Solar system tests versus cosmological constraints for $f(G)$ models

Jacob Moldenhauer[1,2*], Mustapha Ishak[2†]
[1] *Department of Physics and Astronomy, Francis Marion University, Florence, SC 29502, USA* [2] *Department of Physics, The University of Texas at Dallas, Richardson, TX 75083, USA*
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Recently, some $f(G)$ higher order gravity models have been shown to exhibit some interesting phenomenology including a late time cosmic acceleration following a matter-dominated deceleration period with no separatrix singularities in between the two phases. In this work, we compare the models to the solar system limits from the gravitational frequency redshift, the deflection of light, the Cassini experiment, the time delay and the perihelion shift of planets deriving various bounds on the model parameters. We contrast the bounds obtained with the cosmological constraints on these models finding that the models pass simultaneously both types of constraints.

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I. INTRODUCTION

One possible cause of cosmic acceleration can be a modification or an extension to General Relativity that takes effect at cosmological scales of distance. Among these modified gravity models are the higher order gravity models. In addition to the usual Einstein-Hilbert action, these models contain other more general invariants based on the Ricci and Riemann curvature tensors [1]. In some of these models, there is a late cosmic acceleration due to a different coupling between the matter and space-time curvature. Many papers have been written on the $f(R)$ models [2] while a smaller fraction was devoted to models built from the Ricci and Riemann tensors invariants [3]. Higher order gravity models have been shown to have an interesting phenomenology [1, 2] and have also theoretical motivations within unification theories and field quantization on curved space-times, see for example [4, 5, 5–15] and references therein.

In this paper, we consider a composed invariant that is a topological invariant and is called in the literature the Gauss-Bonnet invariant, denoted as G. This invariant leads to a theory free of unphysical states [6, 16, 17]. Specifically, we consider models where the action is made of the Einstein-Hilbert action plus a function $F(G)$ of the Gauss-Bonnet invariant. We also restrict our study to models that have been shown in previous works to have no ghost instabilities and no superluminal propagations [18] in cosmological homogeneous and isotropic backgrounds. The models that we consider have also been shown to be cosmologically viable including radiation/matter domination followed by a late time self acceleration period [19] with no separatrix in between. Here we compare the models to the solar system tests including the bending of light, the Cassini effect, time delay, the perihelion shift, and the gravitational redshift. We then compare the solar system constraints to those from cosmology in order to put more stringent combined bounds on the models.

II. $f(G)$ MODELS

Several $f(G)$ models have been proposed in the literature [18] and were found [19] to have a cosmological transition from a radiation/matter dominated decelerating period followed by a period dominated by a cosmic acceleration, with no separatrix in between the two periods, as it should. The models are in general derived from varying the action

$$I = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + f(G)\right] + \int d^4x\sqrt{-g}L^{\text{sources}} \tag{1}$$

with respect to the metric $\delta g_{\alpha\beta}$, where

$$G = R^2 - 4R^{\alpha\beta}R_{\alpha\beta} + R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} \tag{2}$$

is the Gauss-Bonnet invariant, R is the Ricci scalar, $R_{\alpha\beta}$ is the Ricci tensor, $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor and L^{sources} Lagrangians corresponding to the sources of spacetime. Units with reduced Planck mass $M_{pl}^2 = (8\pi G_N)^{-1} = 1$ are

* Electronic address: jmoldenhauer@fmarion.edu, jam042100@utdallas.edu
† Electronic address: mishak@utdallas.edu

used. The corresponding field equations read

$$8[R_{\alpha\gamma\beta\delta} + R_{\gamma\beta}g_{\delta\alpha} - R_{\gamma\delta}g_{\beta\alpha} - R_{\alpha\beta}g_{\delta\gamma} + R_{\alpha\delta}g_{\beta\gamma} + \frac{1}{2}R(g_{\alpha\beta}g_{\delta\gamma} - g_{\alpha\delta}g_{\beta\gamma})]\nabla^\gamma\nabla^\delta f_G$$
$$+ (Gf_G - f)g_{\alpha\beta} + R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T^{\text{sources}}_{\alpha\beta}, \tag{3}$$

where $T^{\text{sources}}_{\alpha\beta}$ is the energy momentum tensor of the source(s).

III. EXPANDING AROUND THE STATIC SPHERICALLY SYMMETRIC VACUUM SPACETIME

First, we can use for the models considered in this paper the notation for the action $R + F(G) = R + \alpha f(G)$ where it was found from cosmological constraints that $\alpha \ll 1$ [19]. Next, we consider the field equations in vacuum and use the α parameter so we can write the equations as

$$G_{\alpha\beta} + \alpha H_{\alpha\beta} = 0, \tag{4}$$

where we used the short notation

$$\alpha\big\{8[R_{\alpha\gamma\beta\delta} + R_{\gamma\beta}g_{\delta\alpha} - R_{\gamma\delta}g_{\beta\alpha} - R_{\alpha\beta}g_{\delta\gamma} + R_{\alpha\delta}g_{\beta\gamma} + \frac{1}{2}R(g_{\alpha\beta}g_{\delta\gamma} - g_{\alpha\delta}g_{\beta\gamma})]\nabla^\gamma\nabla^\delta f_G + (Gf_G - f)g_{\alpha\beta}\big\} = \alpha H_{\alpha\beta}. \tag{5}$$

