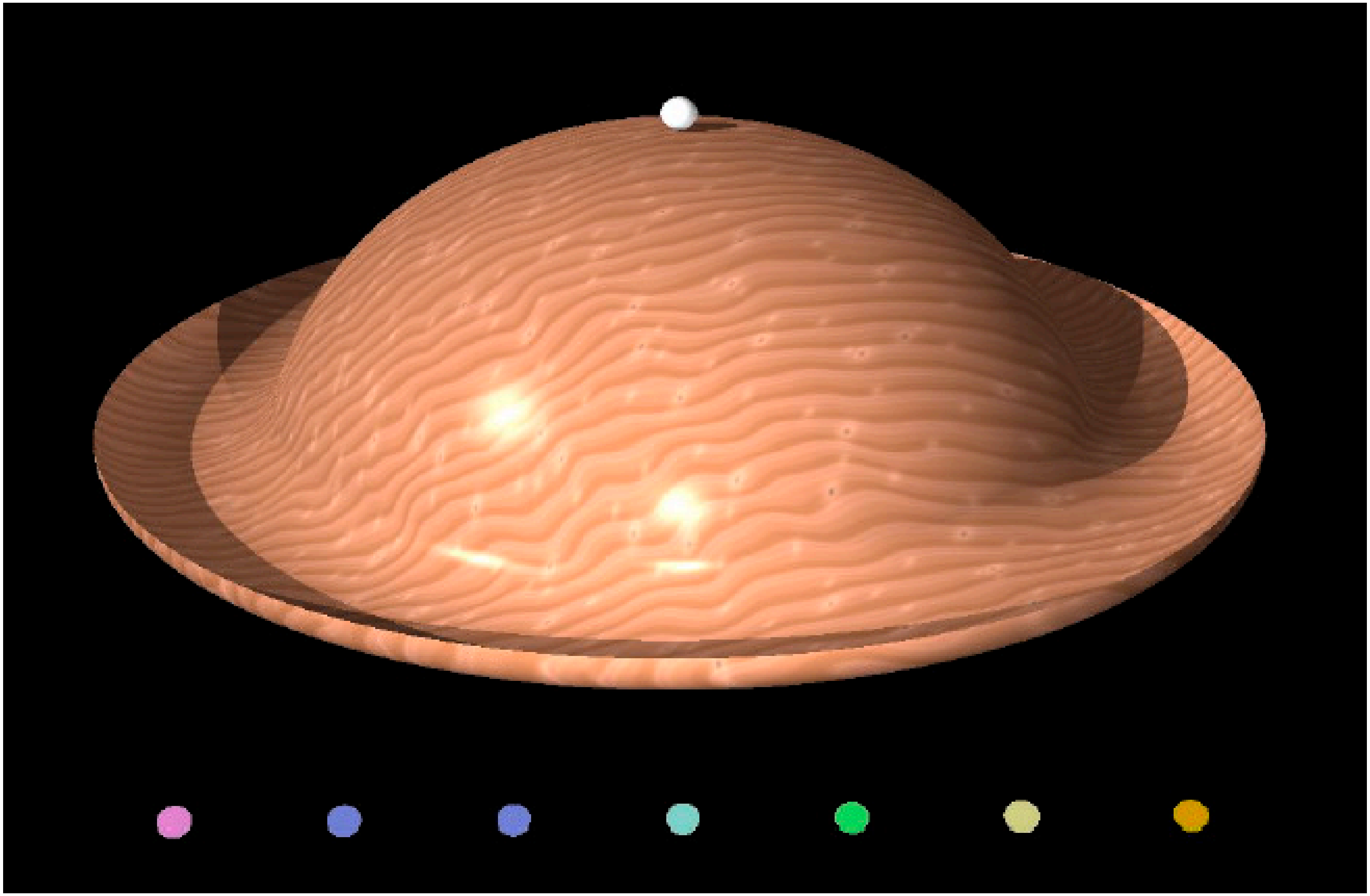
Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

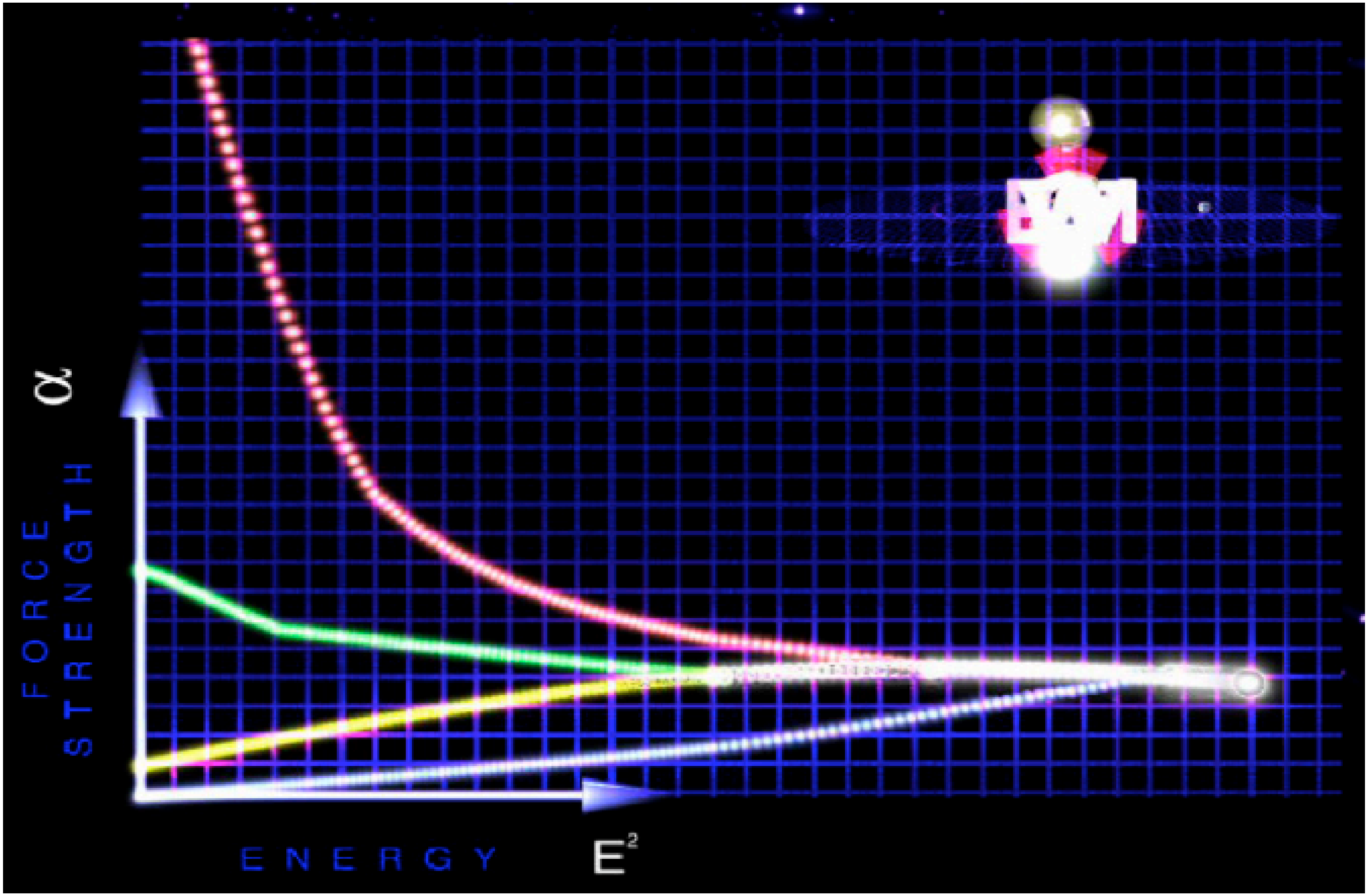
PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong Fundamental	Strong Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	Not applicable to quarks
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

