Hadronic Loop Corrections to Charmonium Decays

Di-Lun Yang (with Thomas Mehen)
Duke University

Outline

- The puzzle of M1 radiative transitions between S-wave charmonia.
- The discrepancy between the experimental data and LO chiral perturbation theory for $\psi' \to J/\psi\pi^0(\eta)$ decays.
- Charmed meson loop correction in the non-relativistic heavy hadron chiral perturbation theory(NRHHχ PT).
- Primary goal:

  fitting both hadronic and radiative decays to extract the coupling constant of S-wave charmonia to charmed mesons.
- The coupling is important for studying the decays of charmed meson molecule such as X(3872) to lower excited states of charmonia.

The Puzzle of M1 Radiative Transitions

- The M1 radiative transitions $J/\psi \to \gamma\eta_c$ and $\psi' \to \gamma\eta_c$ can be computed by using the quark model

$$\Gamma(n^3S_1 \to \gamma m^1S_0) = \frac{4}{3} e_c^2 \alpha \frac{|\vec{q}|^3}{m_c^2} |I_{mn}|^2$$

M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008)

- Orthonality condition $I_{nm} = \delta_{nm}$ leads to

$\Gamma(J/\psi \to \gamma\eta_c) \approx 3.3\ keV$ and $\Gamma(\psi' \to \gamma\eta_c) \approx 0$

Relativistic correction: $\Gamma(J/\psi \to \gamma\eta_c) \approx 2.4\ keV$ $\Gamma(\psi' \to \gamma\eta_c) \approx 9.8\ keV$

- Contradicted to experimental measurements

$\Gamma_{\exp}(J/\psi \to \gamma\eta_c) = 1.58 \pm 0.37\ keV$ and $\Gamma_{\exp}(\psi' \to \gamma\eta_c) = 0.97 \pm 0.14\ keV$

- NLO correction is essential.
- Recent works by using pNRQCD

N. Brambilla et.al.,
Phys.Rev. D73 (2006) 054005
hep-ph/0512369
arXiv:1012.0773

Useful Tool to Analyze the Transitions Systematically

- Due to the small velocity of interacting charm quarks, the charmonium transitions can be treated by NRHHχ PT.
- The theory preserves approximate chiral symmetry and heavy quark spin symmetry.
- The power counting is manifested in terms of $|\vec{q}|$.
- The building blocks of EFT:

  charmonium multiplets

  charmed meson multiplets

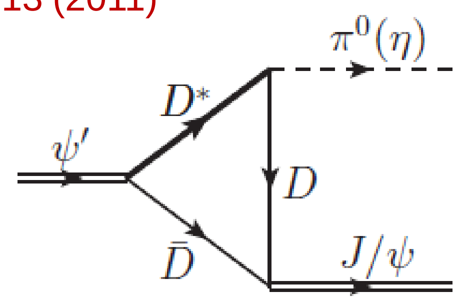
  Goldstone bosons

  electromagnetic field.

Charmed Meson Loop Corrections for Hadronic Decays $\psi' \to J/\psi\pi^0(\eta)$

- Extracted quark mass ratio contradicts to LO ChPT.
- The corrections for $\psi' \to J/\psi\pi^0(\eta)$ were studied by using NRHHχ PT. (F.-K.Guo et al. Phys. Rev. Lett. 103, 08200 3 (2009) Phys. Rev. D83, 034013 (2011))
- Interaction terms for loop corrections:

$$i\frac{g_2}{2}Tr[J^\dagger H_a \vec{\sigma}\cdot\overleftrightarrow{\partial}\bar{H}_a]-\frac{g}{2}Tr[H_a^\dagger H_b \vec{\sigma}\cdot\vec{u}_{ab}]+H.C.$$



- Simple power counting : 1/v enhancement
- Assuming the saturation of loop diagrams.
- Fitting

$$\Gamma(\psi' \to J/\psi\eta) = 10 \pm 0.4\ keV$$
$$\Gamma(\psi' \to J/\psi\pi^0) = 0.4 \pm 0.03\ keV$$

$\Rightarrow$ $\sqrt{g_2 g_2'} \approx 2\ GeV^{-3/2}$

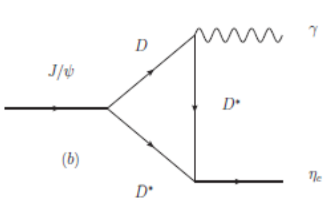
- Does 1/v~2 enhancement results in the dominant contribution?
- What would be numerical values of g_2 and g_2'?