Now, since $|\alpha| \ll 1$, we choose to use it as the expansion parameter in our Taylor series of the metric functions $A(r, \alpha)$ and $B(r, \alpha)$ below. Following previous work [20–22] for solar system tests, we will consider the contribution from the Gauss-Bonnet term to be very small compared to the General Relativity (or Newtonian) term and will expand around the latter. We will see later that this approximation is correct. For that we consider the general static spherically symmetric metric of the form,

$$ds^2 = -A(r, \alpha)dt^2 + \frac{dr^2}{B(r, \alpha)} + r^2 d\Omega^2, \tag{6}$$

where $A(r, \alpha)$ and $B(r, \alpha)$ are the Taylor expanded functions

$$A(r,\, \alpha) = A_0(r) + \alpha A_1(r) + \alpha^2 A_2(r)..., \tag{7}$$

$$B(r,\, \alpha) = B_0(r) + \alpha B_1(r) + \alpha^2 B_2(r)..., \tag{8}$$

and the Schwarzschild solution corresponds to $A_0(r) = B_0(r) = 1 - 2G_N M/r$. While the additional terms $\alpha A_1(r)$, $\alpha^2 A_2(r)$, $\alpha B_1(r)$ and $\alpha^2 B_2(r)$ are due to the contribution from the higher order terms in the action. We only consider solutions up to order α, so the metric becomes,

$$ds^2 = -\Big(1 - \frac{2G_N M}{r} + \alpha A_1(r)\Big)dt^2 + \Big(1 - \frac{2G_N M}{r} + \alpha B_1(r)\Big)^{-1} dr^2 + r^2 d\Omega^2, \tag{9}$$

and that we can write also as

$$ds^2 = -\Big(1 - \frac{2G_N M}{r} + \Phi_{HOG}(r)\Big)dt^2 + \Big(1 - \frac{2G_N M}{r} + \Psi_{HOG}(r)\Big)^{-1} dr^2 + r^2 d\Omega^2. \tag{10}$$

As can be seen in the next section, solving the field equations using this metric allows us to write $A_1(r)$ and $B_1(r)$ in terms of α and also in powers of $\rho \equiv r/(2G_N M)$. We can then select the leading term that makes the next contribution after the GR term, $2G_N M/r$ in the potential.

IV. SOLAR SYSTEM CONSTRAINTS ON COSMOLOGICALLY VIABLE $f(G)$ MODELS

We will now study some of the cosmologically viable $f(G)$ models considered in [19, 23, 24] and

i) place constraints from the solar system tests as described in the appendix

ii) contrast the solar system constraint to those from cosmological observations looking for what parameter ranges, if any, the models pass both constraints. The models we consider are:

1. The models proposed by Zhou, Copeland and Saffin (ZCS) [23]: The authors of [23] performed a thorough phase space analysis of $f(G)$ models and analyzed some specific models that satisfy some cosmological viability conditions. Following [25] for $f(R)$ models, the authors of [23] expressed the viability conditions as constraints on the derivatives of the function $f(G)$. We consider here some of their models as follows:

$$\text{Model ZCS-A:} \quad f(G) = \alpha\sqrt{G} + \beta\sqrt[4]{G}, \tag{11}$$

$$\text{Model ZCS-B:} \quad f(G) = \alpha(G^{3/4} - \beta)^{2/3}, \tag{12}$$

$$\text{Model ZCS-C:} \quad f(G) = \sqrt{\alpha}\exp(\beta/G). \tag{13}$$

where α and β are constants.

2. The models proposed by De Felice and Tsujikawa (DFT) [24]: In Ref. [24], the authors imposed certain conditions on the function $f(G)$ and its derivatives. The most important condition being $d^2f/dG^2 > 0$ in order to ensure the stability of a late-time de Sitter solution as well as the existence of a radiation/matter dominated epochs preceding a late-time accelerating phase. Other additional regularity and viability conditions in [24] provide the following $f(G)$ functions

$$\text{Model DFT-A:} \quad f(G) = \lambda\frac{G}{\sqrt{G_0}}\arctan\left(\frac{G}{G_0}\right) - \frac{\lambda}{2}\sqrt{G_0}\ln\left(1+\frac{G^2}{G_0^2}\right) - \alpha\lambda\sqrt{G_0}, \tag{14}$$

$$\text{Model DFT-B:} \quad f(G) = \lambda\frac{G}{\sqrt{G_0}}\arctan\left(\frac{G}{G_0}\right) - \alpha\lambda\sqrt{G_0}, \tag{15}$$

where λ, G_0 and n are positive constants and α is a constant. These functions were derived from the integration of the following second order derivatives that satisfy the condition $f_{,GG} > 0$ for all values of G along with other regularity conditions [24].

We derive new solar system constraints for the ZCS models and verify those for the DFT models. We then compare the solar constraints with the cosmological ones for all the models.

In the following sections, we use the form of the metric as given by equation (10).

A. Deriving solar system limits on the ZCS-A model

For the solar system tests of the ZCS models, we will use the approximate solutions for first order in α, the model parameter that has been well constrained to be very small, $\alpha \sim 10^{-4}$. We write the ZCS-A model as

$$f(G) = \alpha(\sqrt{G} + \gamma\sqrt[4]{G}), \tag{16}$$

where $\gamma = \beta/\alpha$. From the form of the model (16), the expansion parameter contribution will be of the form $\Phi_{HOG}(r) = \alpha\phi(r)$ and $\Psi_{HOG}(r) = \alpha\psi(r)$. We assume $\rho = r/(2G_N M)$ to be positive and derive the [0, 0] component of the $\psi(\rho)$ differential equation at first order in α as

$$\frac{d\psi}{d\rho}\rho + \psi(\rho) = -\frac{1}{4}\sqrt{3}\sqrt{\rho}\Big(- 18\sqrt{2}3^{1/4}\gamma - 8\sqrt{3}\Big(\frac{1}{\rho^6}\Big)^{1/4} -8\sqrt{3}\Big(\frac{1}{\rho^6}\Big)^{3/4} + 15\sqrt{2}3^{1/4}\gamma\rho + 16\sqrt{3}\Big(\frac{1}{\rho^6}\Big)^{3/4}\rho\Big), \tag{17}$$

with the particular solution

$$\psi = -\frac{2}{\rho^4} + \frac{6}{\rho^3} - \frac{3}{\sqrt{2}} \times 3^{3/4}\gamma\rho^{3/2} + 3\sqrt{2} \times 3^{3/4}\gamma\sqrt{\rho} + 6\frac{1}{\rho}\ln\rho, \tag{18}$$

where we have neglected the homogeneous solution. Now, the [1, 1] component to first order in α for $\phi(\rho)$ is given by

$$(\rho - \rho^2)\frac{d\phi}{d\rho} + \phi(\rho) - \psi(\rho)\rho = \frac{1}{4}\sqrt{3}(-1+\rho)\sqrt{\rho}\Big(8\sqrt{3}\Big(\frac{1}{\rho^6}\Big)^{1/4} +3\sqrt{2} \times 3^{1/4}\gamma(-1+2\rho) + 8\sqrt{3}\Big(\frac{1}{\rho^6}\Big)^{3/4}(-3+2\rho)\Big), \tag{19}$$

which, after substituting (18), the particular solution is given by

$$\phi = \frac{6}{\rho^4} - \frac{2}{\rho^3} + 2\sqrt{2}\times 3^{3/4}\gamma\rho^{3/2} - \frac{3^{3/4}}{\sqrt{2}}\gamma\sqrt{\rho} + 6\frac{1}{\rho}\ln\rho. \tag{20}$$