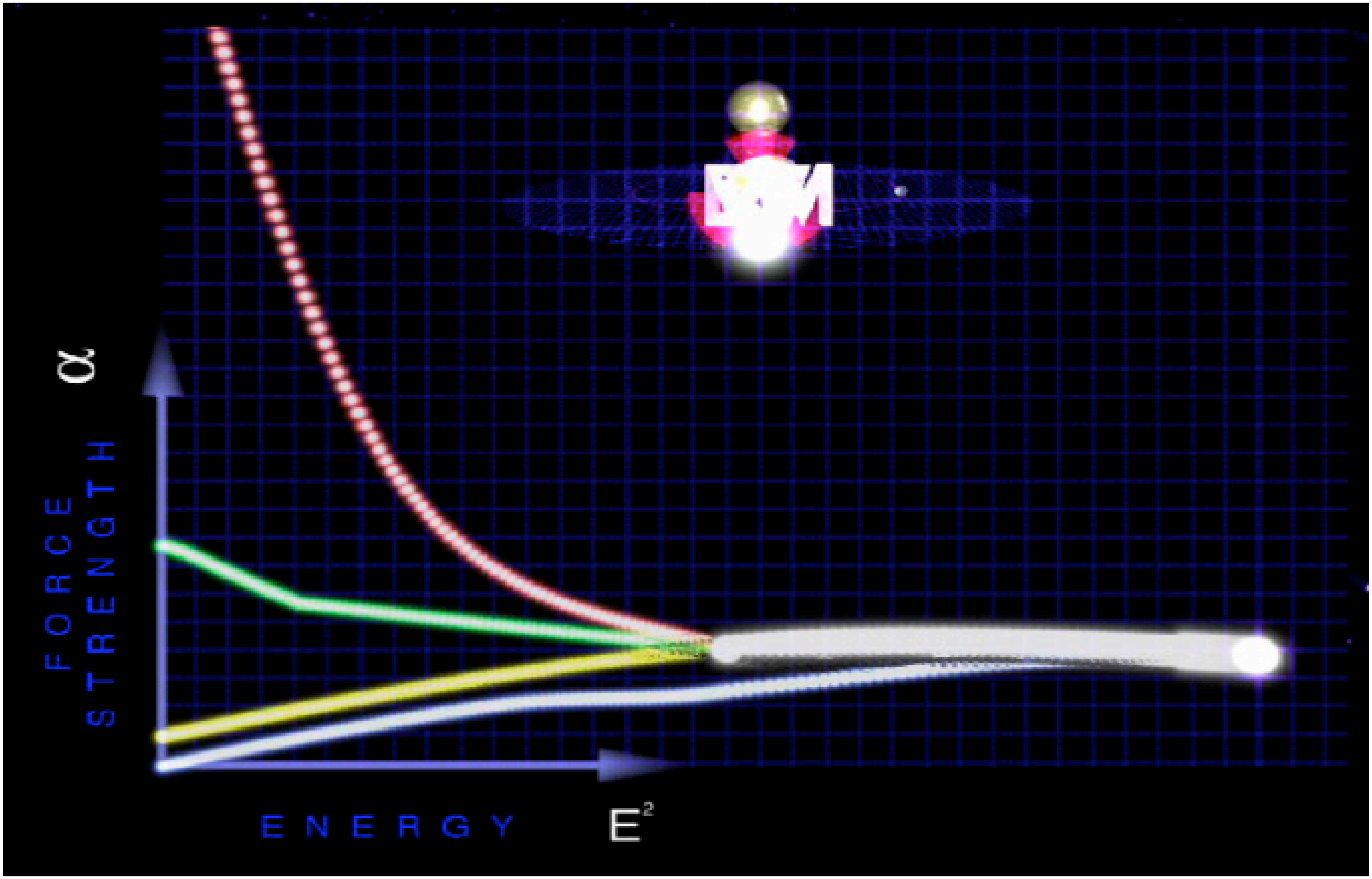
Introducing Superpartners







FORCE
STRENGTH
α
ENERGY
E^2



FORCE
STRENGTH
α
ENERGY
E^2

$$D_a K = \zeta_a$$

$$D_a M = \Lambda_a - \frac{1}{2}(\gamma^\nu)_a{}^d \partial_\nu \zeta_d$$

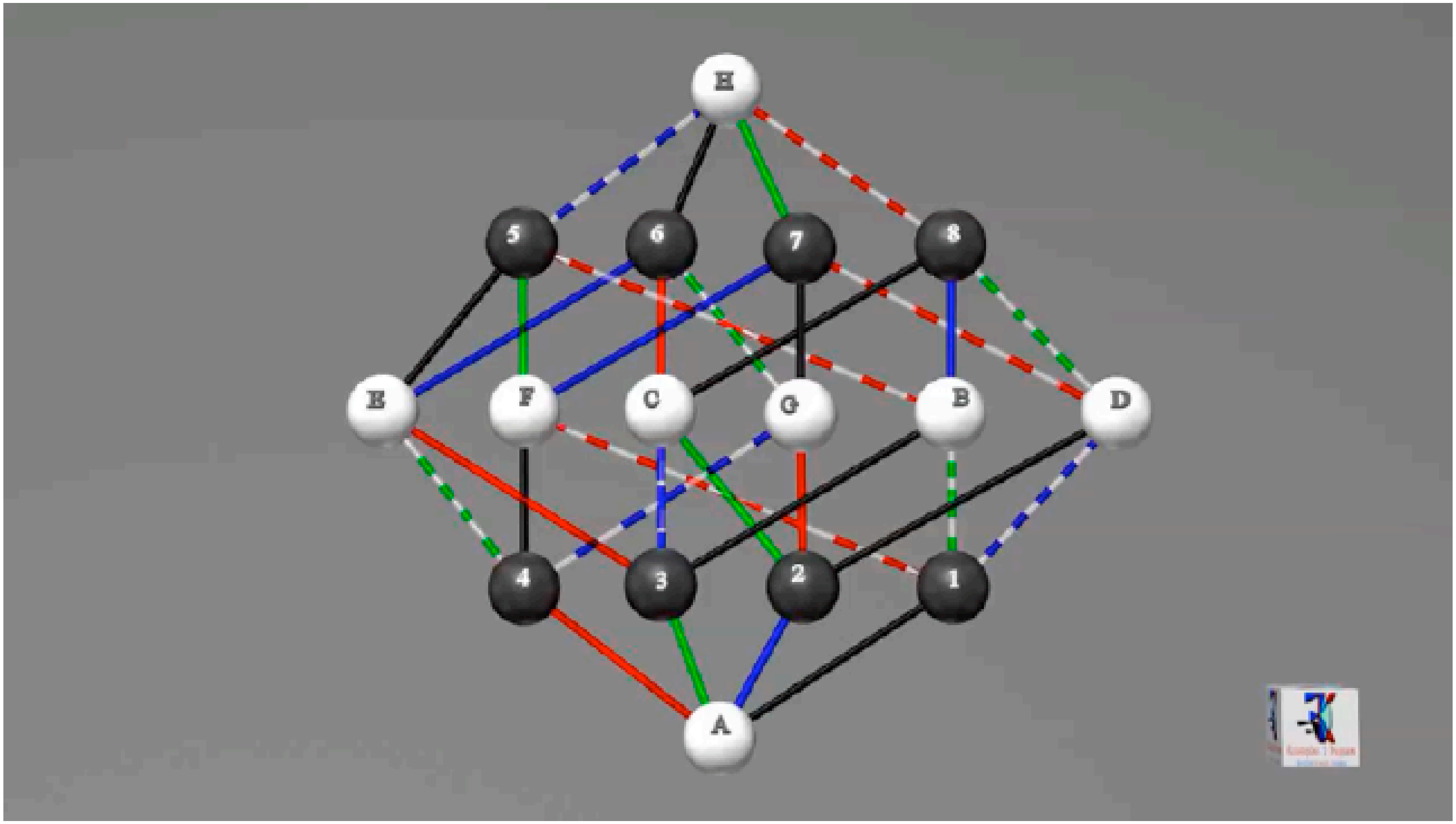
$$D_a N = -i(\gamma^5)_a{}^d \Lambda_d + i\frac{1}{2}(\gamma^5\gamma^\nu)_a{}^d \partial_\nu \zeta_d$$

$$D_a U_\mu = i(\gamma^5\gamma_\mu)_a{}^d \Lambda_d - i\frac{1}{2}(\gamma^5\gamma^\nu\gamma_\mu)_a{}^d \partial_\nu \zeta_d$$

