From Finite Nuclei to Neutron Stars

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Motivation

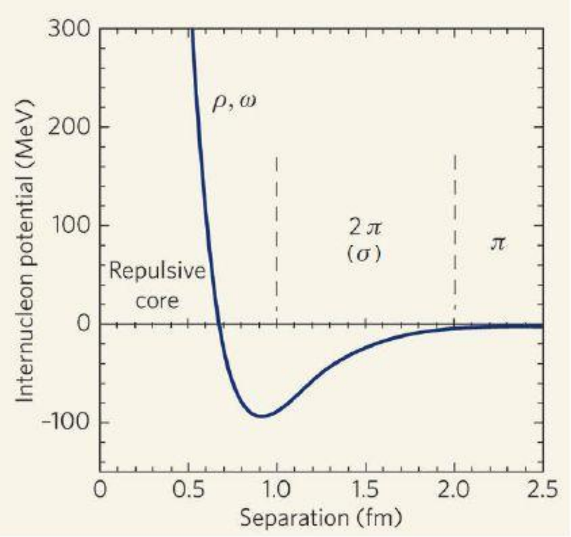
- ➢ Current *relativistic mean-field* (RMF) model for nuclear matter are calibrated by ground-state properties of finite nuclei only.

- Reproduce ground-state properties of finite nuclei very well.
- When extrapolated to high nuclear densities and/or large isospin asymmetry, e.g. neutron stars, results differ from model to model.

- ➢ Hence, we plan to construct a new RMF model involving…

- *Ground-state properties* of finite nuclei
- *Collective excitation modes* of finite nuclei
- *Neutron star observables*
- The calibration will be done by doing *chi-square/covariance analysis*.

Formulation : Relativistic Mean-Field Theory



- The nuclear many-body system is composed of nucleons interacting via the exchange of mesons and photons.

- *Spherical symmetric*
- *Static*
- *Mean-field limit*

$$\begin{aligned}\mathcal{L}_0 &= \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}\,\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\,m_s^2\phi^2 - \frac{1}{4}\,V^{\mu\nu}V_{\mu\nu} + \frac{1}{2}\,m_v^2V^\mu V_\mu \\ &- \frac{1}{4}\,\mathrm{b}^{\mu\nu}\cdot\mathrm{b}_{\mu\nu} + \frac{1}{2}\,m_\rho^2\mathrm{b}^\mu\cdot\mathrm{b}_\mu - \frac{1}{4}\,F^{\mu\nu}F_{\mu\nu}\end{aligned}$$

$$\mathcal{L}_{int} = \bar{\psi}\left[g_s\phi - \left(g_vV_\mu + \frac{g_\rho}{2}\tau\cdot\mathrm{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi - U_{eff}(\phi, V^\mu, \mathrm{b}^\mu)$$

$$U_{eff}(\phi, V^\mu, \mathrm{b}^\mu) = \frac{\kappa}{3!}\,(g_s\phi)^3 + \frac{\lambda}{4!}\,(g_s\phi)^4 - \frac{\zeta}{4!}\left(g_v^2V_\mu V^\mu\right)^2 - \Lambda_v\left(g_\rho^2\mathrm{b}_\mu\cdot\mathrm{b}^\mu\right)\left(g_v^2V_\nu V^\nu\right)$$

Finite Nuclei and their Ground-State Properties

➢ Euler-Lagrange equation:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu q)} - \frac{\partial \mathcal{L}}{\partial q} = 0$$

➢ Field equations:

- Dirac equation:

$$\left[i\gamma_\mu \partial^\mu - (M - g_s\phi_0) - \gamma^0 \left(g_v V_0 + \frac{g_\rho}{2}\tau_3 b_0 + e\tau_p A_0\right)\right]\psi = 0$$

