Quantum Gravity=Gravitization of the Quantum: Cosmology and Fundamental Parameters

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Outline

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What is Quantum Gravity (QG)?

I will introduce **the problem of QG** and **emphasize the need for empirical probes**. I will argue for "**gravitization of quantum theory (GQ)**", with specific experimental signatures (**triple and higher-order interference**) at low energies. As an existing empirical probe I will discuss the **observed cosmological constant (the first measured quantum gravity phenomenon)** [arXiv:2202.06890 [hep-th]], [arXiv:2203.17137 [gr-qc]], [arXiv:2303.15645 [gr-qc]], [arXiv:2212.00901 [hep-th]], [arXiv:2212.06086 [hep-th]], [arXiv:2003.00318 [hep-th]], [arXiv:2307.16712 [hep-th]]. $M = 10^{-34} eV$; $M_P = 10^{19} GeV$
Cosmology and Fundamental Parameters
(Work done with Freidel, Leigh (FLM), Kowalski-Glikman (FKLM), Berglund, Hübsch (BHM), Mattingly and Geraci.)

QG = GQ and Experiment - Results 1:

CC: $m_\Lambda \sim \sqrt{MM_P} \sim 10^{-3} eV$; $m_H \sim \sqrt{m_\Lambda M_P} \sim 125 GeV$; m_H and RG: m_t, m_b and m_τ (SM criticality). $M_{BZ}^3 \sim MM_P^2 \sim (7 MeV)^3$.

Then: $m_c \sim \sqrt{M_{BZ}\, m_t} = M_{BZ}\sqrt{\frac{m_t}{M_{BZ}}} \sim 1.10\ (1.27)$ GeV.

$m_s \sim \sqrt{M_{BZ}\, m_b} = M_{BZ}\sqrt{\frac{m_b}{M_{BZ}}} \sim 171\ (93.4)$MeV.

$m_u \sim M_{BZ}^2/m_c \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_t}} \sim 10^{-2} M_{BZ} \sim 10^{-1}\ (2.16)$MeV.

$m_d \sim M_{BZ}^2/m_s \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_b}} \sim 10^{-1} M_{BZ} \sim 1\ (4.67)$MeV.

$m_\mu \sim \sqrt{M_{BZ}\, m_\tau} = M_{BZ}\sqrt{\frac{m_\tau}{M_{BZ}}} \sim 112\ (106)$MeV.

$m_e \sim \frac{M_{BZ}^2}{m_\mu} \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_\tau}} \sim 464\ (511)$keV. **Prediction for: (Neutrino masses)** $m_3 \sim m_H^2/M_{SM} \sim (10^{-2} - 10^{-1})$eV. $m_2 \sim \sqrt{m_\Lambda m_3} \sim (10^{-2.5} - 10^{-2})$eV. $m_1 \sim \frac{{m_\Lambda}^2}{m_3} \sim (10^{-4})$eV.

QG = GQ and Experiment - Results 2:

CKM matrix (quark mixing matrix)

$$|V_{cb}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_d}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_d}} \sim 0.050 \quad (0.041), (\rightsquigarrow \theta_{23})$$

$$|V_{td}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_s}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_s}} \sim 0.011 \quad (0.008)(\rightsquigarrow \theta_{12})$$

$$|V_{ub}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_b}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_b}} \sim 0.002 \quad (0.003)(\rightsquigarrow \theta_{13})$$

PMNS (neutrino mixing matrix: $M_{BZ} \to m_\Lambda$)

$$|U_{\mu 3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_1}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_1}} \sim 0.50, \quad (0.63)$$

$$|U_{\tau 1}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_2}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_2}} \sim 0.13, \quad (0.26)$$

$$|U_{e3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_3}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_3}} \sim 0.06, \quad (0.14) \text{ (Similar structure!)}$$

What is Quantum Gravity?

To **motivate the problem of quantum gravity** let us start with **the fundamental constants** ("magic cube of physics"): **Newton's gravitational G, speed of light c, Planck constant** $\hbar$.

1) **c** relates space and time - feature of spacetime "stuff" ("structure constant" of spacetime that converts from measurement of time to measurement of space) and of the causal structure in spacetime

2) **G** measures the response ("elasticity") of spacetime geometry to the presence of matter and makes causal structure dynamical (gravity tilts light cones)

3) $\hbar$ distinguishes between possible and actual (observed with some probability) histories of time evolution of matter degrees of freedom (also, atom of action - atomistic nature of matter)

What is Quantum Gravity?

Note the fundamental dichotomy: **spacetime is classical** (where events are observed), **matter is quantum** (atomistic).

(*Spacetime as "index set" for quantum matter.*)

Spacetime is a (dynamical) arena for physics; matter makes physics happen in spacetime.

Each fundamental constant defines an axis of the "cube of theories/frameworks".

Eight fundamental theories/frameworks - six theories/frameworks realized so far:

Classical mechanics, Newtonian gravity (G), Classical Field theory (c), General relativity (GR - G, c), Quantum mechanics ($\hbar$), Quantum field theory (QFT - $\hbar$, c)

Need QG - relativistic ($\hbar$, G, c) and non-relativistic ($\hbar$, G)

What is Quantum Gravity?

Empirically we see (currently) **NO departures from canonical QFT (including quantum mechanics) as well as GR (including special theory of relativity)**.
(*However, the discovery of dark energy (cosmological constant,* Λ*/vacuum energy) is the first observable quantum gravity effect!*)
What is obviously missing in this list is a theory/framework defined with all three fundamental constants (and its non-relativistic contraction).
What is missing is quantum spacetime (gravity/cosmology)!

What is Quantum Gravity?

$\mathbf{G}, \mathbf{c}, \hbar$ - **relativistic, quantum theory of gravity (QG)** and its non-relativistic limit/contraction $\mathbf{G}, \hbar$ - a non-relativistic quantum theory of gravity. (**QG - "under construction"**.)

This theory/framework should contain, in appropriate limits, QFT, quantum theory, classical field theory (like EM), Newtonian gravity and Galilean physics. However, as everyone learned in kindergarten: $l_P = 10^{-35} m$, $t_P = l_P/c = 10^{-43} s$ **and** $E_P = \hbar/t_P = 10^{19} GeV$.

Seems hopeless to probe empirically! (**Is QG irrelevant? NO!**)

Cosmologically, QG represents the "original/initial" physics that gets diversified in various physical limits.

From this point of view, **everything in** the magic cube of **physics is a left-over of quantum gravity**: *Quantum spacetime, quantum matter, origin of space, time, inertia and the quantum!*

What is Quantum Gravity?

There are at least two dozen (plus) approaches to QG:
EFT approach, asymptotic safety, supergravity, string theory, holography (AdS/CFT and generalizations), Euclidean quantum gravity, topological (and categorical) quantum field theory, canonical quantum gravity, loop quantum gravity (Hamiltonian/spin networks and covariant/spin foams), causal sets, quantum cosmology, group field theory, emergent gravity in condensed matter, Regge calculus, (causal) dynamical triangulation, non-commutative geometry, twistor theory, Horava-Lifshitz, analog gravity models, modified/massive gravity, gauge theories of gravity, non-local theories of gravity, shape dynamics, entropic gravity, quantum gravity phenomenology, quantum graphity, gravitizing the quantum etc.

