Epidemic spreading on preferred degree adaptive networks

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NSF-DMR Materials Theory

Motivation

- Most epidemic models are based on regular lattices, *Erdös-Renyi or scale free networks* , with active nodes and static links.
  - ➢ Active nodes: Links influence the nodes (epidemics, opinions, Ising model)
  - ➢ Active links: network topology changes

- Preferred degree networks: Nodes (individuals) have a preferred number of links (social connection). Make more realistic networks.

- Static epidemic model: active nodes, static links
- Adaptive model: active nodes and active links.
  - ➢ Feedback between Dynamics <u>on</u> the network and Dynamics <u>of</u> the network.

- Two community Coupled networks: Epidemic propagation from one community to other.

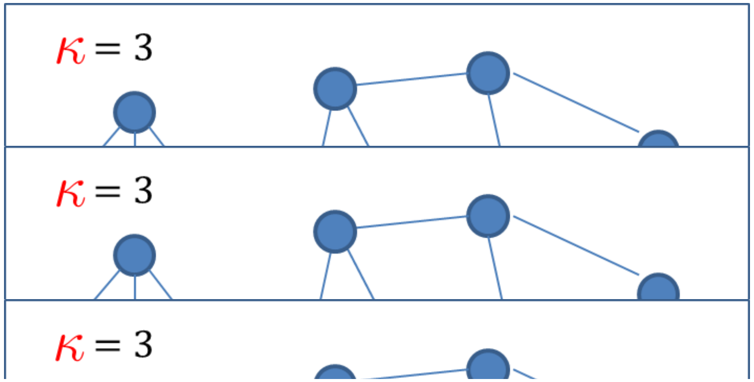
Preferred degree networks

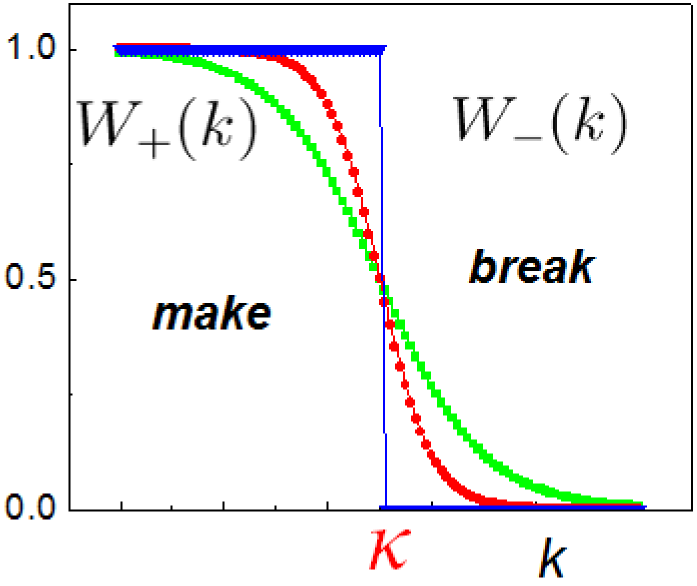
- Nodes have preferred degree κ (Initially homogeneous).

- Select a random node j with degree k_j
  - Make a link: $W_+(k_j)$
  - Break a link : $W_-(k_j)$

