What do we know about the viscosity of QCD matter?

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• Near-Ideal Fluids & Elliptic Flow
• Shear-Viscosity of QCD Matter
• \( \eta/s \) of a Hadron Gas
• Improved Constraints on \( \eta/s \)
Near-Ideal Fluids & Elliptic Flow
• on April 18th, 2005, BNL announced in a press release that RHIC had created a new state of hot and dense matter which behaves like a nearly perfect liquid.

• how does one measure/calculate the properties of a near ideal liquid?

• are there any other near ideal liquid systems found in nature?
Viscosity:
- shear and bulk viscosity are defined as the coefficients in the expansion of the stress tensor in terms of the velocity fields:

\[ T_{ik} = \varepsilon u_i u_k + P (\delta_{ik} + u_i u_k) - \eta \left( \nabla_i u_k + \nabla_k u_i - \frac{2}{3} \delta_{ik} \nabla \cdot u \right) + \zeta \delta_{ik} \nabla \cdot u \]

- viscous RFD requires solving an additional 9 eqns. for the dissipative flows

Note:
- for quasi-particulate matter, viscosity decreases with increasing cross section
- for viscous RFD, the microscopic origin of viscosity is not relevant!
Collision Geometry: Elliptic Flow

- two nuclei collide rarely head-on, but mostly with an offset:
  - only matter in the overlap area gets compressed and heated up

Reaction plane

elliptic flow \( (v_2) \):
- gradients of almond-shape surface will lead to preferential emission in the reaction plane
- asymmetry out- vs. in-plane emission is quantified by 2\(^{nd}\) Fourier coefficient of angular distribution: \( v_2 \)

vRFD: good agreement with data for very small \( \eta/s \)
Elliptic flow: early creation

Most model calculations suggest that flow anisotropies are generated at the earliest stages of the expansion, on a timescale of $\sim 5 \text{ fm}/c$ if a QGP EoS is assumed.

shear-viscosity of QCD matter

Viscosity at RHIC

- Viscosity of QCD matter @ RHIC changes strongly with temperature & phase
- How can we quantify the viscosity of QCD matter?

Approximations not valid

• initial state
• pre-equilibrium
• QGP and hydrodynamic expansion
• hadronization
• hadronic phase and freeze-out

Large elliptic flow: near ideal fluid w/ small viscosity
Parton recombination: quasi-particle d.o.f.
Expanding hadron gas w/ increasing m.f.p.: large viscosity

η/s from Lattice QCD

The confines of the Euclidean Formulation:
• extracting η/s formally requires taking the zero momentum limit in an infinite spatial volume, which is numerically not possible...

preliminary estimates:
caution:
systematic errors are O(1)!

<table>
<thead>
<tr>
<th>T</th>
<th>1.58 T_c</th>
<th>2.32 T_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>η/s</td>
<td>0.2</td>
<td>0.26</td>
</tr>
</tbody>
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• calculating QCD transport coefficients on the Lattice has been identified as a Priority Research Direction by the DOE Office of Nuclear Physics and the Office of Advanced Scientific Computing Research (ASCR) in their report on Extreme-Scale Computing

Harvey B. Meyer: arXiv:0809.5202 [hep-lat]
AdS/CFT correspondence

• calculating viscosity and viscosity/entropy ratio too difficult in full QCD
• quantities are calculable in a related theory using string theory methods

model for QCD:

\[
N = 4 \text{ Super-Yang-Mills theory} \quad \longleftrightarrow \quad \text{a string theory in 5d AdS}
\]

\[
\text{finite temperature} \quad \longleftrightarrow \quad \text{black hole in AdS}_5
\]

\[
\text{large } N_C \text{ and strong coupling limit} \quad \longleftrightarrow \quad \text{classical gravity limit}
\]

› YM observables at infinite \(N_C\) and infinite coupling can be computed using classical gravity
› technique can be applied to dynamical and thermodynamic observables

in all theories with gravity-duals one finds: \(\frac{\eta}{s} = \frac{\hbar}{4\pi}\) (very small number!)