What is Quantum Gravity?

Questions: Spacetime sector: *resolution of singularities (black hole and cosmological), quantum black holes (including BH entropy, information puzzle), astrophysics of the (resolved) BH singularity, cosmology of the (resolved) initial singularity, fine tuning of the initial state, early universe cosmology, relevance of quantum gravity for structure formation, quantum structure of spacetime and its phenomenology, the vacuum energy problem...*
Matter sector: *hierarchy of scales, origin of the Standard Models of particle physics and cosmology, masses and couplings of fundamental particles, dark matter and dark energy, baryogenesis...,*
Conceptual: *emergence of quantum theory, quantum/classical transition, quantum measurement problem, topology change, problem of time, CTCs, emergence of spacetime, inertia...*

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QG=Gravitization of Quantum Theory (GQ)

A tall order indeed! **To address these questions** we **need** theoretically **clear conceptual ideas** and methodology. Experimentally, we need **clear empirical phenomena**. (*Relativity as a good example!*) In what follows I will concentrate on a particular approach we call **"gravitization of the quantum" (GQ), in which the fixed geometry of quantum theory is made dynamical.** In the context of **QG=GQ** I will discuss specific empirical implications (Sorkin's **triple interference** etc). $QG = GQ$ synergizes with *string theory*, *loop quantum gravity* (via **quanta of spacetime**), *matrix models*, as well as *non-commutative geometry* and *twistor theory*, and the idea of **relative locality** and the program of *quantum gravity phenomenology* and *quantum foundations*. (Explains **observed** Λ**; Masses of elementary particles.**)

QG=Gravitization of Quantum Theory

Central intuition: **Quantum Relativity** in analogy with **Classical Relativity**

Classical relativity:

a) special relativity - motivated by EM - (*Minkowski spacetime/geometry, relativity of simultaneity*),

b) relativistic field theory (*reps of Lorentz/Poincare, particles/antiparticles*),

c) general relativity (*dynamical classical spacetime*)

Spacetime relativity - first (classical) relativity (both spacetime and matter classical).

QG=Gravitization of Quantum Theory

Quantum relativity: (FLM, '13, '14, '15, '16, '17)
A) QM from quantum spacetime (**modular spacetime, Born geometry, relative locality**),
B) QFT (**metafields/metaparticles**),
C) **gravitized quantum theory** (**dynamical quantum spacetime**, **dynamical Born geometry**, **metastrings**, metaparticles - dark matter, geometry of dual spacetime - dark energy)
QM/QFT - second (quantum) relativity (matter quantum, spacetime classical).
QG=GQ - third (QG) relativity (spacetime/matter quantum).
(Third relativity - Finkelstein; Wheeler)

QG=Gravitization of Quantum Theory

Main insight (FLM): suppose we define our physics on a lattice (lattice QFT). Continuum via Wilsonian RG (via path integral). Lattice is classical, physics is quantum. In quantum theory we need a **lattice (l) and a dual lattice ($\tilde{l}$)**! (as noticed by Zak)
Instead of considering the standard commutation relations between the position and momentum operators, $[q,p]=i\hbar$, take the generators of translations in *phase space*

$$\hat{U}_a = e^{\frac{i}{\hbar}\hat{p}a}, \quad \hat{V}_{\frac{2\pi\hbar}{a}} = e^{\frac{i}{\hbar}\hat{q}\frac{2\pi\hbar}{a}}, \quad \Longrightarrow [\hat{U}_a, \hat{V}_{\frac{2\pi\hbar}{a}}] = 0 \qquad (1)$$

In terms of **modular variables** a la Aharonov et al,

$$[\hat{q}]_a \equiv \hat{q} \bmod a \quad [\hat{p}]_{\frac{2\pi\hbar}{a}} \equiv \hat{p} \bmod \frac{2\pi\hbar}{a} \quad \Longrightarrow [[\hat{q}]_a, [\hat{p}]_{\frac{2\pi\hbar}{a}}] = 0 \qquad (2)$$

QG=Gravitization of Quantum Theory

Note that **modular variables are covariant** (modular energy, modular time as well).
Take fundamental length λ and energy ϵ, so that $\lambda\epsilon \equiv \hbar$. **Modular variables are non-local (but consistent with causality - origin of the uncertainty principle)**.
Contextuality: *in a double slit experiment the parameters λ and ϵ are contextual to the experiment*. This will be an important point when we discuss the cosmological constant problem! (FKLM)

QG=Gravitization of Quantum Theory

Explicit non-locality: Take $H = \frac{p^2}{2m} + V(q)$ and write the Heisenberg equation of motion for $e^{ipR/\hbar}$, or equivalently $[p]_R$ (R - contextuality parameter, such as the distance between two slits in the double slit interference experiment). (Aharonov et al)

$$\frac{d[p]_R}{dt} = \ldots \frac{V(q+R/2) - V(q-R/2)}{R} \tag{3}$$

Quantum mechanics=non-locality (from modular variables) plus causality (compatibility with Lorentz).
Now, reformulate quantum mechanics (QM) using (covariant) modular variables via modular spacetime. (**Quantum theory tells us something new about quantum spacetime!**)

QG=Gravitization of Quantum Theory

What is **modular space**? (FLM, '16)
Modular space is the space of all commuting subalgebras of the Heisenberg-Weyl algebra.
Note $[q, p] = i\hbar$ - Heiseinberg-Weyl algebra, whereas
$[[q]_a, [p]_{2\pi\hbar/a}] = 0$ - commuting subalgebra of Weyl-Heisenberg.
Theorem of Mackey: *the space of all commuting subalgebras of the Heisenberg-Weyl algebra is a self-dual phase space lattice lifted to Heisenberg-Weyl.*
Use covariant modular variables - **modular spacetime** of d spacetime dimensions.

Note (FLM): **phase space - symplectic structure** $Sp(2d)$ - ω_{ab}.
Self-dual lattice (l plus $\tilde{l}$) - **doubly-orthogonal** $O(d,d)$ - η_{ab}.
To **define the vacuum** on this self-dual lattice - **need doubly metric structure** $O(2,2d-2)$ - H_{ab}.
ω, η, H **define Born geometry.** (FLM, '13, '14, '15, '16)
Their triple intersection gives the Lorentz group.
Thus **QM follows from non-locality (fundamental length/time) consistent with causality.**
Note: can be localized (local QFT possible!) in a particular phase space cell, but can't tell in which one (uncertainty principle)!