$$D_a d = -(\gamma^\nu)_a{}^d \partial_\nu \Lambda_d$$

$$D_a \zeta_b = i(\gamma^\mu)_{ab} \partial_\mu K + (\gamma^5\gamma^\mu)_{ab} U_\mu + iC_{ab} M + (\gamma^5)_{ab} N$$

$$D_a \Lambda_b = i\frac{1}{2}(\gamma^\mu)_{ab} \partial_\mu M + \frac{1}{2}(\gamma^5\gamma^\mu)_{ab} \partial_\mu N + \frac{1}{2}(\gamma^5\gamma^\mu\gamma^\nu)_{ab} \partial_\mu U_\nu + iC_{ab} d$$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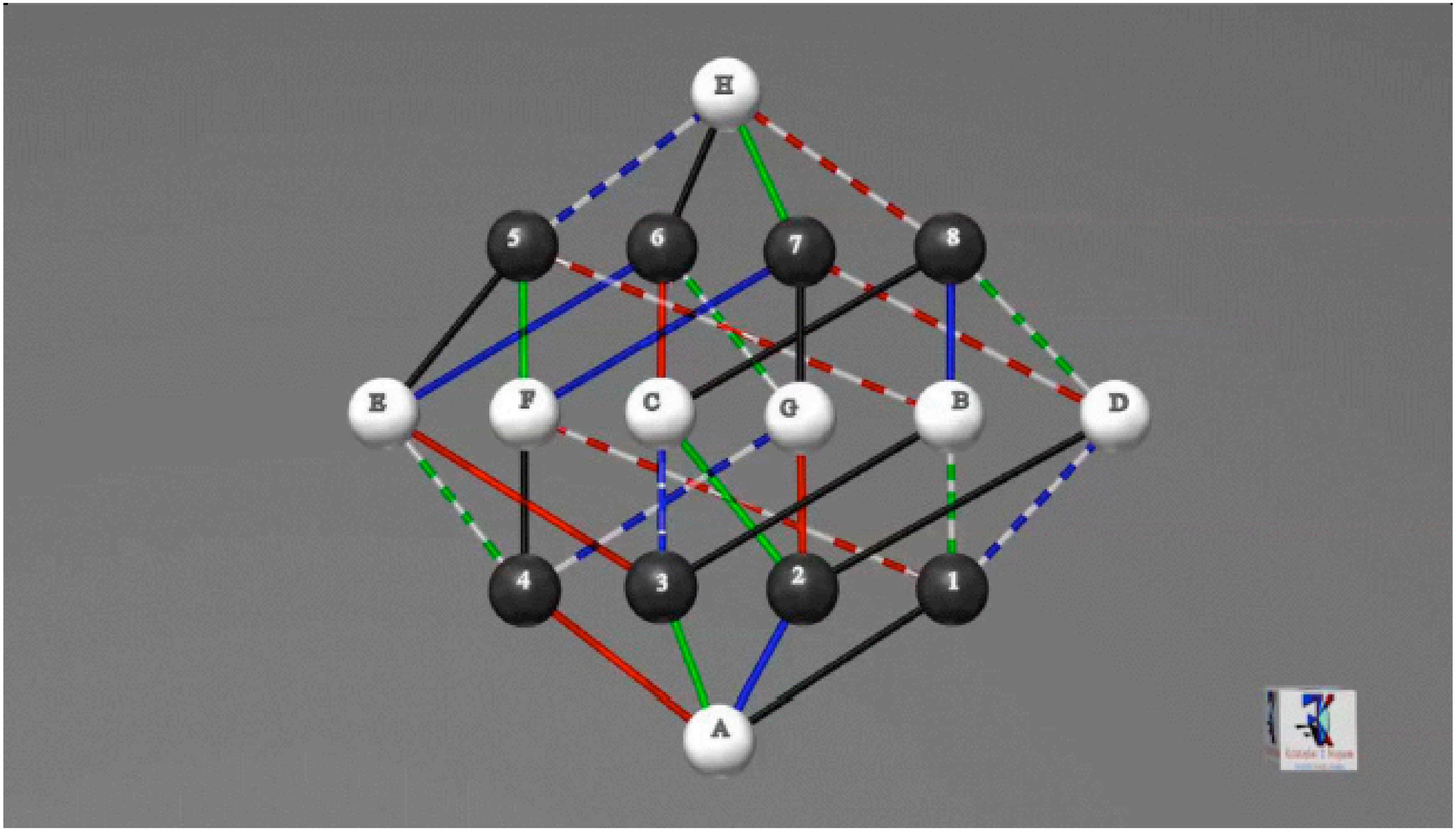
1
A