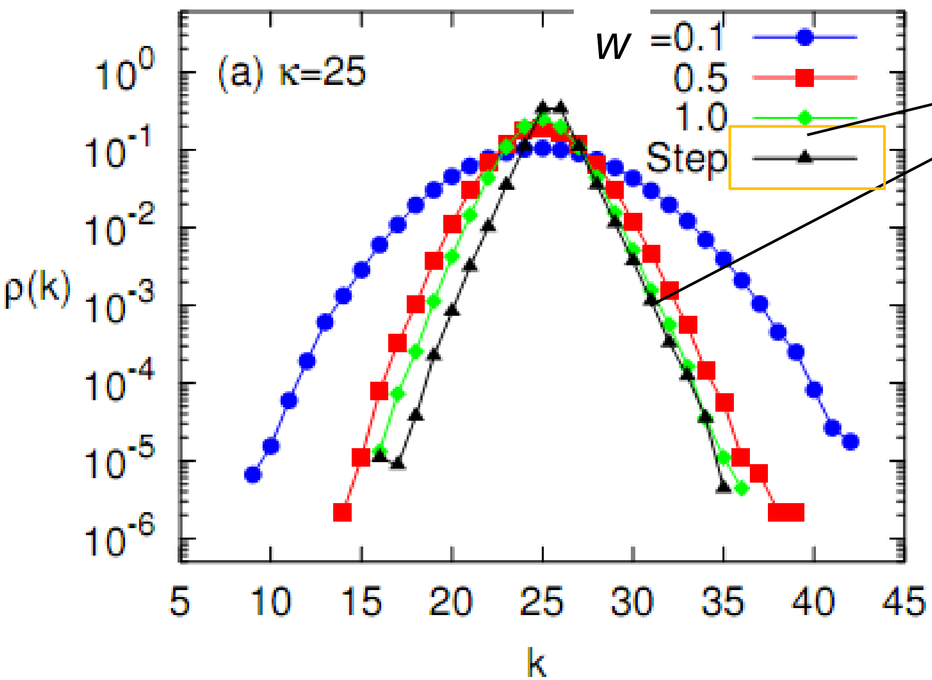
  $W_-(k_j) = 1 - W_+(k_j)$

- $W_+(k_j) = \frac{1}{1+\exp\left(\frac{k-\kappa}{w}\right)}$





Degree distribution



Two tailed exponential (Laplace) distribution

$$\rho(k) = \frac{\exp[-(k-\kappa)/\sigma]}{2\sigma}$$

All have same preferred degree κ

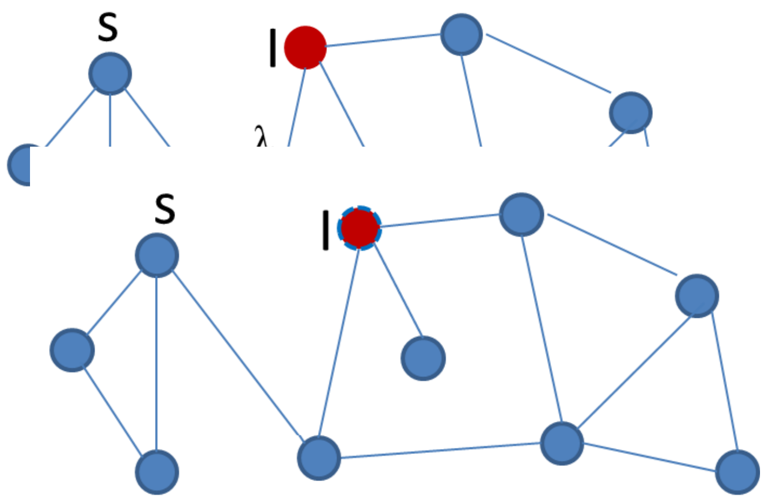
SIS disease dynamics

- Individuals are either Susceptible (S) or Infected (I)

- Susceptible meets an Infected and gets infected with rate λ

$$S + I \xrightarrow{\lambda} I + I$$

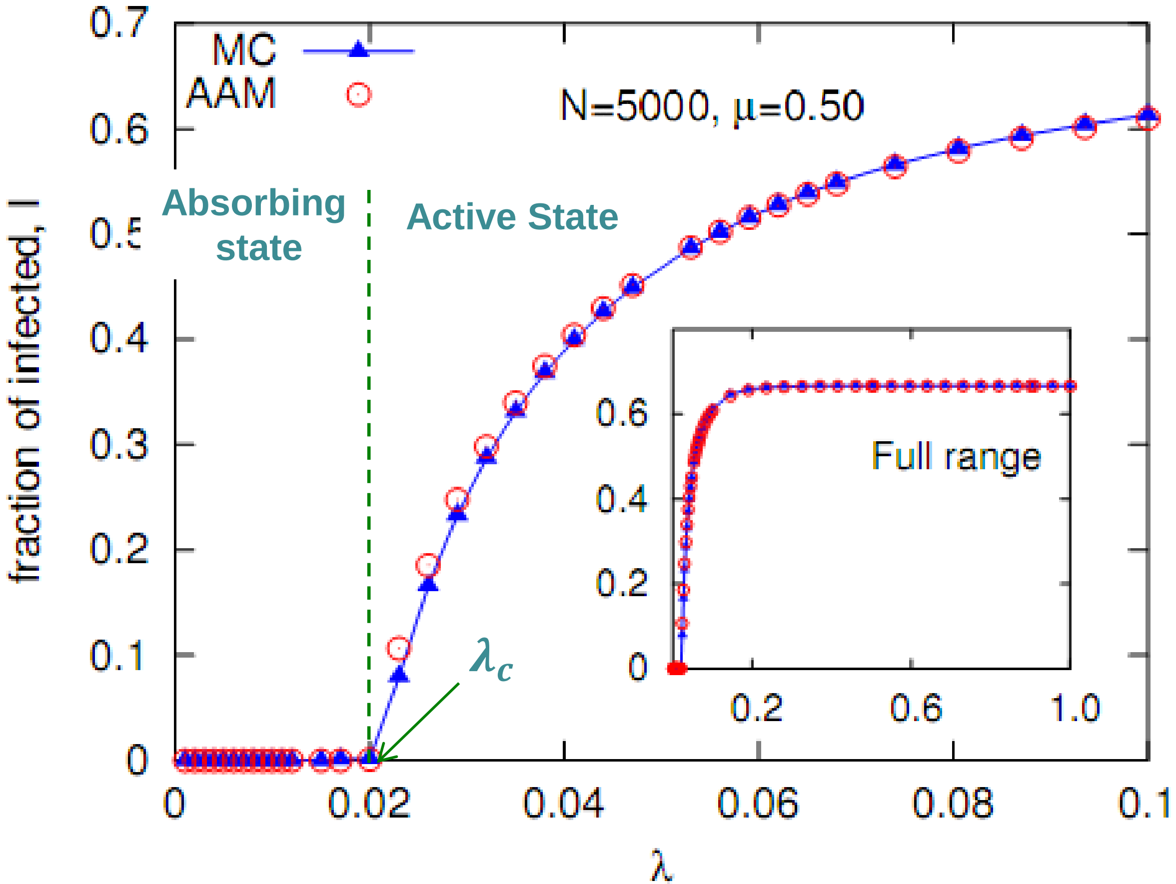
- All infected reco



- Neglect all spatial structure.

SIS dynamics on static preferred degree network

Stationary Infection : $I(t \to \infty; \lambda)$



$$\frac{\lambda_c}{\mu} \approx \frac{\langle k \rangle}{\langle k^2 \rangle}$$

Heterogeneous mean field theory

Adaptive behavior

- People exhibit 'social distancing' when there is a raging epidemic.