So, we have arrived at the two solutions for the additional terms $\phi(\rho)$ and $\psi(\rho)$, equations (20) and (18). To find the largest contribution to the metric from these solutions, we can write the full form of the contribution to the metric as, $\Psi_{HOG}(\rho) = \alpha\psi(\rho)$,

$$\alpha\psi = -\alpha\frac{2}{\rho^4} + \alpha\frac{6}{\rho^3} - \frac{3}{\sqrt{2}}\times 3^{3/4}\beta\rho^{3/2} + 3\sqrt{2}\times 3^{3/4}\beta\sqrt{\rho} + 6\alpha\frac{1}{\rho}\ln\rho, \tag{21}$$

and $\Phi_{HOG}(\rho) = \alpha\phi(\rho)$,

$$\alpha\phi = \alpha\frac{6}{\rho^4} - \alpha\frac{2}{\rho^3} + 2\sqrt{2}\times 3^{3/4}\beta\rho^{3/2} - \frac{3^{3/4}}{\sqrt{2}}\beta\sqrt{\rho} + 6\alpha\frac{1}{\rho}\ln\rho, \tag{22}$$

We use the best-fit values from cosmology on α and β, see Table I in [19], to determine the largest contribution will come from the largest power of ρ, which is term three in (22) and (21). These dominant terms are confirmed in the general analysis of [22]. This allows us to set bounds on β from solar system constraints, below. Now, we read off from equations (21) and (22), for the ZCS-A model, the terms for the contribution to the metric as

$$\Phi_{HOG} = 2\sqrt{2}\times 3^{3/4}\beta\rho^{3/2},\ \Psi_{HOG} = -\frac{3}{\sqrt{2}}\times 3^{3/4}\beta\rho^{3/2}, \tag{23}$$

for placing bounds on solar system tests.

Using the solution for the contribution for the expansion contribution into the constraints (A2) and (A3) we arrive at the bound for the gravitational redshift as

$$\frac{\rho_2\rho_1(\Phi_{HOG}(\rho_2) - \Phi_{HOG}(\rho_1))}{\rho_2 - \rho_1} < 2\times 10^{-4}. \tag{24}$$

For the situation described for the gravitational frequency redshift, see [20], $\rho_1 = 7.18\times 10^8$ is the radius of the Earth and $\rho_2 = 1.84\times 10^9$ is the distance of the rocket in units of the Schwarzschild radius. So, with the value from (23) the bound is

$$\beta < 4.41\times 10^{-28}. \tag{25}$$

For the measurement of the deflection of light, the radius of the Sun is $\rho_0 = 2.35\times 10^5$ and the distance of the Earth to the Sun is $\rho = 5.08\times 10^7$ [20]. For the ZCS models, we numerically integrate the contribution for $\vartheta_{\Phi_{HOG},\Psi_{HOG}}$ term in equation (A11) for light bending tests of ZCS models now with the dominant contributions of Φ_{HOG} and Ψ_{HOG} from equations (23) and the system for ρ_0, ρ known. This solution is used in the constraint (A13) to give the bound

$$\beta < 2.37\times 10^{-20}. \tag{26}$$

We also numerically integrate (A11) for conditions of the Cassini experiment. The experiment gives the speed of Earth $v_{Earth} = 9.93\times 10^{-5}$ in units of speed of light, and the distance of Saturn as $\rho_{Cass} = 4.85\times 10^8$. This gives the contribution to the bound from equations (A14) and (A15) using $y_\alpha < 10^{-14}$, as

$$\beta < 6.69\times 10^{-22}. \tag{27}$$

For the constraint on time delay, we numerically integrate the contribution for $t_{\Phi_{HOG},\Psi_{HOG}}$ term in equation (A17). We use the dominant contributions of Φ_{HOG} and Ψ_{HOG} from equations (23) and the system for the Viking mission to Mars with $\rho_{Mars} = 7.71\times 10^7$ and the values for ρ_0 and ρ_{Earth} given earlier. This solution combined with the constraint from equation (A19) gives the bound

$$\beta < 3.31\times 10^{-22}. \tag{28}$$

Finally, for the perihelion shift of the planets, we see from the solutions (23) that the form of the expansion contribution can be written as $\Phi_{HOG} = c_1\beta\rho^p$ and $\Psi_{HOG} = c_2\beta\rho^p$. We can expand (A23) to first order in β as

$$\frac{d^2u}{d\varphi^2} + u - \frac{G_NM}{h^2} = \frac{3}{2}u^2 + \frac{\beta u^{-p-1}G_NM}{h^2\rho_0^3}\{c_1p(\rho_0-1)(h^2(2G_NM)^{-1} + \rho_0^2)\}, \tag{29}$$

where the subdominant term in c_2, see [20] has been omitted as long as $c_1 \neq 0$. Using the solution (A25), we can derive the condition

$$\frac{h^2}{G_N M} \approx \rho_0(1+\delta), \tag{30}$$

for the right hand side, and in the limit $\rho_0 \gg 1$, equation (29) becomes

$$\frac{d^2u}{d\varphi^2} + u - \frac{G_N M}{h^2} = \frac{\beta[\rho_0(1+\delta)]^p}{(\delta+1)\rho_0^2(\delta\cos\varphi+1)^{p+1}}\{2c_1 p(1+\delta)\rho_0^2\}. \tag{31}$$