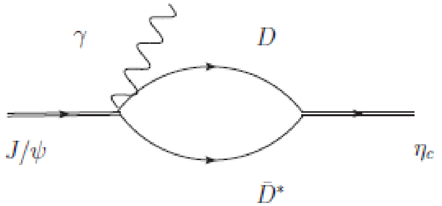
Charmed Meson Loop Corrections for M1 radiative Decays

- Introduce the interaction terms:

$$\frac{\rho}{2}Tr[J\vec{B}\cdot\vec{\sigma}J^{\dagger}]+\frac{e\beta}{2}Tr[H_a^{\dagger}H_b\vec{B}\cdot\vec{\sigma}Q_{ab}]+\frac{e}{2m_c}Q'Tr[H_a^{\dagger}\vec{B}\cdot\vec{\sigma}H_b]+g_2eTr[J^{\dagger}H_a\vec{A}\cdot\vec{\sigma}\bar{H}_a]$$

- Two kinds of loop diagrams:





$iM_{0(c)}\propto\varepsilon_{ijk}\vec{q}_i\varepsilon_j^{\gamma}\varepsilon_k^{J/\psi}$ $\quad iM_t\propto\varepsilon_{ijk}\vec{q}_i\varepsilon_j^{\gamma}\varepsilon_k^{J/\psi}\,|\vec{q}\,|^2$

- Investigating $J/\psi(\psi')\to\gamma\eta_c$ and $\psi'\to\gamma\eta_c'$ decays.
- Both diagrams lead to 1/v enhancement from power counting.
- The contact diagrams turn out to be suppressed.
- Global fit: $\psi'\to J/\psi\pi^0(\eta)$ ⇨ $A,\ g_2g_2'$

$$\begin{cases} J/\psi\to\gamma\eta_c & \Rightarrow \rho \\ \psi'\to\gamma\eta_c & \Rightarrow \rho' \\ \psi'\to\gamma\eta_c' & \Rightarrow \rho'' \end{cases}$$

Global Fit

- We also consider the counterterm contribution in $\psi' \to J/\psi\pi^0(\eta)$ (the c.t. interaction $\frac{A}{4}(Tr[J'\sigma^i J^\dagger] - Tr[J^\dagger \sigma^i J'])\partial^i(\chi_-)_{aa}$ for $(\chi_-)_{aa}$ encoding the light quark masses and Goldstone bosons by Guo et al.)
- We thus have six parameters to be fitted $(A,\ g_2,\ g_2'\ \rho,\ \rho',\ \rho'')$
- First step : extract $A,\ g_2 g_2'$ from $\psi' \to J/\psi\pi^0(\eta)$
- Two branches: $g_2 g_2' = 0.731\ GeV^{-3}$

  $g_2 g_2' = 3.43\ GeV^{-3}$
- Assuming the relation: $g_2 = \sqrt{m_{\psi'}/m_{J/\psi}}\, g_2'$
- Second step: fit $J/\psi(\psi') \to \gamma\eta_c$ and $\psi' \to \gamma\eta_c'$ decays by using the two branches to extract the counterterm couplings. $(\rho,\ \rho',\ \rho'')$

Comparison with the quark model

- The set with larger product leads to considerable discrepancy with QM, which suggests a fine-tuned cancellation between the loop diagrams and counterterm interactions.
- QM suggests the equivalent couplings of counterterm $\rho = \rho''$
- Cannot be satisfied by using the g_2 and g_2' extracted from the hadronic decays.

	A' (GeV^{-2})	g_2 ($GeV^{-3/2}$)	g_2' ($GeV^{-3/2}$)	ρ (GeV^{-1})	ρ'' (GeV^{-1})	ρ' (GeV^{-1})
$g_2 g_2' = 3.43\ GeV^{-3}$	$-0.163^{+0.001}_{-0.001}$	$1.93^{+0.03}_{-0.03}$	$1.77^{+0.03}_{-0.03}$	$2.21^{-0.03}_{+0.02}$	$0.944^{-0.09}_{+0.09}$	$1.82^{-0.0002}_{+0.0016}$
	$-0.163^{+0.001}_{-0.001}$	$1.93^{+0.03}_{-0.03}$	$1.77^{+0.03}_{-0.03}$	$2.61^{+0.03}_{-0.02}$	$1.43^{+0.09}_{-0.09}$	$1.84^{+0.013}_{-0.091}$
$g_2 g_2' = 0.731\ GeV^{-3}$	$0.652^{+0.016}_{-0.018}$	$0.893^{+0.031}_{-0.034}$	$0.819^{+0.028}_{-0.031}$	$0.312^{-0.0227}_{+0.0246}$	$0.042^{-0.0900}_{+0.0904}$	$0.379^{-0.0004}_{+0.090}$
	$0.652^{+0.016}_{-0.016}$	$0.893^{+0.031}_{-0.034}$	$0.819^{+0.028}_{-0.031}$	$0.720^{+0.023}_{-0.024}$	$0.476^{+0.09}_{-0.09}$	$0.406^{+0.0008}_{-0.001}$
QM				0.275	0.263	0.0417

