The effects of mobility on the one-dimensional four-species cyclic predator-prey model

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Why Study This?

- Stochastic fluctuations are very strong in low-dimensional systems. This leads to wide-ranging behaviors and makes the system's behaviors almost impossible to predict.
- Micro-fabricated habitats can be engineered to house bacterial species in certain geometries which are very similar to this type of model.

The Model

$A + B \rightarrow B + A$ with rate *s*

$A + B \rightarrow A + A$ with rate *k*

and so on…

- One-dimensional lattice with N sites and periodic boundary conditions (very much like a ring of chain links with one occupant per link)
- Interactions between nearest neighbor pairs can be swapping or consumption/conversion processes
- One time-step occurs when N interactions have been made. On average every link is updated once per time-step
- Mobility is realized as the swapping mechanism, with emphasis on the fact that there are no empty links
- A and C species do not interact, likewise B and D species do not interact

Space-Time Diagrams

- Visual picture of what the system is doing

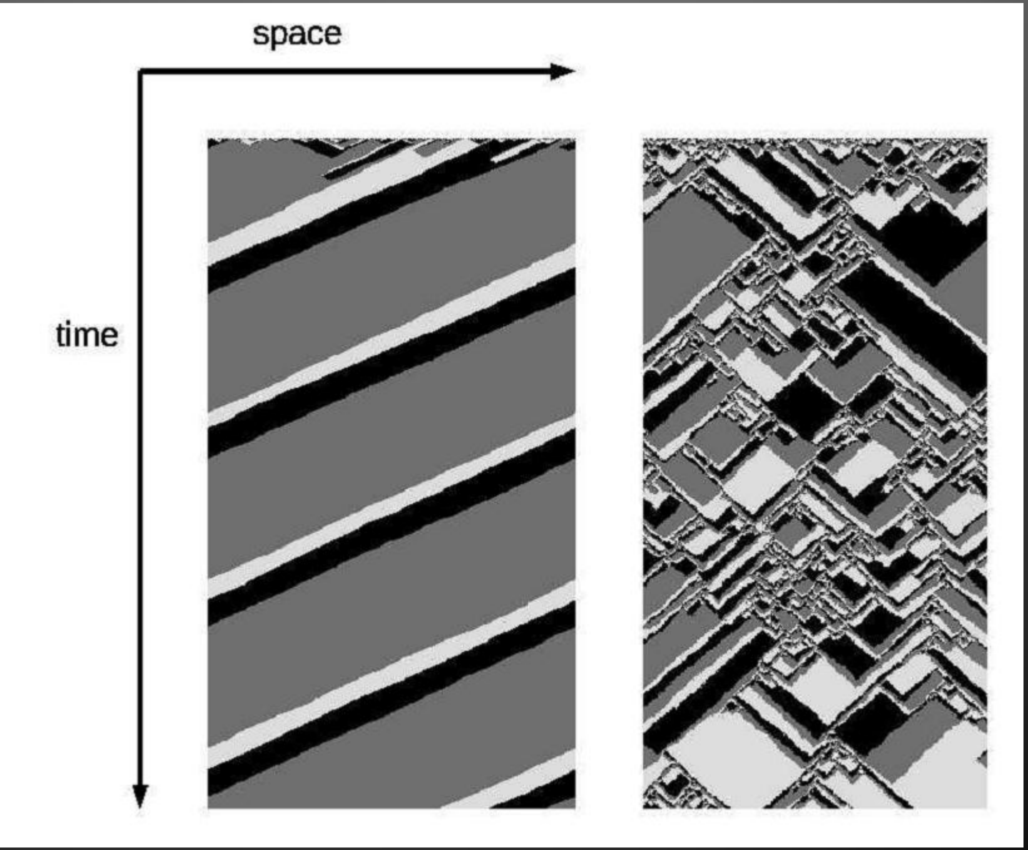
- Many different behaviors are clearly shown in these plots.

- Recall that the lattice is in the shape of a ring, so a better visualization of these plots is in the form of a "tube."