The solution to equation (31) is found

$$u = \frac{G_N M}{h^2}[1+\delta\cos\varphi] + \frac{3\delta\varphi\sin\varphi}{(1+\delta)^2\rho_0^2} \tag{32}$$

for the Newtonian (first term) and GR (second term) contributions, and the β modification term is the solution

$$u_\beta \approx -\frac{1}{2}\delta\beta c_1 p(p+1)\rho_0^p \varphi\sin\varphi, \tag{33}$$

to lowest order in δ. Now we use (30), to write the approximate solutions above as

$$u \approx \frac{G_N M}{h^2}[1+\delta\cos(\varphi-\sigma\varphi)] \tag{34}$$

with

$$\sigma \equiv \frac{3}{2}\frac{1}{(1+\delta)\rho_0} - \frac{1}{2}\beta c_1 p(p+1)(1+\delta)\rho_0^{p+1}. \tag{35}$$

In the same way, [20, 26], we see that in one cycle, the angle between two perihelia is larger than 2π by $2\pi\sigma$, so

$$\begin{aligned}\Delta\varphi - 2\pi &= \frac{3\pi}{(1+\delta)\rho_0} - \pi\beta c_1 p(p+1)(1+\delta)\rho_0^{p+1}\\ &\approx \frac{3\pi}{(1+\delta)\rho_0}\Big(1 - \frac{1}{3}c_1\beta p(p+1)(1+2\delta)\rho_0^{p+2}\Big).\end{aligned} \tag{36}$$

The experimental bound on the perihelion shift for Earth comes from hundreds and hundreds of planetary observations, [27] as $\Delta\varphi - 2\pi = 5 \pm 1$ arcsec/cty. The modification due to β term must be less than the relative experimental error, so from equation (36), we have the constraint

$$\Big|\frac{\Delta\varphi_\beta}{\Delta\varphi_{GR}}\Big| = \frac{1}{3}|c_1|\beta p(p+1)(1+2\delta)\rho_0^{p+2} < \frac{1}{5}. \tag{37}$$

So, for the perihelion shift of the Earth, we apply the constraint

$$\frac{1}{3}|c_1|\beta p(p+1)(1+2\delta_{Earth})\rho_{Earth}^{p+2} < \frac{1}{5}, \tag{38}$$

to the ZCS-A model, with eccentricity $\delta_{Earth} = 0.02$ and the perihelion distance as $\rho_{Earth} = 4.98 \times 10^7$ to give the bound

$$\beta < 2.55 \times 10^{-29}. \tag{39}$$

These are all satisfied for the constraints on α and β from cosmological distances.

B. Deriving solar system limits on the ZCS-B model

For the high density region, where these solar system tests are measured, the model ZCS-B has the asymptotic form, see for example [20],

$$f(G) = \alpha\Big(\sqrt{G} - \frac{2}{3}\gamma\Big(\frac{1}{G}\Big)^{1/4}\Big) + \mathcal{O}(\alpha^2), \tag{40}$$

where $\gamma = \beta/\alpha$. We will only consider the terms first order in α. From the form of the model (40), the expansion parameter contribution will be of the form $\Phi_{HOG}(r) = \alpha\phi(r)$ and $\Psi_{HOG}(r) = \alpha\psi(r)$. We assume $\rho = r/(2G_N M)$ to be positive and derive [0, 0] component of the $\psi(\rho)$ differential equation at first order in α as

$$\frac{d\psi}{d\rho}\rho + \psi(\rho) = \frac{5\sqrt{2}\times 3^{1/4}\gamma(28-33\rho)\rho^{12} - 72(\rho-1)(\rho^6)^{3/4}}{6\rho^4(\rho^6)^{3/4}}, \tag{41}$$

with the particular solution

$$\psi = -\frac{4}{\rho^4} + \frac{6}{\rho^3} - \frac{140\sqrt{2}\gamma\rho^{7/2}}{9\times 3^{3/4}} - 5\sqrt{2}\times 3^{1/4}\gamma\rho^{9/2}. \tag{42}$$

Now, the [1, 1] component to first order in α for $\phi(\rho)$ is given by

$$(\rho-\rho^2)\frac{d\phi}{d\rho} + \phi(\rho) - \psi(\rho)\rho = \frac{(\rho-1)\Big(72(\rho-1)(\rho^6)^{3/4} + 5\sqrt{2}\times 3^{1/4}\gamma\rho^{12}(6\rho-11)\Big)}{6\rho^4(\rho^6)^{3/4}}, \tag{43}$$

which, after substituting (42), the particular solution is given by

$$\phi = \frac{4}{\rho^4} - \frac{2}{\rho^3} + \frac{55\sqrt{2}\gamma\rho^{7/2}}{9\times 3^{3/4}} + \frac{20\sqrt{2}\gamma\rho^{9/2}}{3\times 3^{3/4}}. \tag{44}$$

Again, we have arrived at the two solutions for the additional terms $\phi(\rho)$ and $\psi(\rho)$, equations (44) and (42). We find the largest contribution to the metric from these solutions, by writing the full form of the contribution to the metric as, $\Psi_{HOG}(\rho) = \alpha\psi(\rho)$,

$$\alpha\psi = -\alpha\frac{4}{\rho^4} + \alpha\frac{6}{\rho^3} - \frac{140\sqrt{2}\beta\rho^{7/2}}{9\times 3^{3/4}} - 5\sqrt{2}\times 3^{1/4}\beta\rho^{9/2}, \tag{45}$$

and $\Phi_{HOG}(\rho) = \alpha\phi(\rho)$,

$$\alpha\phi = \alpha\frac{4}{\rho^4} - \alpha\frac{2}{\rho^3} + \frac{55\sqrt{2}\beta\rho^{7/2}}{9\times 3^{3/4}} + \frac{20\sqrt{2}\beta\rho^{9/2}}{3\times 3^{3/4}}. \tag{46}$$

We use the best-fit values from cosmology on α and β, see Table I in [19], to determine the largest contribution will come from the last term in (46) and (45). These dominant terms are confirmed in the general analysis of [22]. This allows us to set bounds on β from solar system constraints, below. Now, we read off from equations (45) and (46), for the ZCS-B model, the terms for the contribution to the metric as

$$\Phi_{HOG} = \frac{20\sqrt{2}\beta\rho^{9/2}}{3\times 3^{3/4}},\ \Psi_{HOG} = -5\sqrt{2}\times 3^{1/4}\beta\rho^{9/2}, \tag{47}$$

for placing bounds on solar system tests.