Adinkra Folding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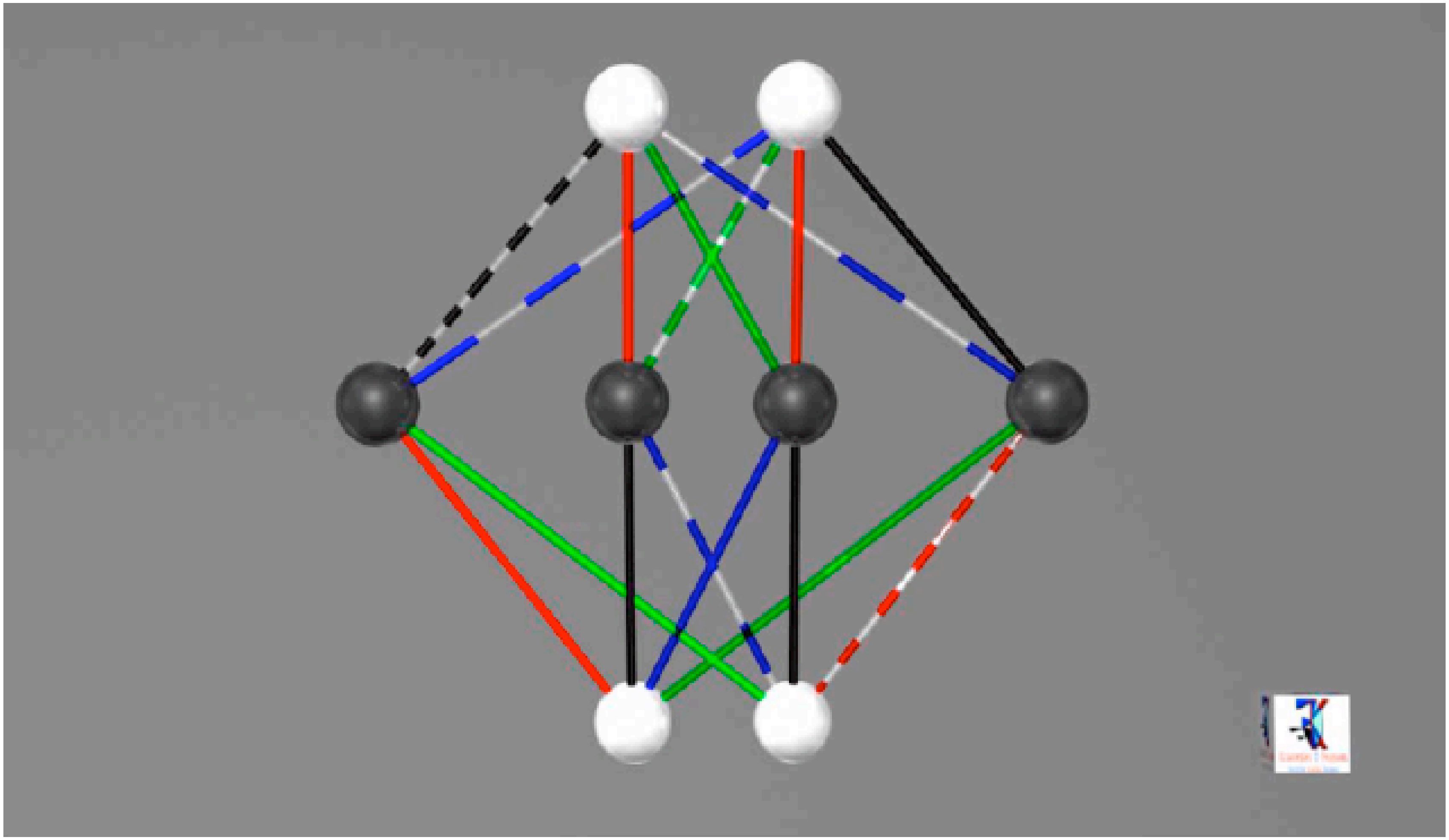


$$\mathrm{D}_a A = \psi_a \quad ,$$

$$\mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \quad ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A - (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B - i\,C_{a\,b}F + (\gamma^5)_{a\,b}G \quad ,$$

