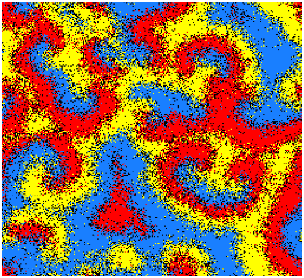
Quenched Spatial Disorder in Cyclic Three-Species Predator-Prey Models

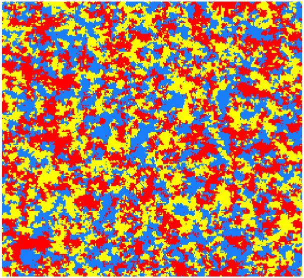
Qian He and Uwe C. Täuber

Department of Physics, Virginia Tech

Stochastic Rock–Paper–Scissors Game and Lotka–Volterra Model

- Two-dimensional Rock–Paper–Scissors(RPS) systems: coexistence of species, species clustering.





- Spatial variability of predation rates enhances the fitness of both predator and prey in two-species Lotka–Volterra model.

U. Dobramysl and U. C. Täuber, *Phys. Rev. L* **101** 258102 (2008), T. Reichenbach *et.al*, *Nature* **448** (2007),

Q. He, M. Mobilia, U. C. Täuber, *Phys. Rev. E* **82**, 051909 (2010), Q. He, M. Mobilia, U. C. Täuber, *Eur. Phys. J. B* **82**, 97 (2011).

Stochastic RPS model with conservation law

- Rock–Paper–Scissors game with conservation law (zero-sum):

$$\begin{aligned} A+B &\to A+A \quad \text{with rate } k_a \,, \\ B+C &\to B+B \quad \text{with rate } k_b \,, \\ C+A &\to C+C \quad \text{with rate } k_c \,. \end{aligned}$$

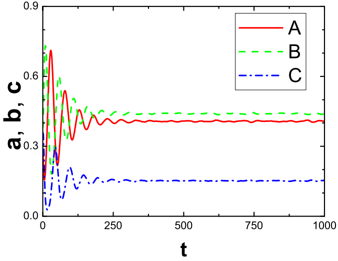
- coexistence state: $\rho(k_b, k_c, k_a)/(k_a + k_b + k_c)$, where ρ is the overall population density; conservation law holds $\Rightarrow$ no spiral patterns.

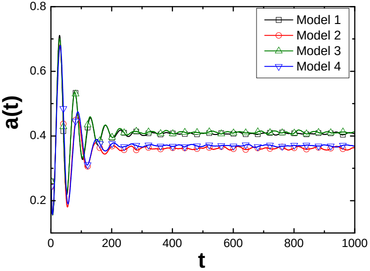
M. Pletomäki *et.al*, Phys. Rev. E. 78, 031906(2008).

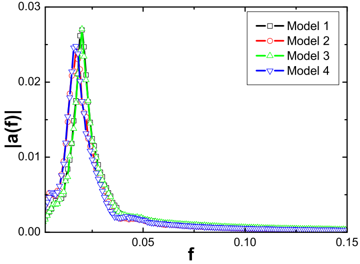
- $k_a \sim k_b \sim k_c \Rightarrow$ coexistence state far away from “corners” of configuration space.
- Model variants:

Model 1	$k_a = 0.2, k_b = 0.5, k_c = 0.8$; no site restriction
Model 2	$k_a = 0.2, k_b = 0.5, k_c = 0.8$; at most one particle per site
Model 3	$k_a \in [0, 0.4]$; no site restriction
Model 4	$k_a \in [0, 0.4]$; at most one particle per site

2-D RPS systems with asymmetric rates

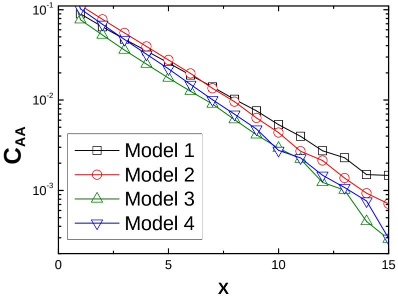


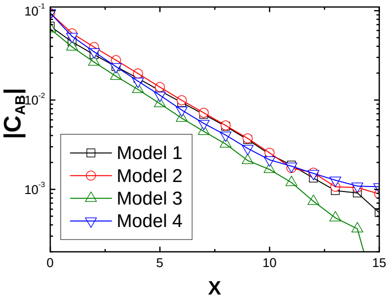




Static correlation functions

- In the state far away from the "corners", spatial disorder reduces correlation length l_{AA} and separation length l_{AB}, more localized population clusters; however, such enhancement in species fitness is minor. $C_{AB}(x,t) = \langle n_A(j+x,t)\, n_B(j,t)\rangle - a(t)\, b(t)$