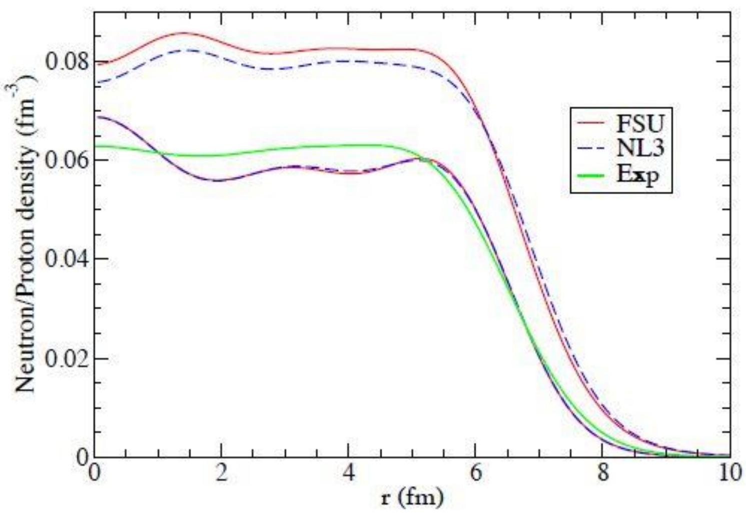
- Klein-Gordon equations:

$$\begin{aligned}
\nabla^2\phi_0 - m_s^2\phi_0 - \partial_{\phi_0} U_{eff}(\phi_0, V_0, b_0) &= -g_s\rho_s \\
\nabla^2 V_0 - m_v^2 V_0 + \partial_{V_0} U_{eff}(\phi_0, V_0, b_0) &= -g_v\rho_v \\
\nabla^2 b_0 - m_\rho^2 b_0 + \partial_{b_0} U_{eff}(\phi_0, V_0, b_0) &= -\frac{g_\rho}{2}\rho_3 \\
\nabla^2 A_0 &= -e\rho_p
\end{aligned}$$

Finite Nuclei and their Ground-State Properties

- Ground-state properties of various (closed-shell) nuclei.
- Neutron/proton density of Pb208.

Nucleus	Observable	Exp	NL3	FSU
^{40}Ca	B/A (MeV)	8.55	8.544	8.540
	$R_n - R_p(fm)$	-	-0.049	-0.051
^{48}Ca	B/A (MeV)	8.67	8.643	8.586
	$R_n - R_p(fm)$	-	0.226	0.197
^{90}Zr	B/A (MeV)	8.71	8.686	8.677
	$R_n - R_p(fm)$	-	0.114	0.088
^{132}Sn	B/A (MeV)	8.36	8.372	8.341
	$R_n - R_p(fm)$	-	0.344	0.270
^{208}Pb	B/A (MeV)	7.87	7.880	7.889
	$R_n - R_p(fm)$	$0.34^{+0.15}_{-0.17}$	0.278	0.206



Infinite Nuclear Matter and its Bulk Properties

- Energy-momentum tensor:

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \partial_\nu q \, \frac{\partial \mathcal{L}}{\partial(\partial^\mu q)}$$

- Energy density and pressure:

$$\begin{aligned}
\mathcal{E} &= 2\int_0^{k_F^{(p)}} \frac{d^3k}{(2\pi)^3} \left(k^2 + M^{*2}\right)^{1/2} + \rho_p \left(g_v V_0 + \frac{g_\rho}{2} b_0\right) \\
&+ 2\int_0^{k_F^{(n)}} \frac{d^3k}{(2\pi)^3} \left(k^2 + M^{*2}\right)^{1/2} + \rho_n \left(g_v V_0 - \frac{g_\rho}{2} b_0\right) \\
&+ \frac{1}{2} m_s^2 \phi_0^2 - \frac{1}{2} m_v^2 V_0^2 - \frac{1}{2} m_\rho^2 b_0^2 + U_{eff}(\phi_0, V_0, b_0)
\end{aligned}$$