Again, the bounds from the gravitational redshift can still be applied as in the general case with equation (24), similar to model ZCS-A, but with the value from (47) to give the bound

$$\beta < 8.48\times 10^{-56}. \tag{48}$$

For the ZCS-B model, we numerically integrate the contribution for $\vartheta_{\Phi_{HOG},\Psi_{HOG}}$ term in equation (A11) for light bending tests of the ZCS-B model now with the dominant contributions of Φ_{HOG} and Ψ_{HOG} from equations (47) and the system for ρ_0, ρ known, (see section above with same value of ρ_0, ρ). This solution is used in the constraint (A13) to give the bound

$$\beta < 1.27 \times 10^{-42}. \tag{49}$$

We use conditions of the Cassini experiment to numerically integrate (A11) to give the contribution to the bound from equation (A14). Using $y_\alpha < 10^{-14}$, and the values from equations (47) we find

$$\beta < 5.57 \times 10^{-47}. \tag{50}$$

Next, we numerically integrate equation (A17), for the Viking mission to Mars with values from equations (47) and the situation described earlier. We use the solution to apply the constraint from equation (A19) to give the bound

$$\beta < 1.76 \times 10^{-45}. \tag{51}$$

For the perihelion shift of the Earth, we use equation (38) to find the bound

$$\beta < 4.60 \times 10^{-53}. \tag{52}$$

Again, all constraints are satisfied for the bounds on α and β from cosmological distances.

C. Deriving solar system limits on the ZCS-C model

Similar to the ZCS-B model, the model ZCS-C has the asymptotic form

$$f(G) = \alpha\Big(\sqrt{G} + \gamma\sqrt{\frac{1}{G}}\Big) + \mathcal{O}(\alpha^2), \tag{53}$$

where $\gamma = \beta/\alpha$. We will only consider the terms first order in α. From the form of the model (40), the expansion parameter contribution will be of the form $\Phi_{HOG}(r) = \alpha\phi(r)$ and $\Psi_{HOG}(r) = \alpha\psi(r)$. We assume $\rho = r/(2G_N M)$ to be positive and derive [0, 0] component of the $\psi(\rho)$ differential equation at first order in α as

$$\frac{d\psi}{d\rho}\rho + \psi(\rho) = \frac{3(4+\rho(-4+\gamma\rho^{11}(7\rho-6)))}{\rho^4}, \tag{54}$$

with the particular solution

$$\psi = -\frac{4}{\rho^4} + \frac{6}{\rho^3} - 2\gamma\rho^8 + \frac{21\gamma\rho^9}{10}. \tag{55}$$

Now, the [1, 1] component to first order in α for $\phi(\rho)$ is given by

$$(\rho - \rho^2)\frac{d\phi}{d\rho} + \phi(\rho) - \psi(\rho)\rho = -\frac{3(\rho-1)(4+\rho(-4+\gamma(\rho-2)\rho^{11}))}{\rho^4}, \tag{56}$$

which, after substituting (55), the particular solution is given by

$$\phi = -\frac{\gamma}{10} + \frac{4}{\rho^4} - \frac{2}{\rho^3} + \frac{\gamma}{10\rho} + \frac{2\gamma\rho^8}{3} - \frac{17\gamma\rho^9}{30}. \tag{57}$$

These are the two solutions for the additional terms $\phi(\rho)$ and $\psi(\rho)$, equations (57) and (55). The largest contribution to the metric from these solutions can be seen from the full form of the contribution to the metric as, $\Psi_{HOG}(\rho) = \alpha\psi(\rho)$,

$$\alpha\psi = -\alpha\frac{4}{\rho^4} + \alpha\frac{6}{\rho^3} - 2\beta\rho^8 + \frac{21\beta\rho^9}{10}. \tag{58}$$

and $\Phi_{HOG}(\rho) = \alpha\phi(\rho)$,

$$\alpha\phi = -\frac{\beta}{10} + \alpha\frac{4}{\rho^4} - \alpha\frac{2}{\rho^3} + \frac{\beta}{10\rho} + \frac{2\beta\rho^8}{3} - \frac{17\beta\rho^9}{30}. \tag{59}$$

Constraints from Observations	α Solar System	λ Solar System	β Solar System	Ω_m Cosmological	H_0 Cosmological	α Cosmological	λ Cosmological	β Cosmological
$ZCS-A$	$< 8.11 \times 10^{-8}$	–	$< 2.55 \times 10^{-29}$	$0.25^{+0.03}_{-0.02}$	$72.06^{+1.63}_{-2.14}$	$0.00084^{+0.00016}_{-0.01632}$	–	$-0.03498^{+0.03595}_{-0.02399}$
$ZCS-B$	$< 8.11 \times 10^{-8}$	–	$< 8.48 \times 10^{-56}$	$0.25^{+0.02}_{-0.02}$	$71.12^{+1.81}_{-1.37}$	$-0.00014^{+0.000083}_{-0.001439}$	–	$0.00031^{+0.03288}_{-0.76993}$
$ZCS-C$	$< 8.11 \times 10^{-8}$	–	$< 1.24 \times 10^{-96}$	$0.27^{+0.02}_{-0.02}$	$81.51^{+1.85}_{-1.74}$	$-0.00012^{+0.000004}_{-0.00295}$	–	$-0.0000017^{+0.00012}_{-0.00142}$
$DFT-A$	$> 0.65 \times 10^{-5}$ from $\alpha\lambda = 1$	$< 1.53 \times 10^{5}$	–	$0.25^{+0.06}_{-0.02}$	$68.11^{+11.38}_{-16.28}$	$54.3437^{+20.6530}_{-44.9701}$	$0.0832318^{+1.81255}_{-0.058080}$	–
$DFT-B$	$> 0.80 \times 10^{-14}$ from $\alpha\lambda = 1$	$< 1.25 \times 10^{14}$	–	$0.25^{+0.05}_{-0.02}$	$70.61^{+8.94}_{-17.11}$	$9.99891^{+39.9510}_{-6.78136}$	$0.313781^{+1.49185}_{-0.288712}$	–

TABLE I: Summary of solar system constraints and best-fit parameters for $f(G)$ models from cosmological constraints (supernovae, baryon acoustic oscillations, Hubble Space Telescope Key Project, and CMB surface) as derived in [19]. The solar system constraints are only listed for the strongest constraints from section IV. For the DFT-A and DFT-B the limit for α is derived from that of λ.