- Modifies the model parameters: Social distancing causes infection probability to decrease ($\lambda \downarrow$)

- Modifies the contact structure (Network).

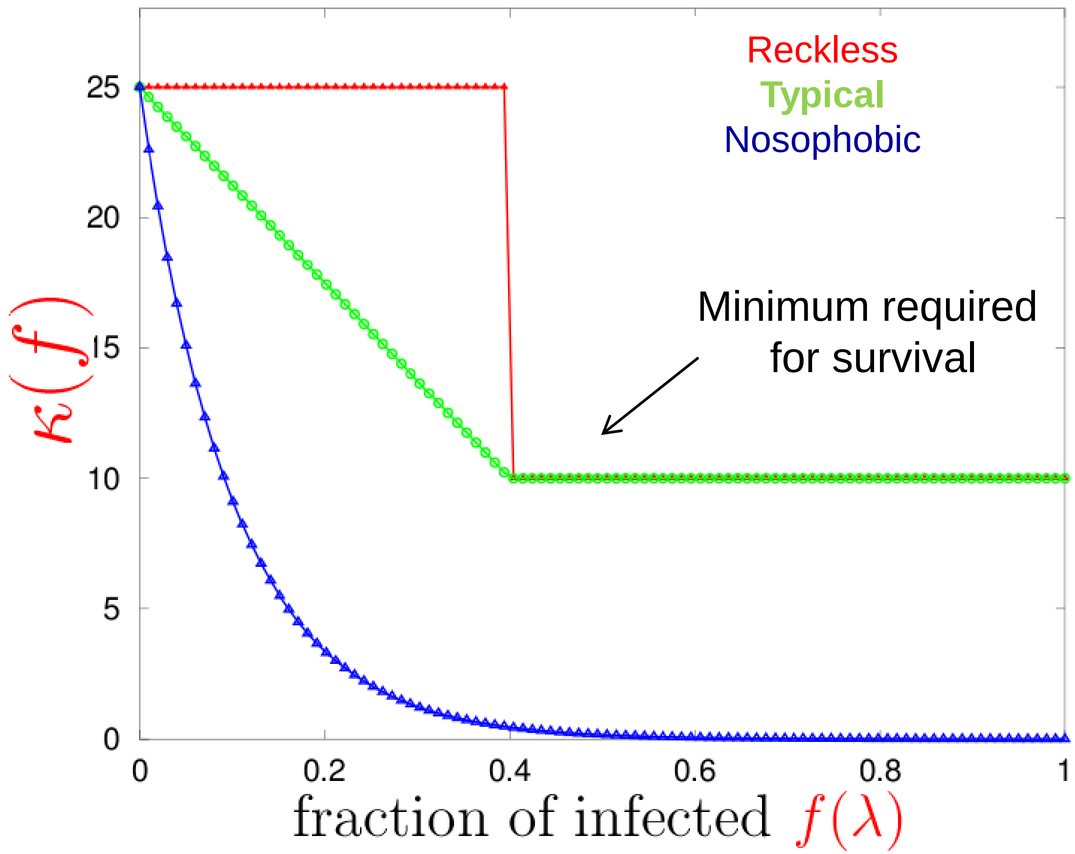
  Adaptive preferred degree $\kappa(f)$

- Our model: κ depends on global information directly linked to disease prevalence. (Flu, SARS etc).