QG=Gravitization of Quantum Theory

How can fundamental length/time be consistent with Lorentz? (One of the main puzzles of QG.)
This is possible because of **relative (observer dependent) locality**. (Amelino-Camelia, Freidel, Kowalski-Glikman, Smolin)
Different observers see different spacetimes (slices of modular spacetime). **Different spacetimes are in linear superposition, and so fundamental length/time is consistent with Lorentz.**
(Similar to spin: the superposition of up and down spin gives the Bloch sphere which is consistent with rotation symmetry, even though spin is discrete.) (FLM)

QG=Gravitization of Quantum Theory

Generic quantum polarization (FLM, '16) - **modular polarization** (defined via the Zak transform). Given Schrödinger's $\psi_n(x)$

$$\psi_\lambda(x, \tilde{x}) \equiv \sqrt{\lambda} \sum_n e^{-2\pi i n \tilde{x}} \psi_n(\lambda(n + x)) \qquad (4)$$

($x \equiv q/\lambda$, $\tilde{x} \equiv p/\epsilon$, so $[x, \tilde{x}] = i$, $\lambda\epsilon = \hbar$). Note, from the point of view of modular polarization, Schrödinger's polarization is very singular. Introduce $\mathbb{X}^A \equiv (x^a, \tilde{x}_a)^T$, so that $[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b] = i\omega^{AB}$. We can write the translations operators in phase space covariantly $W_{\mathbb{K}} \equiv e^{2\pi i \omega(\mathbb{K}, \mathbb{X})}$, where $\mathbb{K}$ stands for the pair $(\tilde{k}, k)$ and $\omega(\mathbb{K}, \mathbb{K}') = k \cdot \tilde{k}' - \tilde{k} \cdot k'$. (Note W - **Aharonov-Bohm phases - prototypical example of modular variables**.)

QG=Gravitization of Quantum Theory

So far we have discussed covariant quantum phase space as an example of modular space, and so we are ready to discuss modular spacetime. Consider (FKLM, '18) a **metaparticle** (mp) propagating in a modular space defined by Born geometry - ω, η, H. The metaparticle world-line action $S_{mp} = \int d\tau L_{mp}$ (canonical particle - $\mu \to 0$ and $\tilde{p} \to 0$)

$$L_{mp} = p_\mu \, \dot{x}^\mu + \tilde{p}^\mu \, \dot{\tilde{x}}_\mu + \lambda^2 p_\mu \, \dot{\tilde{p}}^\mu - \frac{N}{2} \left(p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu - m^2 \right) + \tilde{N} \left(p_\mu \tilde{p}^\mu - \mu \right), \tag{5}$$

where ω is in ("Berry-phase") $p_\mu \, \dot{\tilde{p}}^\mu$, and η in the diffeo constraint $p_\mu \tilde{p}^\mu = \mu$ and H in the Hamiltonian constraint $p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu = m^2$. **Dual spacetime** $\tilde{x}$, $[x, \tilde{x}] = i\lambda^2$, and **dual momentum space** $\tilde{p}$, $[p, \tilde{p}] = 0$. (Also, $[x, p] = i\hbar = [\tilde{x}, \tilde{p}]$.)

QG=Gravitization of Quantum Theory

The metaparticle can be understood also as follows: If one second quantizes Schrödinger's $\psi(x)$ one naturally ends up with a quantum field operator $\hat{\phi}(x)$. Similarly, the second quantization of the modular $\psi_\lambda(x, \tilde{x})$ would lead to a modular quantum field operator $\hat{\phi}_\lambda(x, \tilde{x})$ (**modular fields - metafields**)

$$\hat{\phi}(x) \to \hat{\phi}_\lambda(x, \tilde{x}). \tag{6}$$

with $[x, \tilde{x}] = i\lambda^2$ - covariant non-commutative field theory. (FLM, '17)
Classical spacetime label x of canonical QFT - choice of (classical spacetime) polarization in modular (quantum) spacetime with a contextuality parameter λ.

QG=Gravitization of Quantum Theory

Quanta of canonical quantum fields $\phi(x)$ - particles (and their antiparticles).
Quanta of modular quantum fields $\phi_\lambda(x, \tilde{x})$ - metaparticles.
First prediction of modular spacetime approach to quantum theory - metaparticles! (FKLM)
(We will argue that dual particles, correlated to visible particles, represent dark matter.)
Note that if we turn on backgrounds $p \to p + \phi$ and $\tilde{p} \to \tilde{p} + \tilde{\phi}$.
Thus we have **"dark matter" fields** $\tilde{\phi}(x)$ in the effective classical spacetime x description (after integrating over the dual spacetime $\tilde{x}$). **Visible ϕ and Invisible (dark matter) $\tilde{\phi}$ do not commute!**

QG=Gravitization of Quantum Theory

Note (FLM): *modular spacetime has double the dimension of spacetime*.

The modular cells are not simply connected (there is a unit flux through each cell - matter). [Also: **Quantum statistics; Quantum-correlations. Double (UV/IR) RG.**]

Modular wavefunctions are quasiperiodic.

Classical spacetime emerges from the process of **extensification** (imagine one unit length in dual direction and many, N, modular cells in the spacetime direction).

Spacetime emerges, in the large N limit, as a natural pointer basis in quantum theory. (Quantum measurement.)

Also, **spacetime and matter - "two sides of the same coin"**.

Cosmology: interplay between visible and dual dof.

(Singularities... Information paradox...)

QG=Gravitization of Quantum Theory

Propagator for the metaparticle (FKLM, '18)

$$G(p, \tilde{p}; p_i, \tilde{p}_i) \sim \delta^{(d)}(p - p_i)\delta^{(d)}(\tilde{p} - \tilde{p}_i)\frac{\delta(p \cdot \tilde{p} - \mu)}{p^2 + \tilde{p}^2 + m^2 - i\varepsilon}. \quad (7)$$

Canonical particle: highly singular $\tilde{p} \to 0$ (and $\mu \to 0$) limit of this expression. Dispersion relation (in a particular gauge $\vec{\tilde{p}} = 0$)

$$E_p^2 + \frac{\mu^2}{E_p^2} = \vec{p}^2 + m^2. \quad (8)$$

For each particle at energy E there exists a dual particle at energy $\frac{\mu}{E}$. (Analogous to the prediction of antiparticles in QFT.)
Dispersion relation - **quantum gravity phenomenology in the IR**! (FKLM, '21) Friedel-like static potential for metaparticles. Quasi-metaparticles (CMP).

QG=Gravitization of Quantum Theory

Dual “particles” (dual fields) - dark matter (to leading order in λ)

$$S_{eff} = -\int \sqrt{g(x)\tilde{g}(\tilde{x})}[R(x)+\tilde{R}(\tilde{x})+L_m(A(x,\tilde{x}))+\tilde{L}_{dm}(\tilde{A}(x,\tilde{x}))], \tag{9}$$

Here the A fields denote the usual Standard Model fields, and the $\tilde{A}$ are their duals, as predicted by the general (modular) formulation of quantum theory that is sensitive to the minimal length.
Note that we need to integrate over the dual space coordinates $\tilde{x}$ to get an effective description of **visible matter,** $A(x)$**, and dark matter,** $\tilde{A}(x)$, in classical x spacetime. (BHM, '21, '22)

QG=Gravitization of Quantum Theory

Dynamical geometry of dual spacetime - dark energy (to leading order in λ) [also: equation of state]

$$S_{eff} = -\int \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x}) + ...], \qquad (10)$$

In this leading limit, the $\tilde{x}$-integration in the first term defines the gravitational constant G_N, and in the second term produces a **positive cosmological constant constant**! (BHM, '21, '22) *In general, visible and dark matter degrees of freedom are correlated (via the minimal length λ) - origin (from dark matter!) of Milgrom's scaling (galaxies, clusters, superclusters) and Milgrom's acceleration $a_0 \sim cH/(2\pi)$.* $(\Lambda \sim H^2.)$