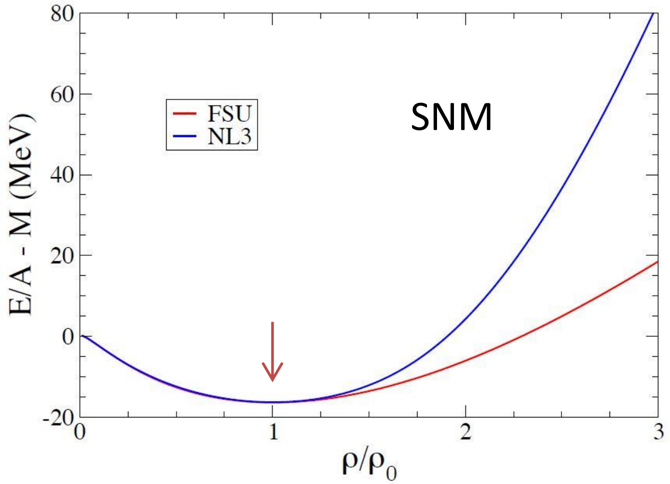
$$\begin{aligned}
p &= \frac{2}{3}\int_0^{k_F^{(p)}} \frac{d^3k}{(2\pi)^3} \frac{k^2}{(k^2 + M^{*2})^{1/2}} + \frac{2}{3}\int_0^{k_F^{(n)}} \frac{d^3k}{(2\pi)^3} \frac{k^2}{(k^2 + M^{*2})^{1/2}} \\
&- \frac{1}{2} m_s^2 \phi_0^2 + \frac{1}{2} m_v^2 V_0^2 + \frac{1}{2} m_\rho^2 b_0^2 - U_{eff}(\phi_0, V_0, b_0)
\end{aligned}$$

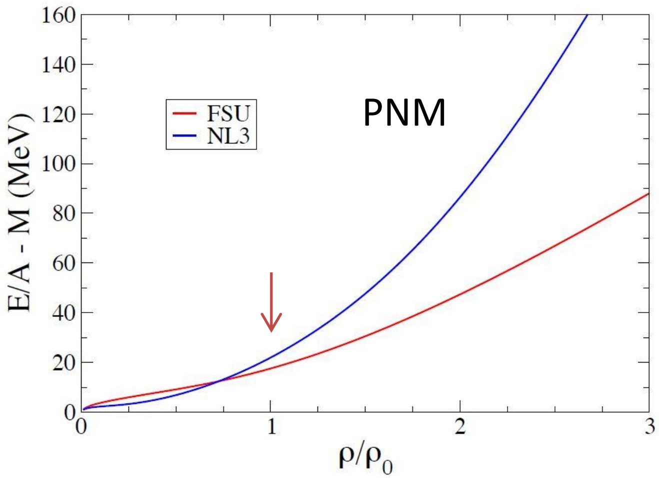
Infinite Nuclear Matter and its Bulk Properties

- Bulk properties of symmetric nuclear matter at saturation density.

Model	$\rho_0\ (fm^{-3})$	$\epsilon_0\ (MeV)$	$K_0\ (MeV)$	$J\ (MeV)$	$L\ (MeV)$
NL3	0.1482	-16.24	271.55	37.28	118.19
FSU	0.1484	-16.30	230.01	32.59	60.52

- Equation of state for symmetric nuclear matter (SNM) and pure neutron matter (PNM).





Neutron Star Observables

➢ Neutron star matter in beta-equilibrium:

$$n \to p + e^- + \bar{\nu}_e \quad ; \quad p + e^- \to n + \nu_e$$

$$n \to p + \mu^- + \bar{\nu}_\mu \quad ; \quad p + \mu^- \to n + \nu_\mu$$

➢ Neutron star structure:

- Tolman-Oppenheimer-Volkoff (TOV) equation:

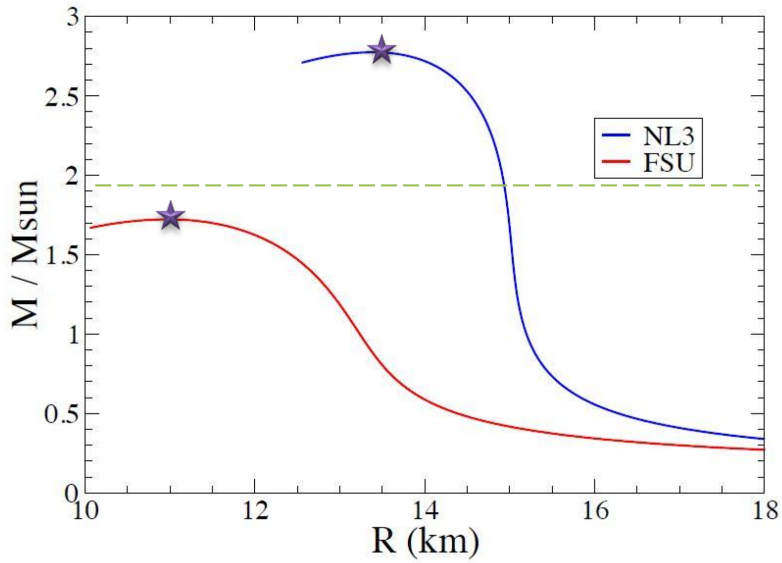
$$\frac{dp}{dr} = -\frac{G\mathcal{E}(p)m}{c^2r^2}\left(1+\frac{p}{\mathcal{E}(p)}\right)\left(1+\frac{4\pi pr^3}{mc^2}\right)\left(1-\frac{2Gm}{c^2r}\right)^{-1}$$