Caution:

• \(N=4\) SUSY YM is not QCD!
• no information on how low \(\eta/s\) is microscopically generated

Elliptic Flow at the LHC

**Implications for \( \eta/s \):**
- Despite rise in temperature, \( \eta/s \) has to remain small, on the same level as observed at RHIC!
- Can low value of \( \eta/s \) be reconciled with its known temperature dependence in the HTL calculations?
- \( T \)-dependence is logarithmic; lack of sensitivity for a 30% rise?
- Physics beyond the HTL limit: color fields?

**First data by the ALICE Collaboration:**
- \( v_2 \) vs. \( p_T \) virtually identical to RHIC data
- Rise in integrated \( v_2 \) vs. centrality due to increase in radial flow
- Charged particle multiplicity suggests a rise in temperature by 30% compared to RHIC (or a factor of approx. 2.9 in energy-density)

The sQGP Challenge: do quasi-particles drive $\eta/s$?

- the success of near ideal hydrodynamics has led the community to equate low viscosity with a vanishing mean free path and thus large parton cross sections: strongly interacting QGP (sQGP)

- does a small viscosity have to imply that matter is strongly interacting?
- consider effects of (turbulent) color fields?

microscopic kinetic theory:
$\eta$ is given by the rate of momentum transport:
$$\eta \approx \frac{1}{3} n \bar{p} \lambda_f = \frac{\bar{p}}{3\sigma_{tr}}$$

- microscopic transport with parton d.o.f. requires either unphysically large cross sections or a very specific implementation of the LPM effect to thermalize & create elliptic flow

Anomalous Viscosity: (see e.g. in Plasma-, Astro-, Biophysics)
- any contribution to the shear viscosity not explicitly resulting from momentum transport via a transport cross section

- can the QGP viscosity be anomalous?
  - soft, turbulent color fields generate anomalous transport coefficients, which may give the medium the character of a nearly perfect fluid even at moderately weak coupling.

Z. Xu & C. Greiner: talk @ QM2008
Anomalous vs. Collisional Viscosity

**Collisional Viscosity:**
- derived in HTL weak coupling limit

\[ \frac{\eta_C}{s} \approx \frac{5}{g^4 \ln g^{-1}} \]

**Anomalous Viscosity:**
- induced by turbulent color fields, due to momentum-space anisotropy

\[ \frac{\eta_A}{s} = \mathcal{O}(1) \left( \frac{N_c^2 - 1}{N_c} \right) \frac{T^6}{g^2 \langle B^2 \rangle \tau_m} \Rightarrow \frac{\eta_A}{s} \sim \frac{1}{\langle B^2 \rangle} \]

- with ansatz for fields:

\[ \frac{\eta_A}{s} = \bar{c}_0 \left( \frac{T}{g^2 |\nabla u|} \right)^{3/5} \]

- for reasonable values of g: \( \eta_A < \eta_C \)

- sum-rule for system w/ 2 viscosities:
  (derived from variational principle)

\[ \frac{1}{\eta} = \frac{1}{\eta_A} + \frac{1}{\eta_C} \]

- total viscosity dominated by \( \eta_A \)
Collisional vs. Anomalous Viscosity

- smaller viscosity dominates in system w/ 2 viscosities!

Temperature evolution:
- anomalous viscosity dominates total shear viscosity during early QGP evolution
- a small viscosity does not necessarily imply strongly interacting matter!
η/s of a Hadron Gas
Shear Viscosity: Linear Transport Equation & Green - Kubo Formalism

Mechanical definition of shear viscosity:
- Application of a shear force to a system gives rise to a non-zero value of the $xy$-component of the pressure tensor $P_{xy}$. $P_{xy}$ is then related to the velocity flow field via the shear viscosity coefficient $\eta$:

$$P_{xy} = -\eta \frac{\partial v_x}{\partial y}$$

- A similar linear transport equation can be defined for other transport coefficients: thermal conductivity, diffusion ...