The best-fit values from cosmology on α and β, see Table I in [19], determine the largest contribution will come from the last term in (58) and (59). These dominant terms are confirmed in the general analysis of [22]. This allows us to set bounds on β from solar system constraints, below. Now, we read off from equations (58) and (59), for the ZCS-C model, the terms for the contribution to the metric as

$$\Phi_{HOG} = -\frac{17\beta\rho^9}{30}, \ \Psi_{HOG} = -\frac{21\beta\rho^9}{10}, \tag{60}$$

for placing bounds on solar system tests.

Again, the bounds from the gravitational redshift can still be applied as in the general case with equation (24), similar to the models ZCS-A and ZCS-B, but with the value from (60) to give the bound

$$\beta < 1.24 \times 10^{-96}. \tag{61}$$

For the ZCS-B model, we numerically integrate the contribution for $\vartheta_{\Phi_{HOG},\Psi_{HOG}}$ term in equation (A11) for light bending tests of the ZCS-B model now with the dominant contributions of Φ_{HOG} and Ψ_{HOG} from equations (60) and the system for ρ_0, ρ known, (see section above with same value of ρ_0, ρ). This solution is used in the constraint (A13) to give the bound

$$\beta < 2.45 \times 10^{-76}. \tag{62}$$

We also numerically integrate (A11) for conditions of the Cassini experiment and the values (60). This solution combined with the contribution to the bound from equation (A14) using $y_\alpha < 10^{-14}$ gives the bound

$$\beta < 4.20 \times 10^{-85}. \tag{63}$$

Again, we numerically integrate equation (A17), for the Viking mission to Mars with values from equations (60) and the situation described earlier. We use the solution to apply the constraint from equation (A19) to give the bound

$$\beta < 1.95 \times 10^{-80}. \tag{64}$$

For the perihelion shift of the Earth, we use equation (38) find the bound

$$\beta < 1.95 \times 10^{-87}. \tag{65}$$

These are all satisfied for the constraints on α and β from cosmological distances. Our results are summarized in Table I.

We also rederived the solar system test constraints on the DFT models and found them in perfect agreement with those of [20]. These limits are also reported on Table I.

Finally, it is worth mentioning that for the ZCS models that we analyzed above, we could have expanded around β, as it is found small from the cosmological observations [19]. We did the analysis around β and arrived at exactly the same limits as above.

V. COMPARISONS AND CONCLUDING REMARKS

We compare here limits from solar system test constraints and cosmological constraints. The results for solar system derived in the previous section are summarized in the comparative Table I, along with the constraints from cosmology (including supernova, baryon acoustic oscillations, Hubble Space Telescope Key Project, and CMB surface distance constraints) [19]. We find that the solar system constraints on the ZCS models are much more stringent than the constraints from cosmology but provide only a single side bound. Indeed, the solar system constraints on the α parameter for the ZCS models are four orders of magnitude tighter than the ones from cosmology. The solar system constraints on the parameter β are extremely tight and the β-term is practically restrained to be negligible compared to the α-term. Thus for the ZCS models the solar system constraints are totally dominant. For the DFT models, the solar system constraints for the parameter λ are 7 to 15 orders of magnitude looser than the ones from cosmology so the cosmology bounds are the significant ones whereas the most stringent constraints on the α parameter are again from solar system tests but again with only one bound.

Finally, the comparisons show that the cosmological constraints are compatible with those from the solar system tests thus the combination of the two cannot rule out the models. While the model parameters are constrained to be small, the Gauss-Bonnet term is very large at cosmological scales so that the resulting terms (except for the β ones) are large enough to take effect at cosmological scales and to possibly mimic cosmic acceleration. In conclusion, we find that probes of the cosmic expansion on large scales combined with solar system tests are not enough to rule out $f(G)$ models.

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Appendix A: Solar system tests

We consider the solar system tests as the constraints from the deviation of General Relativity as discussed, see for example [26] and more recently [28, 29], for the deflection of light, Cassini experiment, perihelion shift, time delay and gravitational redshift. We give here only a brief overview on the test and refer the reader to standard books, see for example [26, 30].

1. Gravitational frequency redshift constraint

For a light signal traveling between two different lattice points [26], such as heights r and r_1, the frequency shift between the two frequencies ν and ν_1 can be read off from (10) as

$$\frac{\nu}{\nu_1} = \sqrt{\frac{A(r)}{A(r_1)}} \approx 1 + \frac{1}{2}(\rho_1^{-1} - \rho^{-1}) + \frac{1}{2}(\Phi_{HOG} - \Phi_{HOG_1}). \tag{A1}$$

For the small expansion parameter contribution,

$$\frac{\Delta\nu_{\Phi_{HOG}}/\nu}{\Delta\nu_{GR}/\nu} = \frac{\rho\rho_1(\Phi_{HOG} - \Phi_{HOG_1})}{\rho - \rho_1}, \tag{A2}$$

where $\Delta\nu = \nu - \nu_1$. $\Delta\nu_{exp}/\Delta\nu_{GR} = 1 \pm 0.0002$ is the bound for this experiment from a hydrogen-maser clock on a rocket launched to a height of 10^7 m, [31]. This gives the bound for the modification from Φ_{HOG} term as

$$\frac{\Delta\nu_{\Phi_{HOG}}/\nu}{\Delta\nu_{GR}/\nu} < 2 \times 10^{-4}, \tag{A3}$$

which we use in the section IV to place gravitational frequency redshift constraints on the ZCS models and confirm the constraints on the DFT models.