Comparing loop contributions with experimental measurements

- We investigate the contributions from triangle loop diagrams of different sets for $J/\psi \to \gamma\eta_c$ and $\psi' \to \gamma\eta_c'$ decays.
- $g_2 g_2' = 3.43\ GeV^{-3}$ ⇒ fine-tuned cancellation
- $g_2 g_2' = 0.731\ GeV^{-3}$ ⇒ close to counterterm contribution

$g_2 = 1.93^{+0.03}_{-0.03}$ $g_2 = 0.893^{+0.031}_{-0.034}$

	$G_2 = 3.43^{+0.10}_{-0.11}$ (GeV^{-3})	$G_2 = 0.731^{+0.031}_{-0.034}$ (GeV^{-3})	QM	PDG and BES III
$\Gamma(J/\psi \to \gamma\eta_c)_{Tri}$	222^{+14}_{-14} keV	$10.2^{+1.5}_{-1.5}$ keV	2.9 keV	1.58± 0.37 keV
$\Gamma(\psi' \to \gamma\eta_c')_{Tri}$	$4.5^{+0.3}_{-0.3}$ keV	$0.20^{+0.03}_{-0.03}$ keV	0.21 keV	0.143± 0.027±0.092 keV

Summary

- We have analyzed the effect of hadronic loop correction for the radiative M1 transition between S-wave charmonia.
- By incorporating the counterterm interaction in hadronic decays, we extract two sets of coupling constants g_2 and g_2' .
- These couplings are important for analyzing other chamonium transitions.
- To avoid the fine-tuning problem for radiative decays, the loop corrections should only lead to sizable effect.
- However, the hadronic decays and radiative decays cannot be well fitted simultaneously, which may suggest the power counting in terms of v is unreliable.

Thank you!

Backup Slices

- Kinetic terms in the Lagrangian:

$$L = Tr[H_a^\dagger(iD_0 + \frac{D^2}{2m_D})H_a] + \frac{\Delta}{4}Tr[H_a^\dagger\vec{\sigma}_i H_a \vec{\sigma}_i]$$

charmed meson multiplet: $H_a = V_a \cdot \vec{\sigma} + P_a$

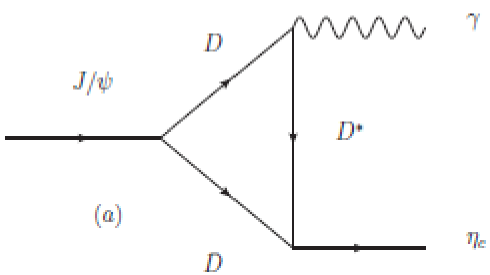
charmonium multiplet: $J = \vec{\sigma} \cdot \vec{\psi} + \eta_c$

$SU(3)_L \times SU(3)_R : H_a \to H_b U_{ba}^\dagger$ Heavy quark spin sym: $H_a \to SH_a$

Power Counting

- We approximate the intermediate momenta as $\vec{l} = mv$ and $l_0 = mv^2$
- The Integral for loop diagram:

$$i\mathcal{M}_a = -2ig_2^2(\lambda_{1(3)})\int\frac{d^4l}{(2\pi)^4}\frac{\epsilon_{\mu\nu\rho}\vec{q}_\mu\epsilon_\nu^\gamma(2\vec{l}-\vec{q})_\rho\epsilon^{J/\psi}\cdot\vec{l}}{8(l_0-\frac{\vec{l}^2}{2m_D}+i\epsilon)(l_0+\frac{\vec{l}^2}{2m_D}+b_{DD}-i\epsilon)(l_0-q_0-\frac{(\vec{l}-\vec{q})^2}{2m_{D^*}}-\Delta+i\epsilon)}$$



numerator $\propto v^2$

denominator $\propto v^6$

measure $\propto v^5$

one diagram $\propto v$

- The cancellation between loop diagrams is proportional to the charmed meson mass difference $\propto \Delta = m_{D^*} - m_D$.
- The full decay amplitude $\propto \frac{v\,\Delta}{mv^2} = \frac{\Delta}{mv}$, where the mv^2 in the denominator is the intrinsic energy scale to balance the dimension.