- Horizontal axis is always spatial, and the vertical axis is always temporal

Three-species model:

The left hand plot is with s = 0.1, and the right hand plot has s = 0.9 with k + s =1 for both plots.



(Venkat/Pleimling 2010)

Four-Species Space-Time Diagrams

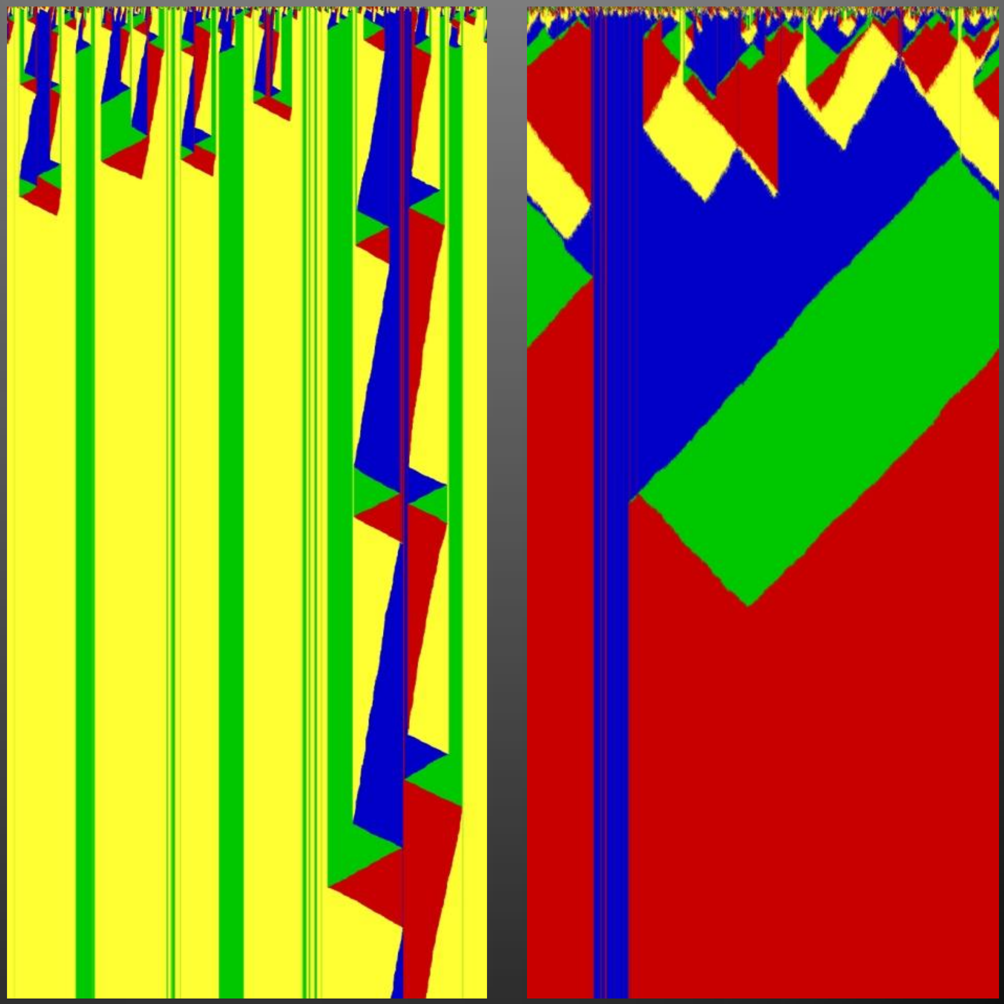
- For the following, all species have the same k, s, and $k + s = 1$

- Outcomes can vary widely in behavior. Some are short lived, some long lived, some seem to freeze quickly, etc.