$$\mathrm{D}_a F = (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad ,$$

$$\mathrm{D}_a G = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad .$$

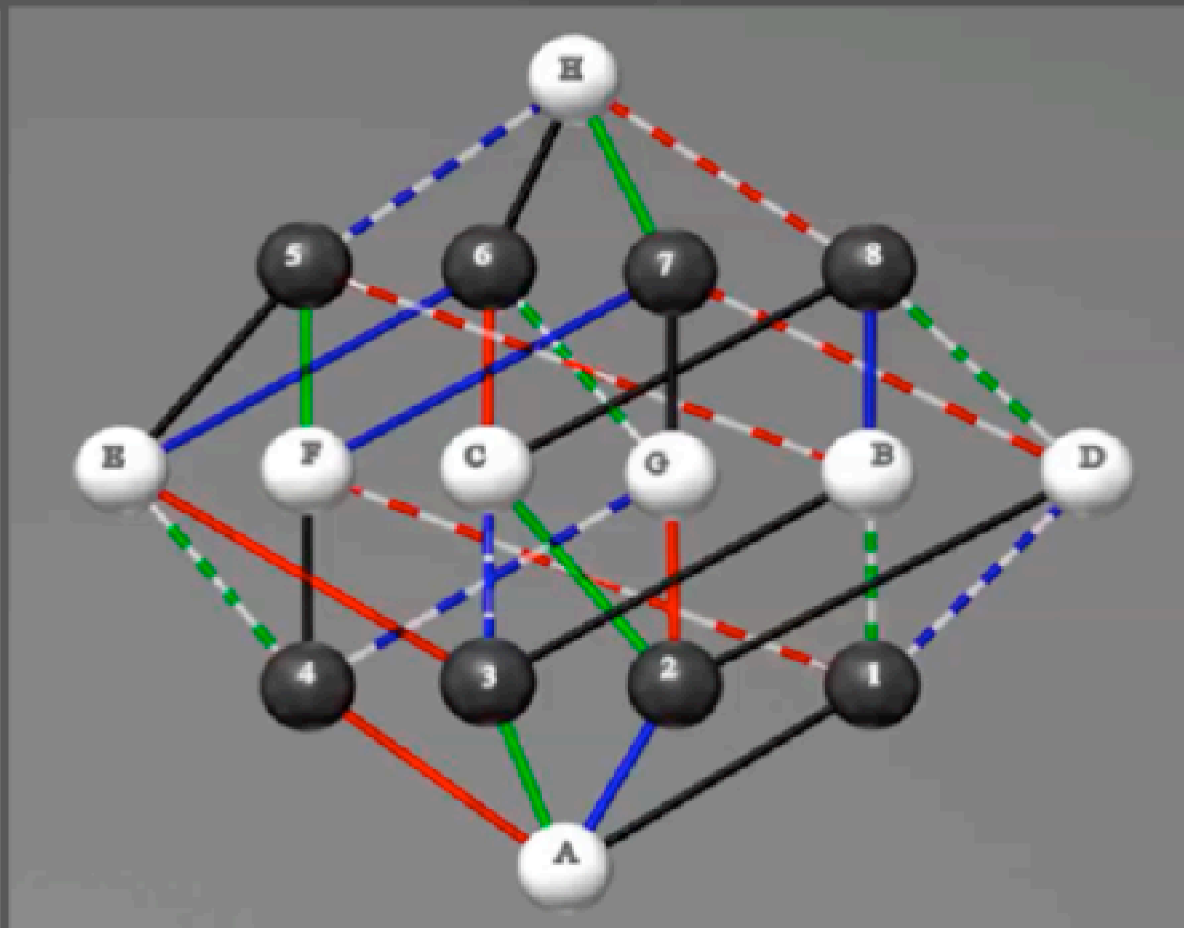
Doubly

Even

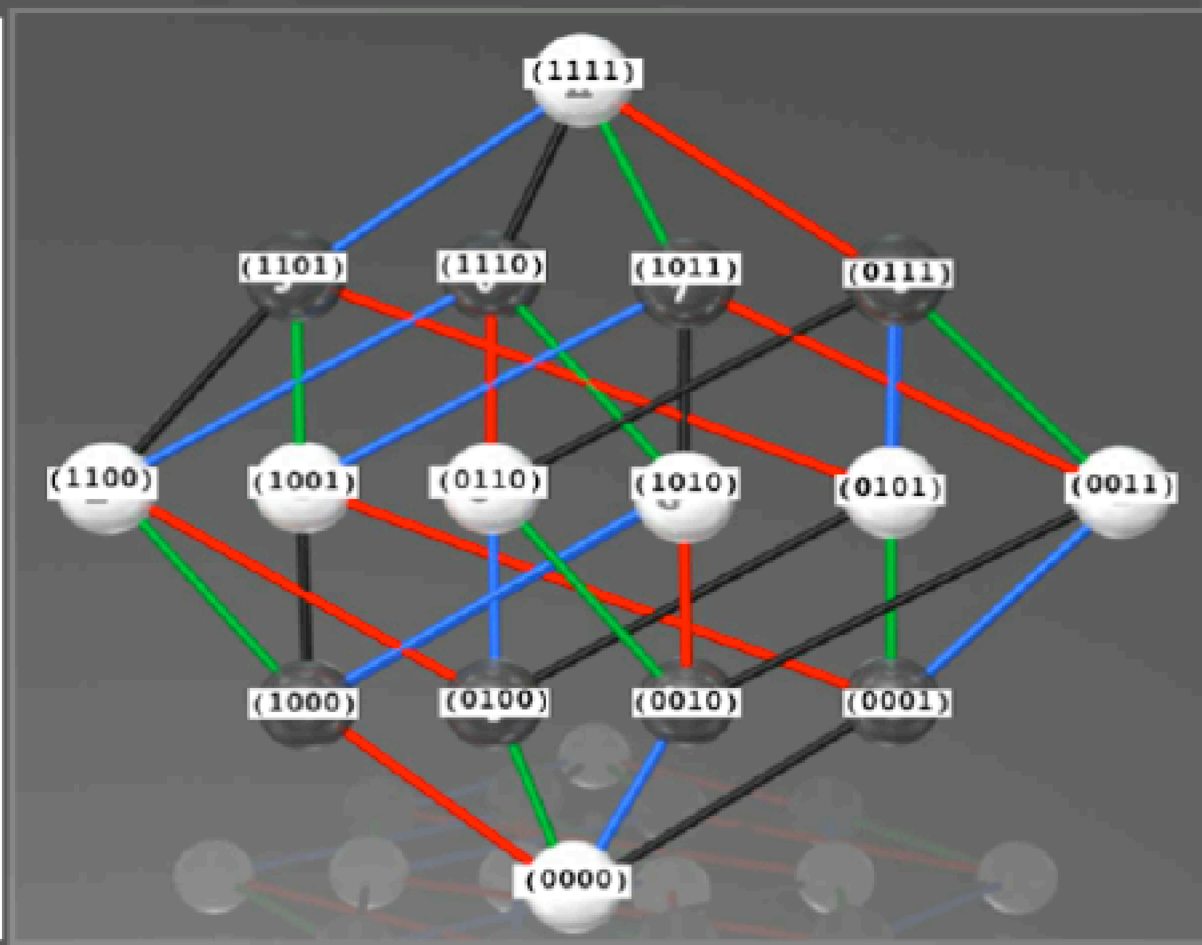
SDEC's

Control

Folding



Doubly Even SDEC's Control Folding



Doubly

Even

SDEC's

Control

Folding

Bits Naturally Arise From The Geometry Of Hypercubes

To a small part, the appearance of the SDEC's is not mysterious. Bits naturally appear in any situation where cubical geometry is relevant. The vertices of a cube can always be written in the form

$$(\pm 1, \pm 1, \pm 1, \ldots, \pm 1)$$

or re-written in the form

$$((\pm 1,)^{p_1}, (\pm 1,)^{p_2}, (\pm 1,)^{p_3}, \ldots, (\pm 1,)^{p_d})$$

where the exponents are bits since they take on values 1 or 0. Thus any vertex has an 'address' that is a string of bits

$$(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \ldots, \boldsymbol{p}_d)$$

the information theoretic definition of a 'word.'

Feynman on Wheeler

Feynman, Wheeler's student in the 1940s, turned to Thorne, Wheeler's student in the 1960s, and said,