QG=GQ - Explicit Realization

Explicit realization in terms of a chiral phase-space reformulation of the bosonic string, the "**metastring**," (FLM, '13, '14, '15) - also a non-perturbative proposal (BHM '21, '22) of QG (matrix model-like, time-asymmetric (?), $\partial_\sigma \cdot \equiv [\hat{\mathbb{X}}, \cdot]$, where $\hat{\mathbb{X}}$ matrix comes from modular world-sheet) - spacetime/matter quanta

$$S^{\mathrm{ch}}_{\mathrm{str}} = \int \mathrm{d}\tau \mathrm{d}\sigma \left[\partial_\tau \mathbb{X}^a \big(\eta_{ab}(\mathbb{X}) + \omega_{ab}(\mathbb{X})\big) - \partial_\sigma \mathbb{X}^a H_{ab}(\mathbb{X})\right] \partial_\sigma \mathbb{X}^b, \tag{11}$$

where $\mathbb{X}^a \equiv (X^a/\lambda, \tilde{X}_a/\lambda)^T$ are coordinates on phase-space like (doubled) target spacetime and η, H, ω are all dynamical. $x^a, \tilde{x}_a$ come from the left and right moving modes of the bosonic string,

$$x^a \equiv x^a_L + x^a_R, \quad \tilde{x}^a \equiv \tilde{x}^a_L - \tilde{x}^a_R \tag{12}$$

In the context of a flat metastring we have constant η_{ab}, H_{ab} and ω_{ab} (zero ω_{ab} - connection to double field theory)

$$\eta_{ab} = \begin{pmatrix} 0 & \delta \\ \delta^T & 0 \end{pmatrix}, \quad H_{ab} = \begin{pmatrix} h & 0 \\ 0 & h^{-1} \end{pmatrix}, \quad \omega_{ab} = \begin{pmatrix} 0 & \delta \\ -\delta^T & 0 \end{pmatrix}, \tag{13}$$

The standard Polyakov action is obtained when setting $\omega_{ab} = 0$ and integrating out the $\tilde{x}_a$,

$$S_P = \int \mathrm{d}\tau \mathrm{d}\sigma \gamma^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b \eta_{ab} + \dots \tag{14}$$

The triplet (ω, η, H) define the Born geometry (FLM, '13, '14) and the metastring propagates in a modular spacetime, a generic phase space like structure of quantum theory (FLM, '16). QFT in modular spacetime - **intrinsically non-commutative**. The *space* of commuting subalgebras of the Heisenberg algebra, $[\hat{x}, \hat{\tilde{x}}] = i\lambda^2$, becomes modular spacetime (FLM, '15, '16)

The new feature in the metastring formulation of the bosonic string is intrinsic non-commutativity and so there is a new Heisenberg algebra (vertex operators reps of Weyl-Heisenberg - no cocycles)

$$[\mathbb{X}^a, \mathbb{X}^b] = i l_s^2 \omega^{ab} \implies [X^a, \tilde{X}^b] = i\delta^{ab} l_s^2 \tag{15}$$

$[\delta q \sim G_N \delta p \to \delta q \sim G_N \delta \tilde{p} \sim G_N \frac{1}{\delta \tilde{q}} \to \delta q \delta \tilde{q} \sim G_N \to [q, \tilde{q}] \sim i l_P^2]$
as well as the standard commutators (with $[\Pi, \tilde{\Pi}] = 0$)

$$[\mathbb{X}^a, \mathbb{P}_b] = i\hbar\delta^a_b \implies [X^a, \Pi_b] = i\delta^a_b\hbar, \quad [\tilde{X}^a, \tilde{\Pi}_b] = i\delta^a_b\hbar \tag{16}$$

Note, if Kalb-Ramond B_{ab} (axion) constant but non-zero, dual coordinates do not commute! In general - non-associativity [SM] (FLM,'17.) **Zero modes of the metastring - metaparticles** (FKLM, '18, '21) - little rigid strings. (Each Standard Model (SM) particle has a correlated "dual" - dark matter pheno (BHM, '20, '21).)

How could we "see" modular spacetime?

Instead of scattering particles, entwine them!

Vertex operators V_K (plane waves - asymptotic particle states) have co-cycles in the Polyakov string if we assume that $[x, \tilde{x}] = 0$

$$V_{\mathbb{P}} V_{\mathbb{P}'} = e^{i(p\tilde{p}' - \tilde{p}p')} V_{\mathbb{P}'} V_{\mathbb{P}} \tag{17}$$

The cocycle factor $e^{i(p\tilde{p}' - \tilde{p}p')}$ indicates the fundamental non-commutativity of x and $\tilde{x}$.

Can this entwining of particles be measured?

(Here we are really talking about the "R-matrix", in the sense of "swapping of particles", instead of the S-matrix - "scattering of particles".)

QG=GQ - Explicit Realization

The metastring has **dynamical Born geometry**, (FLM, '14, '15) $\omega_{ab}(\mathbb{X}), \eta_{ab}(\mathbb{X}), H_{ab}(\mathbb{X})$, but Born geometry is the geometry of the modular spacetime formulation of quantum theory.
Thus **by making Born geometry dynamical we can "gravitize quantum theory"** (that is, **make the geometry of quantum theory dynamical**)! (FLM)
Also, metastring is a theory of quantum gravity, and so we arrive at **"QG = gravitized quantum theory".** - **Triple and higher order interference**
This reasoning is "top-down".
(Note, classical GR gravitizes all of classical physics!)

Consider particle interactions as $0+1$ quantum gravity.
Quantum field theory $=0+1$ quantum gravity/cosmology!
Example: $g\phi^3$ theory. Classical equations:

$$(\partial^2+m^2)\phi+g\phi^2=0 \tag{18}$$

Note ϕ - wave-function of $0+1$ universes.
Thus the above equation: non-linear Wheeler-DeWitt equation.
Interaction vertex: **topology change**.
Classical spacetime viewpoint: decay of a particle (Born rule or S-matrix)
QG=GQ viewpoint from $0+1$ universes (particles) - **triple correlation!**

QG=GQ - Explicit Realization

There is a "bottom-up" reason for "gravitization of quantum theory". (Minic, Tze)
Geometry of quantum theory (review by Ashtekar and Schilling) - maximally symmetric geometry of complex projective spaces (symplectic structure, compatible with the metric structure - the Born rule, and the product of the symplectic and metric structure gives complex structure - responsible for interference).
Quantum clock relates the Born rule (the Fubini-Study metric of complex projective spaces) to infinitesimal time (Aharonov and Anandan)

$$2\hbar ds_{FS} = \Delta E dt \tag{19}$$

where ΔE is the dispersion of energy defined by the Hamiltonian associated with the atomic clock.

QG=GQ - Explicit Realization

In the presence of quantum spacetimes (topology change) - no unique timelike Killing vector - and thus ΔE is state dependent which makes the geometry state dependent, and thus, dynamical. (Minic, Tze) [*Also: dynamical inner product - 2+1 (CS) QG*]
So - for quantum spacetimes - we should expect "gravitized quantum theory". That is, **the geometry of quantum theory is dynamical**. In general - topology change. (Thus the Bloch sphere becomes a Riemann surface of an arbitrary genus.)
In general, not a single Hilbert space, but **observer dependent Hilbert spaces**. Thus, dynamical Born rule and generalized kinematics with new experimental signatures (Sorkin's triple and higher order quantum interference). New observables - beyond the S-matrix etc. (Single Hilbert space = Born rule. Canonical observables, like the S-matrix.)