- Does not behave at all like the three-species model

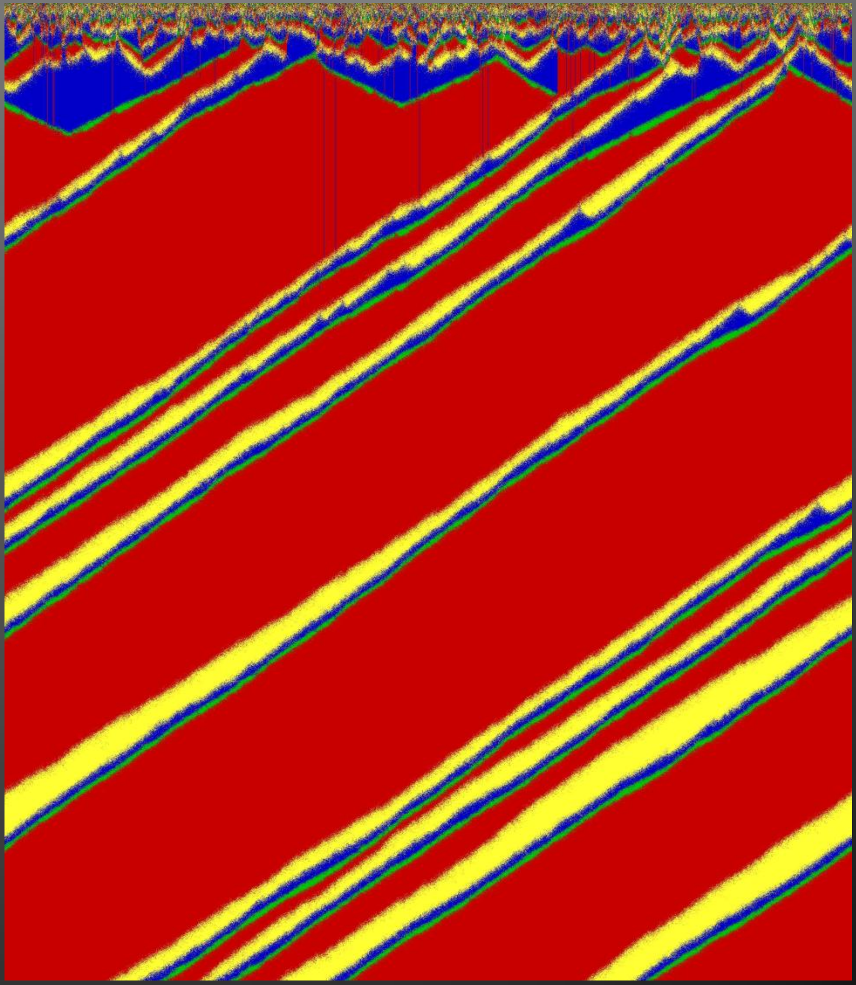
Four-Species Spacetime Plots

- N = 2500
- k + s = 1 for all species, and all species take equal values
- Left hand plot is with s = 0.1 and the right hand plot has s = 0.9



Spacetime Plots (contd.)

- N = 18000
- s = 0.999 for all species
- Only showing every 100th time-step and every 10th lattice site
- Reclamation of coexistence for extremely high mobility; This is the opposite of the three-species model