"This guy sounds crazy. What people of your generation don't know is that he has always sounded crazy. But when I was his student, I discovered that if you take one of his crazy ideas and you unwrap the layers of craziness from it one after another like lifting the layers off an onion, at the heart of the idea you will often find a powerful kernel of truth."

Calculating Kirchoff Bow Ties In Adinkras

Kirchoff's Law: $\mathcal{V} = \oint \vec{\mathcal{E}} \cdot d\vec{\ell}$

$$\oint \vec{\mathcal{E}} \cdot d\vec{\ell} \longrightarrow \sum_{links} h(\mathrm{D_I})\,(\,h_f - h_i\,) \quad ,$$

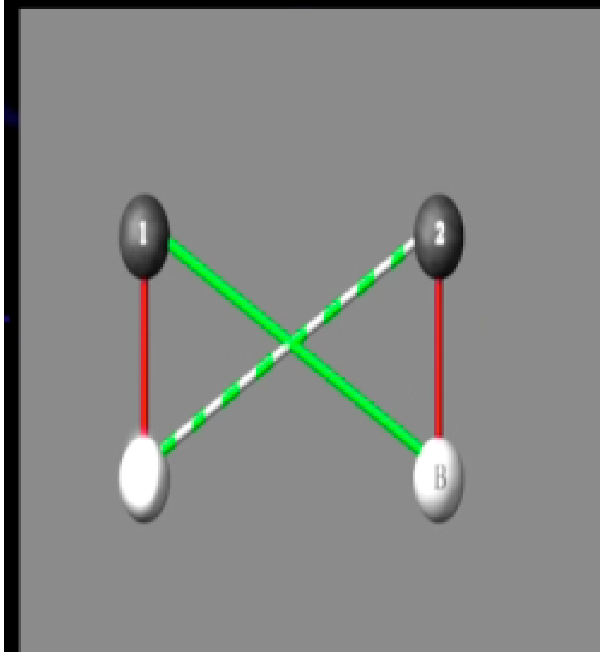
$$\mathcal{V} \longrightarrow \mathcal{B}_N$$

$$\mathcal{B}_N = \sum_{links} h(\mathrm{D_I})\,(\,h_f - h_i\,)$$

$$\begin{aligned}\mathcal{B}_N = \; & h(\mathrm{D}_1)\,(\,h_f - h_i\,)_1 + h(\mathrm{D}_2)\,(\,h_f - h_i\,)_2 \\ & + h(\mathrm{D}_3)\,(\,h_f - h_i\,)_3 + h(\mathrm{D}_4)\,(\,h_f - h_i\,)_4\end{aligned}$$

$$\begin{aligned}\mathcal{B}_N = \; & h(\mathrm{D})\,(\,h_f - h_i\,)_1 + h(\mathrm{D})\,(\,h_f - h_i\,)_2 \\ & + h(\mathrm{D})\,(\,h_f - h_i\,)_3 + h(\mathrm{D})\,(\,h_f - h_i\,)_4\end{aligned}$$

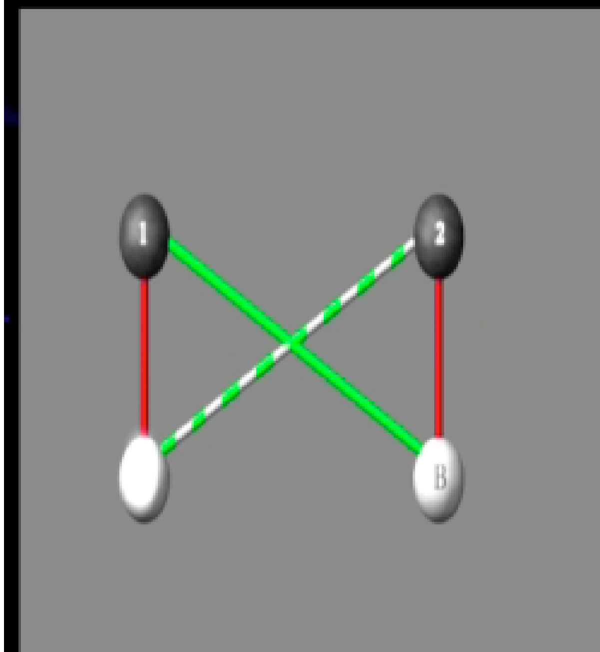
$$h(\mathrm{D}) = h(\mathrm{D}) = \pm \frac{1}{2}$$



Bow-Tie Number Calculations in the "Ferromagnetic Phase"

$$\mathcal{B}_N = \pm \tfrac{1}{2} \{ (h_f - h_i)_1 + (h_f - h_i)_2 + (h_f - h_i)_3 + (h_f - h_i)_4 \}$$