QG=GQ - Explicit Realization

What is the first experimental consequence of "gravitized quantum theory"? (Beglund, Geraci, Hübsch, Mattingly, Minic)
Triple and higher-order interference! (Experiment possible in the next few years.)
Note - the canonical quantum theory does not have intrinsic triple quantum interference (consequence of the Born rule and the fixed geometry of the complex projective space).
Current experimental bounds (photonic) - rather weak (10^{-3})*.*
Neutrino bounds expected to be surprisingly similar (and to be measured at JUNO).(Huber, Minakata, Minic, Pestes, Takeuchi).

QG=GQ - Explicit Realization

In more detail (Sorkin): Classically, we have addition of probabilities

$$P_n(A, B, C, \cdots) = P_1(A) + P_1(B) + P_1(C) + \cdots , \qquad (20)$$

for any number of paths. Quantum mechanically, we have for two paths $P_2(A, B) = |\psi_A + \psi_B|^2$ or more explicitly

$$|\psi_A|^2 + |\psi_B|^2 + (\psi_A^* \psi_B + \psi_B^* \psi_A) \equiv P_1(A) + P_1(B) + I_2(A, B) \quad (21)$$

where the last term

$$I_2(A, B) = P_2(A, B) - P_1(A) - P_1(B) \qquad (22)$$

is the "interference" of the two paths A and B. **Non-vanishing double-path interference, $I_2(A, B) \neq 0$, distinguishes quantum theory from the classical one.**

QG=GQ - Explicit Realization

The **Born rule** dictates that all the superimposed paths only interfere with each other in a pairwise manner. For instance, for three paths we have $P_3(A, B, C) = |\psi_A+\psi_B+\psi_C|^2$

$$P_2(A, B)+P_2(B, C)+P_2(C, A)-P_1(A)-P_1(B)-P_1(C), \tag{23}$$

where only pairwise interferences between the pairs (A, B), (B, C), and (C, A) appear.

It is clear from the above that in order for there to be a non-linear correction in an interference pattern the Born rule must be relaxed.

Consider **a triple slit experiment**: Since only pairwise interferences between the pairs (A, B), (B, C), and (C, A) appear, it makes sense to define any deviation from this relation as the intrinsic triple-path interference $I_3(A, B, C)$ (Sorkin)

$$P_3(A,B,C) - P_2(A,B) - P_2(B,C) - P_2(C,A) + P_1(A) + P_1(B) + P_1(C). \tag{24}$$

(This can be easily generalized for the case of n-paths.) For both classical and quantum theory, this intrinsic triple-path interference is zero for any triplet of paths. **Experimental confirmation of** $I_3 = 0$ **would be a confirmation of the Born rule.**
Weak bounds were placed on the parameter ($\kappa \sim 10^{-3}$) in photonic experiments

$$\kappa = \frac{\varepsilon}{\delta}, \quad \varepsilon = I_3(A,B,C), \quad \delta = |I_2(A,B)| + |I_2(B,C)| + |I_2(C,A)|. \tag{25}$$

The claim of (Berglund, Geraci, Hubsch, Mattingly, Minic) is that **with quantum gravitational degrees of freedom turned on, one can get $I_3 \neq 0$,** but for that one needs gravitized quantum theory, with **observer dependent Hilbert spaces and dynamical Born rule**. Inspired by metastring theory, the generalized probability in this approach to quantum gravity is given by

$$P = g_{ab}(\psi)\,\psi_a\psi_b \equiv \delta_{ab}\,\psi_a\psi_b + \gamma_{abc}\,\psi_a\psi_b\psi_c + \dots, \tag{26}$$

where a, b, c are state-space indices and with (schematically)

$$\frac{\mathrm{d}\psi_a}{\mathrm{d}\tau} = \Gamma_{abc}\,\psi_b\psi_c, \tag{27}$$

where τ is the appropriate evolution parameter. (Here Γ_{abc} is such that one has Schrodinger's evolution for a fixed Hilbert space.)

QG=GQ - Explicit Realization

Effective triple interference - possible in non-linear optical media! (Instead of ψ, non-linear waves; instead of probability P - non-linear/cubic energy density.) **"Smoking gun":** *Talbot effect on a diffraction grating → non-linear Talbot effect.* **Thus intrinsic triple interference with quantum gravity degrees of freedom - analogous to a non-linear "quantum spacetime medium".** (*Note: non-linear quantum theory with fixed Hilbert spaces is NOT GQ!*)
Based on the discussion of the vacuum energy/cosmological constant (the first experimental QG effect?) in what follows we argue for low energy scales: $10^{-4}m$ **or** $10^{-19}m$**.**
Finally, we discuss the possible first experimental QG effect - the observed vacuum energy/cosmological constant.

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The CC Problem in QFT

Cosmological constant Λ (a parameter in Einstein's eqs $G_{\mu\nu} + \Lambda_{cc} g_{\mu\nu} = 8\pi G T_{\mu\nu}$) has been measured (supernovae, CMB, large scale structure). It corresponds to the (quantum) vacuum energy $\Lambda_{cc}/(8\pi G) \sim (10^{-3} eV)^4$. The natural Planckian value is 10^{124} times off $(10^{19} GeV)^4$ - **the cosmological constant problem**. (*Analogous to the stability of atoms.*)

The measurement of Λ_{cc} IS the first QG observation!

Let us start with the QFT vacuum partition function (free scalar):

$$Z_{vac} = \int D\phi e^{-\int \frac{1}{2}\phi(-\partial^2+m^2)\phi} = \sqrt{\frac{\#}{\det(-\partial^2 + m^2)}} \tag{28}$$

which we can rewrite as

$$Z_{vac} = e^{-\frac{1}{2}\mathrm{Tr}\log(-\partial^2+m^2)} \tag{29}$$

The CC Problem in QFT

In momentum space, $-\partial^2 = k^2$, and also

$$-\frac{1}{2}\log(k^2+m^2) = \int \frac{\mathrm{d}l}{2l} e^{-(k^2+m^2)l/2} \tag{30}$$

where the Schwinger parameter l is a worldline parameter associated with a **particle (quantum of the field ϕ)**.

The CC Problem in QFT

Note that after taking the trace we have

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \log(k^2 + m^2) = \int \frac{\mathrm{d}^{D-1} k}{(2\pi)^{D-1}} \frac{\omega_k}{2} \tag{31}$$

because

$$\int \frac{\mathrm{d}l}{2l} \int \frac{\mathrm{d}k^0}{2\pi} e^{-(k^2+m^2)l/2} = \frac{\omega_k}{2} \tag{32}$$

where $\omega_k^2 = k^2 + m^2$ with ω_k equivalent to k_0 on-shell.