- Mass balance inside the star:

$$\frac{dm}{dr} = \frac{4\pi r^2\mathcal{E}(p)}{c^2}$$

Neutron Star Observables

➢ Mass-radius (M-R) relation of neutron stars.



Model	$M_{max}(M_{\odot})$	$R_{M_{max}}(km)$	$R_{1.4M_{\odot}}(km)$
NL3	2.78	13.39	15.05
FSU	1.72	10.97	12.66

- Maximum neutron star mass observed up to date: *1.94 solar mass*

Finite Nuclei and their Collective Excitation Modes*

- Theoretically, it is difficult and time-consuming to calculate the response of finite nuclei, that is, *collective excitations*.

- However, using sum rules the moments of the strength distributions can be evaluated as expectation values of certain commutators in the *ground state*.

- For example, TRK sum rule:

$$S_1 = \int \omega R(\omega) d\omega = \sum_n (E_n - E_0) |\langle n|x|0\rangle|^2 = \langle 0|[x,[H,x]]|0\rangle = \frac{\hbar^2}{2m}$$

Strength distribution

Ground state

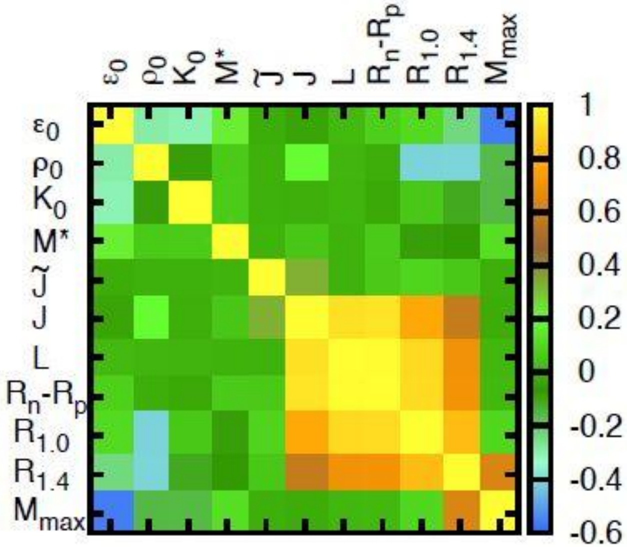
$$S_0 = \int R(\omega) d\omega \qquad\qquad S_{-1} = \int \omega^{-1} R(\omega) d\omega$$

Chi-Square/Covariance Analysis*

- The calibration will proceed through a standard minimization of a chi-square quality measure.

$$\chi^2(\mathbf{p}) = \sum_{n=1}^{N} \left(\frac{\mathcal{O}_n^{(\mathrm{th})}(\mathbf{p}) - \mathcal{O}_n^{(\mathrm{exp})}}{\Delta\mathcal{O}_n^{(\mathrm{exp})}} \right)^2$$

- By studying the parameter landscape around the minimum we will be able to uncover correlations between the physical observables.



F. J. Fattoyev & J. Piekarewicz
arXiv:1109.1576 [nucl-th]

Summary – From Finite Nuclei to Neutron Stars

- ➢ We are constructing a new RMF model for nuclear matter, which will be calibrated using a variety of data from *nuclear experiments* and *astrophysical observations*.

- ✓ Ground-state properties of finite nuclei
- ✓ Bulk properties of infinite nuclear matter
- ✓ Neutron star observables
- Collective excitation modes of finite nuclei
- Chi-square/covariance analysis

- ➢ It will be able to describe numerous phenomena in both *microscopic subatomic* and *macroscopic astrophysical* worlds!