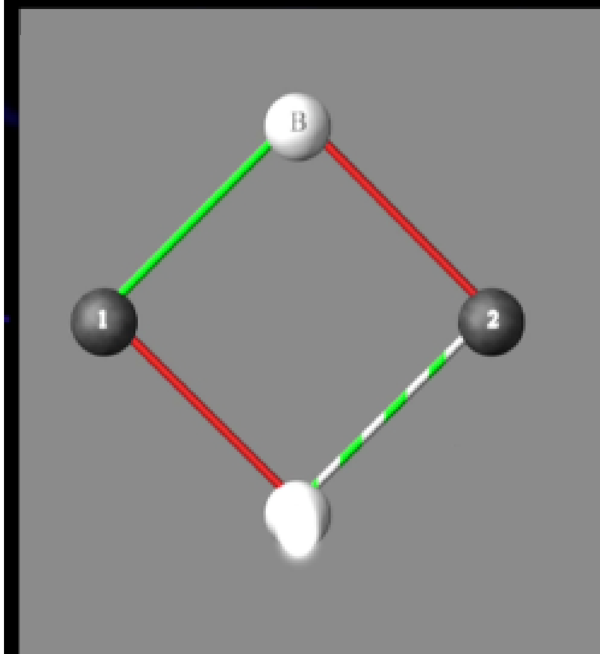
$$h(\mathrm{D}) = h(\mathrm{D}) = \pm\frac{1}{2}$$



$$\begin{aligned}
\mathcal{B}_N &= \pm\frac{1}{2}\{(h_f - h_i)_1 + (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (-\tfrac{1}{2}) + (h_f - h_i)_3 \\
&\qquad + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (-\tfrac{1}{2}) + (\tfrac{1}{2}) \\
&\qquad + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (-\tfrac{1}{2}) + (\tfrac{1}{2}) + (-\tfrac{1}{2})\}
\end{aligned}$$

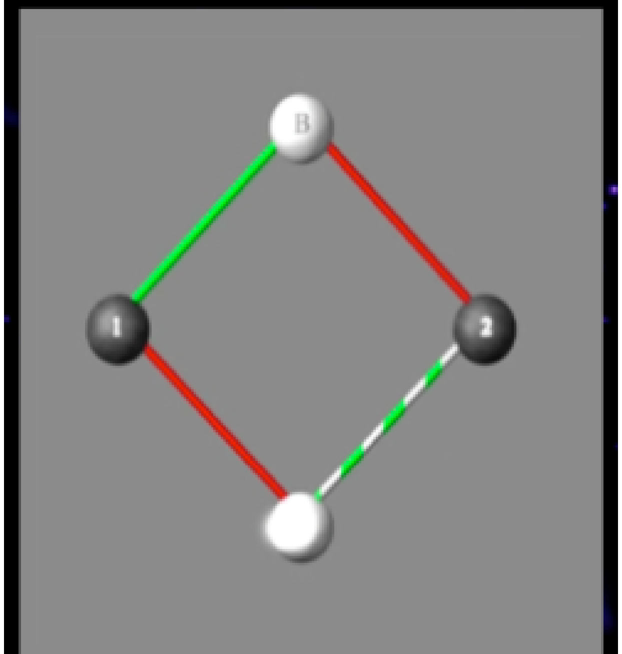
$$\longrightarrow \mathcal{B}_N = 0$$

$$h(\mathrm{D}) = h(\mathrm{D}) = \pm\frac{1}{2}$$



$$\begin{aligned}
\mathcal{B}_N &= \pm\frac{1}{2}\{(h_f - h_i)_1 + (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (\tfrac{1}{2}) + (h_f - h_i)_3 \\
&\qquad + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (\tfrac{1}{2}) + (-\tfrac{1}{2}) \\
&\qquad + (h_f - h_i)_4\} \\
&= \pm\frac{1}{2}\{(\tfrac{1}{2}) + (\tfrac{1}{2}) + (-\tfrac{1}{2}) + (-\tfrac{1}{2})\} \\
&\longrightarrow \mathcal{B}_N = 0 \ .
\end{aligned}$$

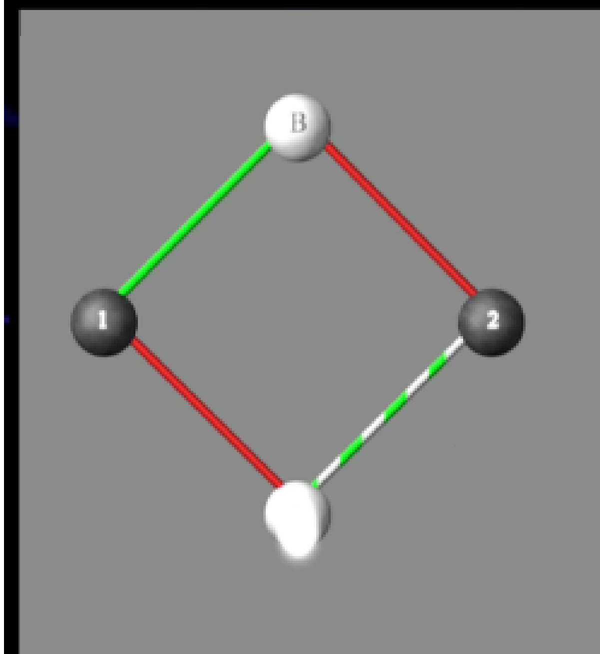
$$h(\mathrm{D}) = -h(\mathrm{D}) = \pm\frac{1}{2}$$



Bow-Tie Number Calculations in the "Anti-Ferromagnetic Phase"

$$\mathcal{B}_N = \pm\frac{1}{2}\{(h_f - h_i)_1 - (h_f - h_i)_2 + (h_f - h_i)_3 - (h_f - h_i)_4\}$$