The CC Problem in QFT

Thus, vacuum energy density in D spacetime dimensions becomes

$$\rho_0 = \int \frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}} \frac{\omega_k}{2} \sim \Lambda_D \tag{33}$$

with Λ_D the volume of energy-momentum space.
This is a divergent expression (to be regularized) that leads to **the cosmological constant problem**. (*Weinberg's classic review.*)
The cosmological constant in 4d is (Einstein's equations $G_{ab} + \Lambda_{cc} g_{ab} = 8\pi G_N T_{ab}$) so that $\Lambda_{cc} \sim \rho_0 G_N \sim \rho_0 l_P^2$.

The CC Problem in QFT

Note that the **vacuum partition function** is also

$$Z_{vac} = \langle 0 | e^{-iH\tau} | 0 \rangle = e^{-i\rho_0 V_D} \tag{34}$$

where V_D is the volume of D-dimensional spacetime, and ρ_o is the vacuum energy density.

Furthermore $Z_{vac} = exp(Z_{S^1})$ where Z_{S^1} is the partition function on S^1 in the world-line formulation

$$Z_{s^1} = V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} \int \frac{\mathrm{d}l}{2l} e^{-(k^2+m^2)\frac{l}{2}} \tag{35}$$

Thus the **vacuum energy density** is given by (scaling as before)

$$\rho_0 = \frac{iZ_{S^1}}{V_d} \sim \Lambda_D \tag{36}$$

The CC Problem in QG/String Theory

For the case of a **bosonic string**, instead of one particle we have an infinite tower of particles with mass spectrum (Polchinski '86; Polchinski's String theory book) - graviton $(h = 1, \bar{h} = 1)$

$$m^2 = \frac{2}{\alpha'}(h + \bar{h} - 2) \tag{37}$$

Thus, summing over the physical string states

$$\sum_{\mathrm{p.s}} Z_{S^1} = \sum_{h,\bar{h}} V_D \int \frac{\mathrm{d}l(2\pi l)^{-D/2}}{2l} \int \frac{\mathrm{d}\theta}{2\pi} e^{i(h-\bar{h})\theta} e^{-\frac{2}{\alpha'}(h+\bar{h}-2)\frac{l}{2}} \tag{38}$$

where we have imposed the level matching $h = \bar{h}$ (or $\delta_{h,\bar{h}}$).

The CC Problem in QG/String Theory

Define $\tau = \theta + i\frac{l}{\alpha'} \equiv \tau_1 + i\tau_2$.
We get the partition function of a bosonic string on T^2

$$Z_{T^2} = V_D \int \frac{\mathrm{d}\tau \mathrm{d}\bar{\tau}}{2\tau_2} (4\pi^2\alpha'\tau_2)^{-D/2} \sum_h q^{h-1}\bar{q}^{\bar{h}-1} \tag{39}$$

where $q \equiv e^{2\pi i \tau}$. This can be derived directly from the Polyakov path integral.

The CC Problem in QG/String Theory

Note that we can rewrite, with $l \equiv \alpha' \tau_2$,

$$(4\pi^2 \alpha' \tau_2)^{-D/2} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} e^{-k^2 \frac{l}{2}} \tag{40}$$

Thus, as in QFT we can write $Z_{T^2} \equiv V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} f(k^2) \sim V_D \Lambda_D$ with

$$\Lambda_D \equiv \int \frac{\mathrm{d}^D k}{(2\pi)^D}; \quad f(k^2) \equiv \int_F \frac{\mathrm{d}^2 \tau}{2\tau_2} e^{-k^2 \alpha' \tau_2 / 2} \sum_h q^{h-1} \bar{q}^{h-1} \tag{41}$$

where F is the fundamental domain. Note that $f(k^2)$ is dimensionless, so it does not contribute to the scaling of Z_{T^2} and the vacuum energy $\rho_0 \sim Z_{T^2}/V_D$.

The CC Problem in QG/String Theory

The only difference is that in QFT the region of integration is

$$|\tau_1| < \frac{1}{2}, \quad \tau_2 > 0. \tag{42}$$

In string theory, because of modular invariance,

$$|\tau_1| < \frac{1}{2}, \quad |\tau| > 1. \tag{43}$$

So, the cosmological constant is UV finite(!) in string theory, but still $\rho_0 \sim Z_{T^2}/V_D \sim \Lambda_D$ (!) - **the cc problem persists in string theory**.

Unbroken SUSY - flat space or AdS. Broken SUSY - still $\rho_0 \sim \Lambda_D$.

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Quantum Gravity, Phase Space and Observation

Following FKLM [2212.00901], [2303.17495] (see also BHM, [2212.06086]) we return to Z_{S^1}, setting $m = 0$ for convenience and with p denoting the momentum,

$$Z_{S^1} = V_D \int \frac{\mathrm{d}\tau}{2\tau} \int \frac{\mathrm{d}^D p}{(2\pi)^D} e^{-\frac{p^2\tau}{2}} \tag{44}$$

But the spacetime volume is given by $V_D = \int \mathrm{d}^D q$. We therefore consider the **phase space** expression

$$Z_{S^1} = \int \frac{\mathrm{d}\tau}{2\tau} Z(\tau), \quad Z(\tau) = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \int \mathrm{d}^D p e^{-\frac{p^2\tau}{2}} \equiv \mathrm{Tr} e^{-\frac{p^2\tau}{2}} \tag{45}$$

where Tr is in *phase space*.

In $D = 4$ we then get

$$Z(\tau) = \prod_{i=1}^{4} \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}q_i \int_{-\infty}^{\infty} \mathrm{d}p_i e^{-\frac{p_i^2 \tau}{2}} \qquad (46)$$

or by discretizing phase space

$$Z(\tau) = \left(\frac{\lambda\epsilon}{2\pi} \sum_{k,\tilde{k} \in \mathbf{Z}} \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}\tilde{x} e^{-\frac{(\tilde{x}+k)^2 \epsilon^2 \tau}{2}} \right)^4 \qquad (47)$$

where $p \to \epsilon\tilde{x}, q \to \lambda x$ with $\lambda\epsilon = \hbar$.

This is divergent but restrict the sum to finite range, **modular regularization** (Following papers on modular polarization in quantum theory and modular spacetime by FLM, '15, '16)

$$Z(\tau) = \left(\frac{\lambda\epsilon}{2\pi} \sum_{k=0}^{N_q-1} \sum_{\tilde{k}=0}^{N_p-1} \int_0^1 \mathrm{d}x \mathrm{d}\tilde{x} e^{-\frac{(k+\tilde{x})^2\epsilon^2\tau}{2}} \right)^4 \tag{48}$$

where N_q, N_p count the number of unit cells in the spacetime and momentum space dimensions, respectively.
Now, define

$$l \equiv N_q\lambda, \quad \text{and} \quad \Lambda \equiv N_p\epsilon \tag{49}$$

where then $l^4 \equiv V_4$ is the size (volume) of spacetime and Λ^4 is the size (volume) of energy-momentum space, and $N = (N_p N_q)^4 \in \mathbf{Z}$.