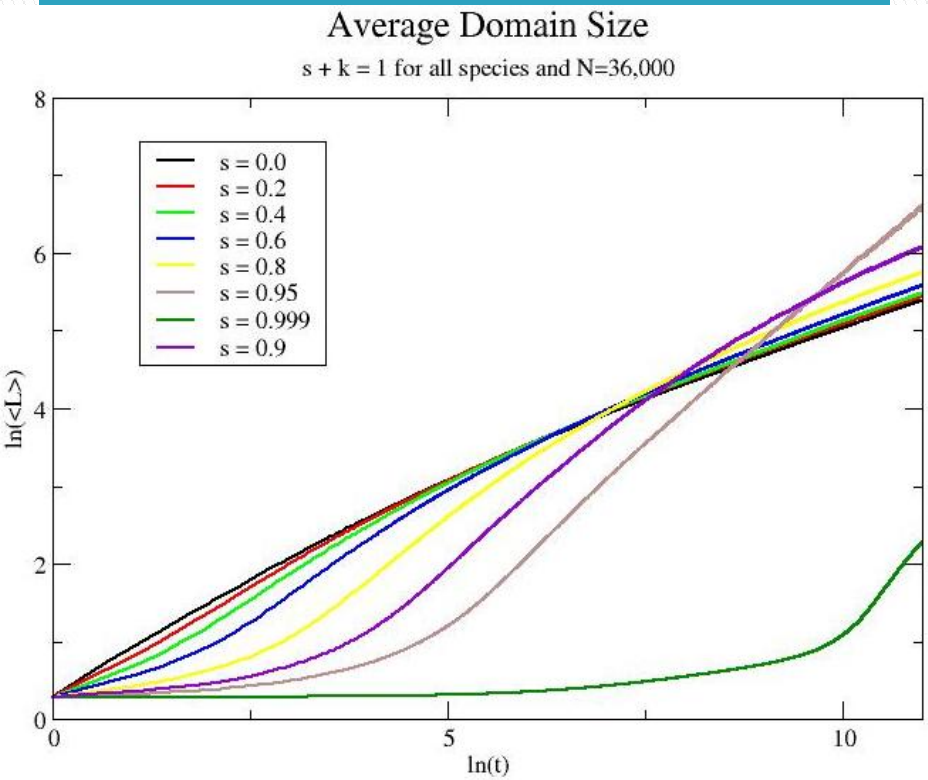


Average Domain Size

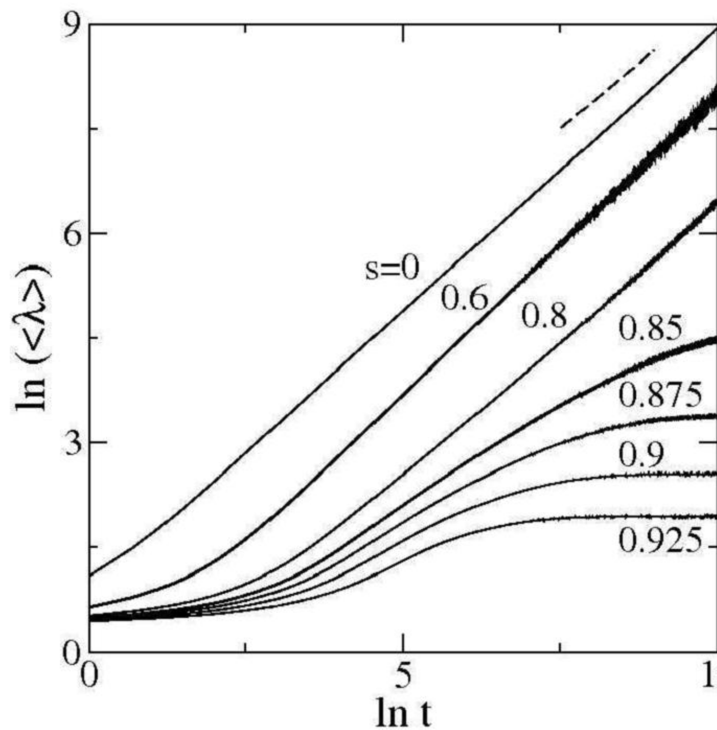
- In all of these runs, species all have equal parameters such that they cannot be distinguished except in name
- Previously shown that for no mobility, domains grow exponentially, with a slope of 1/3 on a logarithmic plot (Frachebourg/Krapivsky/Ben-Naim 1996)
- Unlike the three-species model, with four species the slope increases with higher mobility.
- Lattice size does not appear to have an effect on rate of growth, but finite size effects do limit the accuracy of the measurement