2. Deflection of light constraint

The deflection of light, or the bending of light as it travels close to massive objects, following [26], can be described by the Lagrangian for a photon moving in a geodesic in $\theta = \pi/2$ plane as

$$\mathcal{L} = \frac{1}{2}A\dot{t}^2 - \frac{1}{2}B^{-1}\dot{r}^2 - \frac{1}{2}r^2\dot{\varphi}^2 = 0, \tag{A4}$$

where overdot is differentiation with respect to the affine parameter. From the Euler-Lagrange equations, the two first integrals are

$$A\dot{t} = k, \tag{A5}$$

and

$$r^2\dot{\varphi} = h. \tag{A6}$$

Using these (A5) and (A6), we find

$$\dot{r}^2 = h^2\Big(\frac{k^2}{h^2}\frac{B}{A} - \frac{B}{r^2}\Big), \tag{A7}$$

which for the minimal distance r_0, the condition $\dot{r}(r_0) = 0$ gives

$$\frac{h^2}{(2G_N M)^2 k^2} = \frac{\rho_0^2}{A(\rho_0)}, \tag{A8}$$

with $\rho_0 = r_0/(2G_N M)$ in the notation of [20]. The integration of $d\varphi/dr = \dot{\varphi}/\dot{r}$ using the equations above, will obtain

$$\varphi(\rho) = \pm \int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\rho_0}{\tilde{\rho}}\sqrt{\frac{A(\tilde{\rho})}{B(\tilde{\rho})[A(\rho_0)\tilde{\rho}^2 - A(\tilde{\rho})\rho_0^2]}}, \tag{A9}$$

where ρ and ρ_0 are the upper and lower limits of the integral and $\tilde{\rho}$ is the integration variable. In the domain for first order in the expansion parameter, the above integral can be approximated in the limit $\rho \to \infty$ for the solar system and in the limits ρ, $\rho_0 \gg 1$ to give, see, for example, [20],

$$\begin{aligned}\varphi = &\pm \int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\rho_0}{\tilde{\rho}}\frac{1}{\sqrt{\tilde{\rho}^2 - \rho_0^2}} \pm \int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\tilde{\rho}^2 + \tilde{\rho}\rho_0 + \rho_0^2}{2\tilde{\rho}^2(\tilde{\rho}+\rho_0)\sqrt{\tilde{\rho}^2 - \rho_0^2}} \\ &\pm\frac{1}{2}\rho_0\int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\tilde{\rho}^2(\Phi_{HOG}(\tilde{\rho}) - \Phi_{HOG}(\rho_0)) - \Psi_{HOG}(\tilde{\rho})(\tilde{\rho}^2 - \rho_0^2)}{\tilde{\rho}(\tilde{\rho}^2 - \rho_0^2)^{3/2}}.\end{aligned} \tag{A10}$$

This will give the deflection angle as

$$\begin{aligned}\vartheta(\rho) = &\, 2\int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\rho_0}{\tilde{\rho}}\frac{1}{\sqrt{\tilde{\rho}^2 - \rho_0^2}} + \int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\tilde{\rho}^2 + \tilde{\rho}\rho_0 + \rho_0^2}{\tilde{\rho}^2(\tilde{\rho}+\rho_0)\sqrt{\tilde{\rho}^2 - \rho_0^2}} \\ &+\rho_0\int_{\rho_0}^{\rho} d\tilde{\rho}\frac{\tilde{\rho}^2(\Phi_{HOG}(\tilde{\rho}) - \Phi_{HOG}(\rho_0)) - \Psi_{HOG}(\tilde{\rho})(\tilde{\rho}^2 - \rho_0^2)}{\tilde{\rho}(\tilde{\rho}^2 - \rho_0^2)^{3/2}} - \pi,\end{aligned} \tag{A11}$$

In the limit $\rho \gg \rho_0$, the first term from equation (A11) will cancel with π and the second term will give the GR contribution

$$\vartheta = \frac{4G_N M}{r_0} + \mathcal{O}(r_0/r) = \frac{2}{\rho_0} + \mathcal{O}(\rho_0/\rho), \tag{A12}$$

to the deflection of light. The last term of equation (A11) for the contribution from $\vartheta_{\Phi_{HOG},\Psi_{HOG}}$ contains hypergeometric functions, which can have simple forms for some values of Φ_{HOG} and Ψ_{HOG} when the form is known *apriori*, see below, [20]. For general HOG models, we will leave it in this form until the model is solved. However, we can briefly state the experimental measurement for bounding $\vartheta_{\Phi_{HOG},\Psi_{HOG}}$ will come from the Very Long Baseline Interferometry (VLBI) containing combined radio telescope measurements [32]. VLBI places the ratio of the experimental value of ϑ_{exp} relative to the theoretical prediction ϑ_{theor} as $\vartheta_{exp}/\vartheta_{theor} = 1.0001 \pm 0.0001$, meaning there would be no affect in the GR result from $\vartheta_{\Phi_{HOG},\Psi_{HOG}}$ as long as

$$\frac{\vartheta_{\Phi_{HOG},\Psi_{HOG}}}{\vartheta_{GR}} < 10^{-4}. \tag{A13}$$

We use this relationship in the section IV to place constraints on the ZCS and verify the constraints on the DFT models after solving the field equations and finding the forms and value for Φ_{HOG} and Ψ_{HOG}.