Thus,

$$l^4\Lambda^4 = N, \quad \text{or} \quad \Lambda^4 = \frac{N}{l^4} \tag{50}$$

But there is actually an **upper bound on** $\rho_0 \sim \Lambda^4 \leq \frac{N}{l^4}$ in $D = 4$ due to $exp(-p^2\tau/2) \leq 1$.
The same bound also holds in string theory following our earlier calculation of the partition function of the bosonic string on T^2 in $D = 4$. (*The same bound holds in QFT; cosmological phase transitions described by an effective potential included.)* (FKLM)

$$\rho_0 \leq \frac{N}{l^4} \tag{51}$$

Quantum Gravity, Phase Space and Observation

We now consider the **Bekenstein bound** (**holography!**) in a space with a cosmological horizon, ie assuming that the cosmological constant is positive and we have a dS spacetime. (*This is a feature of semiclassical gravity, and also of gravitational thermodynamics.*) In static coordinates, dS spacetime metric is

$$\mathrm{d}s^2_{dS} = -(1 - \frac{r^2}{r^2_{CH}})dt^2 + \frac{dr^2}{(1 - \frac{r^2}{r^2_{CH}})} + r^2 \mathrm{d}\omega^2_{S^2} \qquad (52)$$

where $l \equiv r_{CH}$, the cosmological horizon, is the size of the observed spacetime.

By identifying the above quantum number N with the gravitational entropy, the **Bekenstein bound ($S_{grav} = l_P^{-2} Area$)** becomes

$$N \leq \frac{l^2}{l_P^2} \tag{53}$$

(*Experimental consequence for black holes - gravitational wave "echoes" - in the quantum chaos phase, because N is large - quantum scars!*)
Combine the Bekenstein bound with the bound on ρ_0 ($\rho_0 \leq N/l^4$)

$$\rho_0 \leq \frac{1}{l^2 l_P^2} \tag{54}$$

and hence a **mixing of the UV (l_P) and the IR (l) scales**.

With the cosmological constant in $D = 4$ dimensions, $\Lambda_{c.c} = \rho_0 l_P^2$ we then get the bound

$$\Lambda_{c.c} \leq \frac{1}{l^2} \tag{55}$$

Thus, the natural energy scale, $\epsilon_{c.c}$ associated with the vacuum energy density,

$$\rho_0 = \epsilon_{c.c}^4 \sim \frac{1}{l^2 l_p^2} \tag{56}$$

the corresponding natural length scale, $l_{c.c} \simeq 1/\epsilon_{c.c}$, we get the **see-saw formula** (FKLM) - also in full QFT (cosmological phase transitions etc) [probe - gravitational interferometry]

$$l_{c.c} \simeq \sqrt{l\, l_p} \tag{57}$$

Quantum Gravity, Phase Space and Observation

Note (integration over the Schwinger parameter can be absorbed in the Newton constant). Also:

- with $l \sim 10^{28} m$ we get $l_{c.c} \simeq 10^{-4}$ m or 10^{-3} eV in agreement with observations!
- natural with $\rho_0 \to 0$ when $l \to \infty$, and l is the IR scale
- radiativelly stable since no UV dependence
- the cc is small because the universe is filled with stuff (large number of degrees of freedom (dof) $N \sim 10^{124}$)
- N is large because fluctuations scale as $\frac{1}{\sqrt{N}}$ - stability of the universe (Schrödinger's argument "Why are atoms small?")
- N_i (where i is t, x, y, z) is $N^{1/4} \sim 10^{31}$ (not so unreasonable if we recall the Avogadro number 10^{23}, for matter dof).

Note further the **contextuality** of the argument: the measurement of a quantum observable depends on which commuting set of observable are within the same measurement set of observable, ie quantum measurements depend on the *context*!

- First, ϵ is NOT a cut-off, as ϵ and λ can be arbitrary, though have to satisfy $\lambda\epsilon = \hbar$
- Second, ϵ^4 is replaced by N, which is the new quantum number, and the size of spacetime, $l = r_{CH}$
- N is determined by the Bekenstein bound–N is related to l and l_P, which is where gravity enters via $G_N \sim l_P^2$

Effective field theory (EFT) does not "see" N and in particular does NOT know about the Bekenstein bound. Vacuum energy cancels in the computation of EFT correlation functions. Also, EFT lives in classical spacetime.

Note: the QFT partition function is defined as $Z(J) \equiv e^{iW(J)} = \int D\phi \; e^{i[S(\phi)+J\phi]}$, where $W(J)$ is the generating functional of vacuum correlation functions, and it represents a direct analogue of the partition function for a particle on a circle, or a string on a torus!

Given $W(J)$, we can define its Legendre transform to obtain $\Gamma(\phi)$, the effective action, as $\Gamma(\phi) \equiv W(J) - \int \mathrm{d}^4x \; J(x)\phi(x)$. The leading term in the expansion of $\Gamma(\phi)$ is the effective potential, $\Gamma(\phi) \equiv \int \mathrm{d}^4x[-V_{\mathrm{eff}}(\phi) + \dots]$, the minimum of which defines the vacuum energy in QFT.

Then by introducing the Scwhinger parametrizaton we can obtain that $\Gamma(\phi) \sim \int \mathrm{d}^4x \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}r}{r} \; e^{-U(k^2,\phi)\,r/2}$, where the exponent in the above integral is bounded by 1. (Also at finite temperature!)

By applying modular regularization to the crucial phase space factor together with the Bekenstein bound we get the already derived result for the bound on the vacuum energy!

We can repeat this argument for the modular space of the second Heisenberg algebra of the metastring and **apply the logic of the cosmological constant computation to the relation between the Higgs mass and the cosmological constant** (ξ and $\langle \mathcal{X} \rangle$ are defined in Abel & Dienes, and they are of order 1)

$$m_H^2 = \frac{\xi \Lambda^4}{M_P^2} - \frac{g_s^2 M_s^2}{8\pi^2} \langle \mathcal{X} \rangle, \tag{58}$$

We can neglect the first term and use

$$m_H \sim g_s M_s \sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} = g_s^2 M_P \sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} \tag{59}$$

Note that the string length l_s and the Planck length l_P are related via the string coupling g_s $g_s l_s = l_P, M_s = g_s M_P$ where M_P is the Planck (energy) scale and M_s the string (energy) scale. The dual spacetime scale is $\tilde{l} = \frac{l_P}{g_s^2} \left(\frac{l_P}{l_\Lambda} \right)^{1/2}$, where $\tilde{l} \equiv l_P$.

This implies $g_s = \left(\frac{l_P}{l_\Lambda}\right)^{1/4} \equiv \left(\frac{M_\Lambda}{M_P}\right)^{1/4} \to g_s^2 = \left(\frac{M_\Lambda}{M_P}\right)^{1/2} \ll 1$. All this follows from $l_\Lambda^4 \tilde{l}^4 = (l_s^2)^4 N_\Lambda$ analogous to $l^4\Lambda^4 = N$ and the holographic bound for the effective spacetime associated with the vacuum energy is $N_\Lambda = l_\Lambda^2/l_P^2$ and hence

$$l_\Lambda \tilde{l} = l_s^2 \left(\frac{l_\Lambda^2}{l_P^2}\right)^{1/4} = l_s^2 \left(\frac{l_\Lambda}{l_P}\right)^{1/2} \quad (60)$$

Finally - a **seesaw formula for the Higgs mass** (BHM, '22)

$$m_H \sim \sqrt{M_\Lambda M_P}\sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} \quad (61)$$

where M_Λ is the vacuum energy scale (10^{-3} eV) and M_P is the Planck energy (10^{19} GeV). The observed Higgs mass ($125\,GeV \to 10^{-19} m$) - if $\sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} \sim 10^{-2}$

Quantum Gravity, Phase Space and Observation

The upshot of the two calculations: the calculation of the observed vacuum energy (cosmological constant) and the observed mass of the Higgs, is that these two **low energy (IR) scales do know about quantum gravity.**
Thus, in the discussions of triple (and higher order) interference we have mentioned that the scales of $10^{-4}m$ **and** $10^{-19}m$ should be taken as **natural scales of quantum gravity phenomenology**.
Is $I_3 \neq 0$ *in the context of QG=GQ at these scales?*
Experiment possible in the next few years!