"corner" RPS model and Lotka-Volterra model

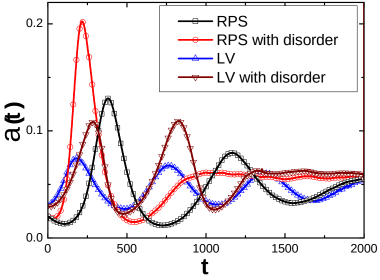
- coexistence state: $(a^*, b^*, c^*) = \rho(k_b\ ,\ k_c\ ,\ k_a)/(k_a + k_b + k_c)$.
- $k_a \gg k_b, k_c \quad \Rightarrow \quad c^* \approx \rho \gg a^*, b^* \quad \Rightarrow$ "corner" of configuration space.

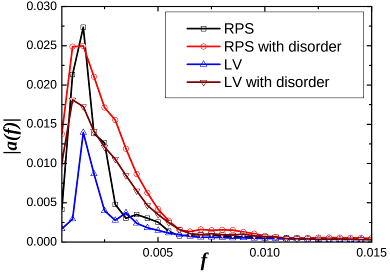
$$
\begin{array}{rcl} A + B & \to & A + A \\ B + C & \to & B + B \\ C + A & \to & C + C \end{array}
\quad \Longrightarrow \quad
\begin{array}{rl} A + B \to A + A & \text{with rate } \lambda = k_a \\ B \to B + B & \text{with rate } \sigma = k_b c^* \\ A \to \emptyset & \text{with rate } \mu = k_c c^* \end{array}
$$

- in spatial Lotka-Volterra model with at most one particle per site, critical predation rate $\lambda_c = \mu$, phase transition from absorbing state to active coexistence state.
- $\lambda \gg \mu, \sigma \quad \Rightarrow \quad \lambda \gg \lambda_c$, deep in the active coexistence state.

"corner" RPS model and Lotka-Volterra model

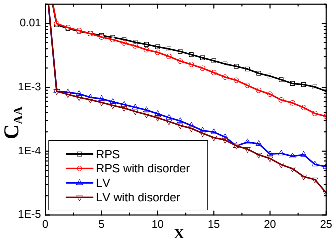
- $k_b = k_c = 0.03, k_a = 0.5 \;\Rightarrow\; \sigma = \mu \approx 0.027,\; \lambda = 0.5;$ disorder: $k_a, \lambda \sim N(0.5, 0.4)$.

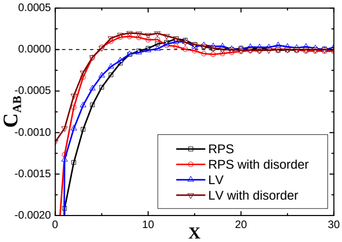




“corner” RPS model and Lotka-Volterra model

- due to quenched spatial disorder, the fitness of both minority species in “corner” RPS system is remarkably enhanced.





	a_s	b_s	$\tau_{A/B}$	l_{AA}	l_{BB}	l_{AB}
RPS model	0.053 ± 0.005	0.056 ± 0.006	$\sim$ 1800 mcs	13.2	16.0	~ 13.0
RPS with disorder	0.060 ± 0.003	0.061 ± 0.005	$\sim$ 800 mcs	9.4	13.0	~ 8.0
LV model	0.052 ± 0.006	0.055 ± 0.004	$\sim$ 2100 mcs	12.8	15.5	~ 13.0
LV with disorder	0.061 ± 0.002	0.063 ± 0.003	$\sim$ 1300 mcs	10.3	11.2	~ 9.0

Conclusion

- In RPS system far away from the “corners” of configuration space, spatial disorder has minor effect on the coexistence state.
- However, in “corner” RPS system with strongly asymmetric rates, spatial rate variability can remarkably enhance the fitness of both minority species as observed in two-species Lotka-Volterra system.
- In both mean-field analysis and numerical simulation, the evolutionary dynamics of coexistence state in two-species Lotka-Volterra model is well approximated by such “corner” RPS system.