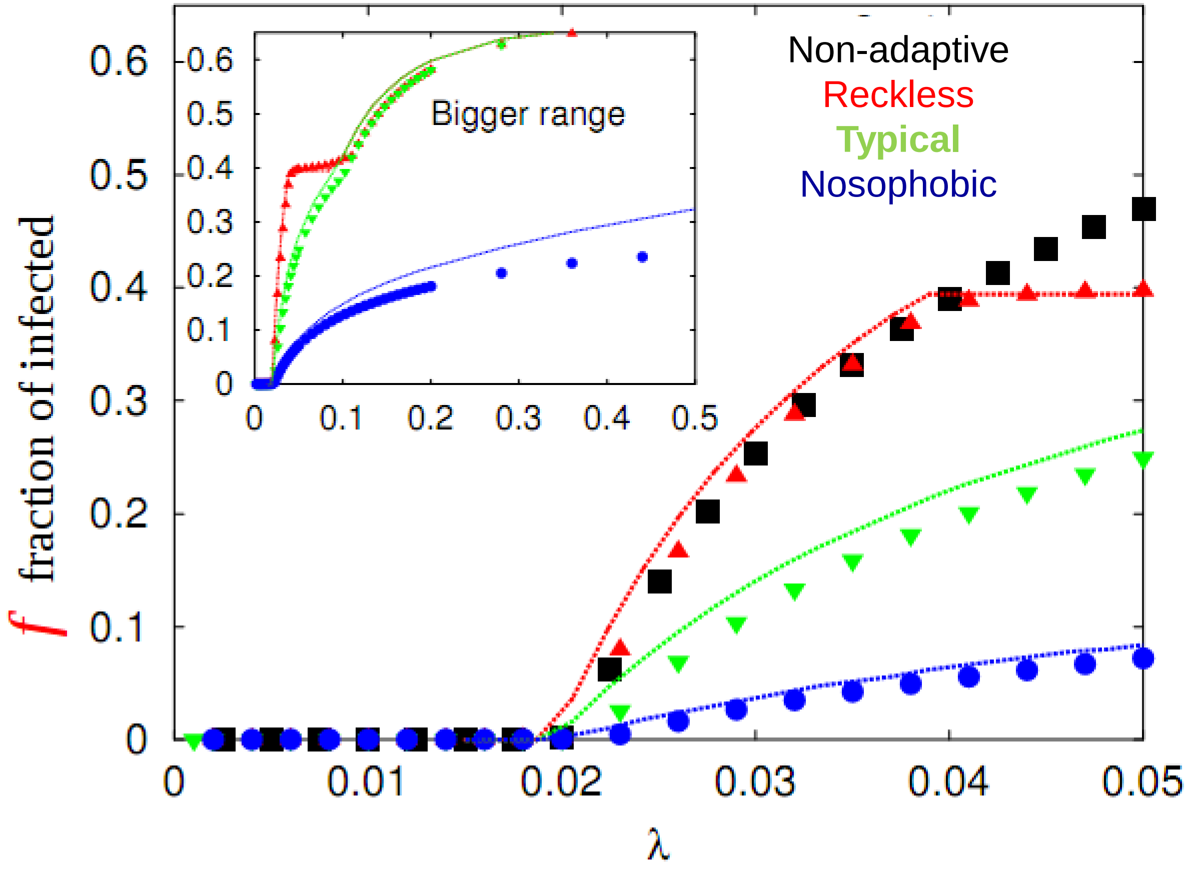
Adaptive preferred degree $\kappa(f)$

$\kappa(f)$: Preferred degree of people depends on the state of the epidemic $f(t)$(fraction of infected at time t).

- Reckless
- Typical
- Nosophobic (highly scared)



Infection phase diagram : Adaptive SIS



Adaptive SIS model with varying preferred degree **doesn't change the epidemic threshold, but changes the level of infection** in the active state.

Mean field analysis

- $\kappa(I) = \kappa_m f(I)$

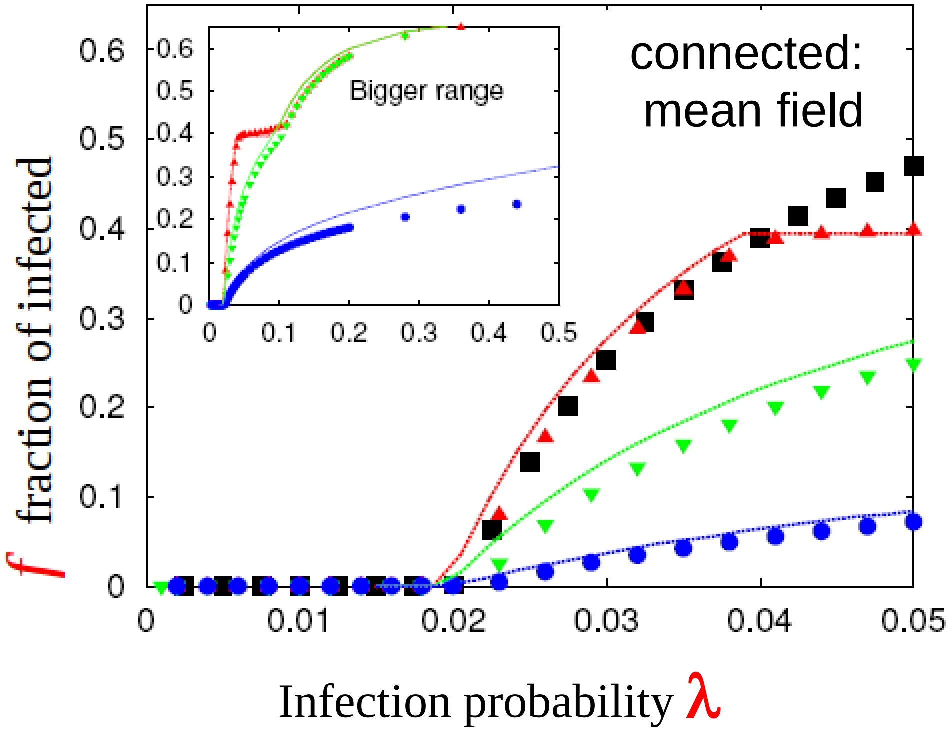
- Mean field equation

$$\frac{dI}{dt} = -\mu I + \lambda_{eff}(I)S$$

- $\lambda_{eff}(I) = 1 - (1-\lambda)^{I\kappa(I)}$

  Assumption: fraction of infected is same around every node.

- Steady state solution $\frac{dI}{dt} = 0$



Mean field predictions closely match simulations

Heterogeneous preferred degree

- Two community network (extroverts and introverts) different preferred degree and coupling
  - Simplest network with heterogeneous preference