3. Cassini experiment constraint

It was shown in [20, 33] that the two-way radio signal from Earth to the Cassini spacecraft and back to Earth shows a fractional frequency shift y, proportional to the deviation angle of light ϑ. Because the distance of the spacecraft from the Sun is much larger than the distance from Earth to the Sun, we consider the expansion contribution to ϑ. Thus $y_{\Phi_{HOG},\Psi_{HOG}}$ is

$$y_{\Phi_{HOG},\Psi_{HOG}} = 2v_{Earth}[\varphi(\rho_0, \rho_{Earth}) + \varphi(\rho_0, \rho_{Cassini})], \tag{A14}$$

where v_{Earth} is the transverse velocity of the Earth. The measurement on $y \sim 10^{-10}$, has experimental error of $\Delta y_{exp} \sim 10^{-14}$. So, the modification for $y_{\Phi_{HOG},\Psi_{HOG}}$ needs to be $\lesssim \Delta y_{exp}$, which gives the condition

$$\frac{y_{\Phi_{HOG},\Psi_{HOG}}}{y_{GR}} < 10^{-4}. \tag{A15}$$

This condition is satisfied by the ZCS and confirmed for the DFT models in the section IV.

4. Time delay of light constraint

For the gravitational time delay of a light signal propagating from ρ_0 to ρ we integrate equation (A7) with respect to ρ and derive

$$t = 2G_N M \int_{\rho_0}^{\rho} d\tilde{\rho}\Big(AB\Big(1 - \frac{A}{A_0}\frac{\rho_0^2}{\tilde{\rho}^2}\Big)\Big)^{-1/2}. \tag{A16}$$

Again, in the domain for first order in the expansion parameter, the above integral can be expanded in the limit for the solar system and in the limits ρ, $\rho_0 \gg 1$ to give, see, for example, [20, 28],

$$\begin{aligned} t(\rho_0, \rho) \approx 2G_N M\Big\{ &\frac{\sqrt{\rho_0 - 1}}{\sqrt{\rho_0}} \int_{\rho_0}^{\rho} d\tilde{\rho} \frac{\tilde{\rho}^2}{(\tilde{\rho} - 1)\sqrt{\tilde{\rho}^2 - \rho_0^2}} \\ &- \frac{1}{2}\int_{\rho_0}^{\rho} d\tilde{\rho}\Big[\rho \frac{\tilde{\rho}^2(\Phi_{HOG}(\tilde{\rho}) + \Psi_{HOG}(\tilde{\rho})) - \rho_0^2(2\Phi_{HOG}(\tilde{\rho}) + \Psi_{HOG}(\tilde{\rho})) + \Phi_{HOG}(\rho_0)\rho_0^2}{(\tilde{\rho}^2 - \rho_0^2)^{3/2}}\Big]\Big\}, \end{aligned} \tag{A17}$$

with ρ for the position of the spacecraft. The term in (A17) that corresponds to the Φ_{HOG} and Ψ_{HOG} contribution again contains hypergeometric functions in its solution which only have simple forms for some values of Φ_{HOG} and Ψ_{HOG}, see [20] and below. For GR, from equation (A17), the difference between two points ρ_1 and ρ_2 can be given by

$$\Delta t_{GR} \approx 2G_N M \ln\Big(\frac{4\rho_1\rho_2}{\rho_0^2}\Big). \tag{A18}$$

The measurement of the ratio of experimental time delay for a light signal from the Viking Mars Mission to the GR predicted value is given by $\Delta t_{exp}/\Delta t_{GR} = 1.000 \pm 0.001$ [34], corresponding to a bound for the Φ_{HOG} and Ψ_{HOG} contribution as

$$\Big|\frac{\Delta t_{\Phi_{HOG},\Psi_{HOG}}}{\Delta t_{GR}}\Big| < 10^{-3}. \tag{A19}$$

We use this constraint in the section IV to put time delay constraints on the ZCS and verify the constraints on the DFT models.

5. Perihelion shift of planets constraint

Now the Lagrangian for a massive particle is given, for example [26], as

$$\mathcal{L} = \frac{1}{2}A\dot{t}^2 - \frac{1}{2}B^{-1}\dot{r}^2 - \frac{1}{2}r^2\dot{\varphi}^2 = \frac{1}{2}, \tag{A20}$$

where overdot is differentiation with respect to the affine parameter. From the Euler-Lagrange equations, the two first integrals are

$$A\dot{t} = k, \tag{A21}$$

and

$$r^2\dot{\varphi} = h. \tag{A22}$$

In the usual way, [26], we define $u = 1/r$, and derive the differential equation,

$$\frac{d^2u}{d\varphi^2} + u - \frac{G_N M}{h^2} = \frac{G_N M k^2}{h^2}\frac{B}{A}\Big(\frac{1}{B}\frac{dB}{du} - \frac{1}{A}\frac{dA}{du}\Big)$$
$$-\frac{1}{2}\frac{dB}{du}u^2 - (B-1)u - \frac{G_N M}{h^2}\Big(\frac{dB}{du} + 1\Big). \tag{A23}$$

Setting the right hand side of equation (A23) is zero, and we can write

$$\frac{d^2u}{d\varphi^2} + u = \frac{G_N M}{h^2} \tag{A24}$$

which has the solution,

$$u = \frac{G_N M}{h^2}(1 + \delta\cos(\phi - \phi_0)), \tag{A25}$$

and δ represents a conic eccentricity of range $0 < \delta < 1$, with focus at the origin of r, therefore all Newtonian orbits are conics, or in the case of planets, ellipses because $\delta < 1$. Again, from the Lagrangian and equations (A21) and (A22) , we find

$$\dot{r} = B\Big(\frac{k^2}{A} - \frac{h^2}{r^2} - 1\Big) \tag{A26}$$

which for the minimum distance $\rho_0 = r_0/(2G_N M)$, we have

$$\frac{G_N M k^2}{h^2} = A(\rho_0)\Big(\frac{1}{2\rho_0^2} + \frac{G_N M}{h^2}\Big) \approx \frac{1}{2\rho_0^2} + \frac{G_N M}{h^2}. \tag{A27}$$

From equation (A27), see [26], we can determine the orbit will be strictly periodic due to the constant values of k and h determining the strict orbit for r_0, together with each cycle r and $\dot{r}$ will have the same initial conditions for the differential equation, so the orbit repeats exactly. We expand (A23) in terms of the expansion parameter term once the form of the Φ_{HOG} and Ψ_{HOG} contributions are known from solving the field equations for the ZCS and DFT models of section IV.

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