Average Domain Size

Four Species



Three Species

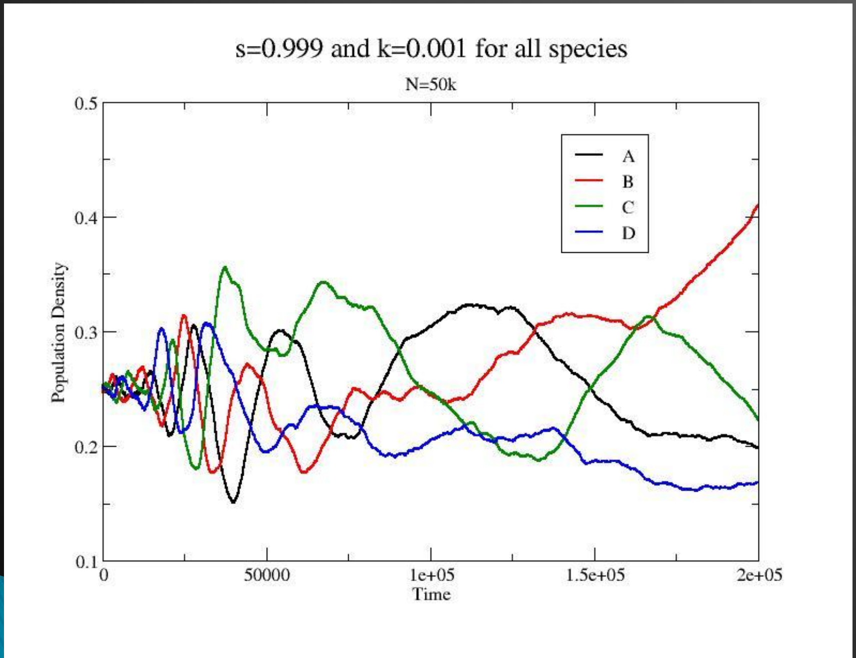


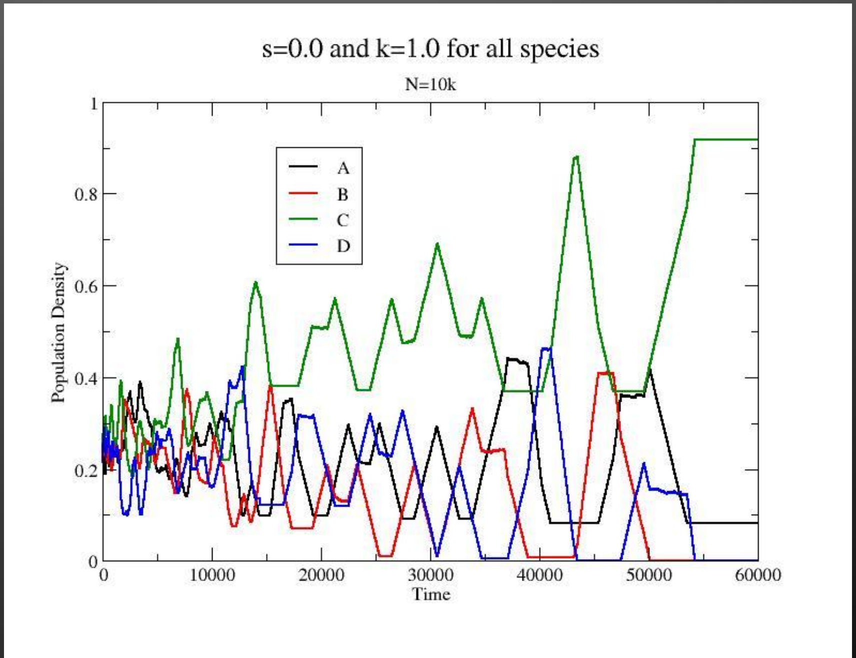
Extinction Processes

•Measuring population density of each species over the entire lattice

•Usually have oscillations near the beginning of a simulation

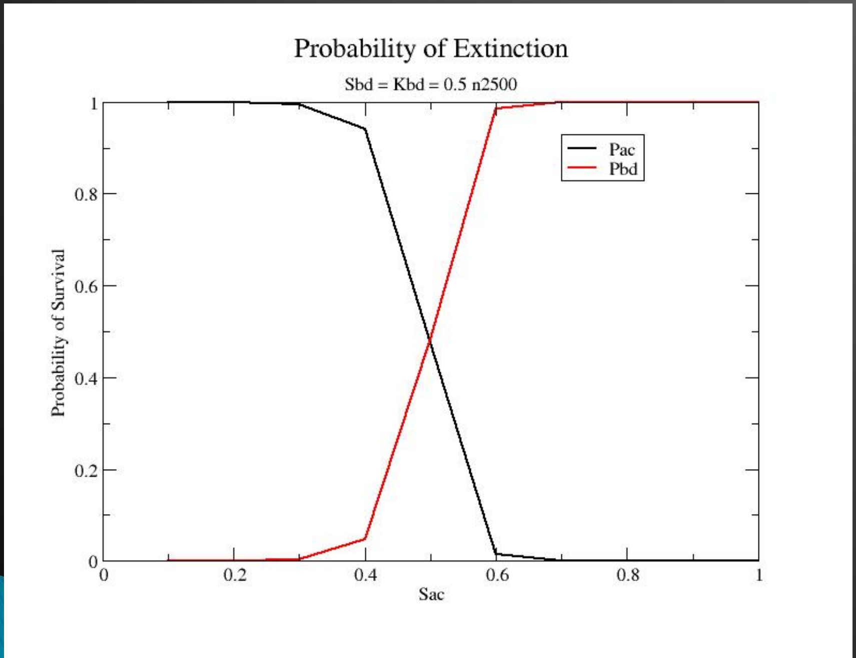
•Multiple possible outcomes: Frozen states, long lived coexistence states, single survivor, etc.

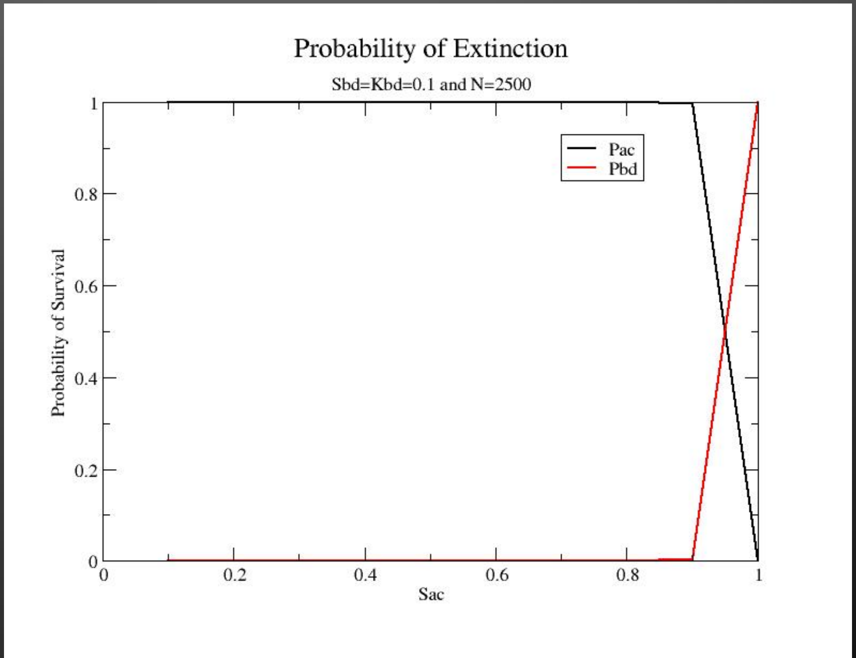




Probabilities of Extinction

- When looking at the likelihood of extinction we see a transition from one pair surviving more than often to the other pair as we vary the mobility
- The transition occurs over longer times when the mobility of all species is larger
- $s_{bd} = k_{bd}$ for all simulations and $s_{ac} + k_{ac} = 1$
- 2000 simulations per data point
- Transition always occurs near when k is equal for all species





Future Research

- Allow swapping between AC/BD pairs
  - What defines a domain?

- Explore more of the parameter space
  - 8 parameters for the current model, 10 if we allow AC/BD interactions
  - Strong predators mixed with weak predators
  - Might we see a “the prey of the prey of the weakest is least likely to survive” type of maxim? Will this maxim hold in one dimension?