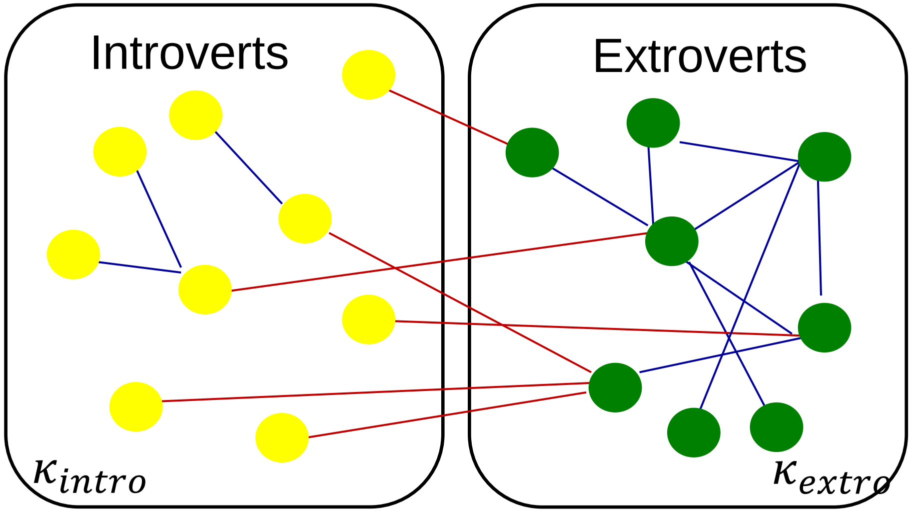
- Questions:
  - How does disease spread across different communities?
  - Are extroverts more prone to contagious diseases?
  - How does disease depend on the interaction/coupling between the communities?