• Using linear-response theory, the Green-Kubo relations for the shear viscosity can be derived, expressing $\eta$ as an integral of an near-equilibrium time correlation function of the stress-energy tensor:

$$\eta = \frac{1}{T} \int d^3 r \int_0^\infty dt \left\langle \pi^{xy}(\vec{0}, 0) \pi^{xy}(\vec{r}, t) \right\rangle_{\text{equil}}$$

with the stress-energy tensor: $\pi^{\mu\nu}(\vec{r}, t) = \int d^3 p \frac{p^\mu p^\nu}{p^0} f(x, p)$.
for particles in a fixed volume, the stress energy tensor discretizes

\[
\pi^{xy} = \frac{1}{V} \sum_{j=1}^{N_{\text{part}}} \frac{p^x(j)p^y(j)}{p^0(j)}
\]

and the Green–Kubo formula reads:

\[
\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(0) \pi^{xy}(t) \rangle
\]

- evaluating the correlator numerically, e.g. in UrQMD, one empirically finds an exponential decay as function of time

- using the following ansatz, one can extract the relaxation time \(\tau_\pi\):

\[
\langle \pi^{xy}(0) \pi^{xy}(t) \rangle \propto \exp\left(-\frac{t}{\tau_\pi}\right)
\]

- the shear viscosity then can be calculated from known/extracted quantities:

\[
\eta = \frac{\tau_\pi}{T} \frac{V}{\langle \pi^{xy}(0)^2 \rangle}
\]

Entropy:

- extract thermodynamic quantities via:

\[
P = \frac{1}{3V} \sum_{j=1}^{N_{\text{part}}} \frac{|\vec{p}|^2(j)}{p^0(j)}
\]

\[
\epsilon = \frac{1}{V} \sum_{j=1}^{N_{\text{part}}} p^0(j)
\]

- use Gibbs relation (with chem. pot. extracted via SM)

\[
S_{\text{Gibbs}} = \left( \frac{\epsilon + P - \mu_i \rho_i}{T} \right)
\]

\( \eta/s \) of a Hadron Gas in & out of Equilibrium

**first reliable calculation of of \( \eta/s \) for a full hadron gas including baryons and anti-baryons:**

- breakdown of vRFD in the hadronic phase?
- what are the consequences for \( \eta/s \) in the deconfined phase?

- RFD freeze-out temperature to reproduce spectral shapes: \( \sim 110 \text{ MeV} \)
- Statistical Model temperature fits to hadron yields/ratios: \( \sim 160 \text{ MeV} \)
  - separation of chemical and kinetic freeze-out in the hadronic phase!
  - confirmed by hybrid models
  - implies non-unit species-dependent fugacities in RFD

- non-unit fugacities reduce \( \eta/s \) by a factor of two to \( \eta/s \approx 0.5 \)
  - **improved constraint:** \( \eta/s \) needs to be significantly lower in deconfined phase for vRFD to reproduce elliptic flow!

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Improved Constraints on $\eta/s$
Viscous Hydro + Micro Model

Viscous RFD
- ideally suited for dense systems
  - model early QGP reaction stage
- well defined Equation of State
- parameters:
  - initial conditions
  - Equation of State including PCE for HG
  - viscosity over entropy-density ratio

Matching condition:
- use same set of hadronic states for EoS as in UrQMD
- generate hadrons in each cell using local T and \( \mu_B \)
- take off-equilibrium distribution functions into account

Micro. transport (UrQMD)
- no equilibrium assumptions
  - model break-up stage
  - calculate freeze-out
  - includes viscosity in hadronic phase
- parameters:
  - (total/partial) cross sections

D. Teaney et al, nucl-th/0110037
**Improved Extraction of $\eta/s$**

### Viscous RFD Improvements:
- use fluctuating initial conditions
- state-of-the-art Lattice EoS, including PCE in hadronic phase prior to $T_{sw}$
- constrain $\tau_0$ and $s$ with fit to data for $dN/dy$ and spectra

### Milestones:
- eccentricity scaling yields same centrality dependence for MC-KLN & MC-Glauber
- centrality dependence agrees with data
- realistic treatment of hadronic phase, including viscosity and freeze-out

#### Slope of $v_2/\varepsilon$
- cannot distinguish between KLN and Glauber initial conditions
- QGP viscosity: $1/(4\pi) < \eta/s < 2/(4\pi)$ [Glauber] & $2/(4\pi) < \eta/s < 3/(4\pi)$ [KLN]
Heavy-Ion collisions at RHIC have produced a state of matter which can be called the **Quark-Gluon-Plasma:**