Quantum Gravity, Phase Space and Observation

Extend this logic to the **observed masses and mixing matrices of quarks and leptons** (Berglund, Hubsch, Minic).
Note, for matter degrees of freedom, entropy extensive l^3/l_{BZ}^3, with l_{BZ} - **Bjorken-Zeldovich scale**
Equate $N \sim l^2/l_P^2$ **to** l^3/l_{BZ}^3**, and find that** $l_{BZ}^3 \sim l l_P^2$**. (BBN)**
$l_{BZ} \sim 10^{-14} m \to M_{BZ} \sim 7 MeV$. (Note: QCD scale $\sim 300 MeV$.)
All observed masses of quarks and charged leptons: seesaw-like expressions (like the ones for CC and the Higgs mass) involving M_{BZ} (Bjorken; Oxford twistor group). For example:
$m_c \sim \sqrt{M_{BZ} m_t}$, $m_s \sim \sqrt{M_{BZ} m_b}$, $m_\mu \sim \sqrt{M_{BZ} m_\tau}$,
$m_e \sim M_{BZ}^2/m_\mu$,
Can be extended to neutrino masses ($M_{BZ} \to m_\Lambda$**)** and the observed mixing matrices of quarks (CKM) and leptons (PMNS).

Repeat the reasoning from the cosmological constant calculation and the Higgs mass calculation. The fermion masses are given by a fermionic analog of the Abel-Dienes formula

$$m_f \sim g_s M_s \tag{62}$$

and use the formula for the string coupling based on the noncommutativity of spacetime and its dual and the holographic bound, as before.

The relevant IR scale is now M_{BZ} and the relevant UV scale is the heaviest fermion scale. Two expressions are possible:

$$m_f \sim M_{IR}\sqrt{\frac{M_{UV}}{M_{IR}}} \sim \sqrt{M_{IR} M_{UV}} \tag{63}$$

or

$$m_f \sim M_{IR}\sqrt{\frac{M_{IR}}{M_{UV}}} \tag{64}$$

Quantum Gravity, Phase Space and Observation

From the Higgs mass and RG deduce m_t, m_b and m_τ (Standard Model criticality). Then: calculated (observed)

$m_c \sim \sqrt{M_{BZ}\, m_t} = M_{BZ}\sqrt{\frac{m_t}{M_{BZ}}} \sim 1.10\ (1.27)$ GeV.

$m_s \sim \sqrt{M_{BZ}\, m_b} = M_{BZ}\sqrt{\frac{m_b}{M_{BZ}}} \sim 171\ (93.4)$MeV.

$m_u \sim M_{BZ}^2/m_c \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_t}} \sim 10^{-2} M_{BZ} \sim 10^{-1}\ (2.16)$MeV.

$m_d \sim M_{BZ}^2/m_s \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_b}} \sim 10^{-1} M_{BZ} \sim 1\ (4.67)$MeV.

$m_\mu \sim \sqrt{M_{BZ}\, m_\tau} = M_{BZ}\sqrt{\frac{m_\tau}{M_{BZ}}} \sim 112\ (106)$MeV.

$m_e \sim \frac{M_{BZ}^2}{m_\mu} \sim M_{BZ}\sqrt{\frac{M_{BZ}}{m_\tau}} \sim 464\ (511)$keV. **Prediction for: (Neutrino masses)** $m_3 \sim m_H^2/M_{SM} \sim (10^{-2} - 10^{-1})$eV.

$m_2 \sim \sqrt{m_\Lambda m_3} \sim (10^{-2.5} - 10^{-2})$eV. $m_1 \sim \frac{{m_\Lambda}^2}{m_3} \sim (10^{-4})$eV.

Quantum Gravity, Phase Space and Observation

CKM matrix (quark mixing matrix), from $\sqrt{\frac{M_{IR}}{M_{UV}}}(\sqrt{\frac{M_{UV}}{M_{IR}}})$ factors

$$|V_{cb}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_d}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_d}} \sim 0.050 \quad (0.041), (\rightsquigarrow \theta_{23})$$

$$|V_{td}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_s}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_s}} \sim 0.011 \quad (0.008)(\rightsquigarrow \theta_{12})$$

$$|V_{ub}| \sim \frac{M_{BZ}}{\sqrt{m_b\, m_b}} \sim \sqrt{\frac{M_{BZ}}{m_b}}\sqrt{\frac{M_{BZ}}{m_b}} \sim 0.002 \quad (0.003)(\rightsquigarrow \theta_{13})$$

PMNS (neutrino mixing matrix), from $\sqrt{\frac{M_{IR}}{M_{UV}}}(\sqrt{\frac{M_{UV}}{M_{IR}}})$ factors

$$|U_{\mu 3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_1}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_1}} \sim 0.50, \quad (0.63)$$

$$|U_{\tau 1}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_2}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_2}} \sim 0.13, \quad (0.26)$$

$$|U_{e3}| \sim \frac{m_\Lambda}{\sqrt{m_3 m_3}} \sim \sqrt{\frac{m_\Lambda}{m_3}}\sqrt{\frac{m_\Lambda}{m_3}} \sim 0.06, \quad (0.14) \text{ (Similar structure!)}$$

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Summary

Quantum gravity (QG) = Gravitization of quantum theory (GQ)
Gravitization of quantum theory = dynamical geometry of quantum theory (dynamical Born geometry)
In principle, not only double, but also triple and higher order interference allowed! "Smoking gun experiment".
Quantum spacetime = modular spacetime (geometry of quantum theory). Spacetime and its dual do not commute!
Consequences: Particles (visible matter) are singular limits of metaparticles - correlated particles and dual (dark) particles
Quantum fields and dual (dark) quantum fields do not commute (fuzzy dark matter). Dark energy - curvature of dual spacetime
Metaparticles - zero modes of the metastring (non-commutative string in a dynamical geometry of modular spacetime) (QG=GQ)

Outlook

Phenomenological Implications of QG=GQ and future work

- Cosmological constant (cc) as the first empirical quantum gravity phenomenon - (also, gravitational interferometry and quantum spacetime)
- Metaparticles (zero modes of the metastring) and dark matter (entangled/correlated SM/dual (DM) particles) - probes?
- Dark energy (cc) as the curvature of the dual spacetime (naturally small by the above argument) - probes?
- Gravitational wave "echoes" - experimental probe of $N \sim l^2/l_P^2$ for black holes (quantum chaos - quantum scars)
- Dynamical Born geometry, "gravitizing the quantum," look at triple (and higher order) quantum interference in QG
- The CC, the Higgs mass and SM fermion masses and mixing. (**Bounds! - Cosmological Attractor?**)