Two community network

Decision to make *internal* (within) or *external* (across) link

$$p_{in} = \frac{k_{ex}}{k_{ex} + \alpha k_{in}} \qquad p_{ex} = 1 - p_{in}$$

k_{ex} external (cross) degree $\qquad$ k_{in} internal degree

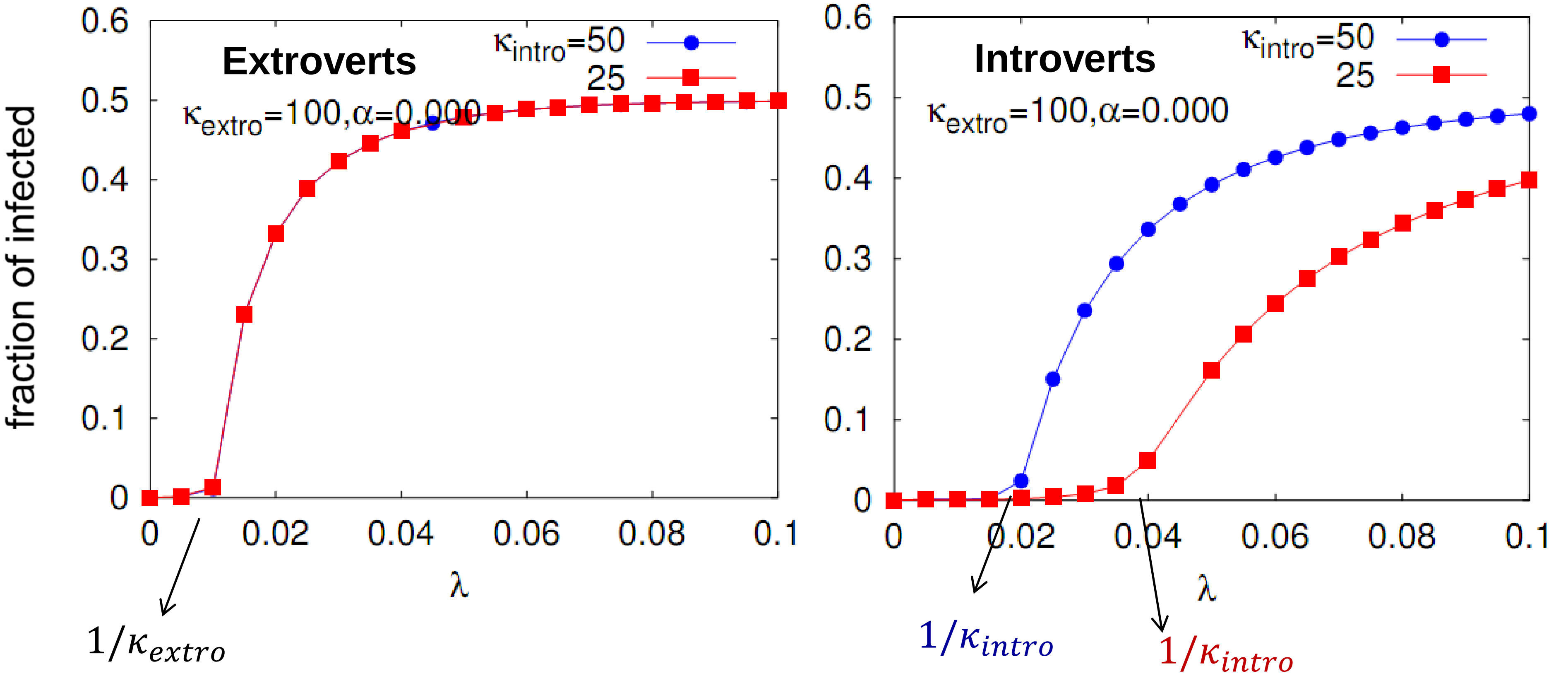


α: controls cross links

$$\alpha = \begin{cases} 0, & p_{in} = 1 \\ \infty, & p_{in} = 0 \end{cases}$$

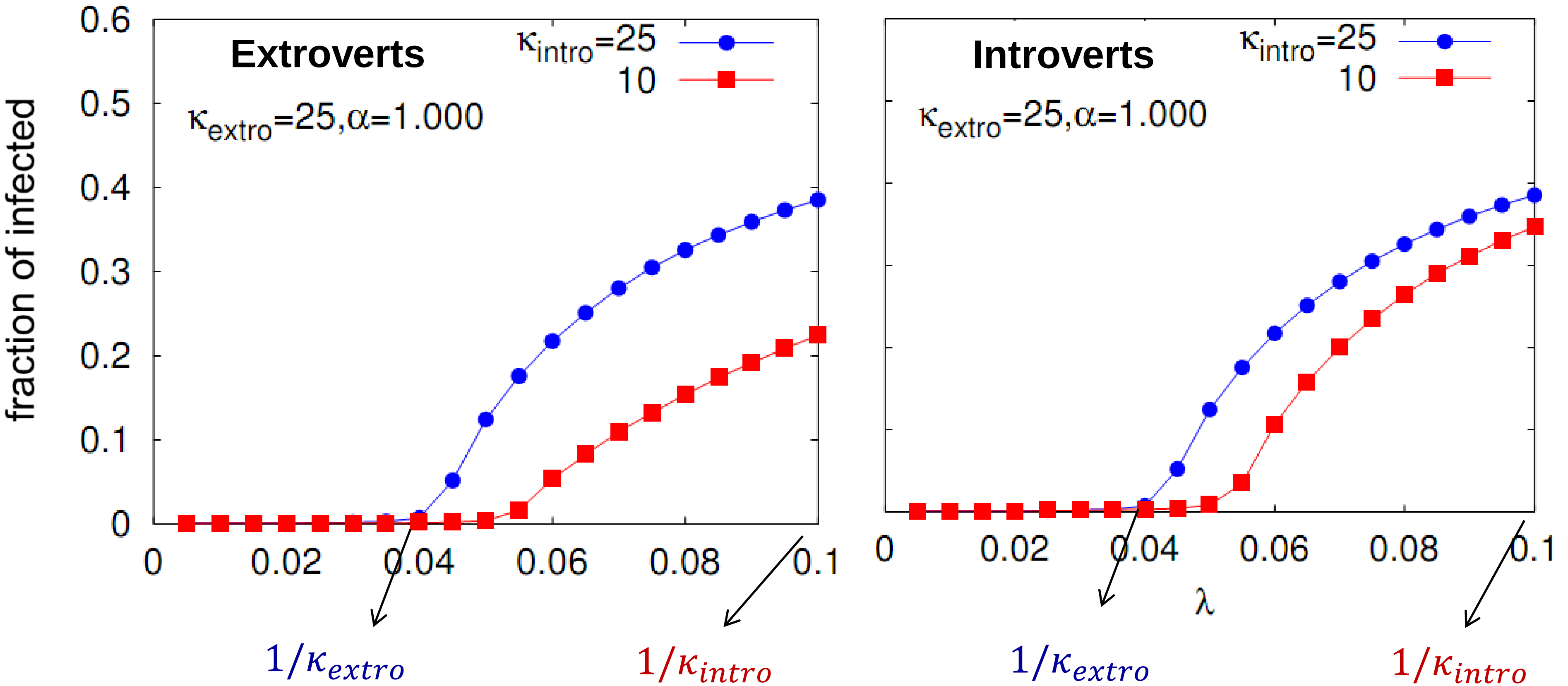
Results- uncoupled communities

$\alpha = 0$ two transition points



Coupled communities

$\alpha \neq 0$ transition point governed by **extroverts** and is not sharp



Summary

- Behavior SIS model with active nodes and active links. Feedback between dynamics on the network and dynamics of the network.

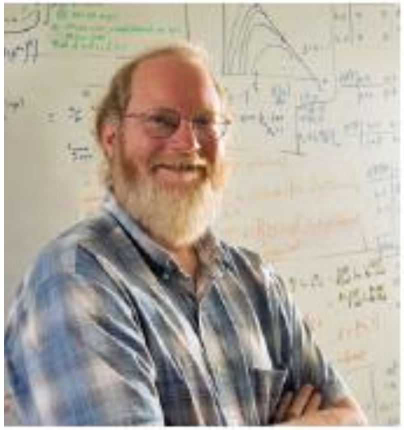
- Adaptive SIS model with varying preferred degree doesn't change the epidemic threshold, but changes the level of infection in the active state.

- Generalizations to more realistic problems:
  - Population with different preferred degrees:
  - Epidemic on a realistic model network with different degree distribution
  - Population with mixed behavior:

$$N = N_{reckless} + N_{typical} + N_{nosophobic}$$

Collaborators

R. K. P. Zia, Beate Schmittmann and Wenjia Liu



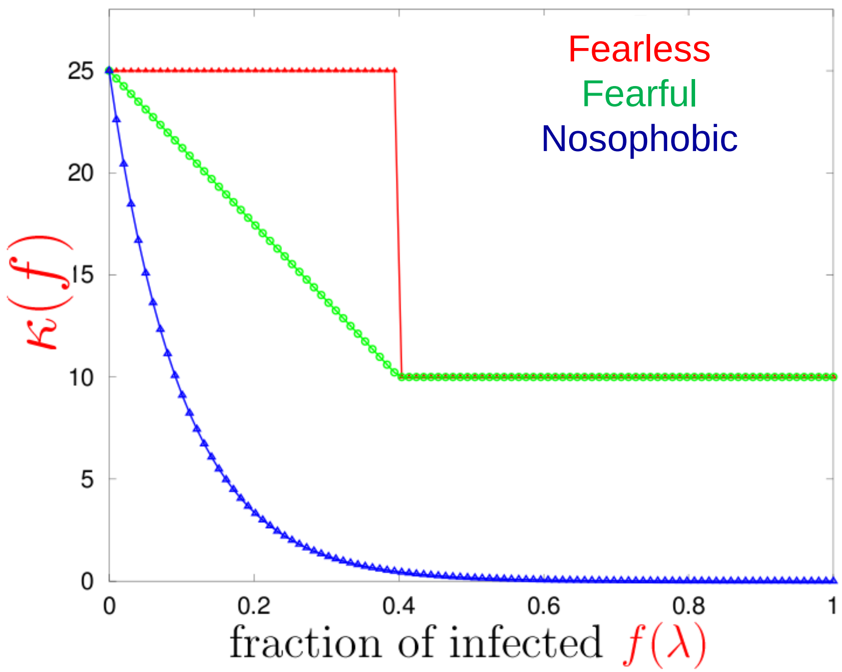
Stephen Eubank , Thierry Platini

Thank you!