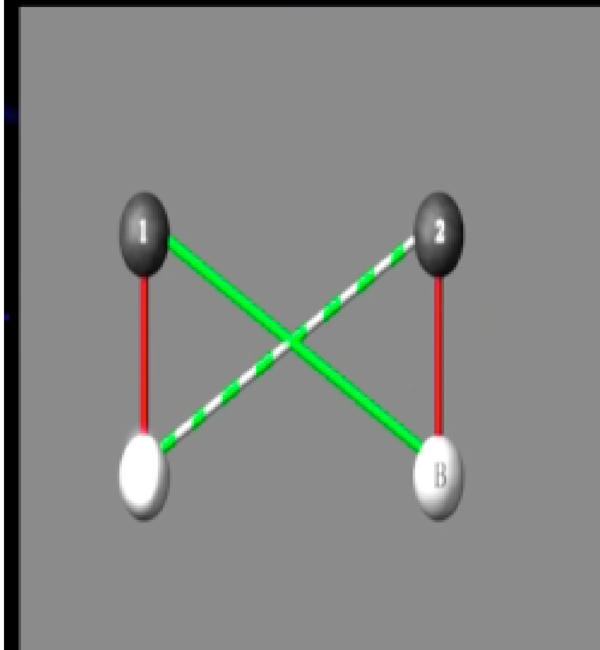
$$h(\mathrm{D}) = -\,h(\mathrm{D}) = \pm\frac{1}{2}$$



$$
\begin{aligned}
\mathcal{B}_N &= \pm\tfrac{1}{2}\{(h_f - h_i)_1 - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (\tfrac{1}{2}) + (h_f - h_i)_3 \\
&\qquad - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (\tfrac{1}{2}) + (-\tfrac{1}{2}) \\
&\qquad - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (\tfrac{1}{2}) + (-\tfrac{1}{2}) - (-\tfrac{1}{2})\}
\end{aligned}
$$

$$\longrightarrow \mathcal{B}_N = 0 \ .$$

$$h(\mathrm{D}) = -h(\mathrm{D}) = \pm\frac{1}{2}$$



$$
\begin{aligned}
\mathcal{B}_N &= \pm\tfrac{1}{2}\{(h_f - h_i)_1 - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (h_f - h_i)_2 \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (-\tfrac{1}{2}) \\
&\qquad + (h_f - h_i)_3 - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (-\tfrac{1}{2}) \\
&\qquad + (\tfrac{1}{2}) - (h_f - h_i)_4\} \\
&= \pm\tfrac{1}{2}\{(\tfrac{1}{2}) - (-\tfrac{1}{2}) + (\tfrac{1}{2}) - (-\tfrac{1}{2})\} \\
&\longrightarrow \mathcal{B}_N = \pm 1 \;.
\end{aligned}
$$

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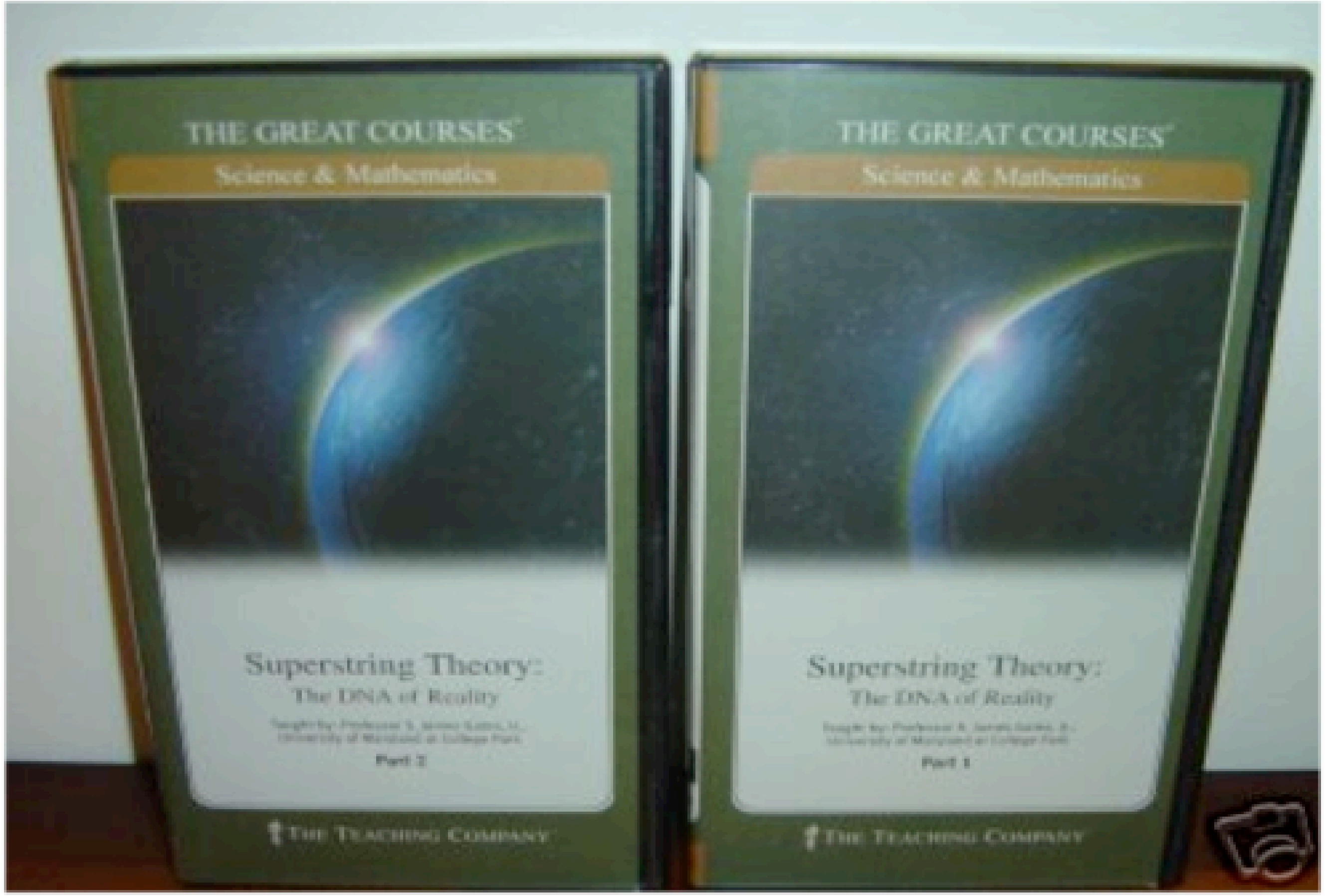

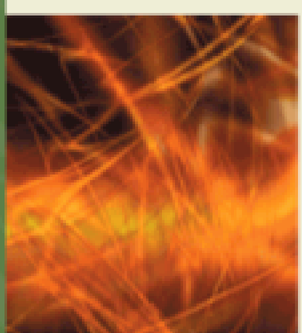

Superstring Theory: The DNA of Reality

Web:
http://www.teach12.com/ttcx/coursedesclong2.aspx?cid=1284

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