- the properties of the QGP can be characterized by its transport coefficients, such as $\eta/s$ and $q$-hat
  - near ideal fluidity: the smallest value of $\eta/s$ observed in nature
  - $\eta/s$ may strongly depend on temperature and phase of QCD matter

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Transition from Discovery Phase to Exploratory Phase and onwards to Precision Spectroscopy of the QGP:

- improved constraints via hybrid viscous RFD + UrQMD calculation, that fully accounts for large viscosity of hadronic phase
- largest uncertainty currently due to lack of knowledge on the structure of the initial conditions
- need to establish the physics driving the small value of $\eta/s$ (e.g. particles vs. fields) in the QGP phase
Thank you! Any questions?
The End
Nonabelian Vlasov equations describe interaction of “hard” (i.e. particle) and “soft” color field modes and generate the “hard-thermal loop” effective theory:

\[
\frac{dp^\mu}{d\tau} = g Q^a F^{a\mu\nu} u_\nu \\
\frac{dQ^a}{d\tau} = g f_{abc} A^{b\nu} u_\nu Q^c \\
D_\mu F^{\mu\nu} = g J^\nu
\]

with \( J^\nu(x) = \sum_i \int d\tau Q_i(\tau) u_i^\nu(\tau) \delta(x - x_i(\tau)) \)

Effective HTL theory permits systematic study of instabilities of “soft” color fields

find HTL modes for anisotropic distribution:
\[
f(\vec{p}, \vec{r}) \approx \left( e^{\beta u \cdot p} - f_1(\vec{p}, \vec{r}) \mp 1 \right)^{-1}
\]

- for most \( f_1 \neq 0 \) there exist unstable modes
- energy-density and growth rate of unstable modes can be calculated:

S. Mrowczynski, PLB 314, 118 (1993)
Anomalous Viscosity Derivation: Sketch

• linear Response: connect \( \eta \) with momentum anisotropy \( \Delta \):

\[
\eta = - \frac{1}{15T} \int \frac{d^3 p}{(2\pi)^3} \frac{\vec{p}^4}{E_P^2} \bar{\Delta}(p) \frac{\partial f}{\partial E_p} \quad \text{with} \quad f(\vec{p}, \vec{r}) \approx \left(e^{\beta u \cdot p - f_1(\vec{p}, \vec{r})} - 1\right)^{-1}
\]

and

\[
f_1(\vec{p}, \vec{r}) = - \frac{\bar{\Delta}(p)}{E_P T^2} p_i p_j (\nabla u)_{ij}
\]

• use color Vlasov-Boltzmann Eqn. to solve for \( f \) and \( \Delta \):

\[
u^\mu \frac{\partial}{\partial x^\mu} f(\vec{r}, \vec{p}, t) + gF^a \cdot \nabla_p f^a(\vec{r}, \vec{p}, t) + C[f] = 0
\]

with

\[
F^a = \mathcal{E}^a + \mathbf{v} \times \mathbf{B}^a
\]

• turbulent color field assumption:

• ensemble average over fields:

\[
\langle B_i^a(x) U_{ab}(x, x') B_j^b(x') \rangle = \langle B_i^a B_j^b \rangle \Phi^{(\text{mag})}(|t - t'|) \Phi^{(\text{mag})}(|x - x'|)
\]

• diffusive Vlasov-Boltzmann Eqn:

\[
u^\mu \frac{\partial}{\partial x^\mu} \bar{f} - \nabla_p \cdot D \cdot \nabla_p \bar{f} + C[\bar{f}] = 0
\]

• example: anomalous viscosity in case of transverse magnetic fields

\[
\eta_A^{(g)} = 16\zeta(6)N_c^2 - 1)^2 \frac{T^6}{\pi^2 N_c g^2 \langle \mathbf{B}^2 \rangle \tau_{\text{mag}}} \quad \eta_A^{(q)} = 62\zeta(6)N_c^2 N_f \frac{T^6}{\pi^2 g^2 \langle \mathbf{B}^2 \rangle \tau_{\text{mag}}}
\]