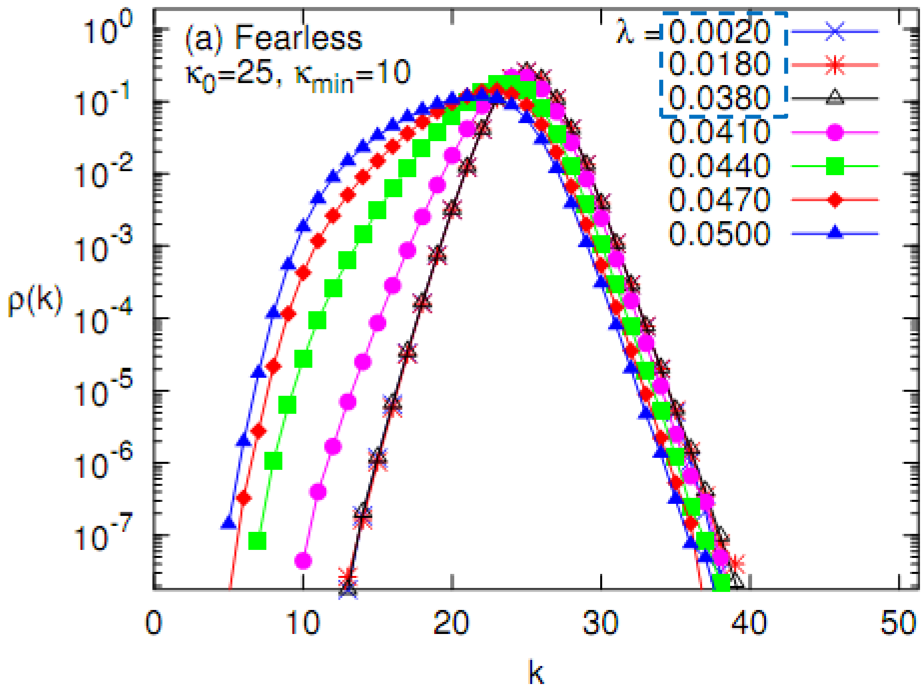
Preferred degree networks

- Make new friends, break old ties (establish/cut contacts) according to some *preference*
- Preferences can be dynamic.
- Preference can vary from person to person

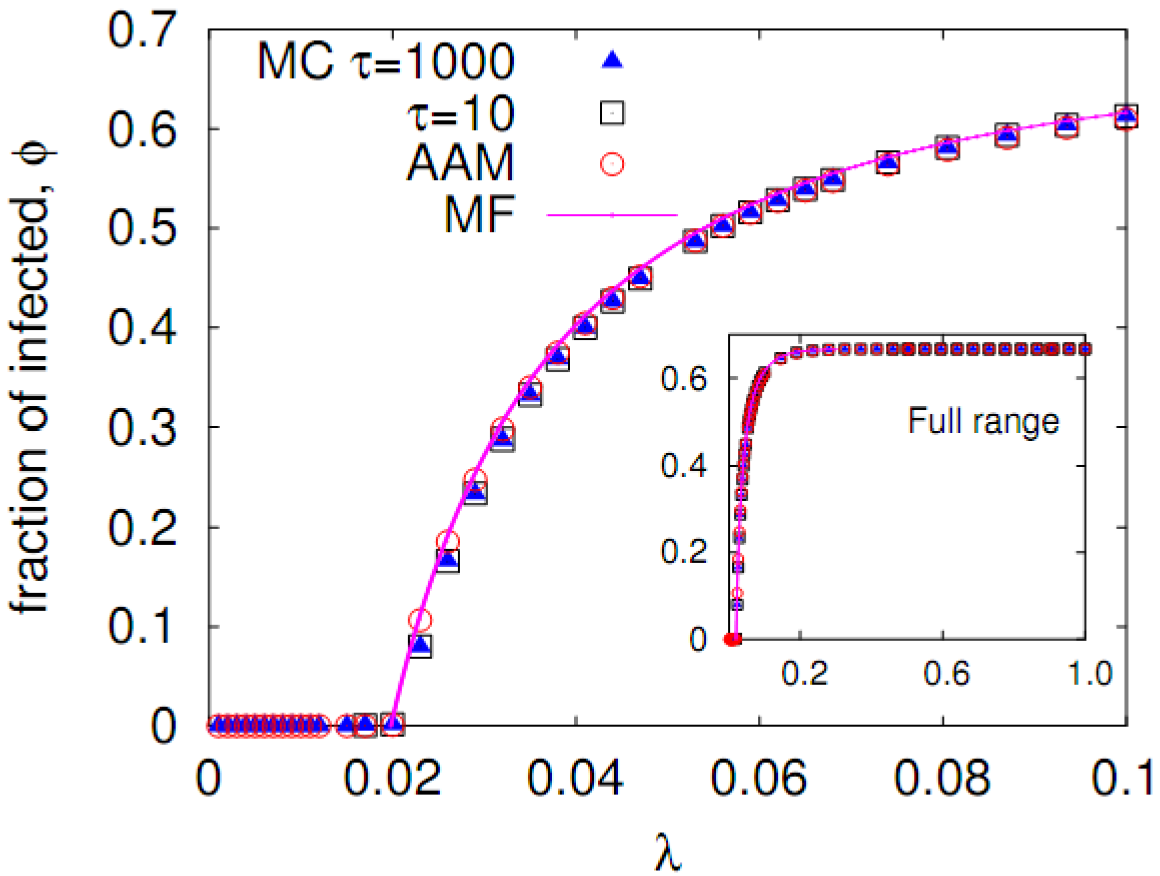
Effect on Network



Fearless



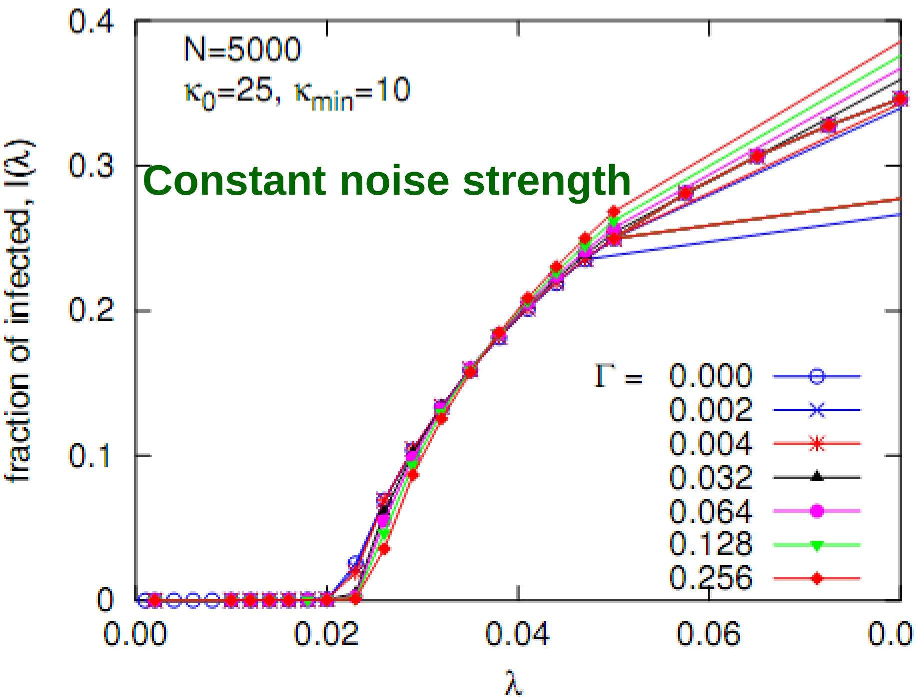
Degree distribution peak centers at $\kappa(f)$, corresponding to fraction of infected $f(\lambda)$.

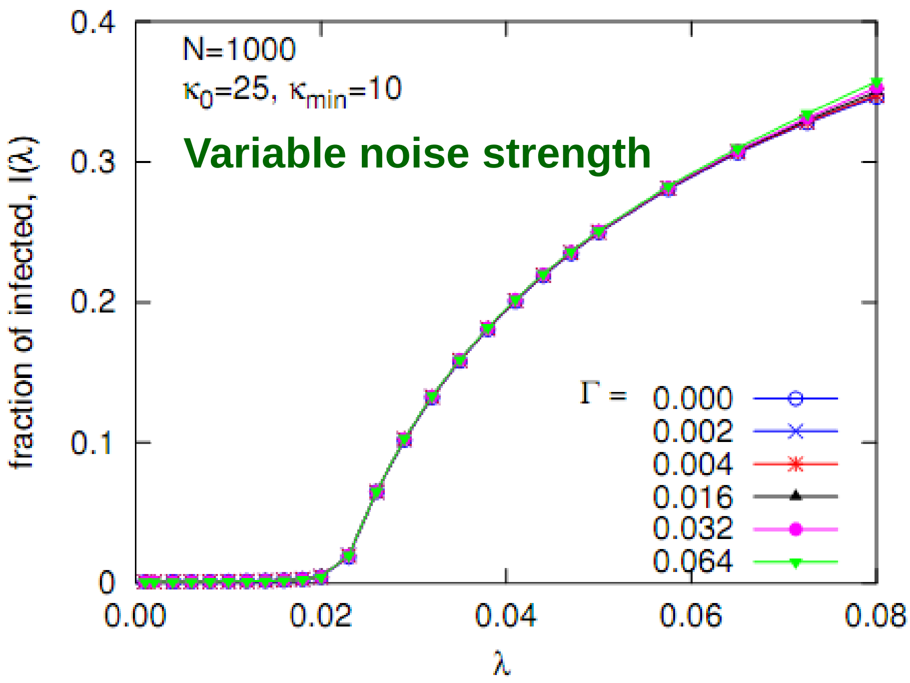


SIS phase diagram near transition point, comparison with mean field theories.

Noisy perception

$$I_{perc}(t) = I(t) + \eta(t)$$





$$\langle \eta(t)\eta(t') \rangle = \Gamma \delta(t-t')$$

$$\langle \eta(t)\eta(t') \rangle = \Gamma I(t) \delta(t-t')$$